Aspects of the Confinement mechanism in Coulomb-gauge QCD

On confinement of colored quark-quark correlations

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Outline

Confinement

Some Selected Theories of Confinement:

see *e.g.*

R.A. and J. Greensite, *Quark Confinement: The Hard Problem of Hadron Physics*, Hadron Physics Focus Issue of J. Phys. G

- chromomagnetic **monopoles**
  't Hooft, diGiacomo, ...
- center **vortices**
  Greensite, Olejnik, ...
- **Coulomb** confinement
  Gribov, Zwanziger, ...
- Landau gauge **Green Functions**
  Smekal, Fischer, ...
- **AdS**$_5$ / QCD correspondence
  Maldacena, Brodsky, ...

Infrared QCD in the Coulomb gauge

RA, M. Kloker, A. Krassnigg, and R. Wagenbrunn, Phys. Rev. Lett. **96** (2006) 022001.

Aim:
understand confinement of quarks and colored composites
(here: quark-quark-correlations, *i.e.* diquarks).

Based on two results in Coulomb gauge QCD:

- No Confinement without Coulomb confinement!
  D. Zwanziger, Phys. Rev. Lett. **90** (2003) 102001
- Infrared regulator cancelation in constituent quark mass function!
  R.A. and P.A. Amundsen, Nucl. Phys. **B306** (1988) 305

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Quark confinement in the Coulomb gauge

D. Zwanziger:
One-gluon-exchange in Coulomb gauge QCD is confining.

- $D_{00}(\vec{x},t) \propto V_C(|\vec{x}|)\,\delta(t) + \text{non} - \text{inst. terms}$
- $\lim_{R\to\infty} V_{Wilson}(R) \leq \lim_{R\to\infty} V_C(R)$
- As $V_{Wilson}(R) = \sigma R + \ldots$ one obtains: $V_C(R) = \sigma_c R$ with $\sigma_c \geq \sigma$.
- OVERCONFINING
- lattice calculations: $\sigma_c \approx 3 \times \sigma$, *i.e.* $\sqrt{\sigma_c} \approx 600 \ldots 750$ MeV.

J. Greensite, S. Olejnik and D. Zwanziger, Phys. Rev. D **69** (2004) 074506 [arXiv:hep-lat/0401003]

A. Nakamura and T. Saito, Prog. Theor. Phys. **115** (2006) 189 [arXiv:hep-lat/0512042].

Gluon confinement in the Coulomb gauge

- Propagator of transverse gluons vanish in the infrared!

see *e.g.* A. Cucchieri and D. Zwanziger, Phys. Rev. D **65** (2002) 014001
A.P. Szczepaniak, Phys. Rev. D **69** (2004) 074031

$\Rightarrow$ **Transverse gluons confined by positivity violation!**

Note similarity to infrared behaviour in Landau gauge!

There: IR behaviour of all YM vertex functions known and unique,
but: quark confinement via IR divergence of quark-gluon vertex,
see talk by *Christian S. Fischer.*

Quark Dyson-Schwinger equation

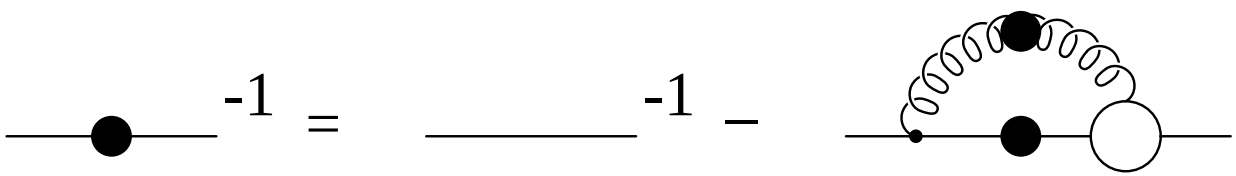
Infrared singular instantaneous interaction:

Quark propagator vanishes, *i.e.* **quarks confined**, nevertheless:

quark mass function and condensate well-defined!

- Quark DS-equation

$$i\,S^{-1}(p) = \not{p} - m - \Sigma(p)$$



- $\Sigma(p), S^{-1}(p) \to \infty$ but $\int dp_0 S(p) \propto \langle \bar{q}q \rangle \propto \sigma_c^{3/2}$

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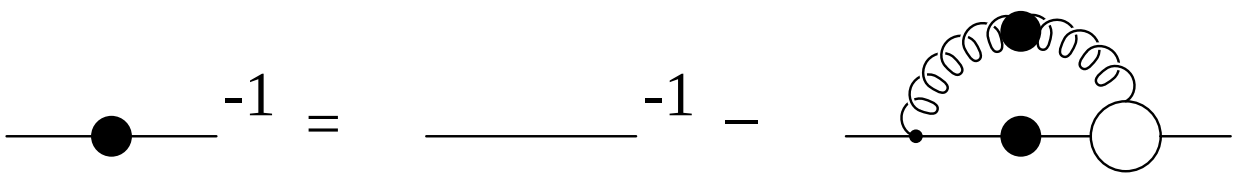

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Dyson-Schwinger & Bethe-Salpeter Eq.

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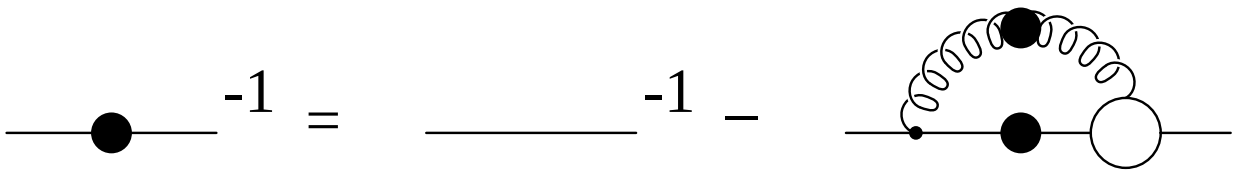


- For the description of quark-(anti)quark bound states: BS-equation

$$\Gamma(P,q) = \int d^4k\; K(q,k,P)\; S(k_+)\; \Gamma(P,k)\; S(k_-)$$

with $K \propto V_C$, consistent with DSE kernel.

Dyson-Schwinger & Bethe-Salpeter Eq.

- **Self-consistent solution:** Involve higher Green functions

  $\longrightarrow$ Truncate the infinite coupled system of integral equations

  In Coulomb gauge: Keeping the time-time component of gluon propagator, employing the instantaneous approx.:

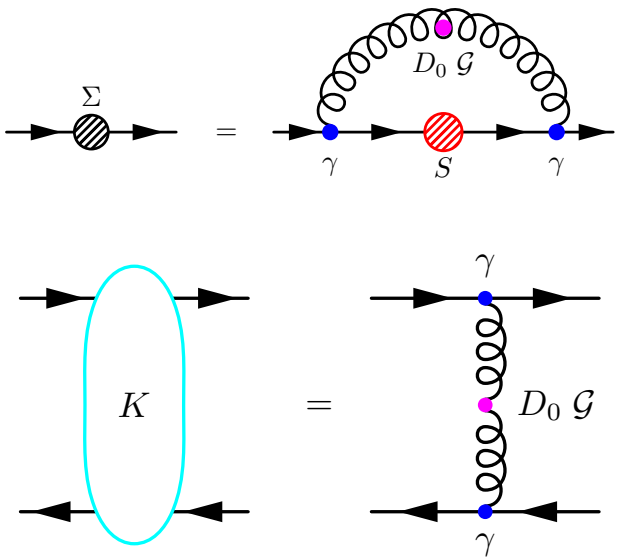
$$D_{00}(\vec{x}, t) \propto V_C(\vec{x})\, \delta(t)$$

- Therefore:

  Qualitative but not quantitative results

Dyson-Schwinger & Bethe-Salpeter Eq.

- Employ the **rainbow-ladder truncation** for the DS- and BS-equations



- **Axial-vector Ward-Takahashi identity** fulfilled

  $\longrightarrow$ Chiral symmetry in Nambu-Goldstone mode

Chiral Symmetry Breaking

- *Self-energy* in our truncation

$$\Sigma(p) = C_f\, 6\pi \int \frac{d^4q}{(2\pi)^4} V_C(\vec{p}-\vec{q})\, \gamma_0\, S(q)\, \gamma_0$$

- *Parametrisation* of the quark propagator:

$$S^{-1}(p) = -i(\gamma_0 p_0 - \vec{\gamma}\cdot\vec{p}\, C(p) - B(p)), \qquad M(p) = B(p)/C(p)$$

- For the quark propagator functions:

$$B(|\vec{p}|) = m + \frac{1}{2\pi^2}\int d^3q\, V_C(|\vec{p}-\vec{q}|)\frac{M(|\vec{q}|)}{\sqrt{M^2(|\vec{q}|)+\vec{q}^2}}\ ,$$

$$C(|\vec{p}|) = 1 + \frac{1}{2\pi^2}\int d^3q\, V_C(|\vec{p}-\vec{q}|)\, \hat{p}\cdot\hat{q}\ \frac{|\vec{q}|}{|\vec{p}|\,\sqrt{M^2(|\vec{q}|)+|\vec{q}|^2}}$$

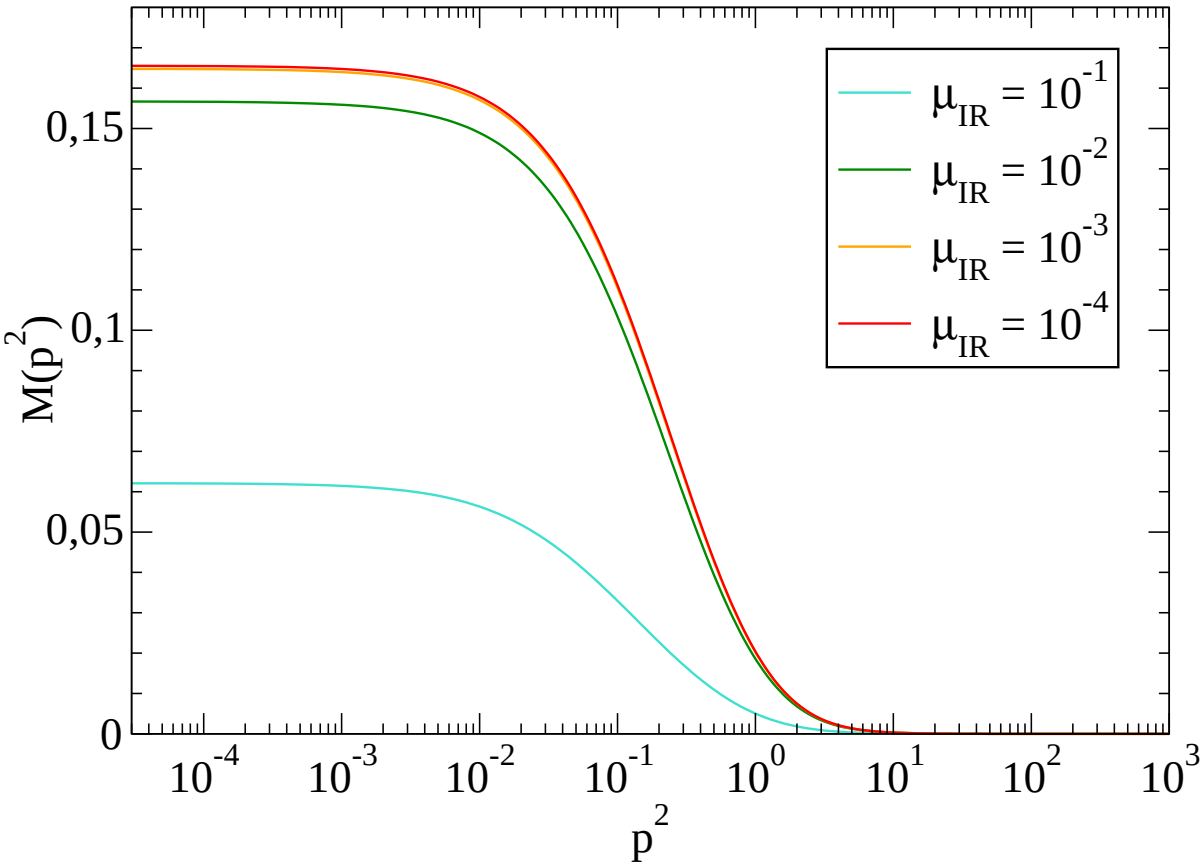
Color Potential: Cancelation of IR divergencies

- V_C is *linearly rising* for large distances: $V_C(R) \sim \sigma_c \cdot R$
- $V_C = \dfrac{\sigma_c}{k^4} \;\rightarrow\; \dfrac{\sigma_c}{(k^2+\mu_{IR}^2)^2}$
- Obtain **final results** by taking the limit $\mu_{IR} \rightarrow 0$
- $M(p) = \dfrac{I(p)M(p)+B_{reg}(p)}{I(p)+C_{reg}(p)} = \dfrac{B_{reg}(p)}{C_{reg}(p)} \quad \text{with} \quad I(p) \propto 1/\mu_{IR}.$

Color Coulomb Potential

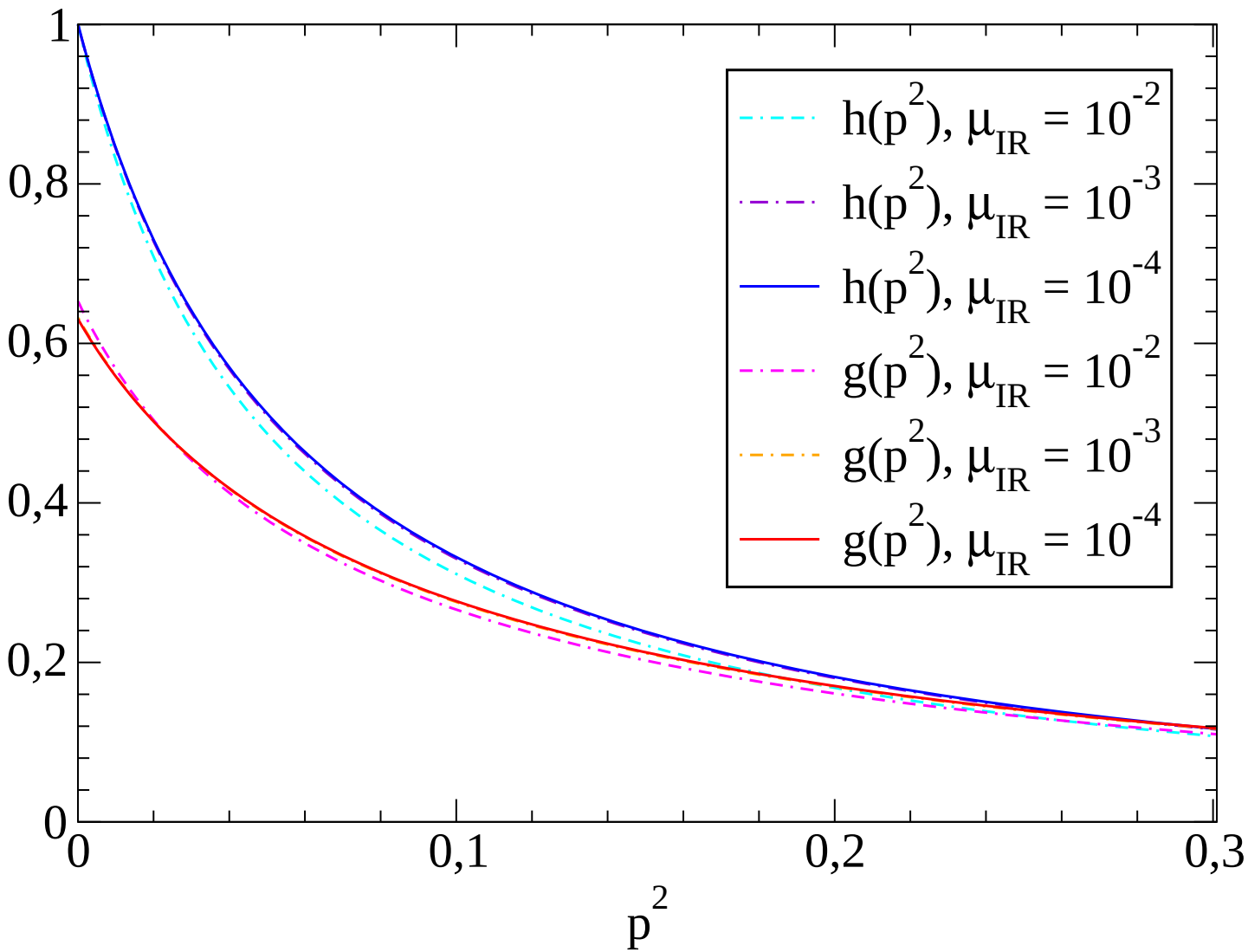
- **Energy of a static $q\bar{q}$ state**
  = Quark self-energies + colour-Coulomb potential
- Cancelation in color singlet state!
- $\langle \bar{q}q \rangle$ and $M(p)$ finite!
- Meson properties well-defined!
- Colored composites???

Quark mass function



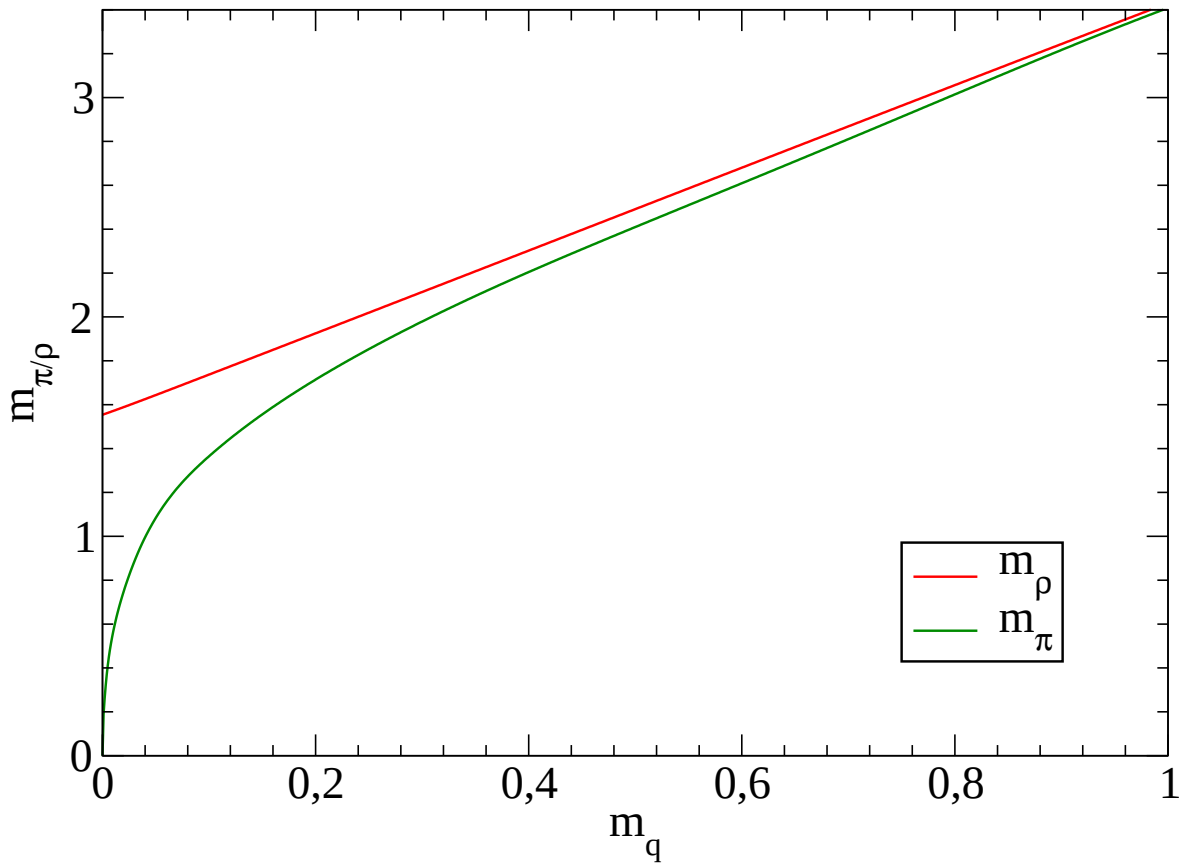
in chiral limit; all quantities in appropriate *units* of $\sqrt{\sigma_c}$.

BS-Amplitudes for the π



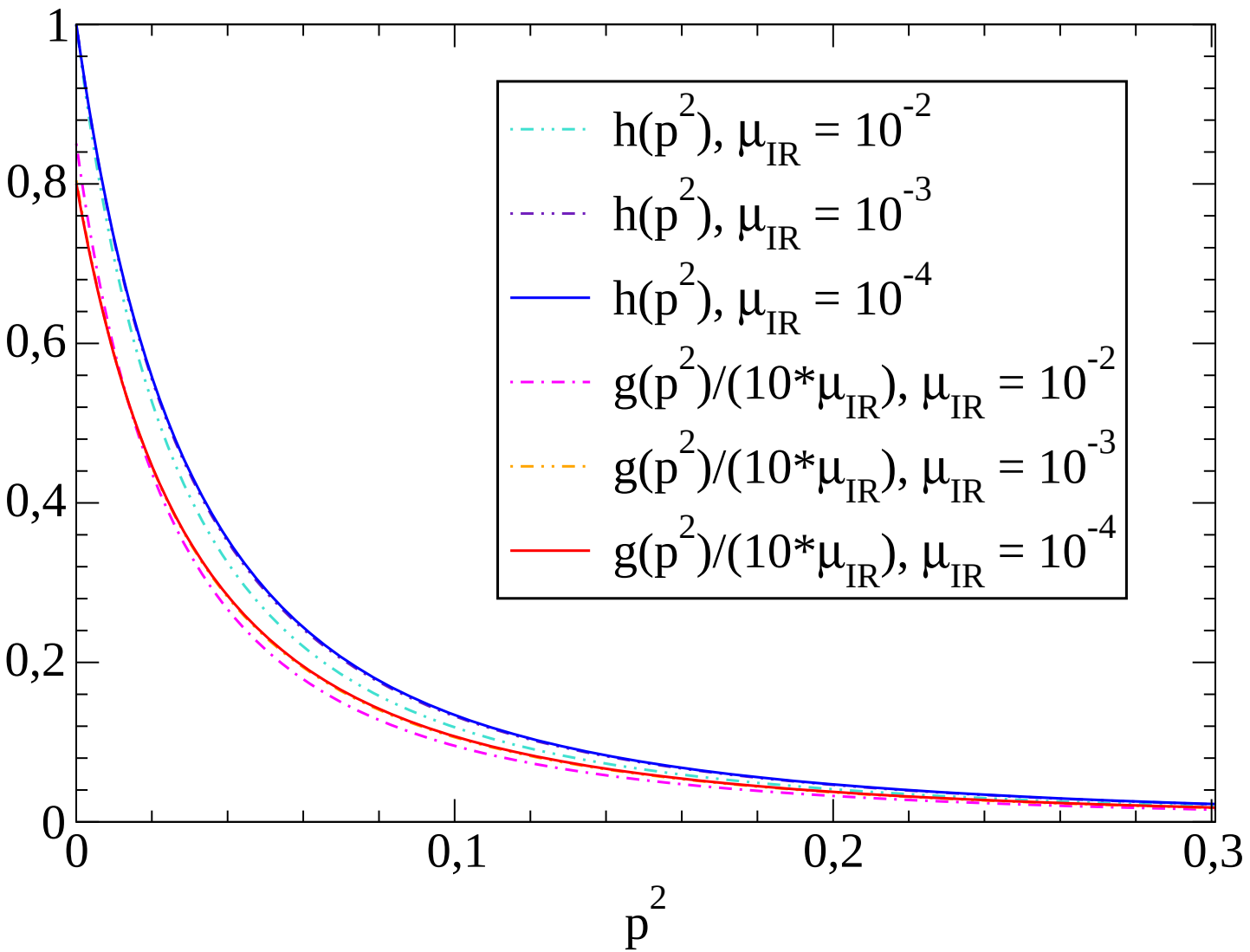
h: pseudoscalar BSA; *g*: axialvector BSA

Pion and Rho Masses



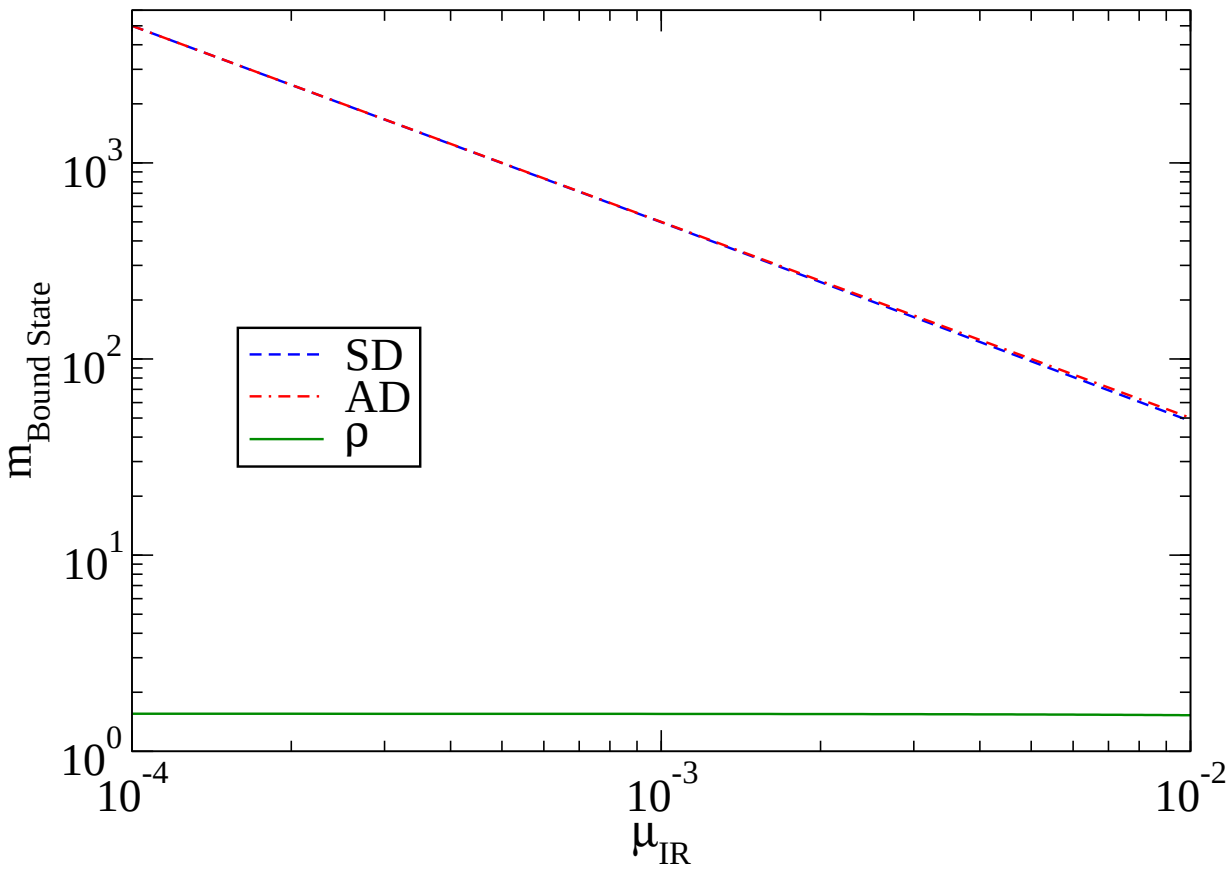
m_π vanishes in the chiral limit as expected!

BS-Amplitudes for the scalar Diquark



h: scalar BSA; *g*: vector BSA

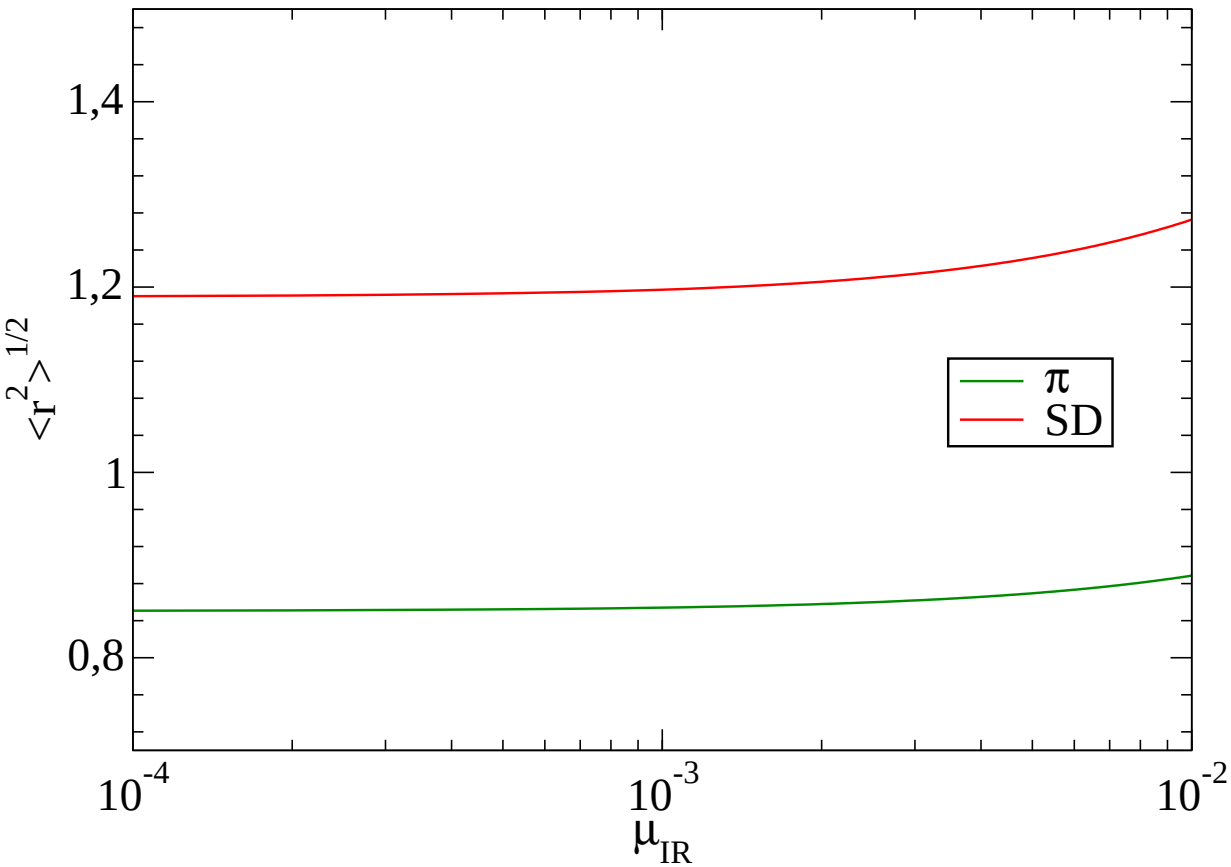
Meson and Diquark Masses



Diquark masses diverge for $\mu_{IR} \to 0$

i.e. DIQUARK CONFINEMENT!

Charge Radii



Meson and diquark radii finite ;
scalar diquark appr. 50% larger than pion, *i.e.* size of the nucleon.

Beyond the instantaneous approximation

- Including transverse gluons in the rainbow truncation does not increase the dynamically generated quark mass!

  M. Kloker, PhD thesis 2007, and R. Krenn, Diploma thesis 2007.

- IR divergent quark-gluon vertex needed!?!

Coulomb gauge: Summary and Outlook

In Coulomb gauge QCD

- one-gluon-exchange (D_{00}) *over*confines!
- 4D quark propagator vanishes and quarks are confined!
- dynamical chiral symmetry breaking;
  color singlet quark condensate well-defined!
- color singlet meson properties well-defined!
- colored diquarks confined!
- nevertheless well-defined size!

This raises the question of

- ? Yang-Mills sector: transverse gluons and ghosts as well as VERTEX FUNCTIONS
- ? quantitative results for mesons and baryons
- ? QCD phase diagram:
  deconfinement??? color superconductors??? ... ???

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