

# Dynamical chiral symmetry breaking and a critical mass

Lei Chang (常 雷)

Department of Physics, Peking University

lei.chiong@gmail.com



# Collaborators

- DSEs group in China
  - ★ Yu-xin Liu, Peking University
  - ★ Xiao-fu Lv, Sichuan University
  - ★ Hong-shi Zong, Nanjing Univeristy
- Craig Roberts, Argonne National Laboratory, USA
  - ★ This talk based on [L. Chang, Y.-X. Liu, M. S. Bhagwat, C. D. Roberts, and S. V. Wright Phys. Rev. C75 \(2007\) 015201](#)

# Outline

- Introduction
- An example
- Closer to QCD
- Summary

# Introduction

- **Dynamical chiral symmetry breaking** is fundamentally important in strong interaction physics (generation of constituentlike masses, realization of Goldstone's theorem, etc)
- We report here novel aspects of the interplay between **explicit** and **dynamical** chiral symmetry breaking
- DSEs  $\int D\bar{q}qG \frac{\delta}{\delta q(x)} e^{-S[\bar{q},q,G]+(\bar{\eta},q)+(\bar{q},\eta)+(J,G)}$ 
  - Euclidean metric, p-space, covariant
  - Truncate to minimum 2-point, 3-point
  - Insist on preserving 1-loop QCD renorm group in UV
  - Constraints for truncation: vector WTI, axial vector WTI

# Introduction

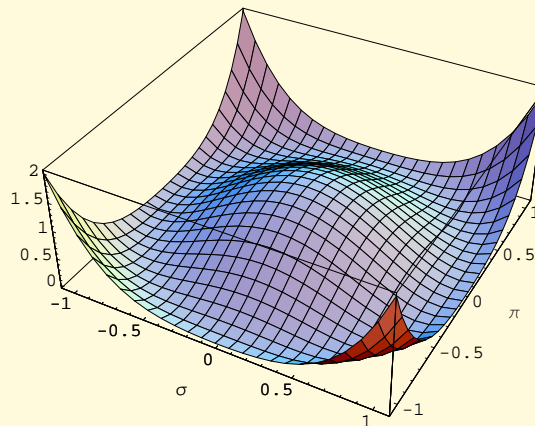
- DSE for quark propagator

$$\text{---}\bullet\text{---}^{-1} = \text{---}\text{---}^{-1} - \text{---}\bullet\text{---}\text{---}\text{---}$$

$$\star S(p) = \frac{Z(p^2, \zeta^2)}{i\gamma \cdot p + M(p^2)}$$

$$\star \text{Perturbation Theory: } M(p^2) = m_0 \left( 1 - \frac{\alpha}{\pi} \ln \left[ \frac{p^2}{m_0^2} \right] + \dots \right)$$

- Dynamical chiral symmetry breaking in the chiral limit



- **Question:** whether the massive gap equation admits solutions other than that which is positive define, the effect of the current-quark mass on such solutions.

## An example

- the model choice

$$g^2 D_{\mu\nu}(p - q) = \delta_{\mu\nu} \frac{1}{m_G^2} \theta(\tilde{\Lambda}^2 - q^2), \quad (1)$$

$$\Gamma_\nu^a(q, p) = \gamma_\nu \frac{\lambda^a}{2}; \quad (2)$$

- the gap equation

$$G(M) := M - m^{\text{bm}} - M \frac{1}{3\pi^2} \frac{1}{m_G^2} C(M^2, 1) = 0; \quad (3)$$

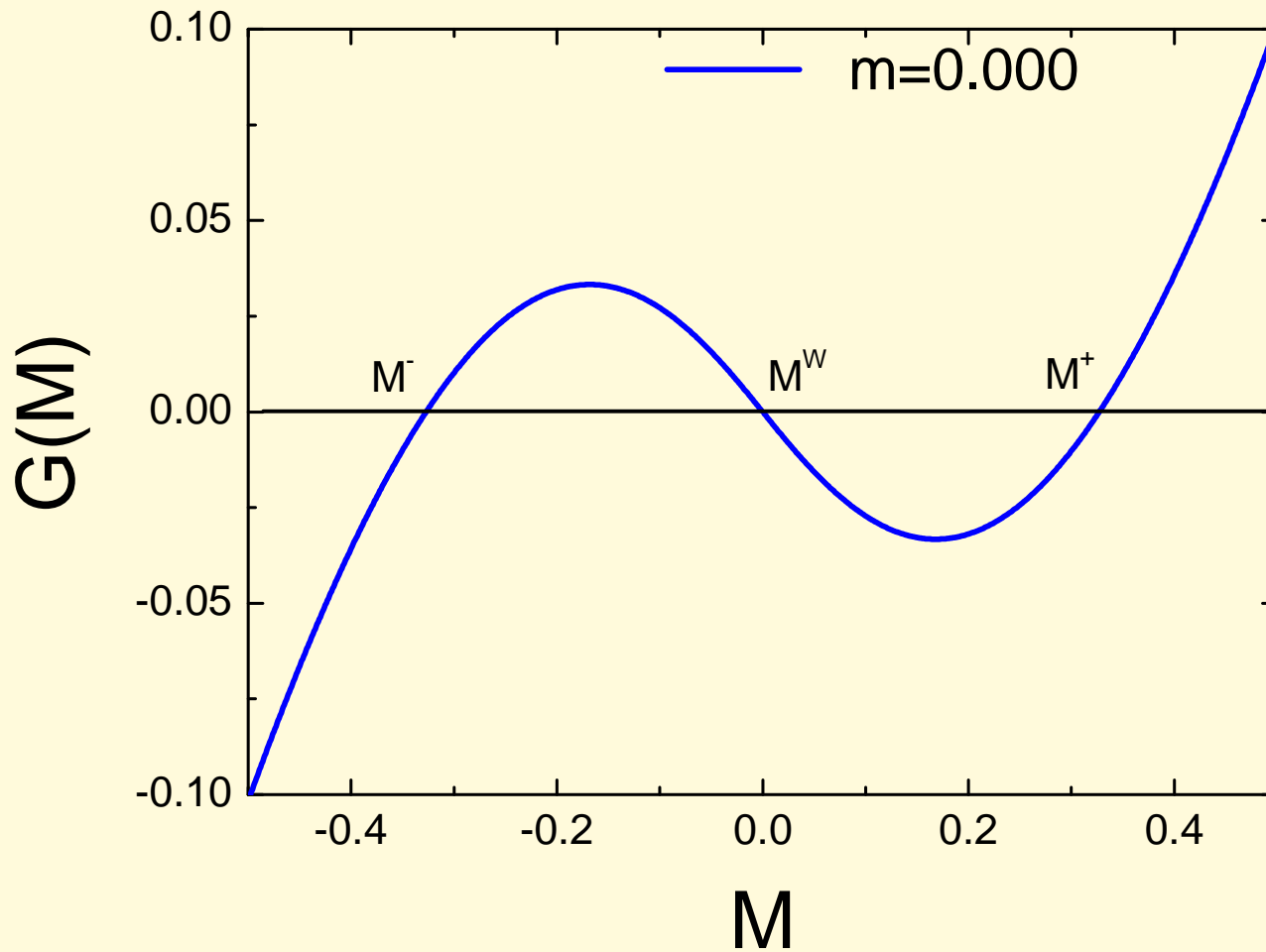
where

$$C(M^2, 1) = 1 - M^2 \ln [1 + 1/M^2]. \quad (4)$$

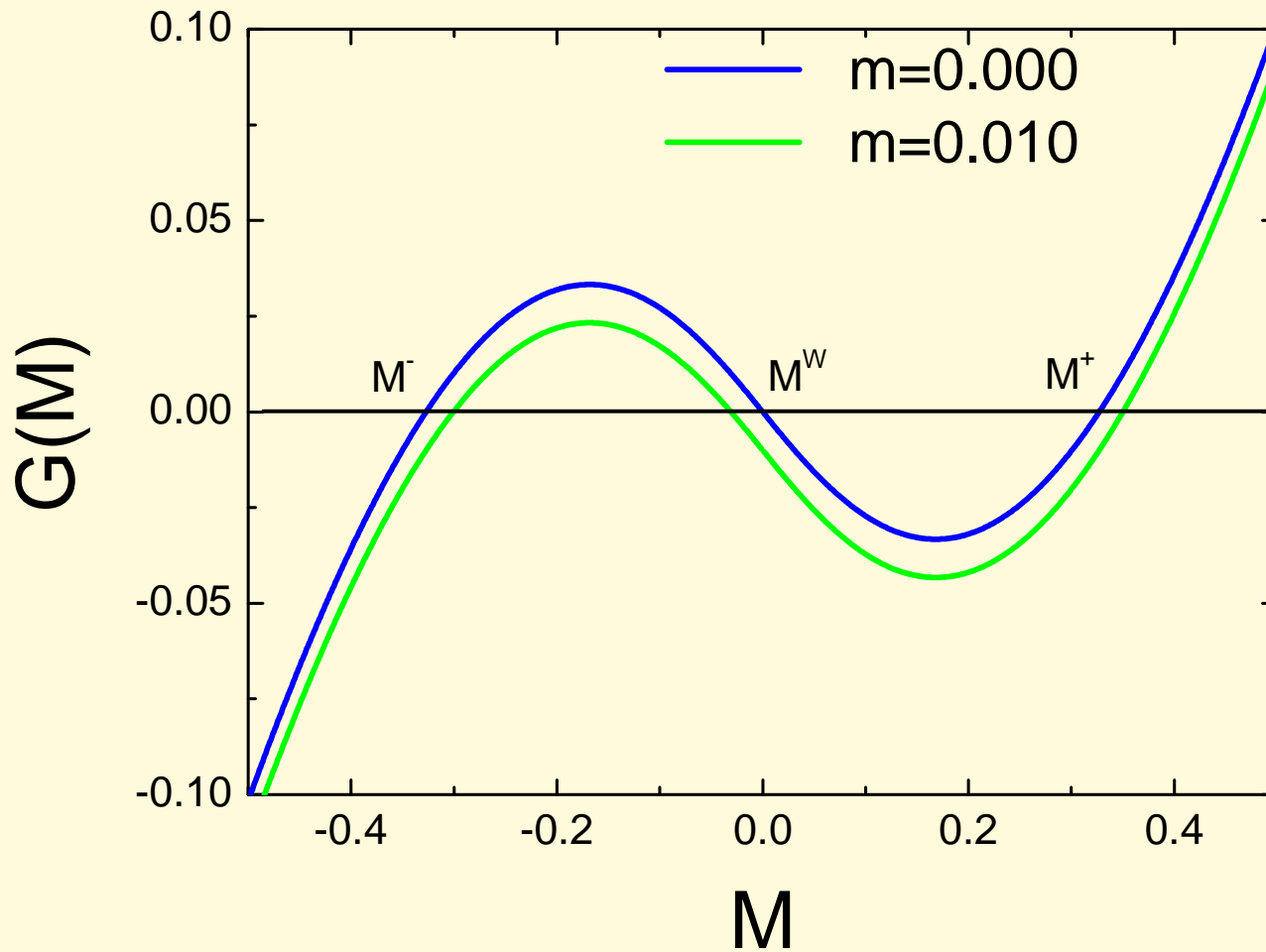
- the critical coupling

$$m_G^2 < (m_G^{\text{cr}})^2 = \frac{1}{3\pi^2}; \quad (5)$$

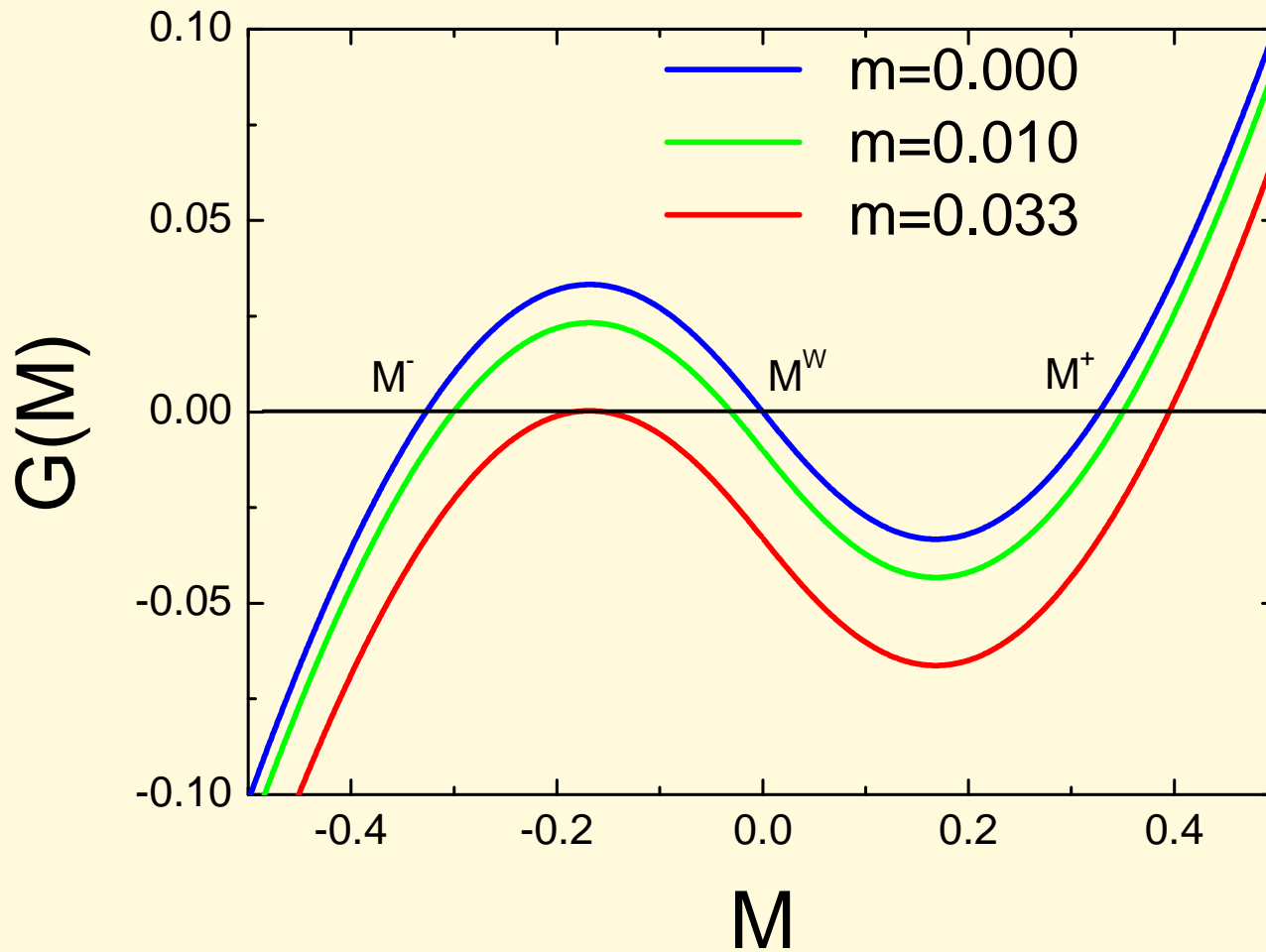
- Description of solutions



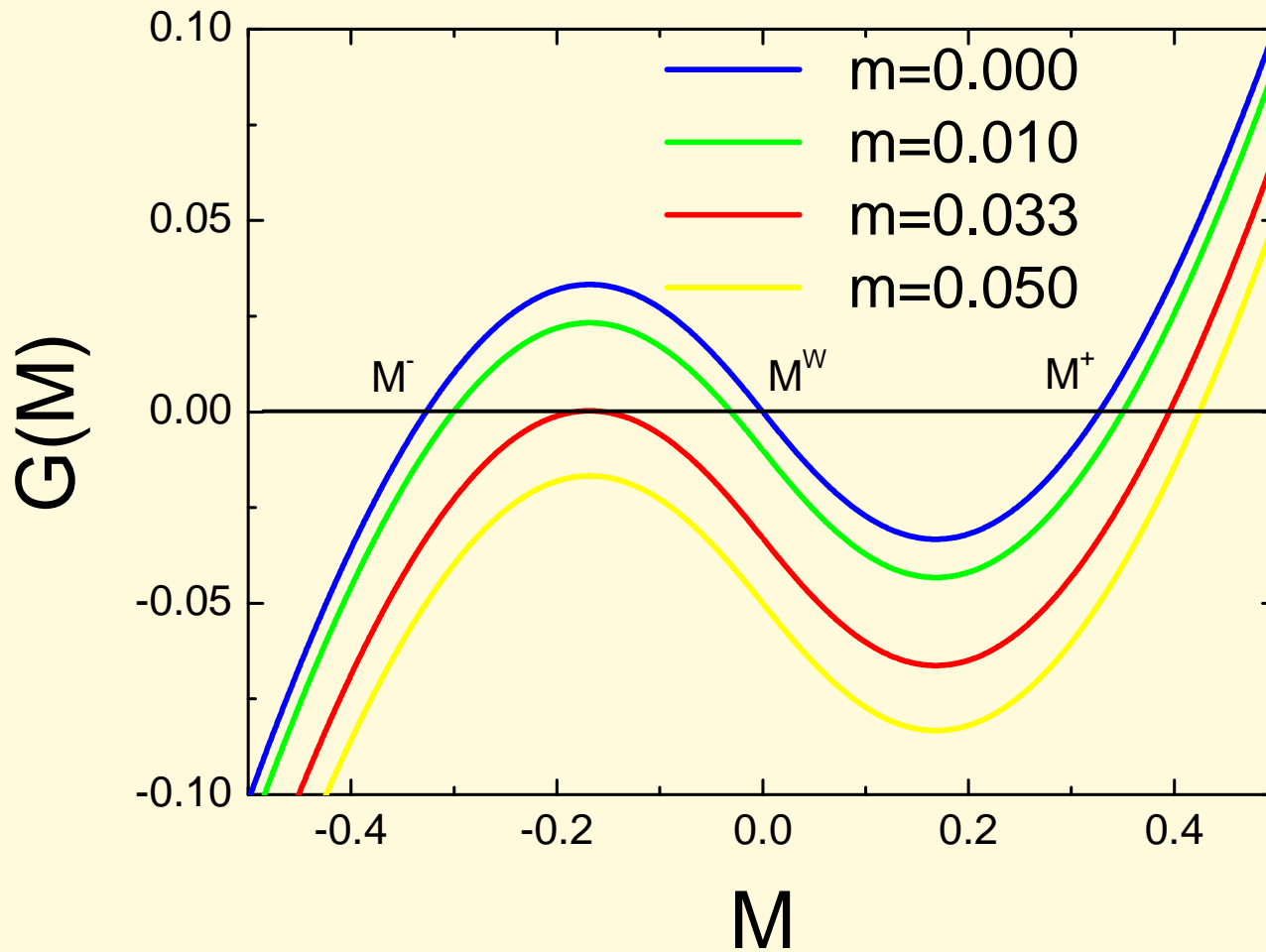
• Description of solutions



• Description of solutions



• Description of solutions



- The radius of convergence for chiral expansion

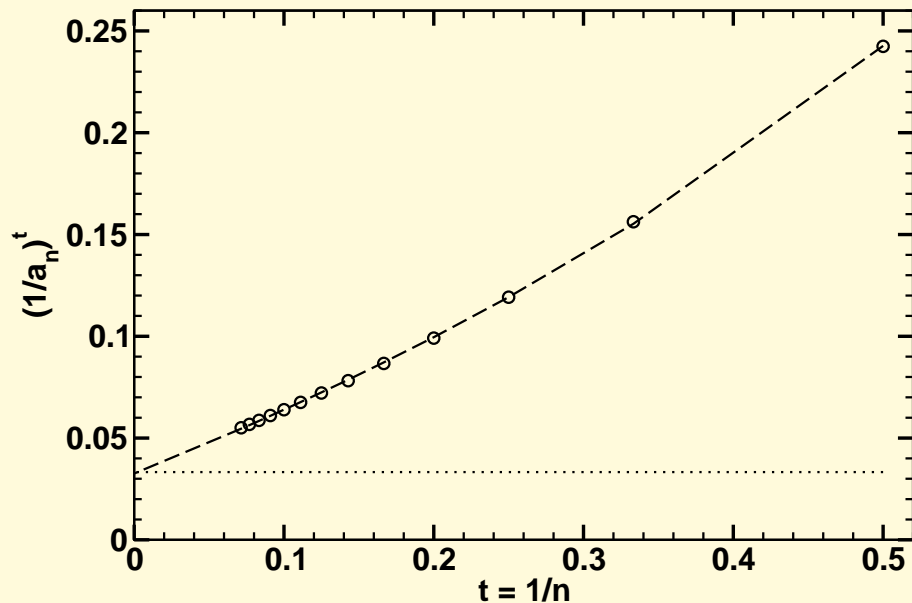
The chiral expansion

$$M_+(m^{bm}) = M_0 + \sum_{n=1}^{\infty} a_n (m^{bm})^n \quad (6)$$

is absolutely convergent, so long as  $\forall n$

$$\left(\frac{1}{|a_n|}\right)^{1/n} > \left(\frac{1}{|a_{n+1}|}\right)^{1/(n+1)} \quad (7)$$

and the radius of convergence is  $m_{rc} := \lim_{n \rightarrow \infty} \left(\frac{1}{|a_n|}\right)^{1/n}$ .



## Closer to QCD

- the model choice

$$\frac{\mathcal{G}(t)}{t} = \frac{4\pi^2}{\omega^6} D t e^{-t/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln \left[ \tau + \left( 1 + t/\Lambda_{\text{QCD}}^2 \right)^2 \right]} \mathcal{F}(t), \quad (8)$$

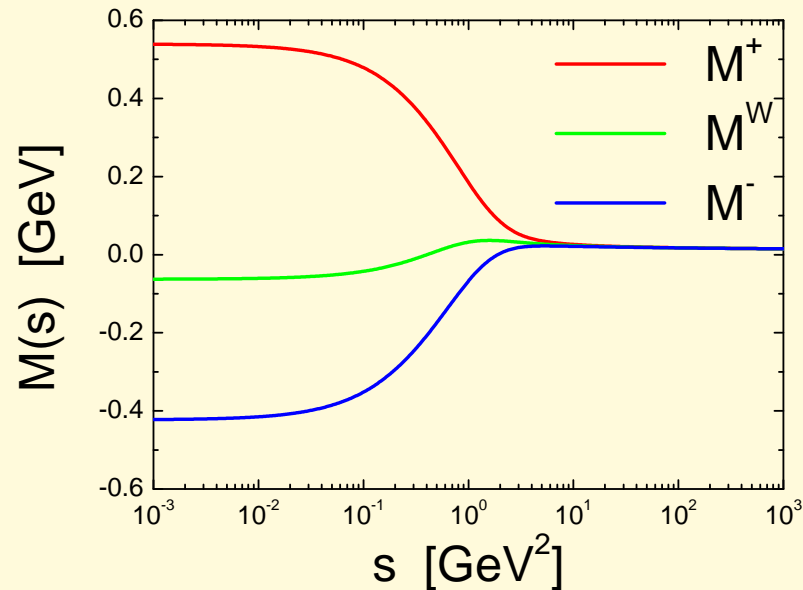
with  $t = k^2$ ,  $\mathcal{F}(t) = [1 - \exp(-t/[4m_{\mathcal{F}}^2])]/t$ ,  $m_{\mathcal{F}} = 0.5 \text{ GeV}$ ,  $\tau = e^2 - 1$ ,  $\gamma_m = 12/25$  and  $\Lambda_{\text{QCD}} = \Lambda_{\overline{MS}}^{(4)} = 0.234 \text{ GeV}$ .

- $(\omega D) = (0.72 \text{ GeV})^3$
- one-loop version of mass function

$$M(p^2) \stackrel{p^2 \gg \Lambda_{\text{QCD}}^2}{=} \frac{\hat{m}}{\left(\frac{1}{2} \ln p^2 / \Lambda_{\text{QCD}}^2\right)^{\gamma_m}}, \quad (9)$$

where  $\hat{m}$  is the renormalisation-group-invariant mass.

- Quark mass functions at  $m(\zeta_{19}) = 16MeV$



★ three different solutions exist at this special current quark mass

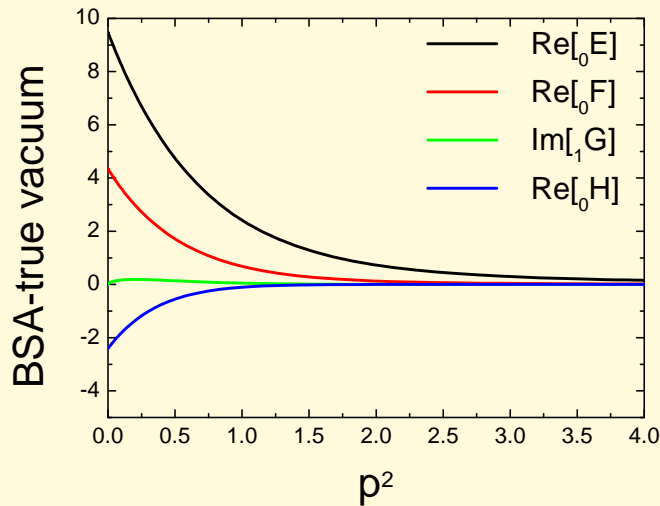
★  $M_+(p^2) \stackrel{p^2 \gg \Lambda_{\text{QCD}}^2}{=} M_-(p^2) \stackrel{p^2 \gg \Lambda_{\text{QCD}}^2}{=} M_W(p^2)$

★  $M_W(p^2)$  is unstable under perturbation of current quark mass

- The pion properties in different vacuum

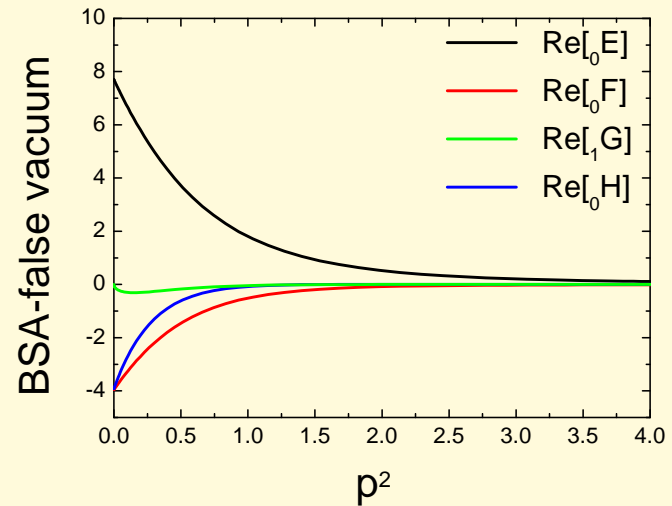
The pion Bethe-Salpeter amplitude is commonly decomposed into

$$\Gamma(q; P) = \gamma_5 [iE + \gamma \cdot P F + \gamma \cdot q G + i[\gamma \cdot P, \gamma \cdot q] H], \quad (10)$$



$$M_\pi^2 = 0.0852 \text{ GeV}^2$$

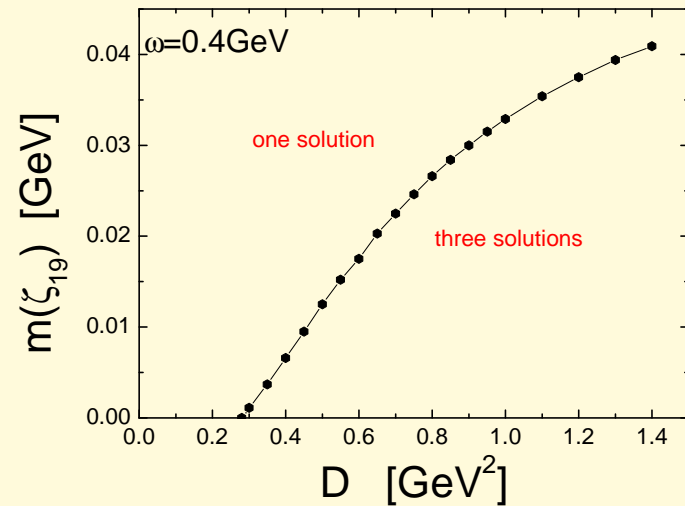
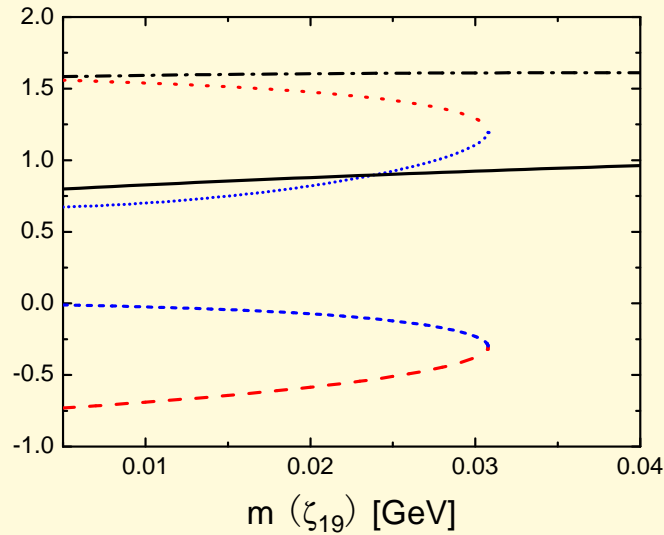
$$f_\pi = 0.104 \text{ GeV}$$



$$M_\pi^2 = -0.0828 \text{ GeV}^2$$

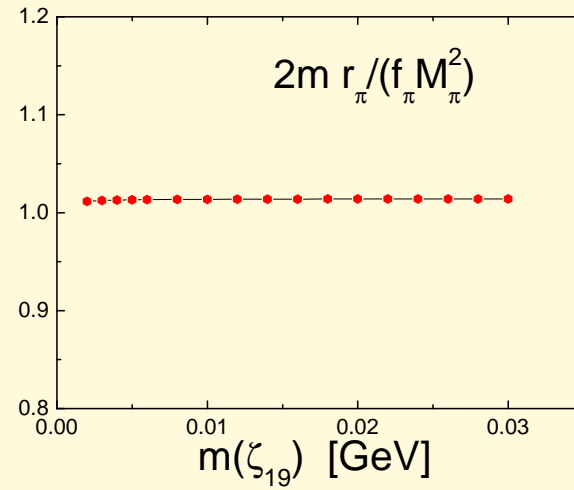
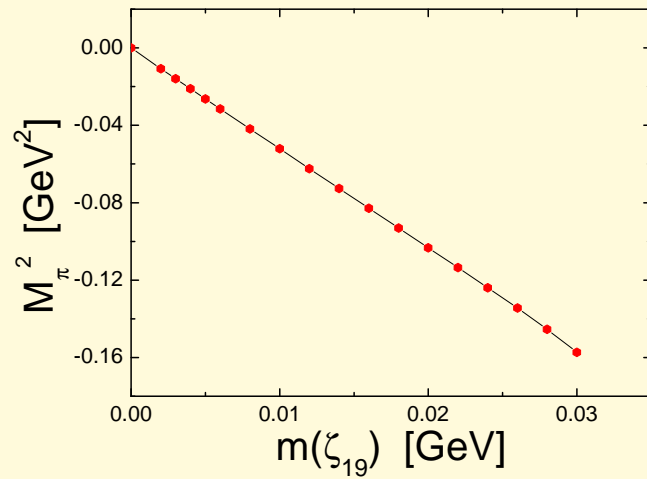
$$f_\pi = -0.079 \text{ GeV}$$

- the critical mass in M-T model



- ★ This model also exhibits a bounded domain of current-quark mass;
- ★ The critical current mass increases with increasing of coupling strength.

- Pion mass in false vacuum on the bounded domain



- Chiral quark condensate beyond the chiral limit-I

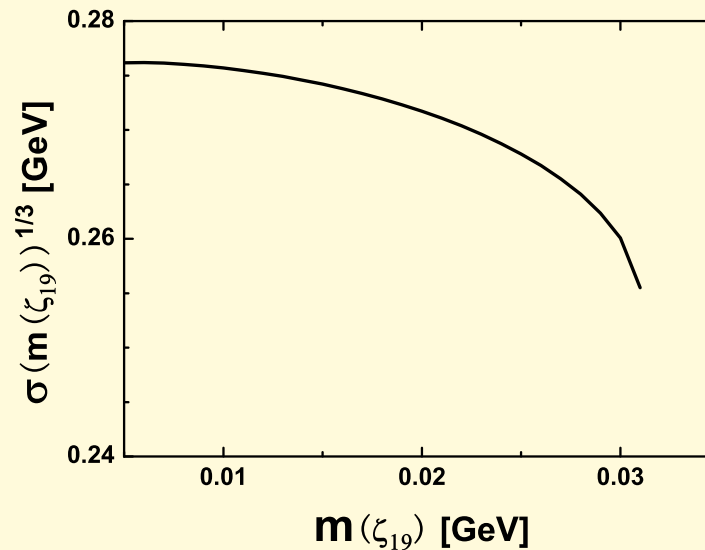
A definition:

$$\bar{\sigma}(m(\zeta)) := \lim_{\Lambda \rightarrow \infty} Z_4(\zeta^2, \Lambda^2) N_c \text{tr}_D \int_q^\Lambda \bar{S}^{m(\zeta)}(q, \zeta), \quad (11)$$

where

$$\bar{S}^{m(\zeta)}(q, \zeta) = \frac{1}{2} \left[ S_+^{m(\zeta)}(q, \zeta) - S_-^{m(\zeta)}(q, \zeta) \right], \quad (12)$$

★ gauge-invariant quantity, well defined, finite and unambiguous



★ the essentially dynamical chiral symmetry breaking decreases with increasing current quark mass

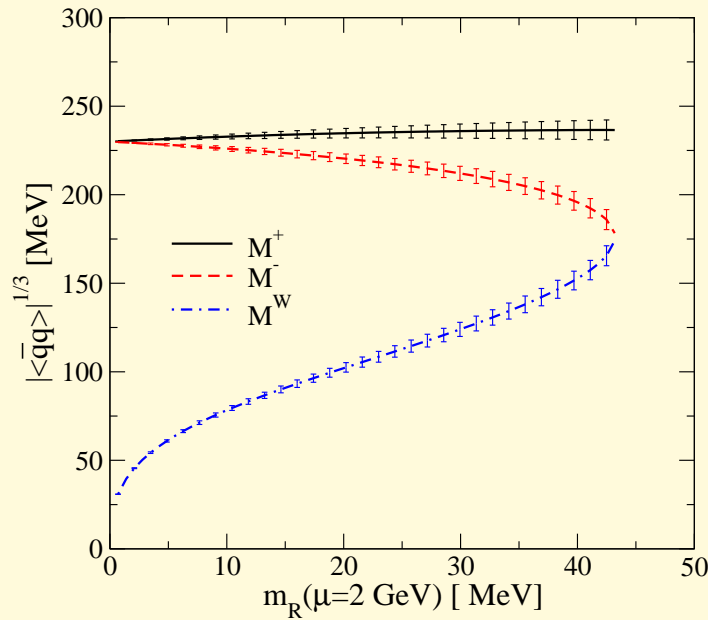
- Chiral quark condensate beyond the chiral limit-II

R. Williams, C. S. Fischer, M. R. Pennington, Phys.Lett. B645 (2007) 167-172.

**Motivation:** the mass function at very large momenta

$$M(p^2)_{asym} = \bar{m} \left[ \log(p^2/\Lambda_1^2) \right]^{-\gamma_m} + \frac{2\pi^2 \mathcal{C}}{3 p^2} \left[ \frac{1}{2} \log(p^2/\Lambda_2^2) \right]^{\gamma_m-1} . \quad (13)$$

**Extracting the Condensates**



## Summary

- On a bounded interval of current-quark mass realistic models of the QCD gap equation can simultaneously admit three different solutions;
- We provide evidence that the upper boundary also defines the radius of convergence for an chiral expansion;
- The pion properties in false vacuum be calculated, the mass square is non-positive and depend on current quark mass linearly;
- A definition of chiral quark condensate beyond chiral limit is elucidated.

# 福娃 Fuwa



福娃贝贝  
Beibei



福娃晶晶  
Jingjing



福娃妮妮  
Nini



福娃迎迎  
Yingying



福娃妮妮  
Nini

## 谢谢