The Beauty of Scalar Mesons?

Confinement: connecting the light- and heavy-quark domains

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Bruno El-Bennich

LPNHE, Theory Group
Université Pierre et Marie Curie, Paris

in collaboration with

Olivier Leitner, Benoît Loiseau & Jean-Pierre Dedonder

The Plot

- The $f_0(980)$ in hadronic charmless *B* decays
- $B \to f_0(980)$ transition form factors in a relativistic quark model
- Dispersion relations & spectral densities
- Parameterization of the $f_0(980)$ wave function
- *D*-decays in QCDF
- Results: transition form factors $F_{\pm}(q^2)$

The $f_0(980)$ in quasi two-body B decays

Examples of *quasi two-body* reactions:

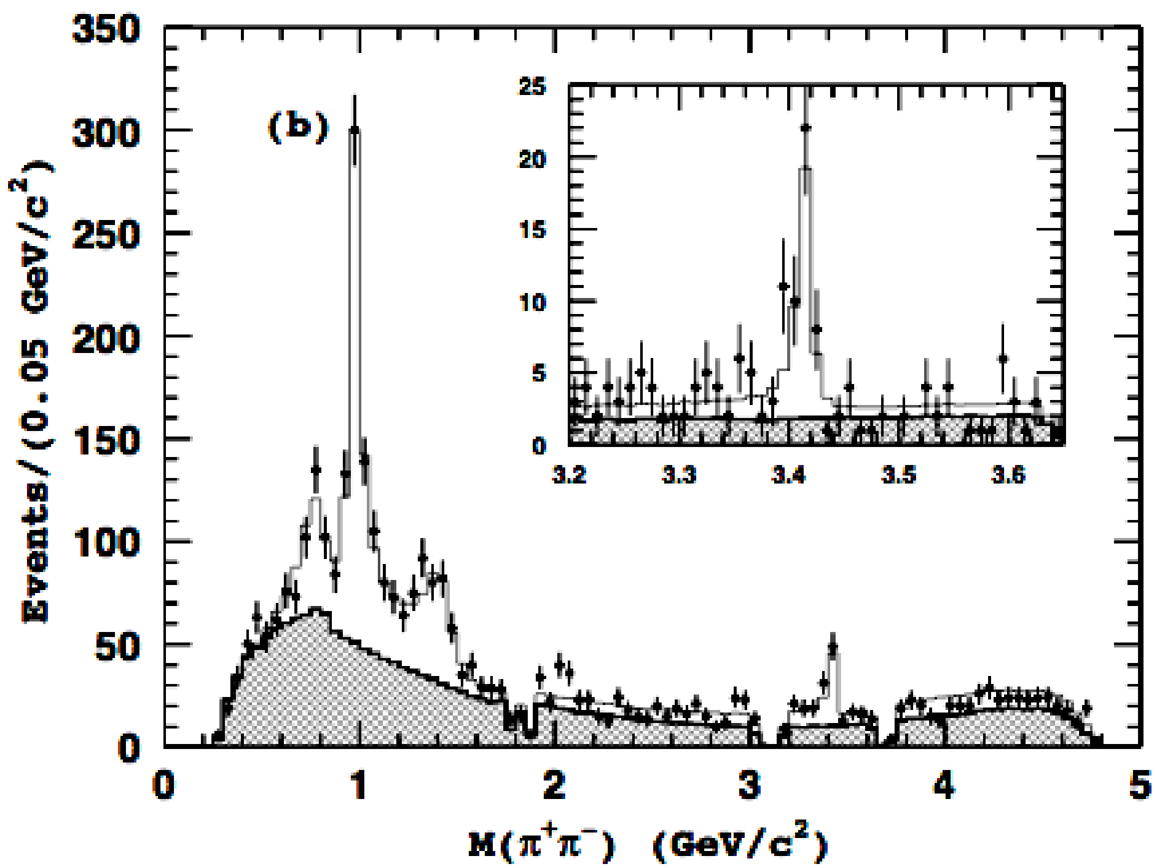
$B^{\pm} \to f_0(980)K^{\pm}$ with subsequent $f_0(980) \to (\pi^+\pi^-)_S$

or $f_0(980) \to (K^+K^-)_S$, where $(\pi^+\pi^-)_S$ and $(K^+K^-)_S$

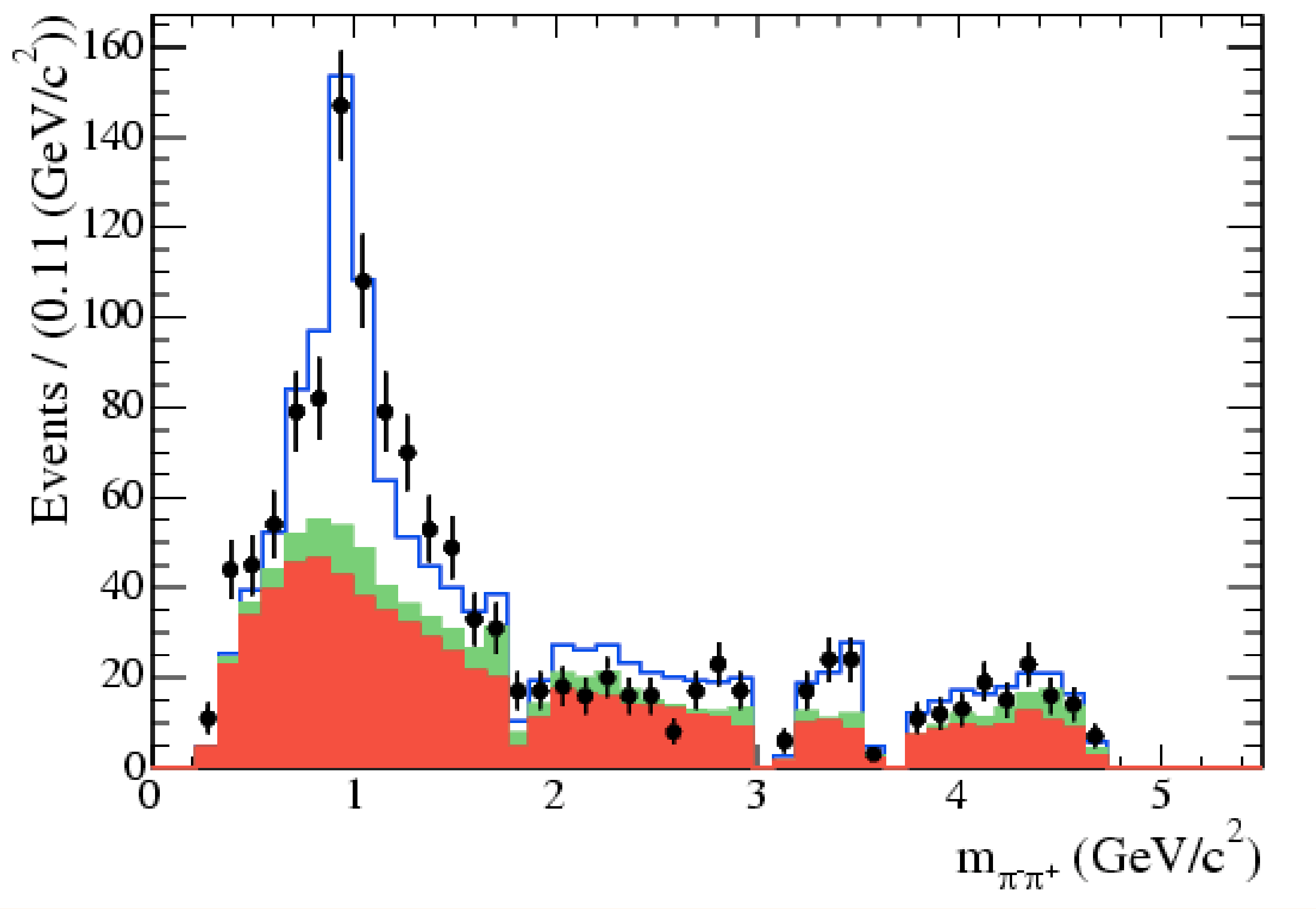
are *S-wave isoscalar* states.

Evidence for scalar resonances in hadronic charmless *B* decays

Belle data (2005)
hep-ex/0509001



BaBar Phys. Rev. D**72** 072003 (2005)



About the anatomy of the scalar-isoscalar $f_0(980)$

Experimentally, there are a few evidences that the $f_0(980)$ can't be either a pure $\bar{u}u(\bar{d}d)$ or $\bar{s}s$ state.

⇨ $BR(J/\Psi \to f_0(980)\phi)$ and $BR(J/\Psi \to f_0(980)\omega)$ measured with almost the same magnitude. Fits give two solutions for *mixing angle*:

$$25^\circ < \theta < 40^\circ \text{ or } 140^\circ < \theta < 160^\circ$$

⇨ First estimate of the mixing angle from $D_S^+ \to f_0(980)\pi^+$ and $D_S^+ \to \phi\pi^+$ decays.

More measurements are necessary!

But is it a meson after all and if yes, what is it made of?

$f_0(980)$?

Other pictures (don't relax yet!)

- In principle one may also consider higher Fock states $\bar{q}qg, \bar{q}^2q^2 \dots$
- The four-quark picture looks plausible for light scalars in low-energy production.
- *But* it is not clear whether energetic $f_0(980)$ in B decays picks up two energetic $\bar{q}q$ pairs.
- Naively, parton distribution amplitudes should be smaller for $\bar{q}^2q^2$ than for $\bar{q}q$.
- *But* for $\bar{q}^2q^2$ <u>twice as many</u> decay diagrams!

(to be continued …)

$B \to f_0(980)$ transition form factors in a relativistic quark model

Calculate the amplitude for $PS \to S\;(M_1 \to M_2)$ transitions:

$$\langle p_2, M_2 | A_\mu | p_1, M_1 \rangle = (p_1 + p_2)_\mu F_+(q^2) + (p_1 - p_2)_\mu F_-(q^2)$$

$p_1^2 = M_1^2,\; p_2^2 = M_2^2;\quad q = p_1 - p_2;\quad A_\mu = \bar{q}_2^i \gamma_\mu \gamma_5 q_1^i\;;\quad i$: color index

Here, q are constituent quarks: $m_u \simeq m_d = 0.35$ GeV, $m_b = 4.95$ GeV

Factorization schematically:

$$\langle M_1 M_2 | Q_k(\mu) | B \rangle \sim \langle M_1 | J_1 | 0 \rangle \langle M_2 | J_2 | B \rangle \left[1 + \sum_n r_n \alpha_s^n + O\left(\Lambda_{QCD} / m_b \right) \right]$$

Decay constant f_M

Transition form factor $F^{B \to M}(q^2)$

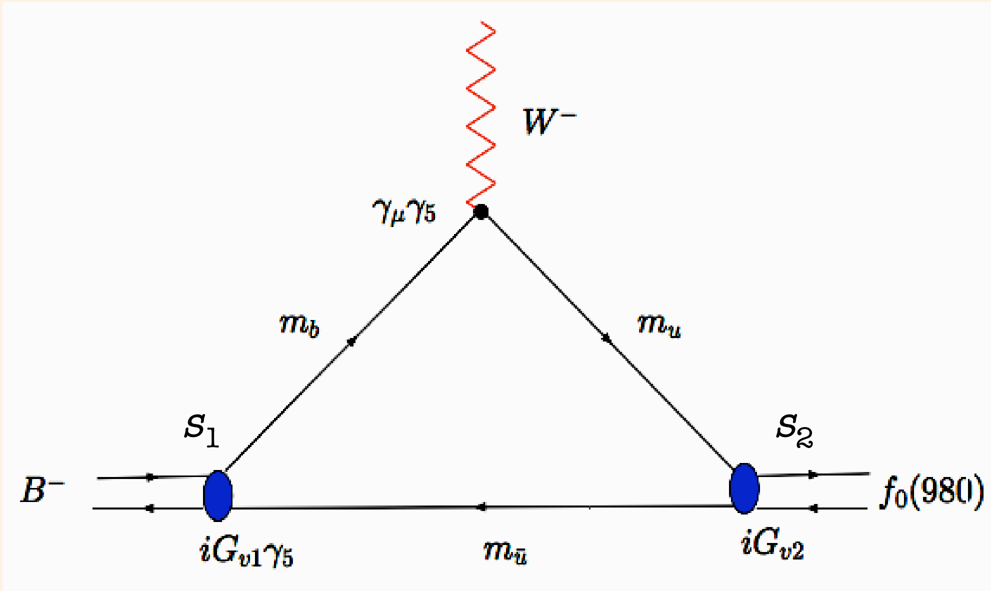
Radiative corrections at weak vertex and final-state hard gluon exchange with spectator quark

Form factors $F_{\pm}(q^2)$ from double spectral densities

The form factor are analytic functions of the external mass variables s_1 and s_2 and can be represented by the double spectral representation:

$$F_{\pm} = \int \frac{ds_1 G_{v1}(s_1)}{\pi(s_1 - M_1^2)} \int \frac{ds_2 G_{v2}(s_2)}{\pi(s_2 - M_2^2)} \Delta_{\pm}(s_1, s_2, q^2 \,|\, m_1, m_2, m_3)$$

where the spectral densities $\Delta_{\pm}(s_1, s_2, q^2 \,|\, m_1, m_2, m_3)$ are obtained from the triangle diagram in a *relativistic quark model*.



The spectral function $\Delta_{\pm}(q^2)$

Use Landau-Cutkosky rules to calculate double discontinuity of the *pseudoscaler to scalar* transition diagram.

$$(\tilde{p}_1+\tilde{p}_2)^\mu \Delta_+(s_1,s_2,q^2;m_1,m_2,m_3)+(\tilde{p}_1-\tilde{p}_2)^\mu \Delta_-(s_1,s_2,q^2;m_1,m_2,m_3)=$$
$$=\frac{1}{8\pi}\int dk_1 dk_2 dk_3\,\delta(k_1^2-m_1^2)\delta(k_2^2-m_2^2)\delta(k_3^2-m_3^2)\delta(\tilde{p}_1-k_2-k_3)\delta(\tilde{p}_2-k_3-k_1)$$
$$\times\,\mathrm{Tr}\left[-(\not{k}_1+m_1)\gamma^\mu\gamma^5(\not{k}_2+m_2)i\gamma^5(m_3-\not{k}_3)i\right].$$

Masses m_1,m_2,m_3 are constituent quark masses

Integration over s_1 and s_2

The double spectral integrals are evaluated for $q^2 < 0$ and can be analytically continued to $q^2 > 0$, where one has square-root and logarithmic cuts in s_1 (fixed s_2) on the physical sheet of the Riemann surface. These cuts give different contributions to the transition form factors.

Decay processes in the $0 < q^2 < (m_2 - m_1)^2$ region

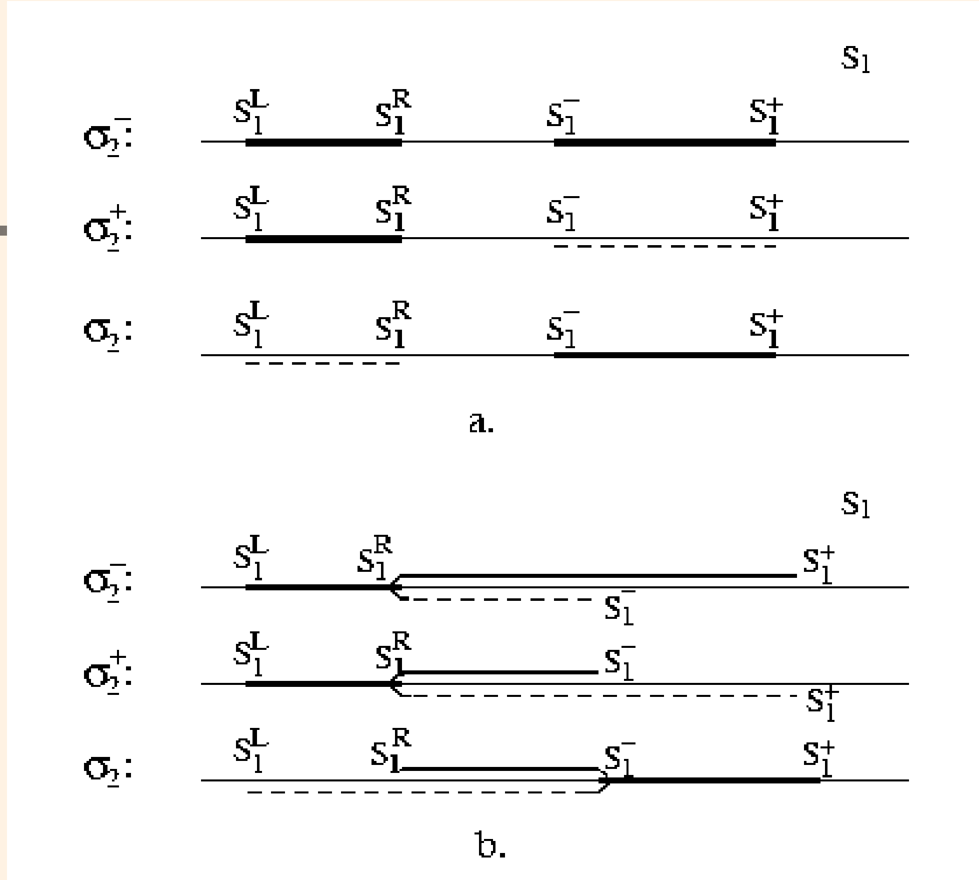
$$F_{\pm} = \int \frac{ds_1 G_{v1}(s_1)}{\pi(s_1 - M_1^2)} \int \frac{ds_2 G_{v2}(s_2)}{\pi(s_2 - M_2^2)} \Delta_{\pm}(s_1, s_2, q^2 \,|\, m_1, m_2, m_3)$$

The $G_v(s)$ vertex functions have no right-hand singularities:

$$\longrightarrow \quad \Gamma(p_1^2, p_2^2, q^2) = \iint \frac{ds_1}{\pi(s_1 - p_1^2)} \frac{ds_2}{\pi(s_2 - p_2^2)} \Delta(s_1, s_2, q^2 \,|\, m_1, m_2, m_2)$$

$$\Gamma(p_1^2, p_2^2, q^2) = \int_{(m_1+m_2)^2}^{\infty} \frac{ds_2}{\pi(s_2 - p_2^2)} \sigma_2(s_1, s_2, q^2)$$

Consider $\sigma_2(s_1, s_2, q^2)$ as the analytic function of $s_1 = p_1^2$ for fixed s_2 and $q^2 > 0$.



$$\sigma_2(p_1^2, s_2, q^2) = \sigma_2^+(p_1^2, s_2, q^2) - \sigma_2^-(p_1^2, s_2, q^2),$$

$$\sigma_2^\pm(s_1, s_2, q^2) = \frac{1}{16\pi\lambda^{1/2}(s_1, s_2, q^2)}$$

$$\times \log\left(-s_2(s_1 + q^2 - s_2 + m_1^2 + m_3^2 - 2m_2^2) - (s_1 - q^2)(m_1^2 - m_3^2) \pm \lambda^{1/2}(s_2, m_1^2, m_3^2)\lambda^{1/2}(s_1, s_2, q^2)\right)$$

Final properly regularized representation for form factors $F(q^2)$

$$F(q^2) = \int\limits_{(m_1+m_3)^2}^{\infty} \frac{ds_2 G_{v2}(s_2)}{\pi(s_2 - M_2^2)} \int\limits_{s_1^-}^{s_1^+} \frac{ds_1 G_{v1}(s_1)}{\pi(s_1 - M_1^2)} \frac{B(s_1, s_2, q^2)}{16\lambda(s_1, s_2, q^2)}$$

$$+ 2\theta(q^2) \int\limits_{s_2^0}^{\infty} \frac{ds_2 G_{v2}(s_2)}{\pi(s_2 - M_2^2)} \int\limits_{s_1^R}^{s_1^-} \frac{ds_1}{16\pi(s_1 - s_1^R)^{3/2}} \left[\frac{G_{v1}(s_1) B(s_1, s_2, q^2)}{(s_1 - s_1^L)^{3/2}(s_1 - M_1^2)} - \frac{G_{v1}(s_1^R) B(s_1^R, s_2, q^2)}{(s_1^R - s_1^L)^{3/2}(s_1^R - M_1^2)} \right]$$

Structure of mesons in terms of vertex/wave functions

The vertices G_{v1} and G_{v2} describe the $\bar{q}q$ bound states:

$$M_1^{PS}: \frac{1}{\sqrt{N_c}} \bar{q}_2(k_2) i\gamma_5 q_3(-k_3) G_{v1}(s_1)$$

$$M_2^{S}: \frac{1}{\sqrt{N_c}} \bar{q}_1(k_1) i q_3(-k_3) G_{v2}(s_2)$$

and the meson wave function $\varphi(s)$ is related to the bound state by:

$$\varphi(s) = \frac{G_v(s)}{s - M^2} = \sqrt{2}\left(\frac{8\pi\alpha}{s\mu^2}\right)^{\frac{3}{4}} \frac{\sqrt{s^2 - (m_1^2 - m_2^2)^2}}{\sqrt{s - (m_1 \pm m_2)^2}} \frac{1}{s^{3/4}} \phi(k)$$

$+$ scalar

$-$ pseudoscalar

with $\mu = \frac{m_1 m_2}{m_1 + m_2}$, $s = (p_1 + p_2)^2$ and α is the size parameter.

The $f_0(980)$ wave function

Assuming mixture of $\bar{u}u$, $\bar{d}d$ and $\bar{s}s$ components:

$$\Psi_{f_0} = \frac{N}{\sqrt{2}}(\bar{u}u + \bar{d}d)\sin\theta_m + \bar{s}s\cos\theta_m = N[\phi^n \sin\theta_m + \phi^s \cos\theta_m]$$

where $n = u, d$ and θ_m is the mixing angle.

Wave function realization: $\phi^q(k) = \frac{1}{\sqrt{2}}\bar{u}(k)v(-k)\exp(-4\,\alpha_q k^2/\mu_{f_0}^2)$

with $q = u, d, s$, $k = \frac{\lambda^{1/2}(s, m_q^2, m_q^2)}{2s^{1/2}}$, $\mu_{f_0} = \frac{m}{2}$ the reduced mass

and α_q the size parameter.

Problem: how to determine parameters of scalar wave function?

Normally, pseudoscalar wave functions can be determined with the knowledge of meson decay constants.

*For scalar meson there are only theoretical estimates with large errors available, so in this case this method is **not** satisfactory!*

▸▸ We employ an alternative method ◂◂

Parameterization of the $f_0(980)$ wave function

Unlike for the B-meson wave function, the size parameter α_q cannot be derived from knowledge of decay constant.

We parametrize the $f_0(980)$ wave function with other observables $\Rightarrow$ Branching ratios of D decays

$$Br(D^+ \to f_0\pi^+) = (2.38 \pm 0.75)\times 10^{-4} \quad \text{(E791)}$$

$$Br(D^+ \to f_0 K^+) = (1.22 \pm 0.17)\times 10^{-4} \quad \text{(FOCUS)}$$

$$Br(D^0 \to f_0 \bar{K}^0) = (3.54 \pm 0.6)\times 10^{-3} \quad \text{(ARGUS, CLEO, BABAR)}$$

$$Br(D_s^+ \to f_0\pi^+) = (9.26 \pm 2.1)\times 10^{-3} \quad \text{(E687, E791, FOCUS)}$$

$$Br(D_s^+ \to f_0 K^+) = (2.55 \pm 1.27)\times 10^{-3} \quad \text{(FOCUS)}$$

D decays in QCDF

At tree level in QCD factorization the amplitude can be written (Penguin contributions very small):

$$A(D \to f_0 M_{PS}) = \frac{G_F}{\sqrt{2}} a_i f_{PS} (m_D^2 - m_{f_0}^2) F^{D \to f_0}(m_{PS}^2) V_{cq} V_{uq}^* \begin{cases} \sin\theta_m / \sqrt{2} \\ \cos\theta_m \end{cases}$$

$$q = d, s$$

QCDF amplitudes from Wilson coeff. $\begin{cases} a_1 = C_1 + C_2 / N_c \\ a_2 = C_2 + C_1 / N_c \end{cases}$

Pseudoscalar decay constant

Transition form factor to be fitted $\Rightarrow$ α_q !

CKM matrix elements

Results

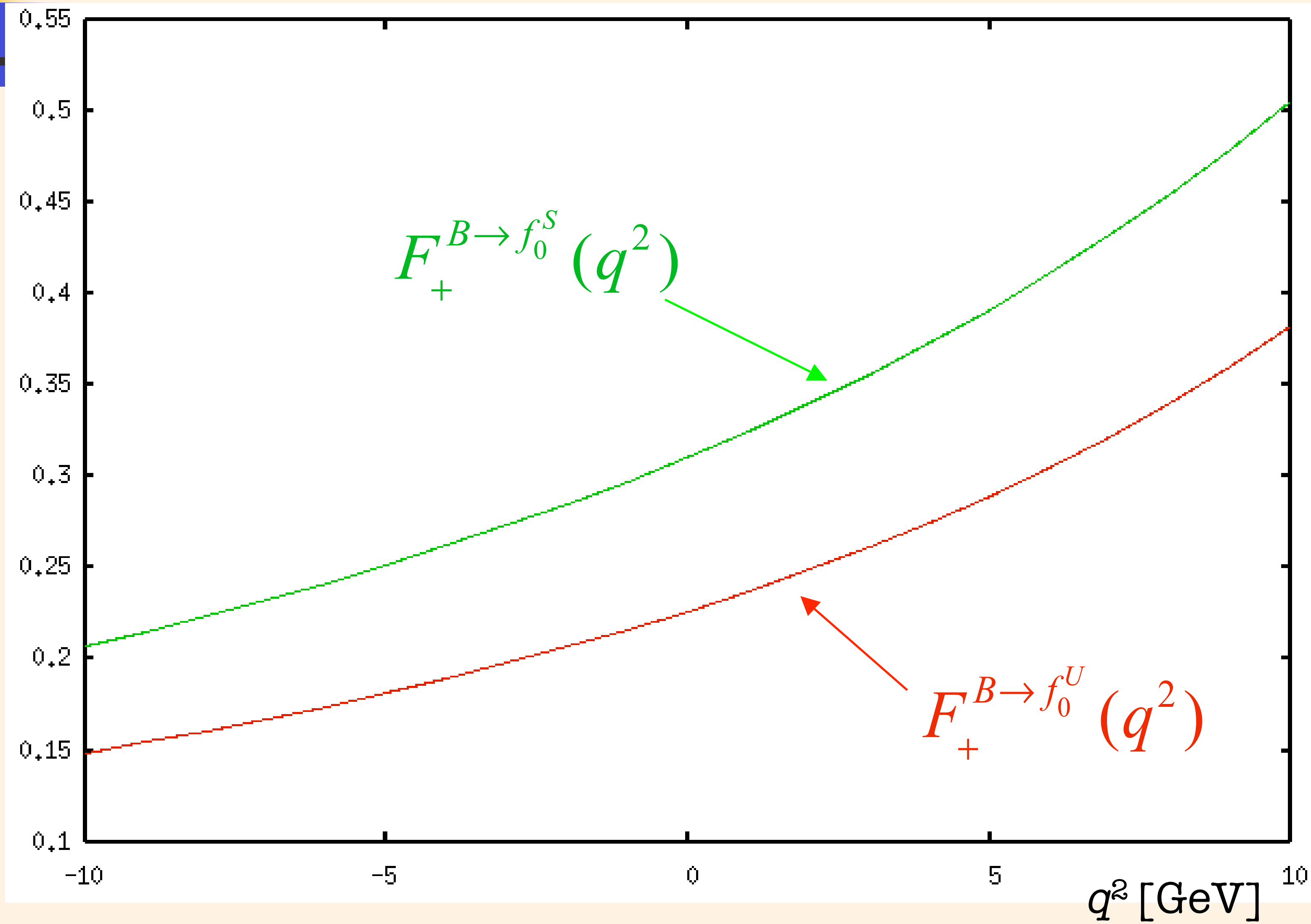
In fitting the D decay branching ratios we obtain a mixing angle: $\theta = 30° \pm 8°$ (strongly data dependent)

At $q^2 = 0$ a recent approximation by Cheng *et al*. PRD **73** (2006) 014017 using an $f_0(980)$ decay constant from QCD sum rules yields:

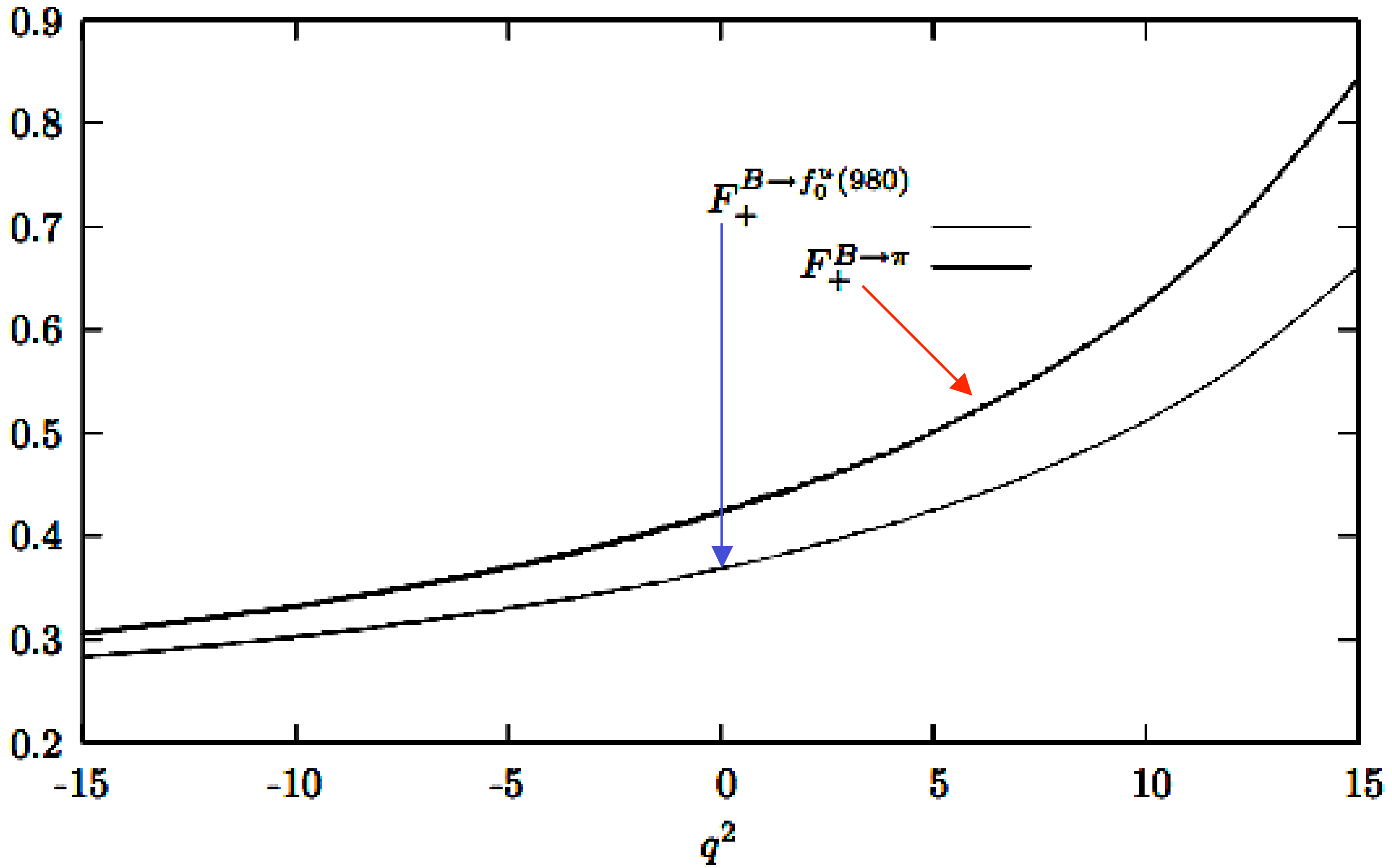
$$F_+^{B\to f_0}(q^2=0) \text{ of order } \approx 0.25$$

$\Rightarrow$ **Our prediction** $F_+^{B\to f_0}(q^2=0) \approx 0.23 - 0.24$

Comparison of $B_u \to f_0^u(980)$ and $B_s \to f_0^s(980)$



Comparison of $F^{+}_{B\to f_0}(q^2)$ and $F^{+}_{B\to\pi}(q^2)$



Conclusions

- We computed the pseudoscalar$\longrightarrow$ scalar transition form factors $F_{\pm}(q^2)$ for B and D decays in a relativistic quark model for $q^2 < 0$ **and** $q^2 > 0$.
- Using branching ratios of D decays we parameterized the scalar $\bar{u}u$, $\bar{d}d$, $\bar{s}s$ components of the $f_0(980)$ wave function.
- Our result for the mixing angle θ_m is in accordance with other theoretical analyses.
- The same calculations in have been performed using the Covariant Light Front Dynamics formalism
  $\rightarrow$ both methods yield very similar results.

THE END

(but not of this "scalar" story)