Hard-exclusive processes and Transition Distribution Amplitudes

Confinement : connecting the light- and heavy-quark domains
March 12-16 2007, ECT*

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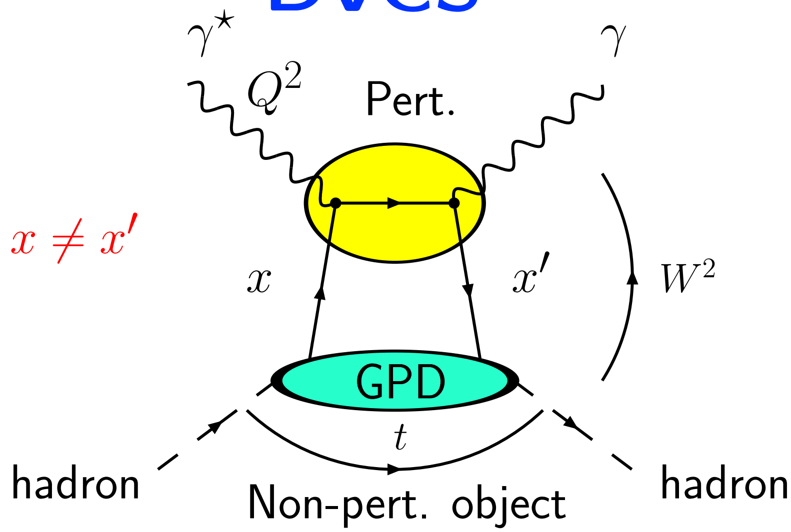
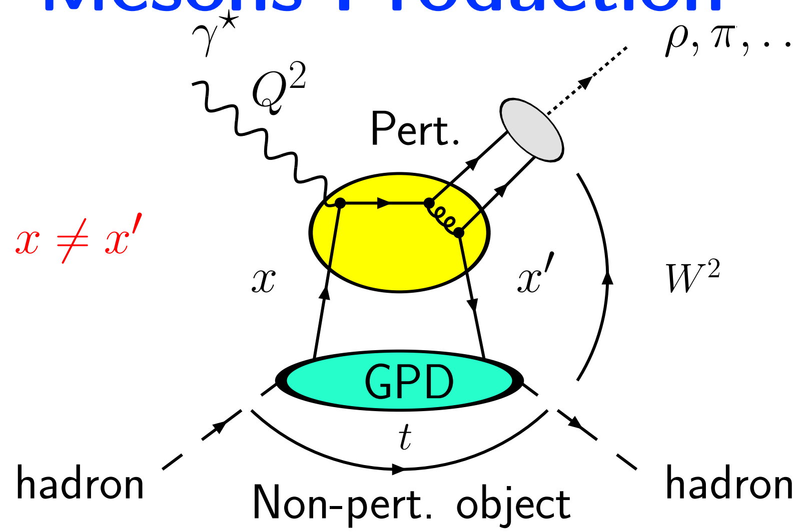
Reminder on Generalised Parton Distributions

DVCS



Mesons Production



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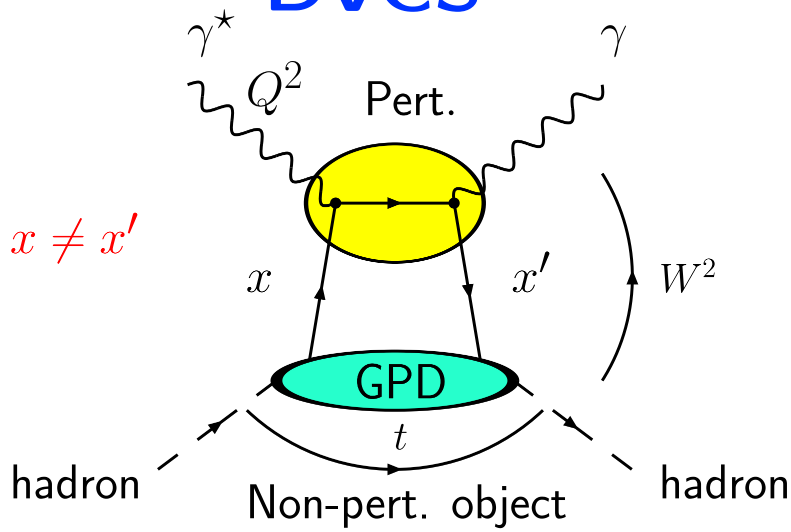

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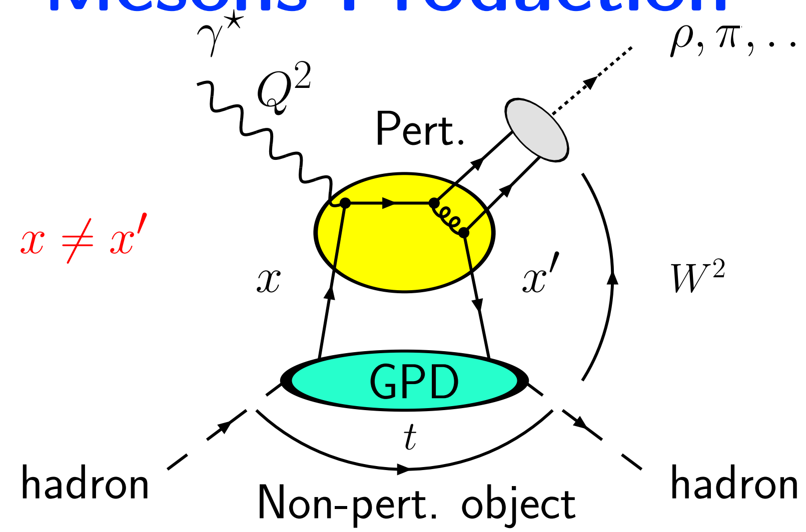

➩ Factorisation between the hard part (perturbatively calculable) and the soft part (non-perturbative) proved by Collins *et al.* for

$$Q^2 \to \infty,\ x_B = \frac{Q^2}{Q^2+W^2} \text{ fixed and } t \ll \text{ fixed}$$

GPDs are universal objects

TDAs : transition distribution amplitudes

B. Pire, L. Szymanowski, PRD 71 :111501,2005 ; PLB 622 :83,2005.

➡ For $u \ll$ DVCS, the non-perturbative part does not describe anymore a $H \to H$ transition, but rather

a hadron-photon or baryon-meson transition.

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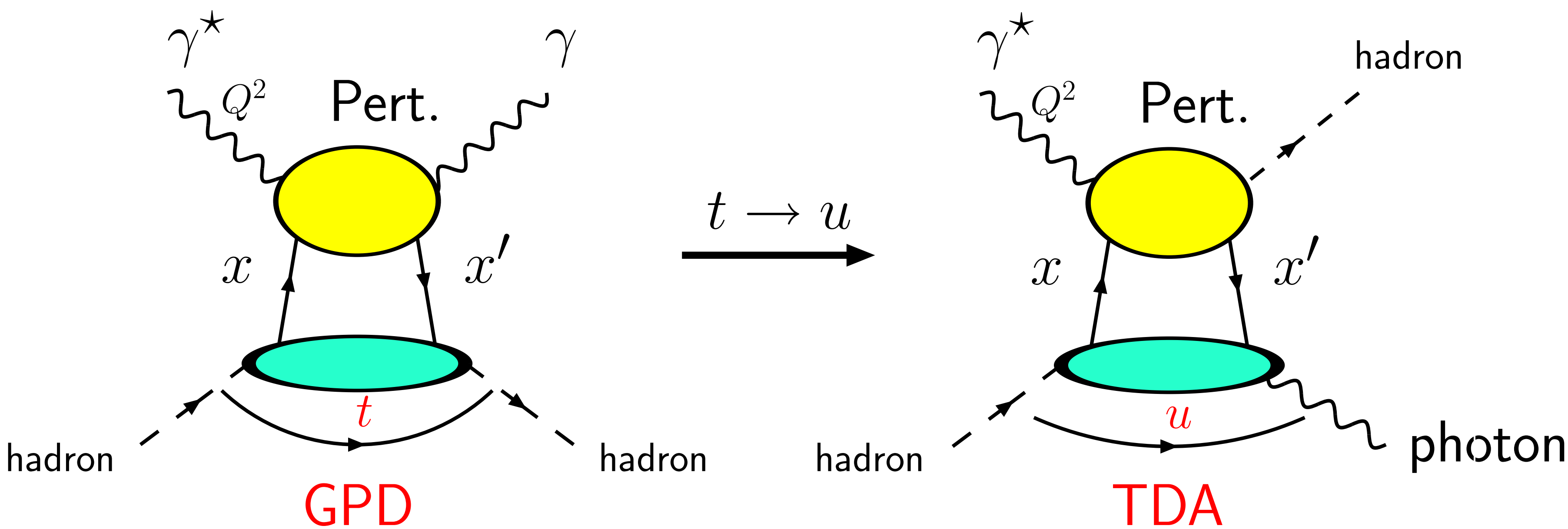


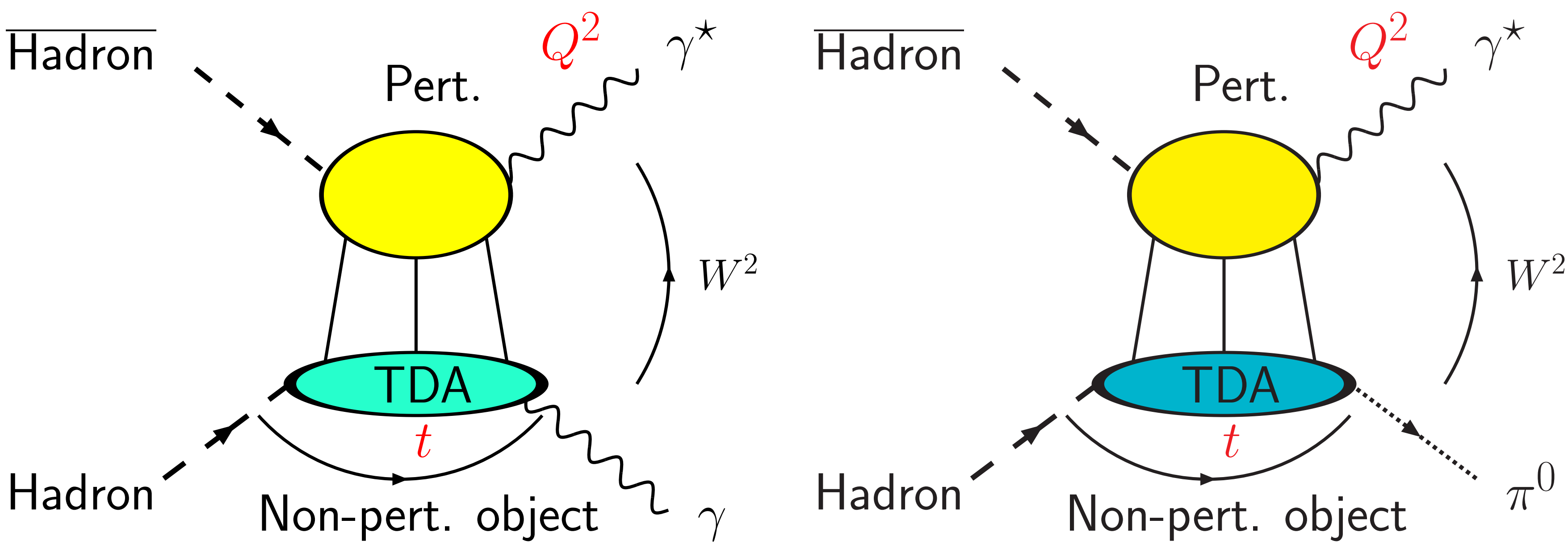
TDAs : transition distribution amplitudes

Also appear in exclusive proton-antiproton annihilations (GSI)

e.g. $p\bar{p} \to \gamma^\star\gamma$ **and** $p\bar{p} \to \gamma^\star\pi^0$ **at** $t \ll$

➯ **Large Q^2 would provide us with a hard scale :**

$\to$ **perturbative expansion**

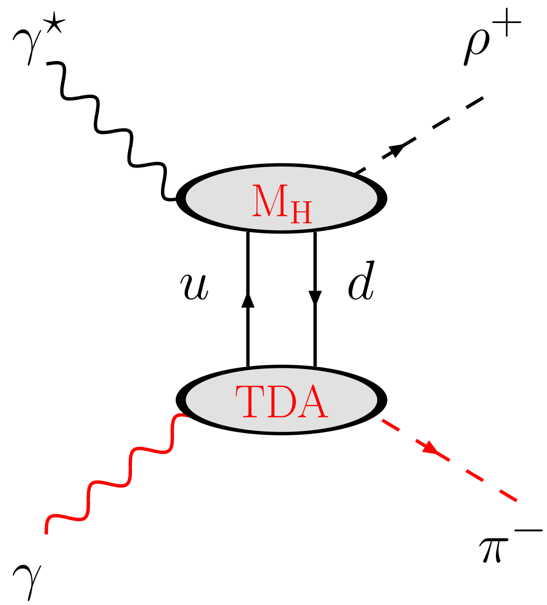


TDAs : transition distribution amplitudes

JPL, B. Pire, L. Szymanowski, PRD 73 :074014,2006.

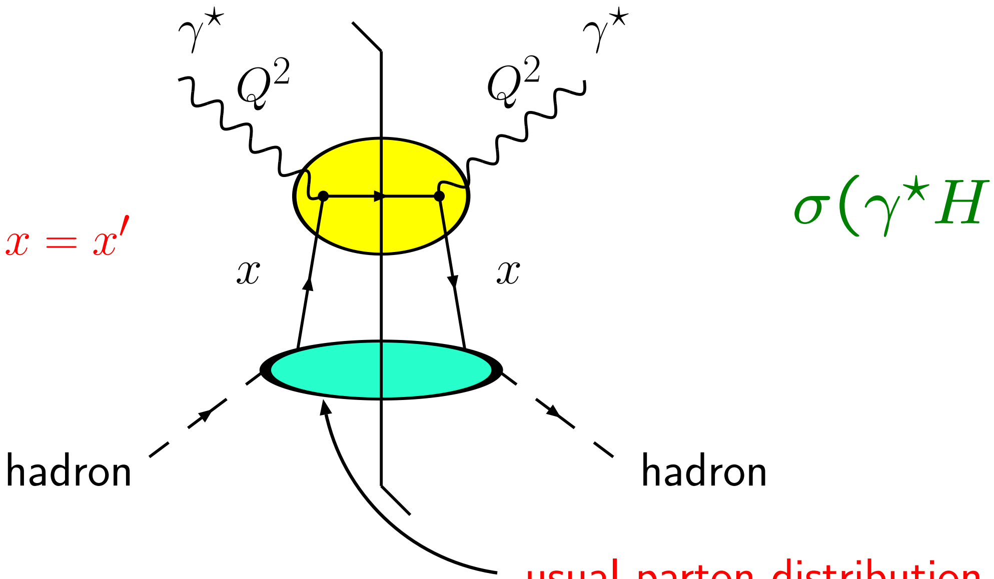
Or at e^+e^- collider (such as B-factories or future facilities)

➩ through $\gamma^\star\gamma$ collisions



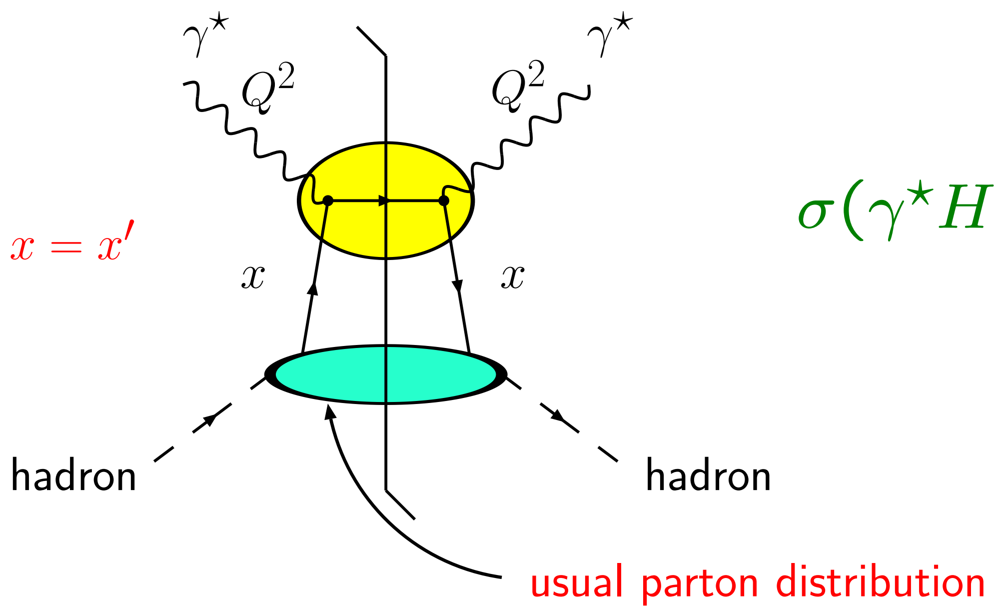
This corresponds to the time-reversed process shown in the previous slide.

GPD in terms of *usual* parton distributions



$$\sigma(\gamma^\star H \to X) \propto \Im m(\mathcal{A}^{diag}(\gamma^\star H \to \gamma^\star H))$$

GPD in terms of *usual* parton distributions



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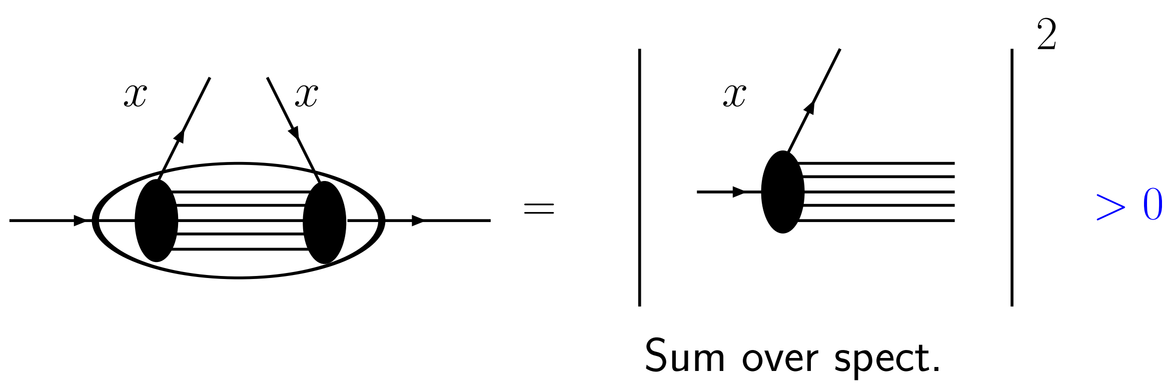
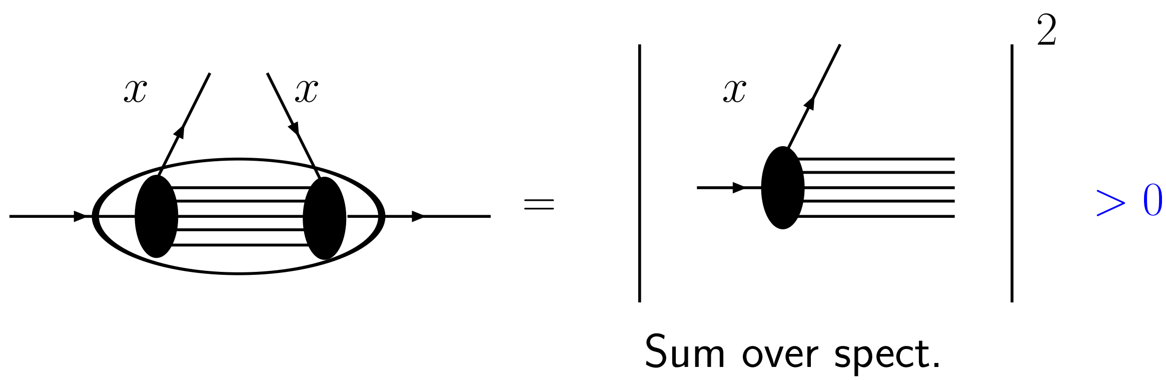


➩ The PDFs give the probability to find, within the hadron, a parton with a momentum fraction x.

GPD in terms of *usual* parton distributions



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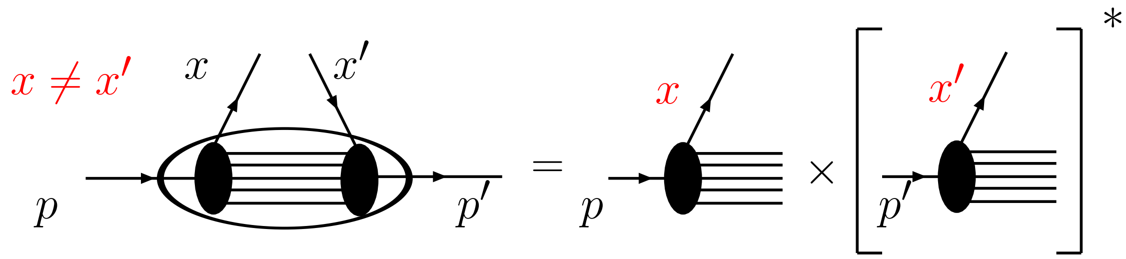
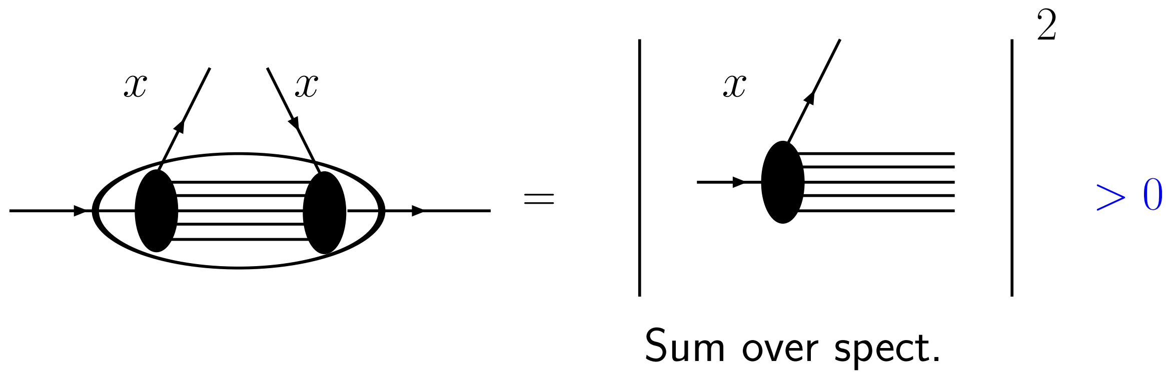
GPD in terms of *usual* parton distributions

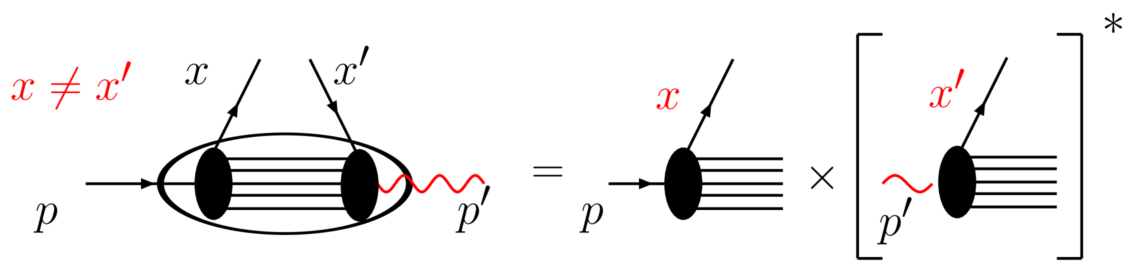


➩ The GPDs possess an interpretation at the amplitude level and provide with information about correlations between quarks



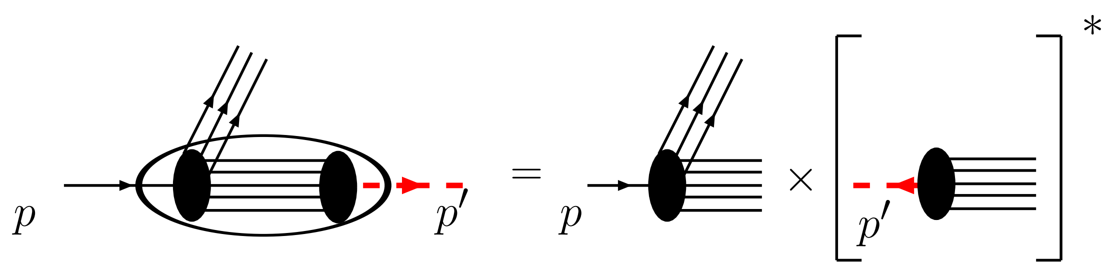
➩ The PDFs give the probability to find, within the hadron, a parton with a momentum fraction x.

Interpretation of the TDAs



➩ The mesonic TDAs possess an interpretation at the amplitude level and provide with information on correlations between a meson DA and a photon DA

whereas



➩ The baryonic TDAs rather provide information on how one can find a meson (a way to study the pion cloud) or a photon in the baryon

TDAs vs GPDs : meson case

	GPDs	TDAs
Matrix elements	$\langle M(p')\vert\Phi^\dagger(z)\Phi(0)\vert M(p)\rangle$	$\langle \gamma(p',\varepsilon)\vert\Phi^\dagger(z)\Phi(0)\vert M(p)\rangle$
Forward limit $\xi \to 0$, $t \to 0$	GPDs $\to$ PDFs $H^q(x,0,0) = q(x)$	N/A
Sum rules : $\int dx$ $\to$ local operator	$\int dx H(x,\xi,t) = F(t)$	$\int dx T(x,\xi,t) = F_{A\to B}(t)$

➯ In view of the sum rules, both GPDs and TDAs are such that their integral on x is independent of ξ !

➟ possible modelling of the TDAs through double distributions (cf. Radyushkin)

➟ we still need non-perturbative inputs !

Example : $\gamma \to \pi$ TDAs

JPL, B. Pire, L. Szymanowski, PRD 73 :074014,2006.

There are 4 leading-twist helicity amplitudes for $\gamma \to q\bar{q}\pi$

$\to$ 4 TDAs : A, V, T_1 and T_2

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \pi^-(p_{\pi^-})|\bar{d}(-\frac{z}{2})[..]\gamma^\mu u(\frac{z}{2})|\gamma(p_\gamma,\varepsilon)\rangle\Big|_{z^+=0,\, z_T=0} = \frac{1}{P^+}\frac{i\,e}{f_\pi}\epsilon^{\mu\varepsilon P\Delta_\perp} V^{\pi^-}(x,\xi,t)$$

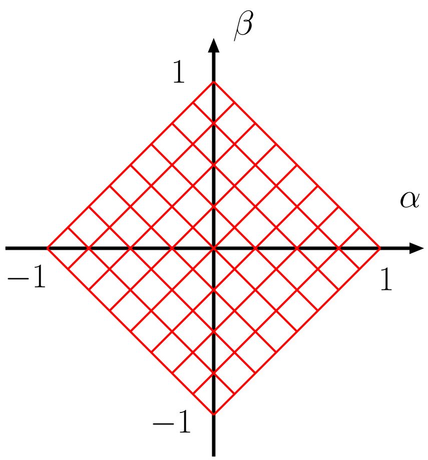
$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \pi^-(p_{\pi^-})|\bar{d}(-\frac{z}{2})[..]\gamma^\mu\gamma^5 u(\frac{z}{2})|\gamma(p_\gamma,\varepsilon)\rangle\Big|_{z^+=0,\, z_T=0} = \frac{1}{P^+}\frac{e}{f_\pi}(\varepsilon\cdot\Delta)P^\mu A^{\pi^-}(x,\xi,t),$$

$$\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \pi^-(p_{\pi^-})|\bar{d}(-\frac{z}{2})[..]\sigma^{\mu\nu} u(\frac{z}{2})|\gamma(p_\gamma,\varepsilon)\rangle\Big|_{z^+=0,\, z_T=0}$$
$$= \frac{e}{P^+}\epsilon^{\mu\nu\rho\sigma}P_\sigma\Big[\varepsilon_\rho T_1^{\pi^-}(x,\xi,t) - \frac{1}{f_\pi}(\varepsilon\cdot\Delta)\Delta_{\perp\rho}T_2^{\pi^-}(x,\xi,t)\Big]$$

Example : $\gamma \to \pi$ TDAs

For $G = (A, V)$, $G^{(0)}(x,\xi) \equiv \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(x-\beta-\xi\alpha) f(\beta,\alpha)$

this insures the correct behaviour with respect to Lorentz invariance



our first choice

- the Double Distribution $f(\beta,\alpha) = q(\beta)h(\beta,\alpha)$ with $q(x)$ a usual pdf
- $h^{(b)}(\beta,\alpha) = \frac{\Gamma(2b+2)}{2^{2b+1}\Gamma^2(b+1)} \frac{[(1-|\beta|)^2-\alpha^2]^b}{(1-|\beta|)^{2b+1}}$: a profile function

Soft limits should give much better inputs

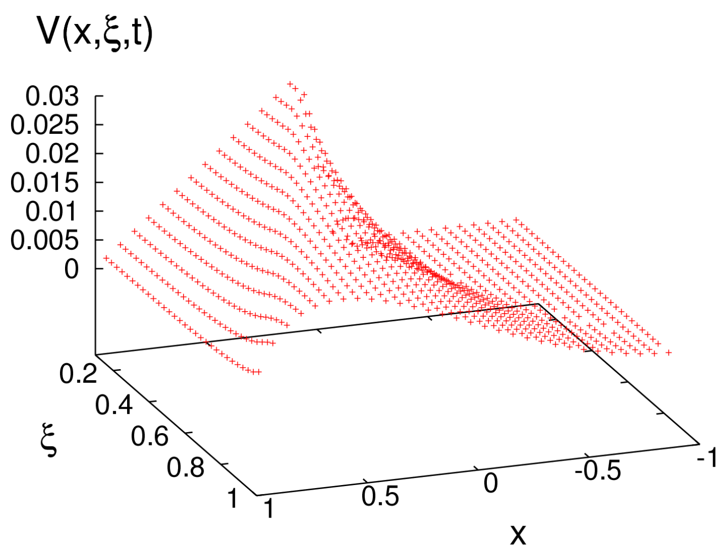
Example : $\gamma \to \pi$ TDAs

The t-dependence is implemented to get the sum rule :

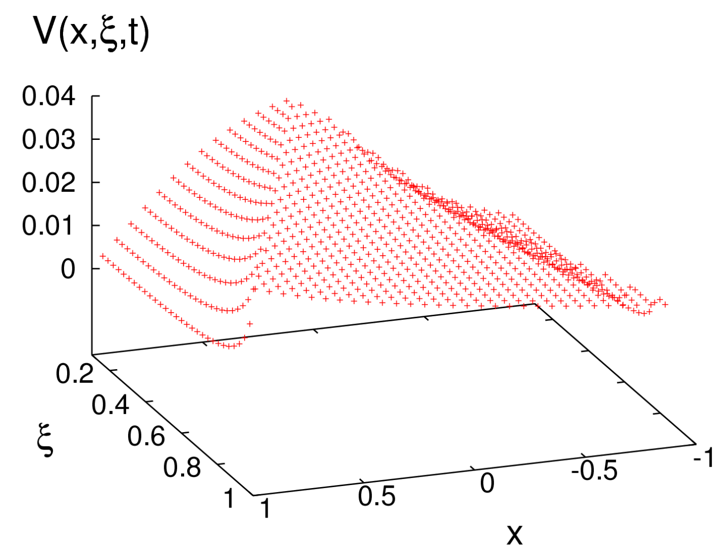
$$G(x,\xi,t) = G^{(0)}(x,\xi) \cdot \frac{f_\pi}{m_\pi} F_G(t) \ .$$

$$\int_{-1}^{1} dx \ G^{(0)}(x,\xi) = 1$$

Setting $b = 1$ and taking $q(\beta) \ = \ 2\,(1-\beta)\,\theta(\beta)$.

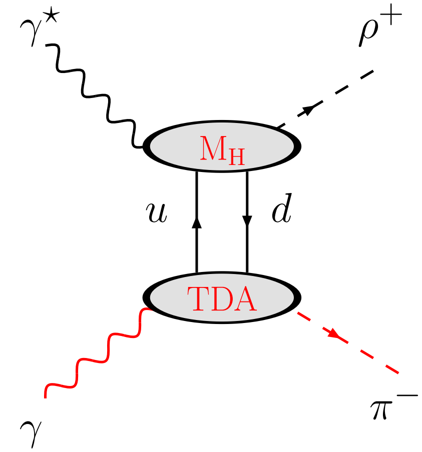


This Model (1)



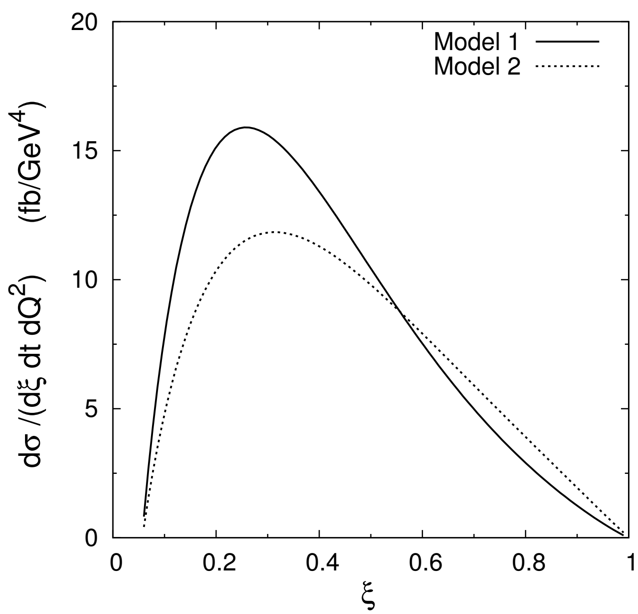
Tiburzi's Model (2)

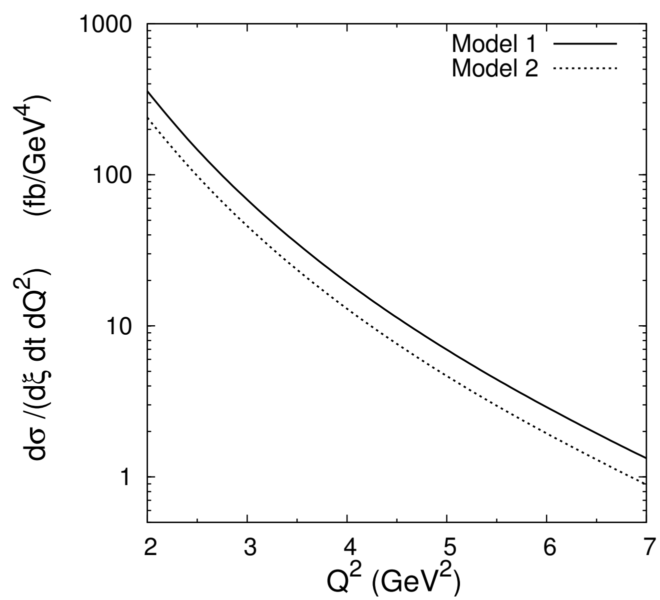
$\gamma \to \pi$ TDAs :Application



- $\rho - \pi$ pair production at small t
  - cross section for $e\ \gamma \to e'\rho\ \pi$
  - π flies in the direction of the γ ($\leftrightarrow \sim 180^\circ$ between ρ and π or large W)
  - Complementary kinematics compared to GDAs or 2π-DAs
  - No Bremsstrahlung

- $s_{e\gamma} = 40$ GeV^2, $t = -0.5$ GeV^2, $Q^2 = 4$ GeV^2 and $\xi = 0.2$





$\gamma \to \pi$ TDAs :Application

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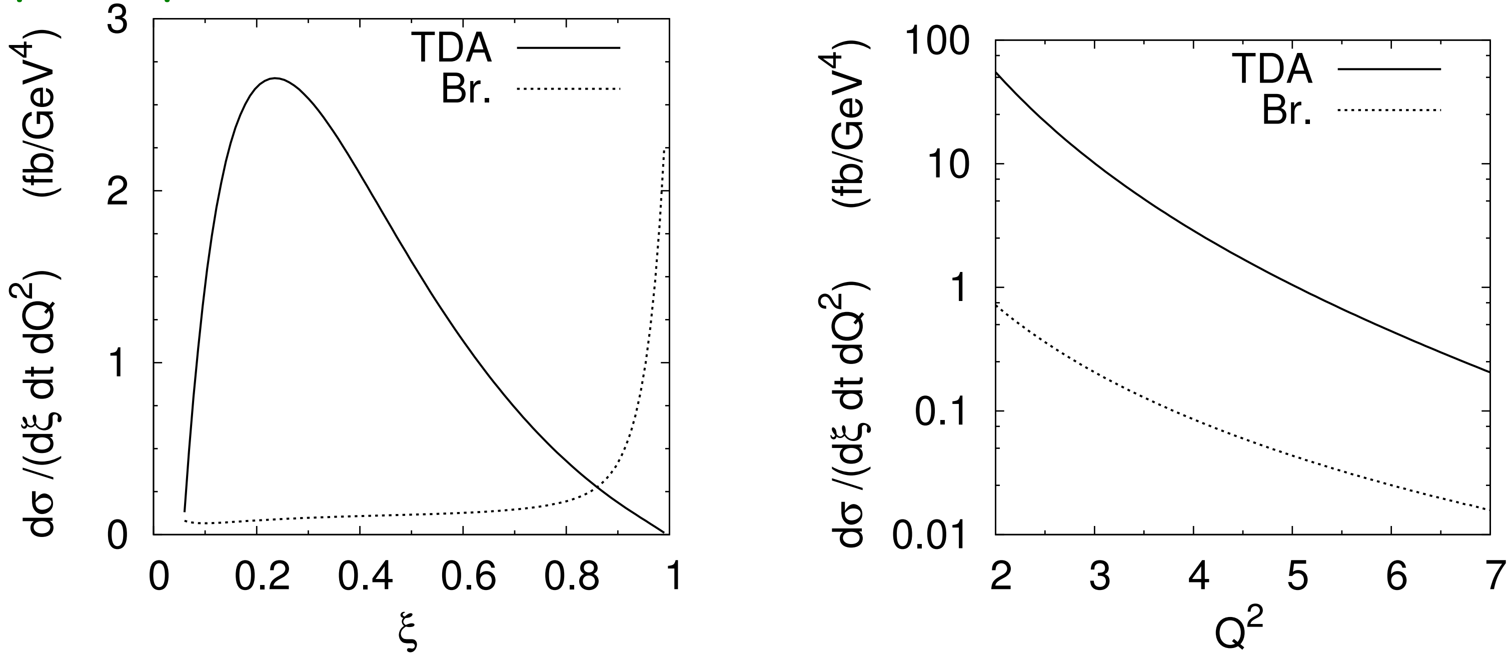

➡ Any measurement of $e\gamma \to e'\pi^0\pi^0$ would provide with information on the Axial Transition Form factor $F_A^{\pi^0}$.

Further Modelling...

- **Spectral Quark Model :**

  W. Broniowski, E. Ruiz Arriola, hep-ph/0701243

- **NJL : S. Noguera *et al.*, on-going work**

- **BSE and DSE : used for PDFs ; previous studies could be extended**

  e.g. **M.B. Hecht, Craig D. Roberts, S.M. Schmidt, PRC 63 :025213,2001**

- **Lattice : as for GPDs, TDA moments are certainly calculable**

  e.g. **QCDSF/UKQCD Collab, D. Brömmel *et al.*, PoS LAT2005 :360,2006.**

TDAs : baryonic case

B. Pire, L. Szymanowski, PLB 622 :83,2005.
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➫ Both for Baryon $\rightarrow$ Meson and Baryon $\rightarrow$ photon,

3 quarks should be exchanged in the t-channel

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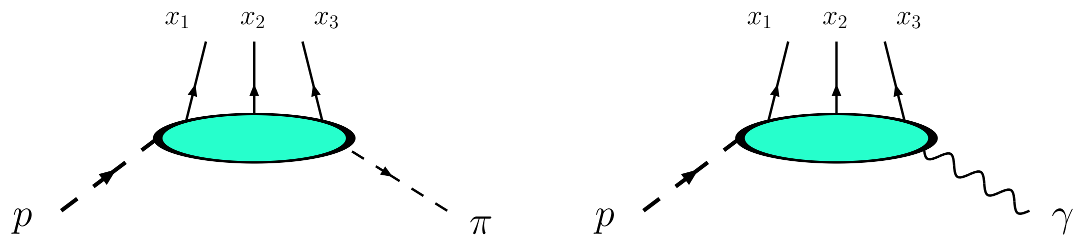

proton $\rightarrow \pi$ proton $\rightarrow \gamma$

- More than the two regions ERBL and DGLAP

- Sum rules

$\rightarrow$ ξ-independence of the moments of the TDA

  - QUADRUPLE distributions : being worked out
  - Diquark picture and double distribution ? would suit some regions only ?

- Closest object : Baryon Distribution Amplitude : $\rightarrow$ **SOFT LIMIT**

$p \to \pi$: parametrisation

- $p \to \pi$ **(at Leading twist accuracy)**
  - $\Delta_T = 0$ **: 3 TDAs** ($3 \times p(\uparrow) \to uud(\uparrow\uparrow\downarrow) + \pi$)

TDA

$$4\langle \pi^0 |\, \epsilon^{ijk} u^i_\alpha(z_1 n) u^j_\beta(z_2 n) d^k_\gamma(z_3 n)\, |p\rangle \propto$$
$$\Big[V_1^{\pi^0}(x_i,\xi,\Delta^2)(\not p\, C)_{\alpha\beta}(N)_\gamma +$$
$$A_1^{\pi^0}(x_i,\xi,\Delta^2)(\not p\, \gamma^5 C)_{\alpha\beta}(\gamma^5 N)_\gamma +$$
$$T_1^{\pi^0}(x_i,\xi,\Delta^2)(\sigma_{\rho p} C)_{\alpha\beta}(\gamma^\rho N)_\gamma \Big]$$

DA (Chernyak-Zhitnitsky)

$$4\langle 0 | \epsilon^{ijk} u^i_\alpha(z_1 n) u^j_\beta(z_2 n) d^k_\gamma(z_3 n) |p\rangle \propto$$
$$\Big[V(x_i)(\not p C)_{\alpha\beta}(\gamma^5 N)_\gamma +$$
$$A(x_i)(\not p \gamma^5 C)_{\alpha\beta} N_\gamma +$$
$$T(x_i)(i\sigma_{\rho p}\, C)_{\alpha\beta}(\gamma^\rho \gamma^5 N)_\gamma \Big]$$

$p \longrightarrow \pi$: parametrisation

➩ $p \longrightarrow \pi$ **(at Leading twist accuracy)**

➟ $\Delta_T = 0$ **: 3 TDAs** ($3 \times p(\uparrow) \rightarrow uud(\uparrow\uparrow\downarrow) + \pi$)

TDA

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$$\Big[V_1^{\pi^0}(x_i, \xi, \Delta^2)(\not p \, C)_{\alpha\beta}(N)_\gamma +$$
$$A_1^{\pi^0}(x_i, \xi, \Delta^2)(\not p \, \gamma^5 C)_{\alpha\beta}(\gamma^5 N)_\gamma +$$
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DA (Chernyak-Zhitnitsky)

$$4\langle 0 | \epsilon^{ijk} u^i_\alpha(z_1 n) u^j_\beta(z_2 n) d^k_\gamma(z_3 n) | p \rangle \propto$$
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$$A(x_i)(\not p \gamma^5 C)_{\alpha\beta} N_\gamma +$$
$$T(x_i)(i\sigma_{\rho p} \, C)_{\alpha\beta}(\gamma^\rho \gamma^5 N)_\gamma \Big]$$

➟ $\Delta_T \neq 0$ **: 8 TDAs** ($\frac{1}{2} \times 2 \times (2 \times 2 \times 2) \times 1$)

$$4\langle \pi^0(p_\pi) | \, \epsilon^{ijk} u^i_\alpha(z_1 n) u^j_\beta(z_2 n) d^k_\gamma(z_3 n) \, | p(p_1, s) \rangle = \frac{i f_N}{f_\pi} \times$$
$$\Big[V_1^{\pi^0}(x_i, \xi, \Delta^2)(\not p \, C)_{\alpha\beta}(N^+)_\gamma + V_2^{\pi^0}(x_i, \xi, \Delta^2)(\not p \, C)_{\alpha\beta}(\not \Delta_T N^+)_\gamma$$
$$+ A_1^{\pi^0}(x_i, \xi, \Delta^2)(\not p \, \gamma^5 C)_{\alpha\beta}(\gamma^5 N^+)_\gamma + A_2^{\pi^0}(x_i, \xi, \Delta^2)(\not p \, \gamma^5 C)_{\alpha\beta}(\gamma^5 \not \Delta_T N^+)_\gamma$$
$$+ T_1^{\pi^0}(x_i, \xi, \Delta^2)(\sigma_{p\mu} C)_{\alpha\beta}(\gamma^\mu N^+)_\gamma + T_2^{\pi^0}(x_i, \xi, \Delta^2)(\sigma_{p\Delta_T} C)_{\alpha\beta}(N^+)_\gamma$$
$$+ T_3^{\pi^0}(x_i, \xi, \Delta^2)(\sigma_{p\mu} C)_{\alpha\beta}(\sigma^{\mu\Delta_T} N^+)_\gamma + T_4^{\pi^0}(x_i, \xi, \Delta^2)(\sigma_{p\Delta_T} C)_{\alpha\beta}(\not \Delta_T N^+)_\gamma \Big] \quad (1)$$

Soft pion limit for proton to pion TDAs

➟ **soft pion limit :** $\xi \to 1$ & $\Delta_T \to 0 \Rightarrow P \to p$

$$\langle \pi^a(k)|\mathcal{O}|p(p,s)\rangle = -\frac{i}{f_\pi}\langle 0|[Q_5^a,\mathcal{O}]|p(p,s)\rangle + \frac{ig_A}{4f_\pi p\cdot k}\sum_{s'}\bar{u}(p,s)\not{k}\gamma_5\tau^a u(p,s')\langle 0|\mathcal{O}|p(p,s')\rangle$$

➟ Using $[Q_5^b,\psi] = -\frac{\tau^b}{2}\gamma^5\psi$, the baryonic DAs appear and we get the following limiting values :

$$V_1^{\pi^0}(2x_1,2x_2,2x_3,\xi\to 1) \to V(x_1,x_2,x_3)$$
$$A_1^{\pi^0}(2x_1,2x_2,2x_3,\xi\to 1) \to A(x_1,x_2,x_3)$$
$$T_1^{\pi^0}(2x_1,2x_2,2x_3,\xi\to 1) \to 3T(x_1,x_2,x_3)$$

➟ Same relations obtained for the proton-pion DAs $\langle 0|\mathcal{O}|\pi(k)p(p,s)\rangle$

V.M Braun *et al.* PRD75 :014021,2007.

Application to backward electroproduction of a pion

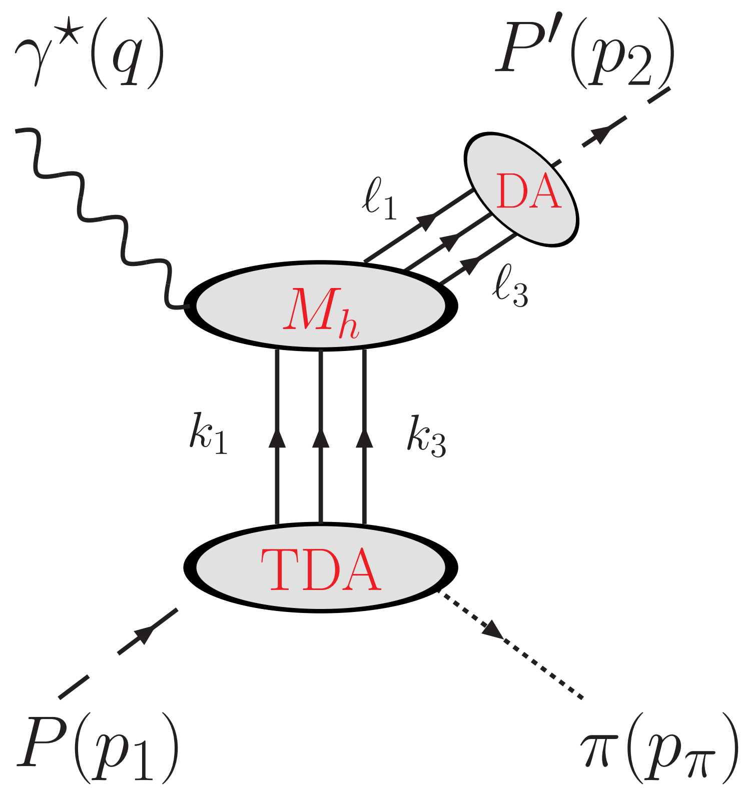
JPL, B. Pire, L. Szymanowski, hep-ph/0701125.

➠ First evaluation : valid at large ξ

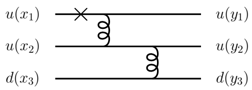
i.e. small pion energy

➠ TDAs extrapolated from their limiting value at $\xi = 1$ ($E_\pi \to 0$)

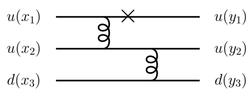
➠ DGLAP contribution neglected : safe for large ξ



Perturbative part : M_h for $\gamma^\star p \to p\pi^0$ at $\Delta_T = 0$



$$\frac{-Q_u(2\xi)^2[(V_1^{p\pi^0}-A_1^{p\pi^0})(V^p-A^p)+4T_1^{p\pi^0}T^p]}{(2\xi-x_1-i\epsilon)^2(x_3-i\epsilon)(1-y_1)^2y_3}$$



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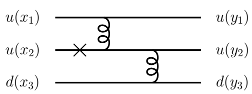
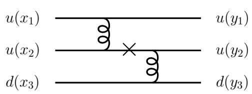
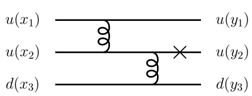


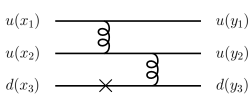
$$\frac{Q_u(2\xi)^2[4T_1^{p\pi^0}T^p]}{(x_1-i\epsilon)(2\xi-x_2-i\epsilon)(x_3-i\epsilon)y_1(1-y_1)y_3}$$



$$\frac{-Q_u(2\xi)^2[(V_1^{p\pi^0}-A_1^{p\pi^0})(V^p-A^p)]}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)(x_3-i\epsilon)y_1(1-y_1)y_3}$$



$$\frac{Q_u(2\xi)^2[(V_1^{p\pi^0}+A_1^{p\pi^0})(V^p+A^p)]}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)(x_3-i\epsilon)y_1(1-y_2)y_3}$$



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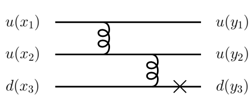
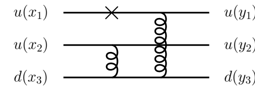


$$\frac{-Q_d(2\xi)^2[2(V_1^{p\pi^0}V^p+A_1^{p\pi^0}A^p)]}{(x_1-i\epsilon)(2\xi-x_3-i\epsilon)^2y_1(1-y_3)^2}$$



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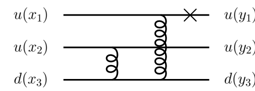
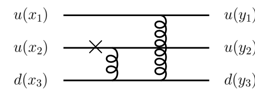
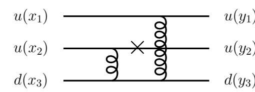


$$\frac{-Q_u(2\xi)^2[(V_1^{p\pi^0}-A_1^{p\pi^0})(V^p-A^p)+4T_1^{p\pi^0}T^p]}{(2\xi-x_1-i\epsilon)^2(x_2-i\epsilon)(1-y_1)^2y_2}$$



$$\frac{-Q_u(2\xi)^2[(V_1^{p\pi^0}+A_1^{p\pi^0})(V^p+A^p)+4T_1^{p\pi^0}T^p]}{(x_1-i\epsilon)(2\xi-x_2-i\epsilon)^2y_1(1-y_2)^2}$$



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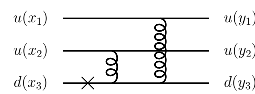
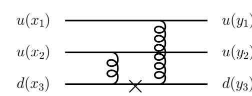
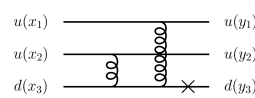


$$\frac{Q_d(2\xi)^2[(V_1^{p\pi^0}+A_1^{p\pi^0})(V^p+A^p)]}{(x_1-i\epsilon)(x_2-i\epsilon)(2\xi-x_3-i\epsilon)y_1(1-y_2)y_2}$$



$$\frac{-Q_d(2\xi)^2[4T_1^{p\pi^0}T^p]}{(x_1-i\epsilon)(2\xi-x_1-i\epsilon)(x_2-i\epsilon)y_1(1-y_2)y_2}$$



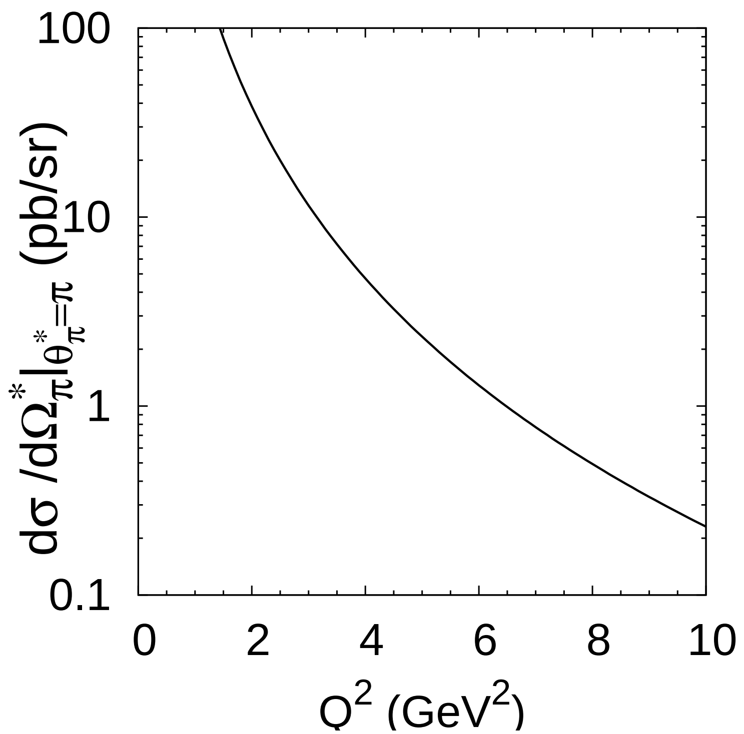
$$\frac{Q_d(2\xi)^2[(V_1^{p\pi^0}-A_1^{p\pi^0})(V^p-A^p)]}{(x_1-i\epsilon)(2\xi-x_1-i\epsilon)(x_2-i\epsilon)y_1y_2(1-y_3)}$$

Application to backward electroproduction of a pion

JPL, B. Pire, L. Szymanowski, hep-ph/0701125.

➟ **The (leading-twist) amplitude reads :**

$$\mathcal{M}^{\lambda}_{s_1 s_2} = -i\frac{(4\pi\alpha_s)^2\sqrt{4\pi\alpha_{em}}f_N^2}{54 f_\pi Q^4}\bar{u}(p_2,s_2)\not{\epsilon}(\lambda)\gamma^5 u(p_1,s_1)\int\limits_{-1+\xi}^{1+\xi} d^3x \int\limits_0^1 d^3y \left(2\sum_{\alpha=1}^{7} T_\alpha + \sum_{\alpha=8}^{14} T_\alpha\right)$$



➟ **Data exist (at least) from JLab (Hall A)**

they will be analysed

➟ **We need more information about the TDAs**

20 years ago...

Gavela, King, Sachrajda, Martinelli

a lattice computation of proton decay amplitudes

Nucl.Phys.B312 :269,1989

➠ **Calculation of the matrix elements for the GUT decays**

$$p \to \pi^0 e^+ \qquad p \to pi^+ \bar{\nu} \qquad p \to K^0 + lepton$$

➠ **Evaluation of the two matrix elements**

$$\epsilon^{ijk} \langle \pi^0 | (u^i C d^j) u^k_\gamma | P \rangle = A_1 N_\gamma \quad \epsilon^{ijk} \langle \pi^0 | (u^i C \gamma_5 d^j)(\gamma_5 u^k)_\gamma | P \rangle = A_2 N_\gamma$$

➠ **Update of this study would be very useful**

Model-independent predictions

- **Scaling law for the amplitude :**

$$\mathcal{M}(Q^2) \propto \frac{\alpha_s^2(Q^2)}{Q^4}$$

- **Approximate Q^2-independence of the ratios**

$$\frac{\mathcal{M}(\gamma^* p \to p\pi)}{\mathcal{M}(\gamma^* p \to p\gamma)}, \quad \frac{\mathcal{M}(\gamma^* p \to p\gamma)}{\mathcal{M}(\gamma^* p \to p)} \textbf{ and } \frac{\frac{d\sigma(p\bar{p}\to\ell^+\ell^-\pi^0)}{dQ^2}}{\frac{d\sigma(p\bar{p}\to\ell^+\ell^-)}{dQ^2}}$$

- **Dominance of γ_T^* emission in $p\bar{p} \to \gamma^* \pi^0 \to \ell^+\ell^-\pi^0$**

  Dilepton angular dependence : $1 + \cos^2\theta$

Conclusions and outlooks

- Quantitative predictions require models
  - **Meson case :**

    double distribution : ok ; models used for GPDs should be suitable
  - **Baryon case :**

    Quadruple distribution : on-going work...
- We still need non-perturbative inputs
  - Soft limits
  - Chiral Perturbation Theory ?
  - DSE/BSE or Fadeev amplitude in Baryonic case ?
- Measurements to come from GSI, JLab and Hermes
  - **Study of the t dependence of** $d\sigma(p\bar{p} \to \gamma^\star \pi^0)$ **and** $d\sigma(\gamma^\star p \to p\pi^0)$

transverse picture on the pion cloud in the proton