# Infrared properties of QCD Greens functions

Christian S. Fischer

TU-Darmstadt

C.F., A. Maas, J. M. Pawlowski and L. v. Smekal, Annals Phys. in print, [hep-ph/0701050].

C.F. and J. M. Pawlowski, Phys. Rev. D **75** (2007) 025012.

R. Alkofer, C.F., F. Llanes-Estrada, hep-ph/0607293.

C.F., J.Phys.G32:R253-R291 (2006).

# Outline

- **Introduction**
- Infrared properties of SU(N) Yang-Mills theory
- Ghost and Glue in a box
- Infrared slavery in quenched QCD
- Light mesons: unquenching effects

# Motivation

QCD Green's functions

- are connected to confinement:
  - Gribov-Zwanziger horizon conditions
  - Positivity
  - Quark-antiquark potential
- encode D$\chi$SB
- are ingredients for hadron phenomenology
  - Bound state equations:
    Bethe–Salpeter equation / Faddeev equation

The Goal:
Ab initio description of hadrons as bound states
in terms of underlying substructure

# Propagators of QCD: Covariant Gauge

- Faddeev-Popov method: quark, gluon, ghost

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[A, c, \Psi] \exp\left\{ \int \bar{\Psi}\left(i\not{D} - m\right)\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial^\mu A^a_\mu)^2}{2\xi} + \bar{c}^a(-\partial^\mu D_\mu)c^a \right\}$$

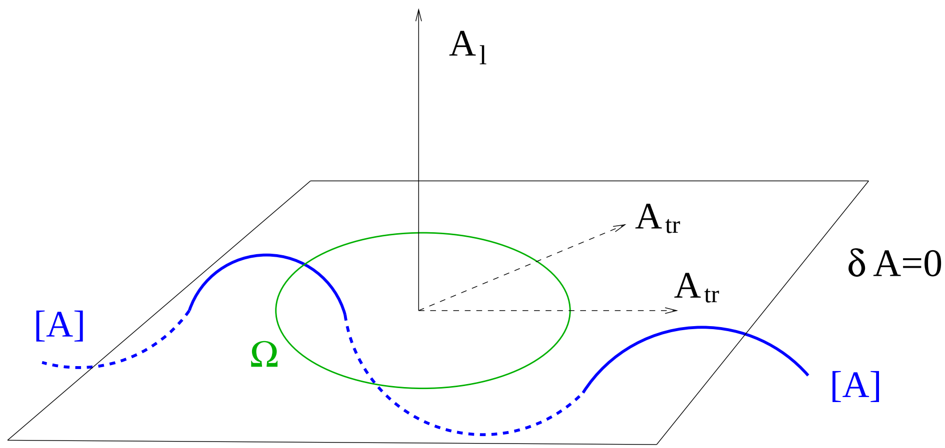
- Landau gauge propagators in momentum space,

$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{\mathbf{Z}(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$D^{\text{Ghost}}(p) = -\frac{\mathbf{G}(p^2)}{p^2}$$

$$S^{\text{Quark}}(p) = \frac{Z_f(p^2)}{-i\not{p} + M(p^2)}$$

# Gauge fixing and Horizon condition



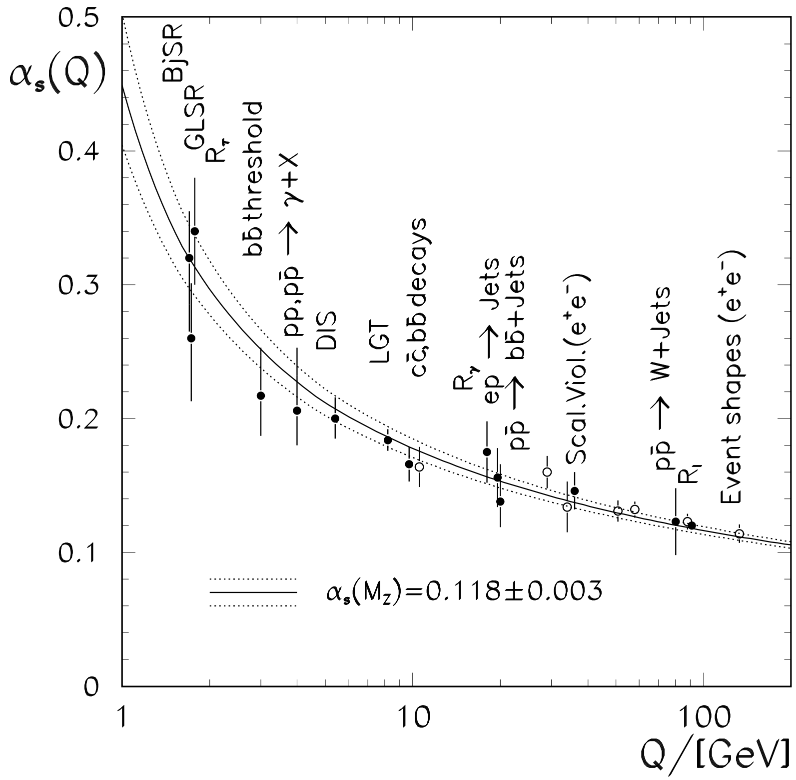

- Problem with gauge fixing: Gribov-copies
- (Partial) Solution: Integrate only over gauge configurations in Gribov region $\Omega$:

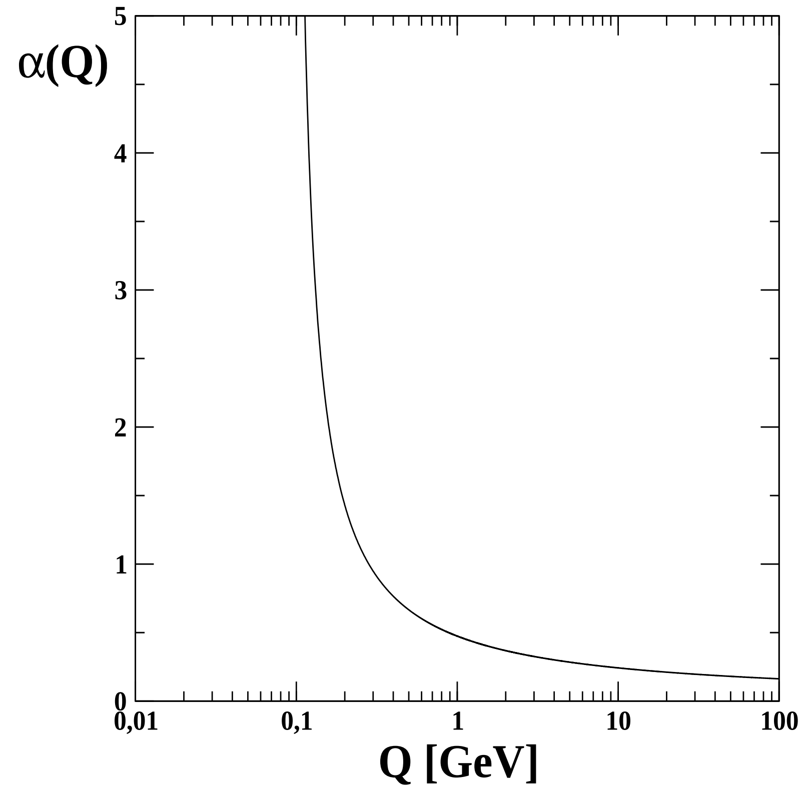
$$\Omega = \{A : \ \partial A = 0 \ \wedge \ -\partial D \geq 0\}$$

- $G(p^2) \stackrel{p^2 \to 0}{\longrightarrow} \infty \, , \qquad \frac{Z(p^2)}{p^2} \stackrel{p^2 \to 0}{\longrightarrow} 0$ Horizon conditions

D. Zwanziger, Phys. Rev. **D69** (2004) 016002.

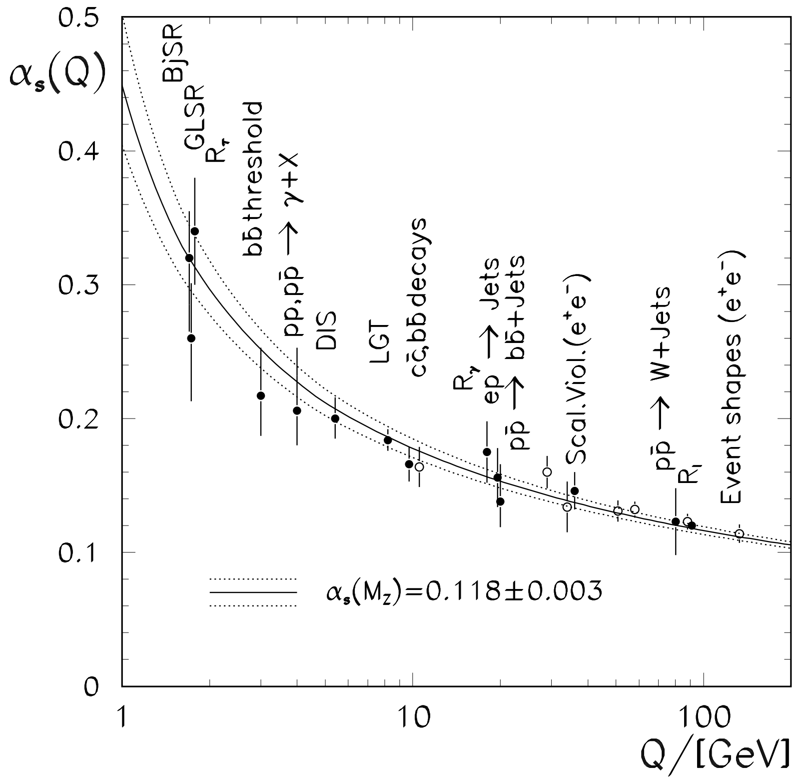
# Running Coupling I

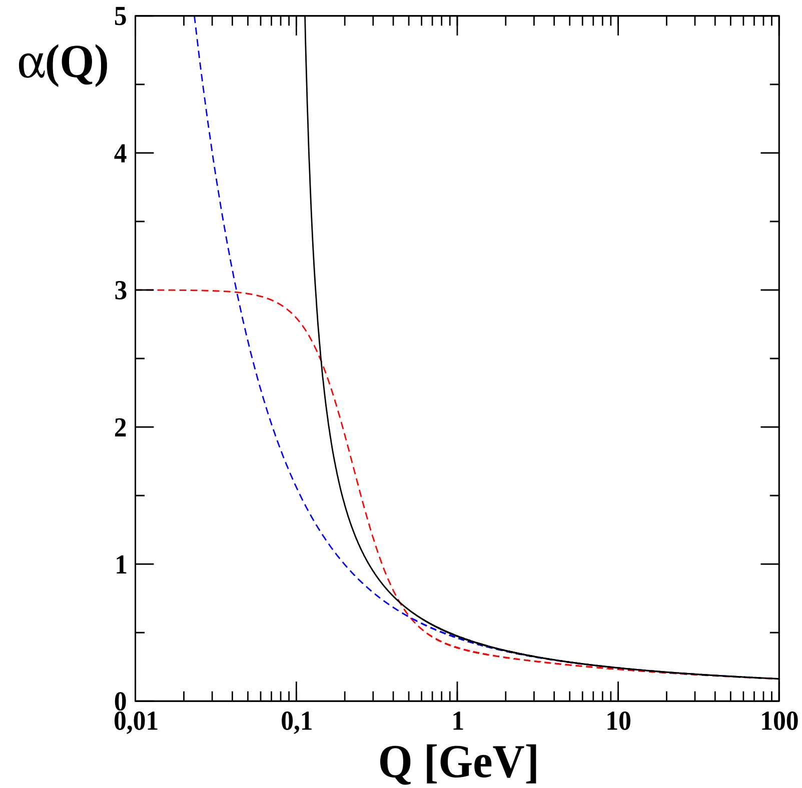



- Perturbation theory: Landau pole

# Running Coupling I



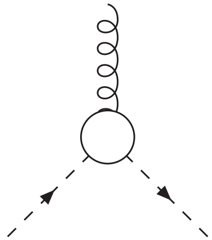




D. J. Gross and F. Wilczek, Phys. Rev. **D8** (1973) 3633–3652.

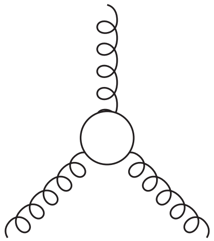
- Nonperturbative definition of running coupling needed!

# Running Coupling II (Nonpert. definition)

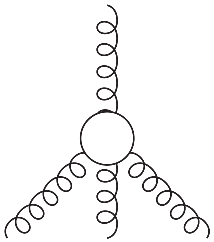
$$\alpha^{gh-gl}(p^2) = \alpha_\mu \, G^2(p^2) \, Z(p^2)$$



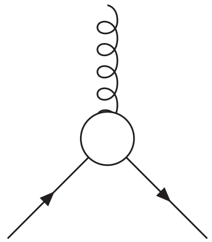
$$\alpha^{3g}(p^2) = \alpha_\mu \, [\Gamma^{3g}(p^2)]^2 \, Z^3(p^2)$$



$$\alpha^{4g}(p^2) = \alpha_\mu \, [\Gamma^{4g}(p^2)] \, Z^2(p^2)$$



$$\alpha^{qg}(p^2) = \alpha_\mu \, [\Gamma^{qg}(p^2)]^2 \, Z_f^2(p^2) \, Z(p^2)$$



# Dynamical Chiral Symmetry Breaking

$$S^{-1}(p) = \left[ i\not{p} + M(p^2) \right] / Z_f(p^2)$$

- **Explicit** breaking of chiral symmetry ($m_{u,d,s,\ldots} \neq 0$)
- **Dynamical** breaking of chiral symmetry:

$$M(p^2) \neq 0 \qquad (m_{u,d,s,\ldots} = 0)$$

Nonperturbative effect!

- Chiral condensate $\langle \bar{\Psi}\Psi \rangle$:

$$M(p^2) \overset{p^2 \to \infty}{\longrightarrow} \frac{-\langle \bar{\Psi}\Psi \rangle}{p^2 \left[ \log(p^2/\Lambda^2) \right]^{1-\gamma}} \qquad (m_{u,d,s,\ldots} = 0)$$

# Introduction: Summary

Dressing functions of propagators of QCD,
$G(p^2)$, $Z(p^2)$, $M(p^2)$, $Z_f(p^2)$

- are connected to Zwanziger's horizon condition and Kugo-Ojima confinement criterion

$$G(p^2) \stackrel{p^2\to 0}{\longrightarrow} \infty \,, \qquad \frac{Z(p^2)}{p^2} \stackrel{p^2\to 0}{\longrightarrow} 0$$

- determine running coupling:

$$\alpha^{gh-gl}(p^2) = \alpha_\mu G^2(p^2) Z(p^2)$$

- indicate dynamical chiral symmetry breaking

$$M(p^2) \neq 0$$

# Lattice vs. DSE/BSE: Complementary!

- Lattice simulations
  - Ab initio ☺
- Green's functions approach:
  Dyson-Schwinger and Bethe-Salpeter-equations
  (DSE/BSE)
  - Analytic solutions at small momenta ☺
  - Chiral symmetry: light quarks and mesons ☺
  - Space-Time-Continuum ☺

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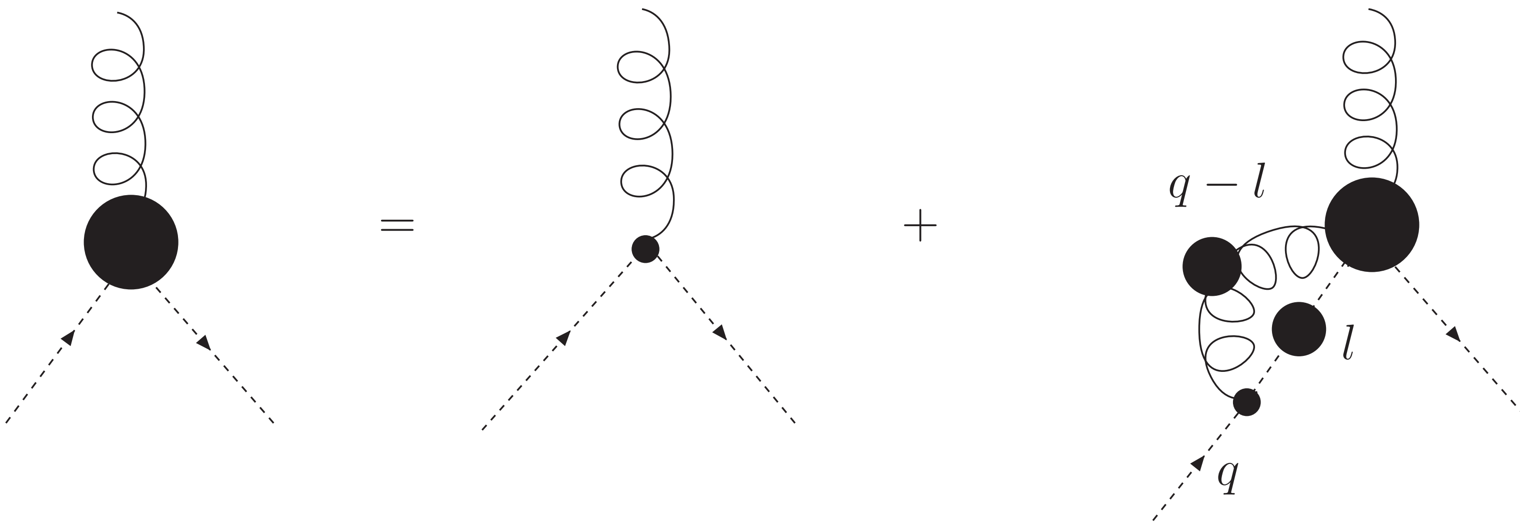
# Infrared Structure of YM-theory I

- Starting point: Ghost-Gluon-Vertex




- $l_\mu D_{\mu\nu}(l-q) = q_\mu D_{\mu\nu}(l-q) \Rightarrow$ Bare Vertex for $q_\mu \to 0$
- No anomalous dimensions in the IR


J. C. Taylor, Nucl. Phys. B **33** (1971) 436.

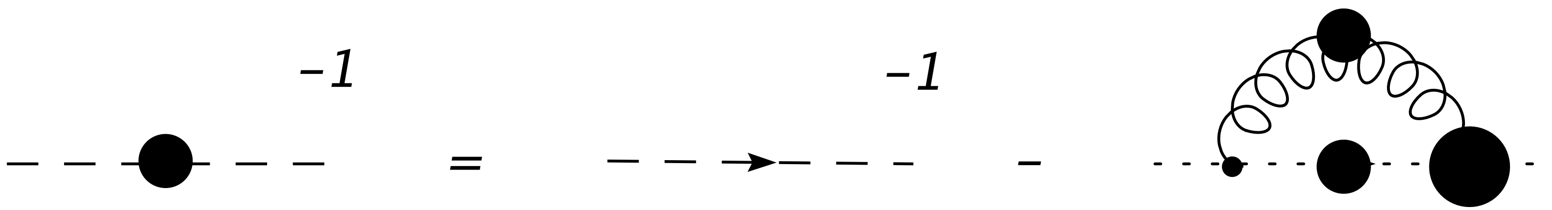
C. Lerche, L. v. Smekal, PRD **65** (2002) 125006.

A. Cucchieri, T. Mendes and A. Mihara, JHEP 0412:012 (2004).

W. Schleifenbaum, A. Maas, J. Wambach and R. Alkofer, Phys.Rev.D72 (2005) 014017.

# Infrared Structure of YM-theory II

- DSE for the ghost-propagator




<u>Ansatz</u> : $Z(p^2) \sim (p^2)^\alpha, \quad G(p^2) \sim (p^2)^\beta$

- Selfconsistency $\Rightarrow \quad -\beta = \alpha + \beta =: \kappa$ i.e.

$$Z(p^2) \sim (p^2)^{2\kappa}\,, \quad G(p^2) \sim (p^2)^{-\kappa}$$

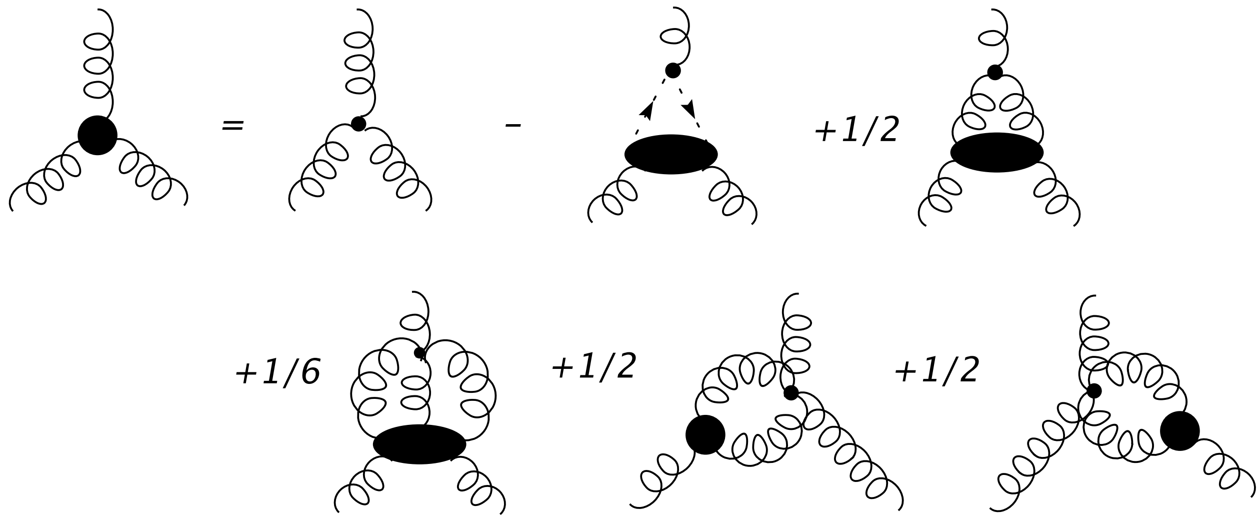
L. v. Smekal, A. Hauck, R. Alkofer, Phys. Rev. Lett. **79** (1997) 3591

- $\kappa > 0 \quad \Rightarrow$ **Well defined global colour charge!**

P. Watson and R. Alkofer, Phys. Rev. Lett. **86** (2001) 5239

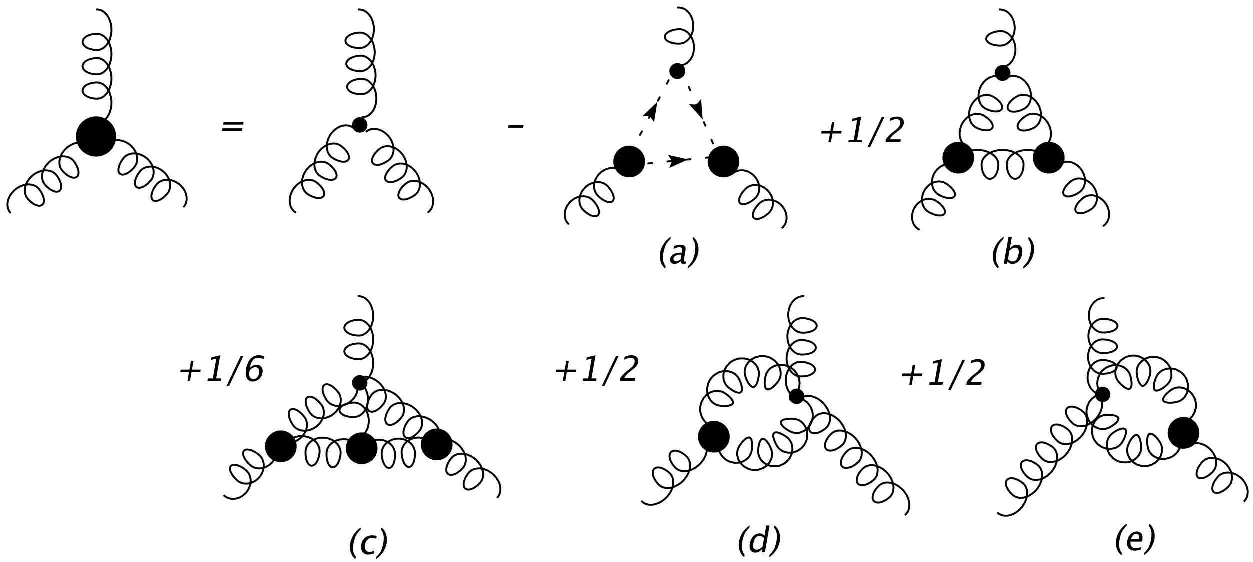
# Infrared Structure of YM-theory III

- Three-gluon vertex:



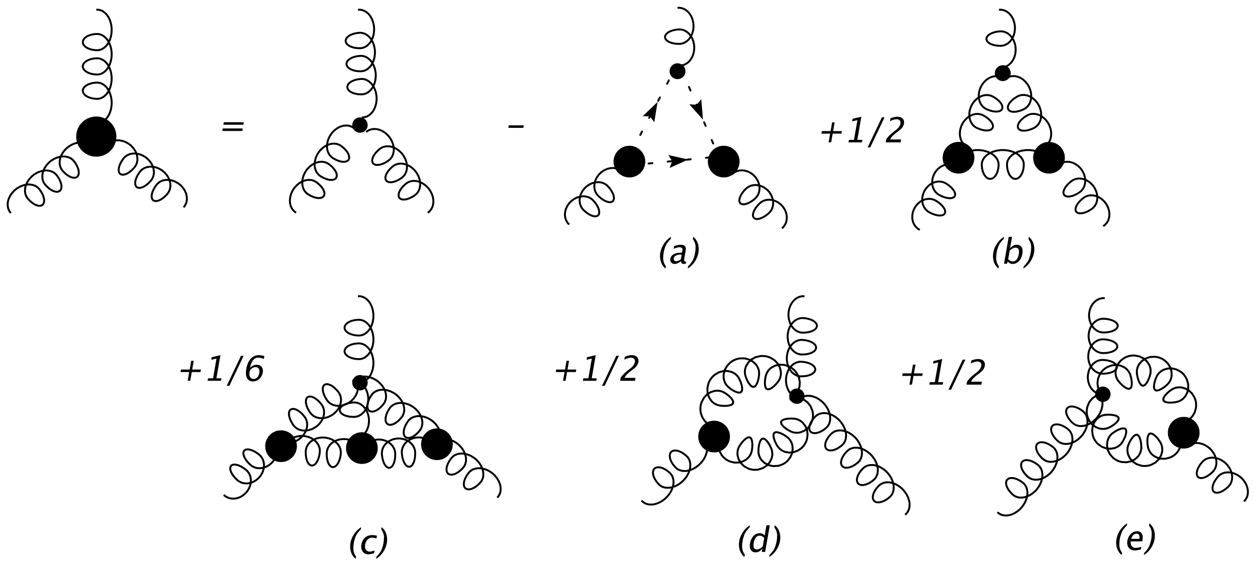
# Infrared Structure of YM-theory III

- Three-gluon vertex: **lowest order** in skeleton expansion



# Infrared Structure of YM-theory III

- Three-gluon vertex: **lowest order** in skeleton expansion

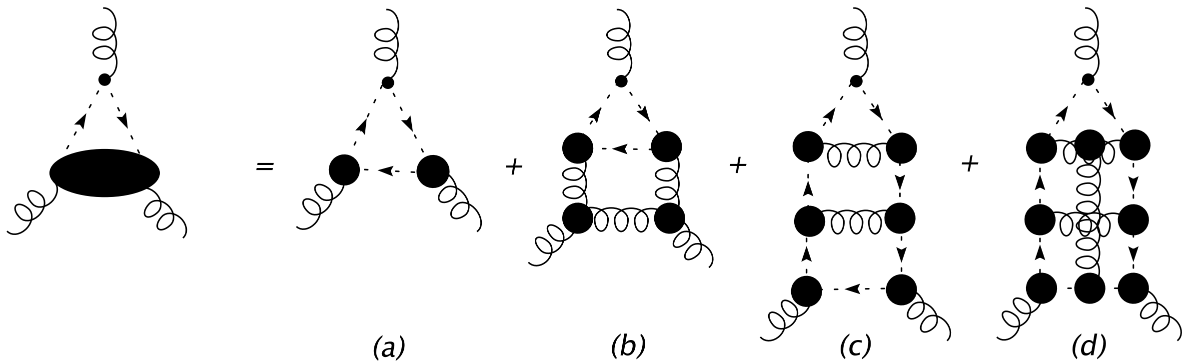



- $Z(p^2) \sim (p^2)^{2\kappa}\,, \quad G(p^2) \sim (p^2)^{-\kappa} \;\Rightarrow$

  Ghost-loop-diagram (a): $\Gamma^{3g}(p^2) \sim (p^2)^{-3\kappa}$
- Selfconsistency: (a) is leading diagram in IR
- IR-singular only if all external scales $\to 0$

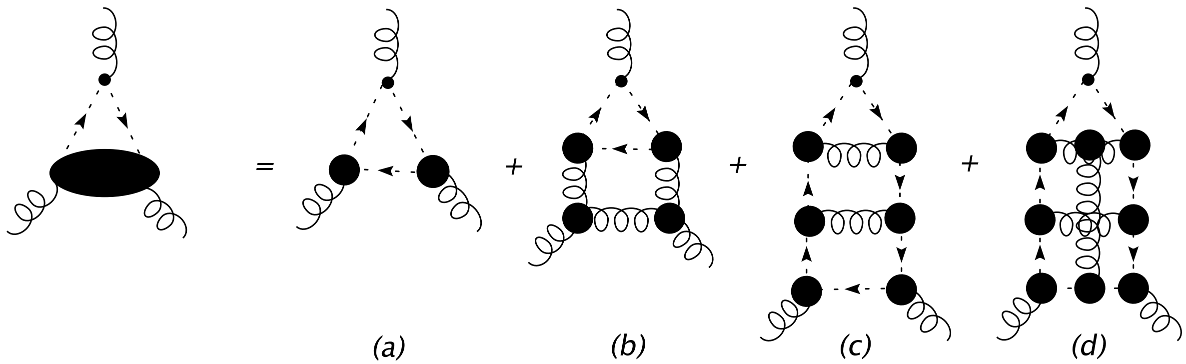
# Infrared Structure of YM-theory III

- Three-gluon vertex: **higher order** in skeleton expansion

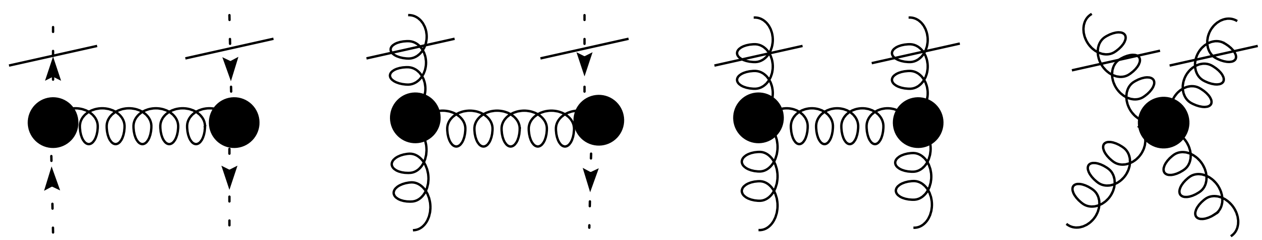


# Infrared Structure of YM-theory III

- Three-gluon vertex: **higher order** in skeleton expansion




- Built from insertions



insertions have zero IR anomalous dimensions ⇒

IR-analysis valid to all orders in skeleton expansion

# General Infrared Structure

- $2n$ external ghost legs and $m$ external gluon legs (one external scale $p^2$; **solves DSEs and STIs**):

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

R. Alkofer, C. F., F. Llanes-Estrada, Phys. Lett. B **611** (2005)

- Ghost sector of YM-theory dominates IR!

IR-asymptotic theory:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[A, c, \bar{c}] \exp\left\{\int \bar{c}^a(-\partial^\mu D_\mu)c^a\right\}$$

D. Zwanziger, Phys. Rev. D **69** (2004) 016002

# Uniqueness of IR-solution I

- DSEs from:

$$\frac{\delta \Gamma}{\delta \phi}[\phi] = \frac{\delta S_{\rm cl}}{\delta \phi}\left[\left(\frac{\delta^2 \Gamma}{\delta \phi \delta \phi_i}\right)^{-1} \frac{\delta}{\delta \phi_i} + \phi\right],$$

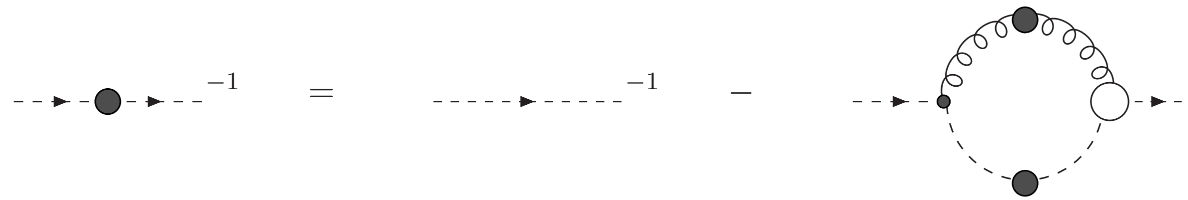
- ERGEs from:

$$\mathcal{Z}_k[\Phi] \;=\; \int \mathcal{D}[\Phi]\; e^{-S-\Delta S_k} \qquad \text{with} \qquad \Delta S_k = \frac{1}{2}\int R_k \phi\phi$$

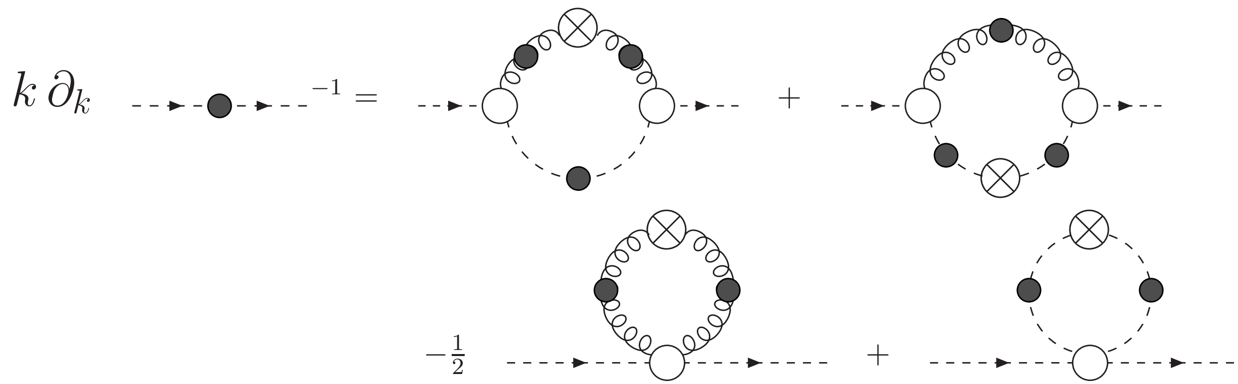
$$\partial_k \Gamma_k[\phi] = \frac{1}{2}\int_q \partial_k R_k \left(\frac{\delta^2 \Gamma_k}{\delta \phi \delta \phi} + R_k\right)^{-1}$$

# Uniqueness of IR-solution II

Ghost-DSE:



Ghost-ERGE:



IR-Analysis of whole tower of equations $\Rightarrow$

$$\Gamma^{2n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

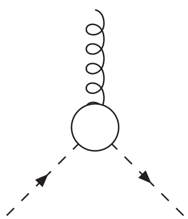
is unique.

C.F. and J. M. Pawlowski, Phys. Rev. D **75** (2007) 025012.

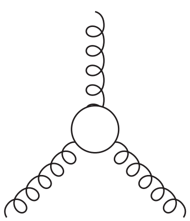
# Running Coupling: IR-Universality

$$G(p^2) \sim (p^2)^{-\kappa}\,, \quad Z(p^2) \sim (p^2)^{2\kappa}$$

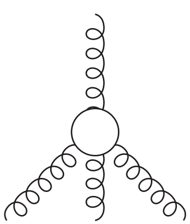
$$\Gamma^{3g}(p^2) \sim (p^2)^{-3\kappa}\,, \quad \Gamma^{4g}(p^2) \sim (p^2)^{-4\kappa}$$



$$\alpha^{gh-gl}(p^2) \;=\; \alpha_\mu\, G^2(p^2)\, Z(p^2) \sim \frac{const_{gh-gl}}{N_c}$$



$$\alpha^{3g}(p^2) \;=\; \alpha_\mu\, [\Gamma^{3g}(p^2)]^2\, Z^3(p^2) \sim \frac{const_{3g}}{N_c}$$



$$\alpha^{4g}(p^2) \;=\; \alpha_\mu\, [\Gamma^{4g}(p^2)]^2\, Z^4(p^2) \sim \frac{const_{4g}}{N_c}$$

# Landau vs. Coulomb gauge YM-theory

- Interpolating gauges: $a\partial_0 A_0 + \partial_i A_i = 0$ with $1 \geq a \geq 0$

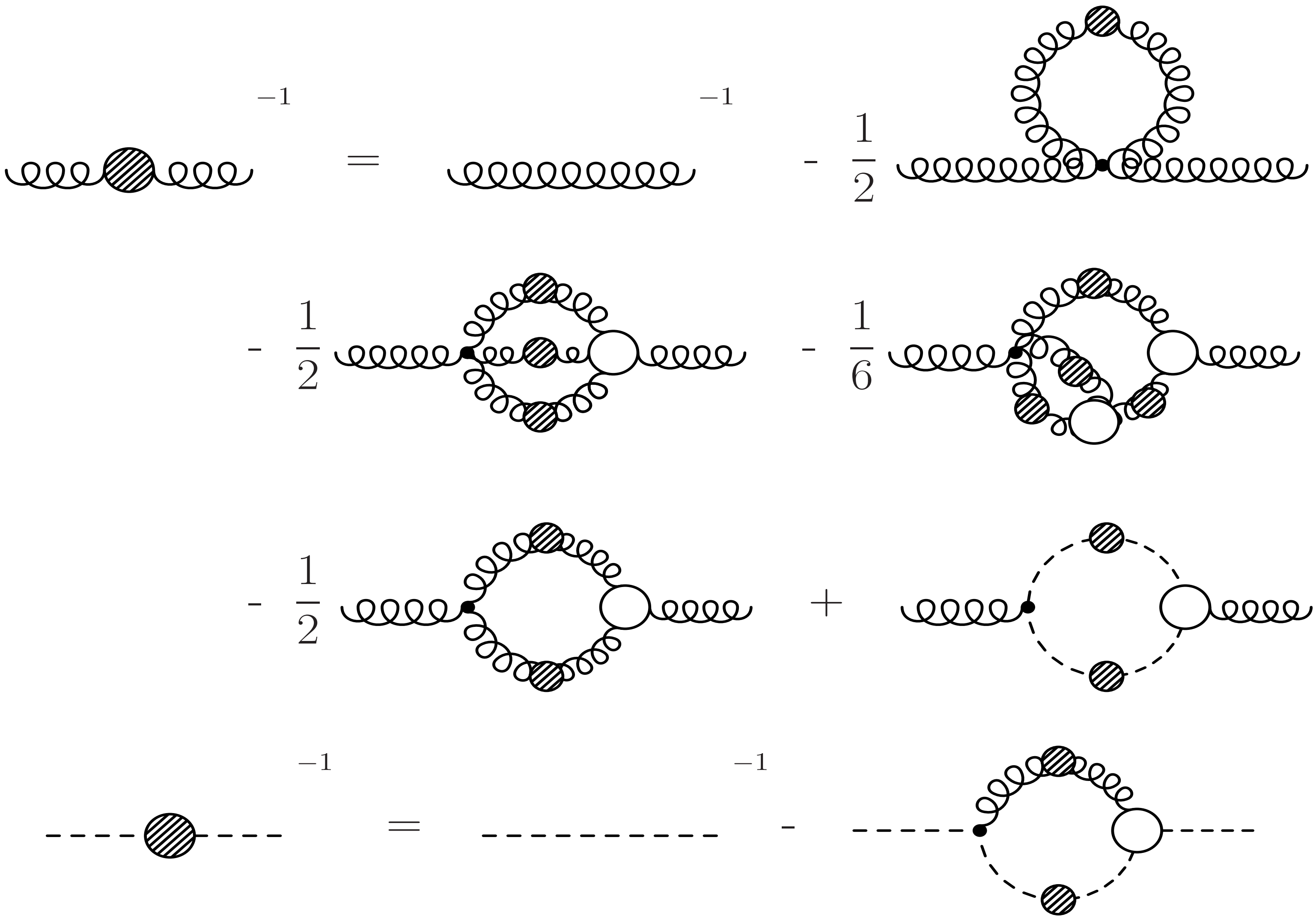
| | Landau gauge | Coulomb gauge |
|---|---|---|
| RG-invariants | $\alpha = \alpha_\mu G^2 Z$ | $\alpha_{\mathrm{coul}} = \alpha_\mu Z_{00}^{inst.}$ <br> $\alpha_I = \alpha_\mu G^2 Z_{ij}$ |
| IR-fixed point | $\alpha(0) \approx 8.92/N_c$ | $\alpha_I(0) \approx 11.99/N_c$ |

C. F., D. Zwanziger, PRD **72** (2005) 054005.

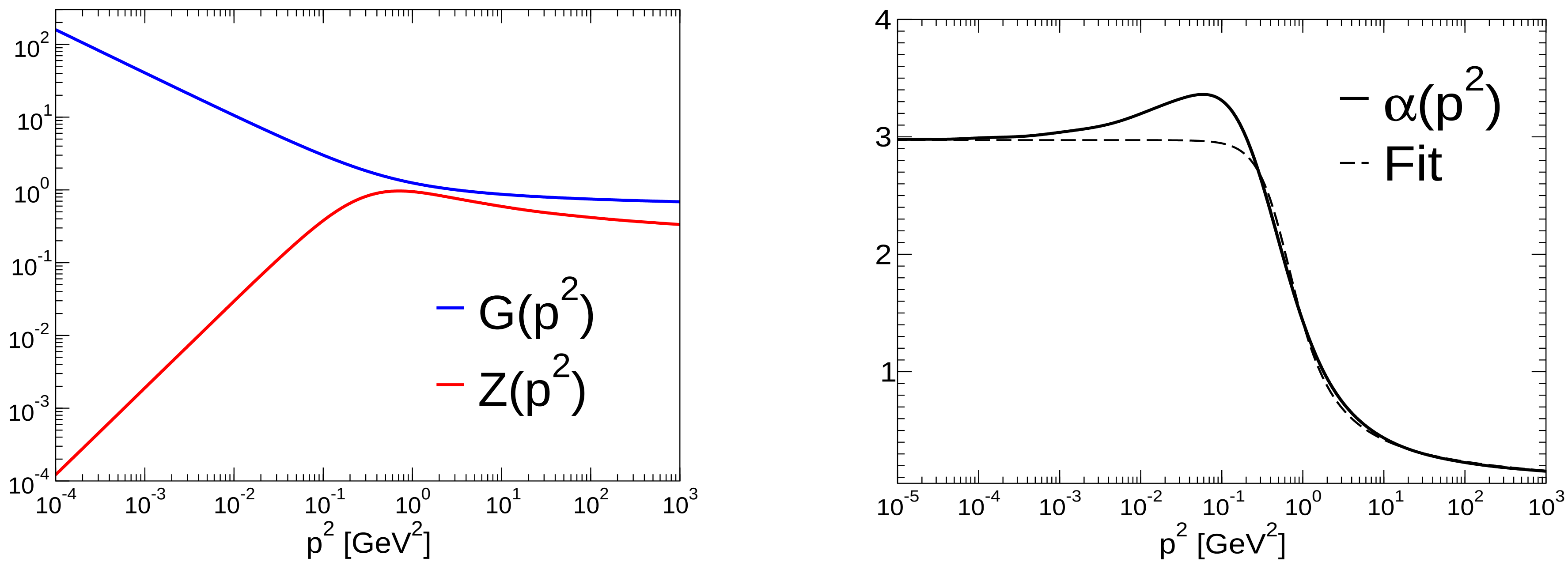
C. Lerche, L. v. Smekal, PRD **65** (2002) 125006.

W. Schleifenbaum, M. Leder and H. Reinhardt, PRD **73** (2006) 125019.

# DSEs for Ghost and Glue



# Quenched Ghost and Glue I




C. F., R. Alkofer, Phys. Lett. **B 536** (2002) 177.

- IR fixed point of the coupling

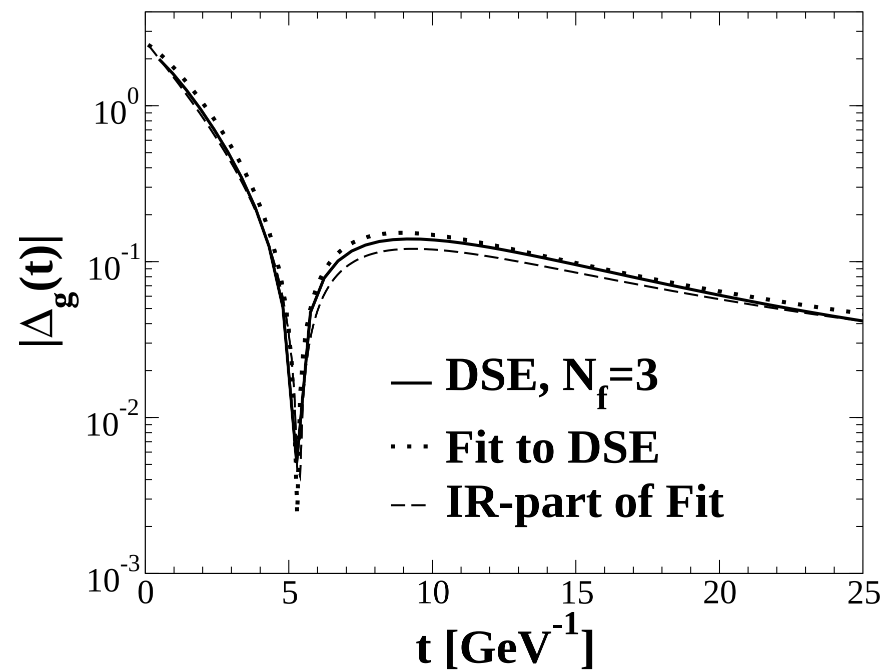
$$\alpha(0) = \frac{4\pi}{6N_c}\frac{\Gamma(3-2\kappa)\Gamma(3+\kappa)\Gamma(1+\kappa)}{\Gamma^2(2-\kappa)\Gamma(2\kappa)} \overset{N_c=3}{\approx} 2.97; \quad \kappa \approx 0.595$$

C. Lerche, L. v. Smekal, PRD **65** (2002) 125006.

# Gluon confinement

- $\int \frac{d^4x}{(2\pi)^4} D(x^2) = \left(\frac{Z(p^2)}{p^2}\right)\Big|_{p^2=0} = \left(\frac{(p^2)^{2\kappa}}{p^2}\right)\Big|_{p^2=0} = 0$ for $\kappa > 1/2$

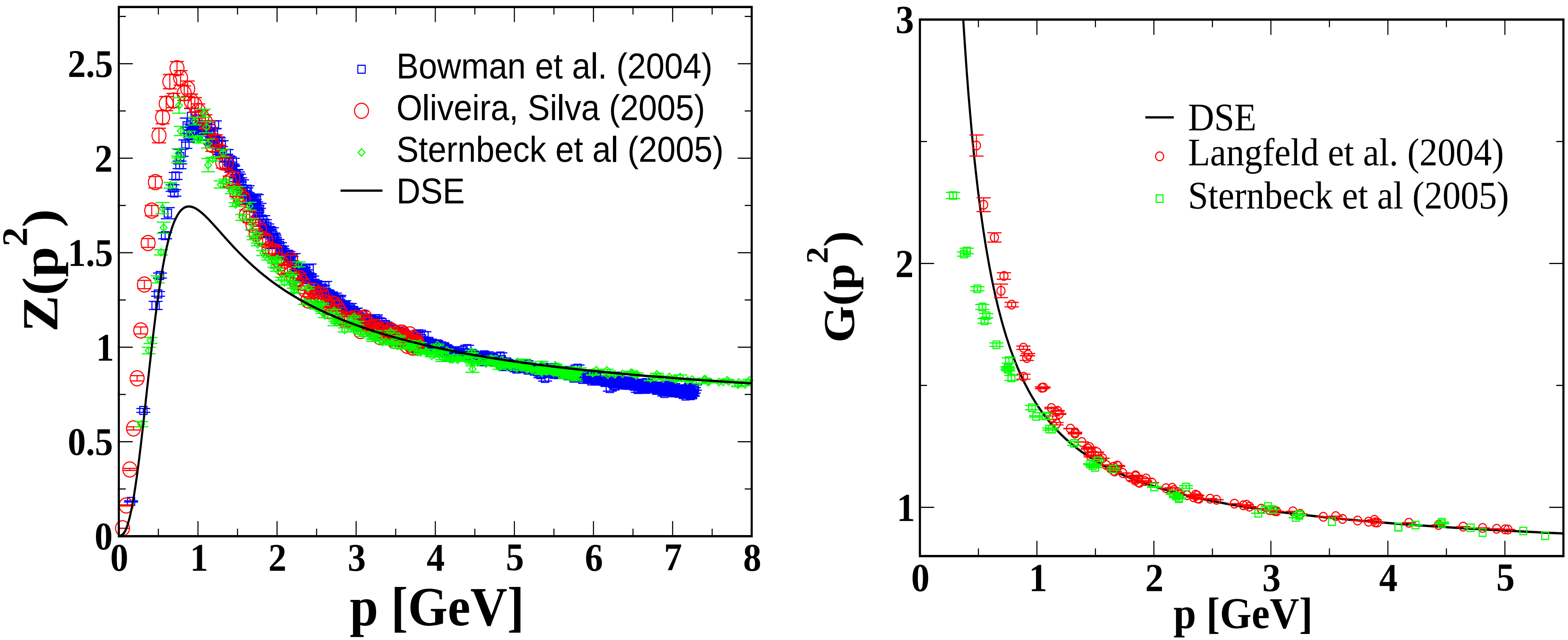
$$\Delta_g(t) := \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(tp_4+\vec{p}\vec{x})} \, \frac{Z(p^2)}{p^2}$$




- Violation of positivity ⇒ Signal for **confined gluons**

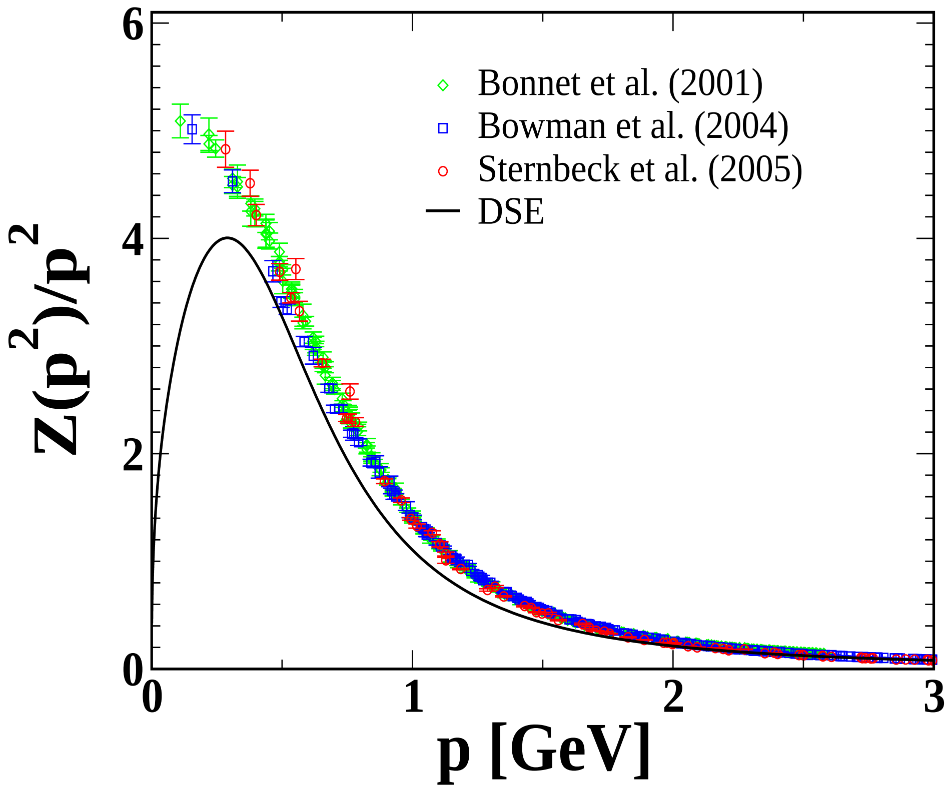
R. Alkofer, W. Detmold, C. F., P. Maris, Phys. Rev. D **70** (2004) 014014

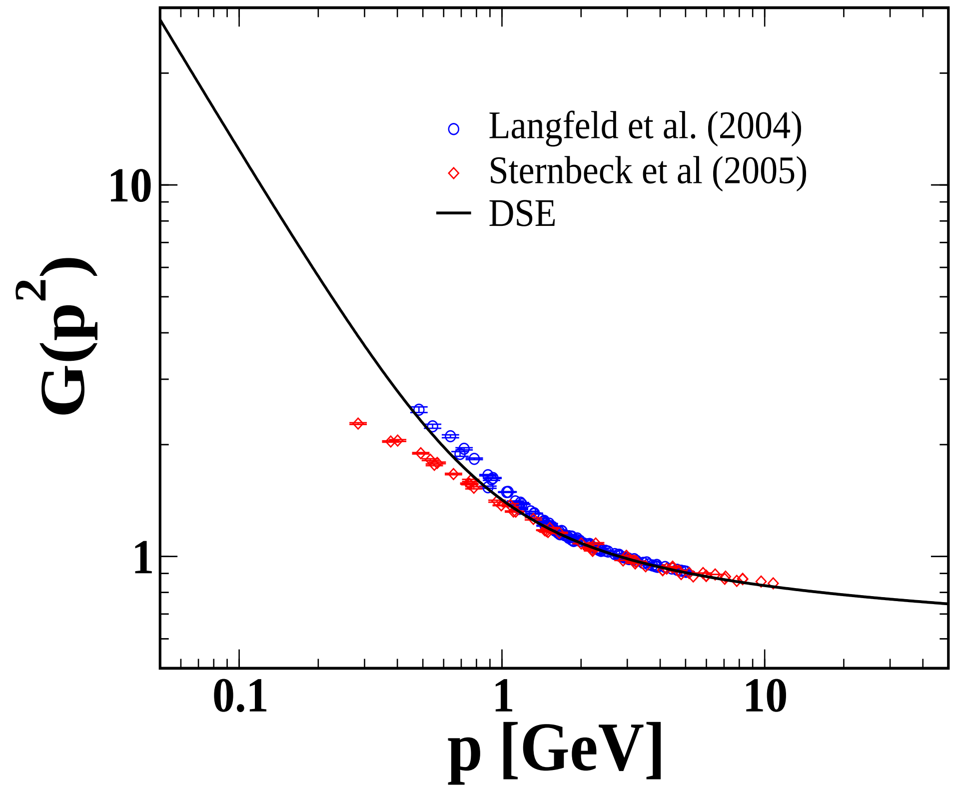
# Quenched Ghost and Glue II




- Kugo-Ojima criterion satisfied: $G(p^2 = 0) \to \infty$

# Ghost and Gluon propagator




- IR: $Z(p^2) \sim (p^2)^{2\kappa}, G(p^2) \sim (p^2)^{-\kappa}, \quad \kappa \approx 0.595$

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# DSEs on the torus

- Periodic boundary conditions

  Integrals become Matsubara sums:

$$\int d^4p \to \left(\frac{2\pi}{L}\right)^4 \sum_{j_1,j_2,j_3,j_4} = \left(\frac{2\pi}{L}\right)^4 \sum_{j,l}$$

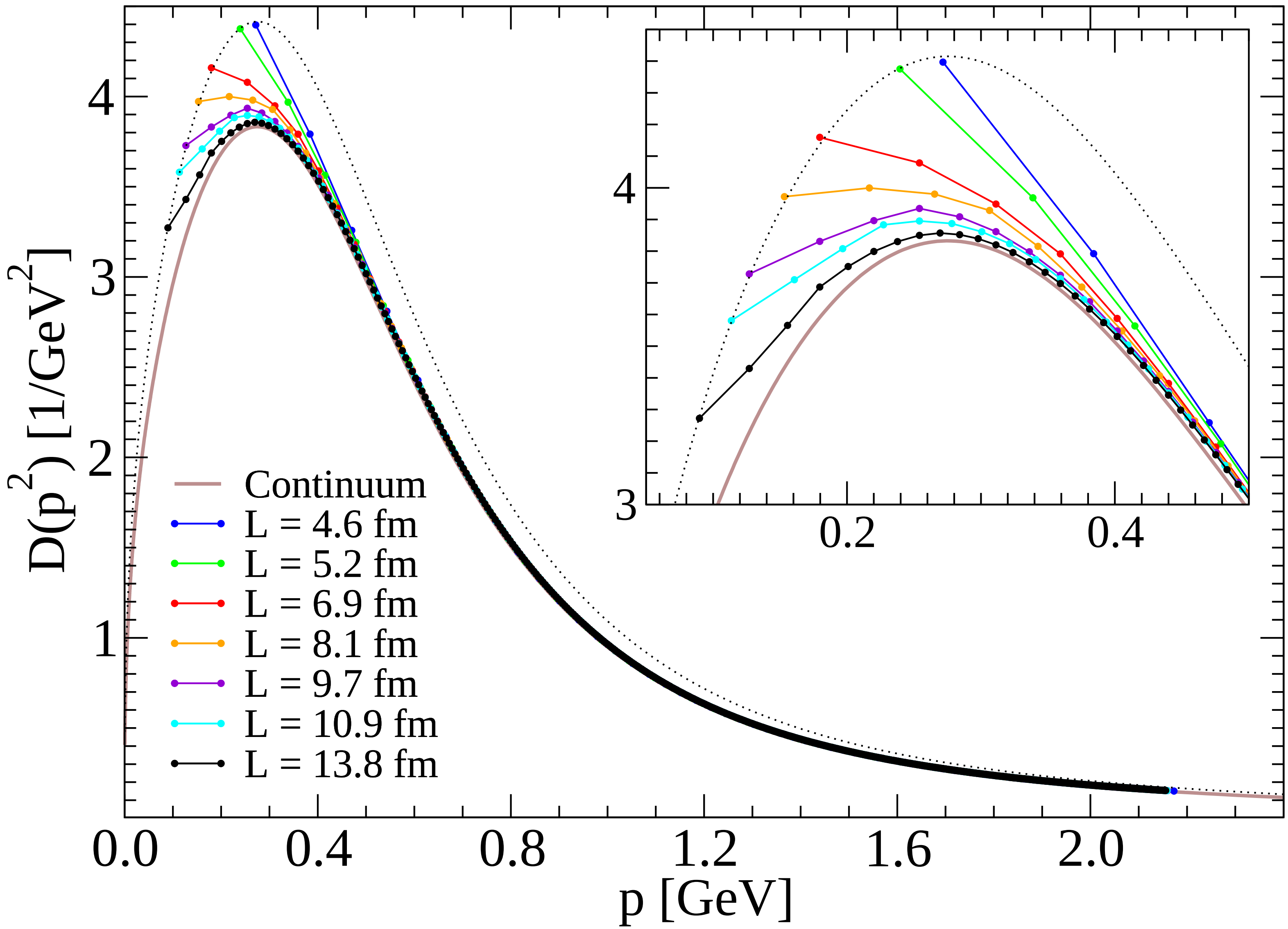
- Same truncation as in continuum!
- Better convergence properties
  but much more time consuming.

C. F., A. Maas, J. Pawlowski and L. v. Smekal, Annals Phys. (2007), [hep-ph/0701050].

C. F., B. Gruter and R. Alkofer, Annals Phys. **321** (2006) 1918

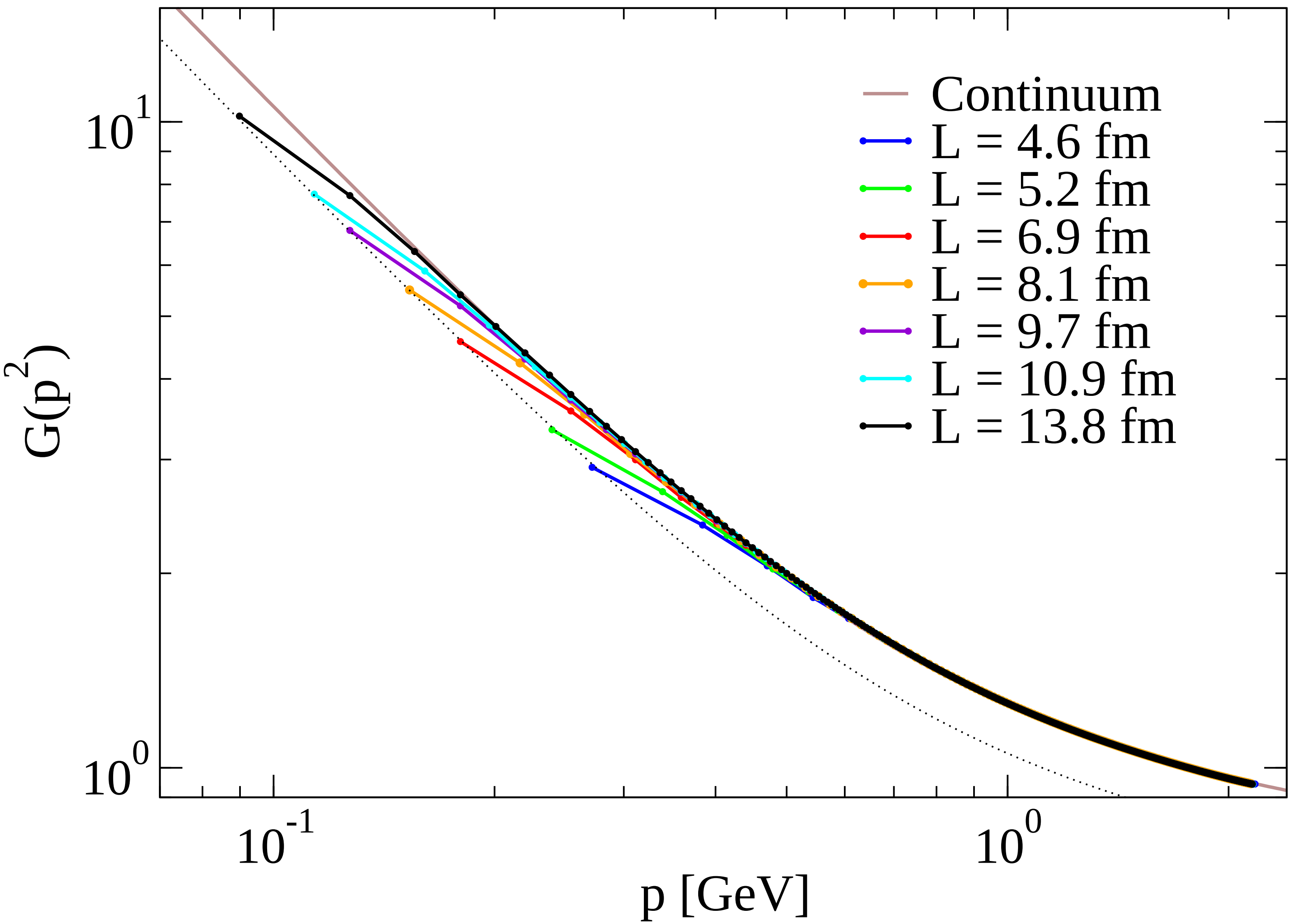
C. F., R. Alkofer and H. Reinhardt, Phys. Rev. **D 65** (2002) 094008

# Gluon



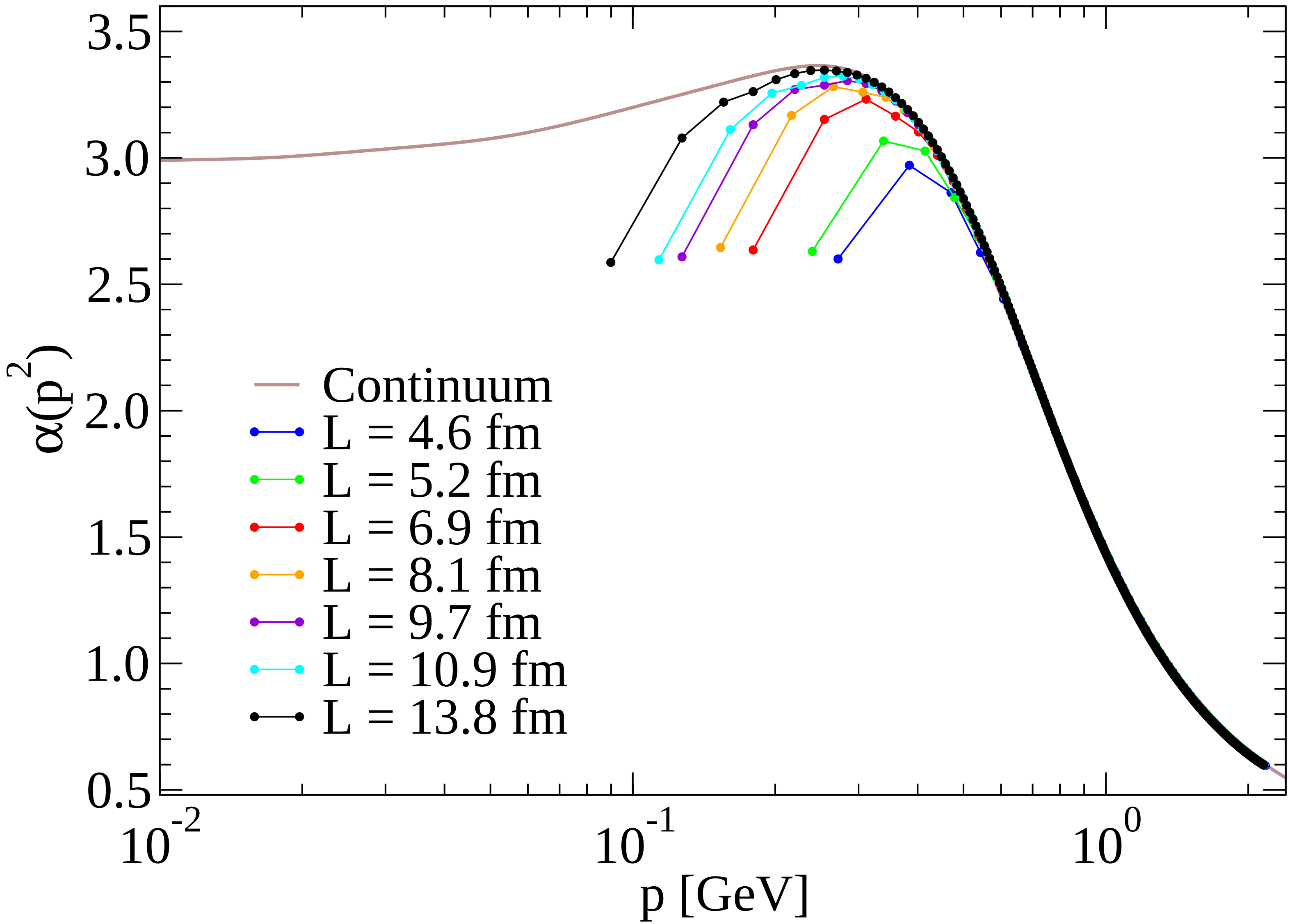
D(p^2) [1/GeV^2]
p [GeV]
0.0
0.4
0.8
1.2
1.6
2.0
1
2
3
4
Continuum
L = 4.6 fm
L = 5.2 fm
L = 6.9 fm
L = 8.1 fm
L = 9.7 fm
L = 10.9 fm
L = 13.8 fm
0.2
0.4
3
4

# Ghost



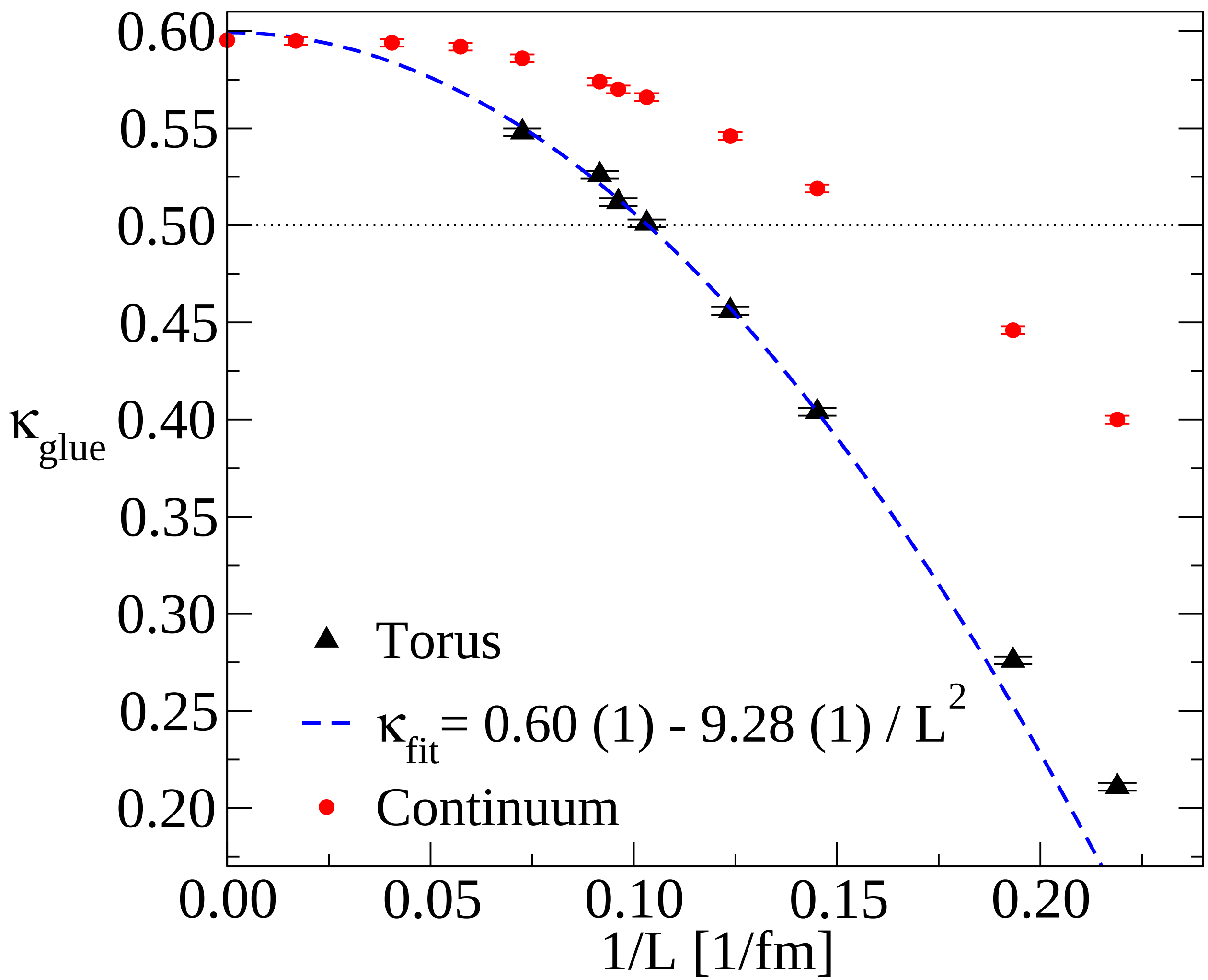
Continuum
L = 4.6 fm
L = 5.2 fm
L = 6.9 fm
L = 8.1 fm
L = 9.7 fm
L = 10.9 fm
L = 13.8 fm
$10^1$
$10^0$
$10^{-1}$
$10^0$
$G(p^2)$
p [GeV]

# Coupling

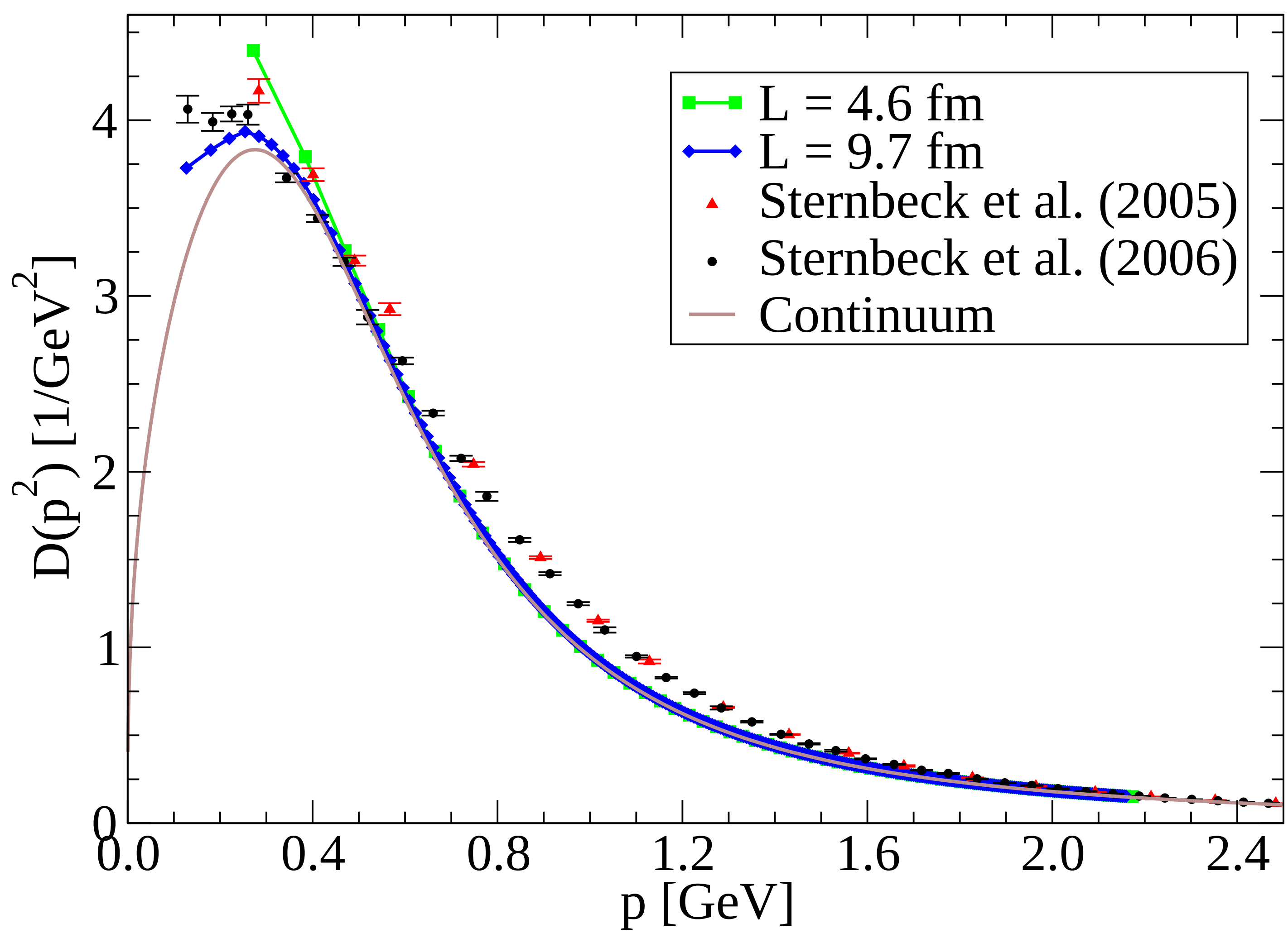


3.5
3.0
2.5
2.0
1.5
1.0
0.5
α(p²)
Continuum
L = 4.6 fm
L = 5.2 fm
L = 6.9 fm
L = 8.1 fm
L = 9.7 fm
L = 10.9 fm
L = 13.8 fm
10⁻²
10⁻¹
10⁰
p [GeV]

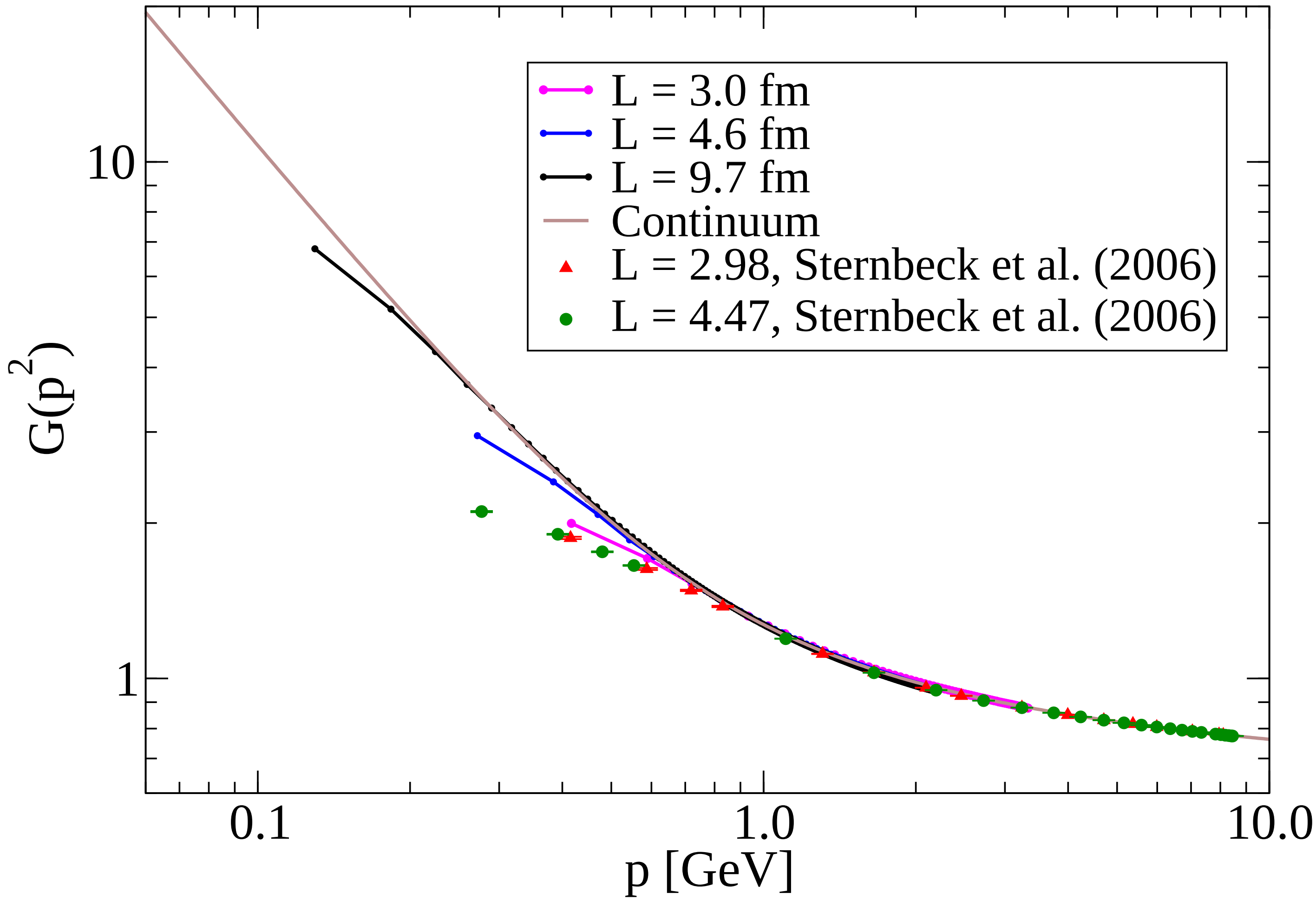
# Emergence of infrared behaviour



# Comparison to lattice QCD: Gluon



# Comparison to lattice QCD: Ghost



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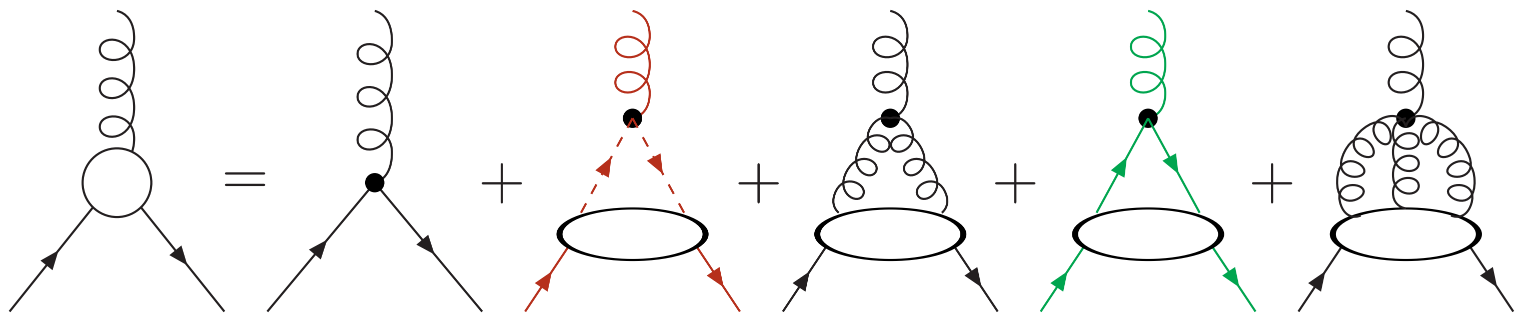
# Infrared Structure of QCD I

Quark-gluon vertex:

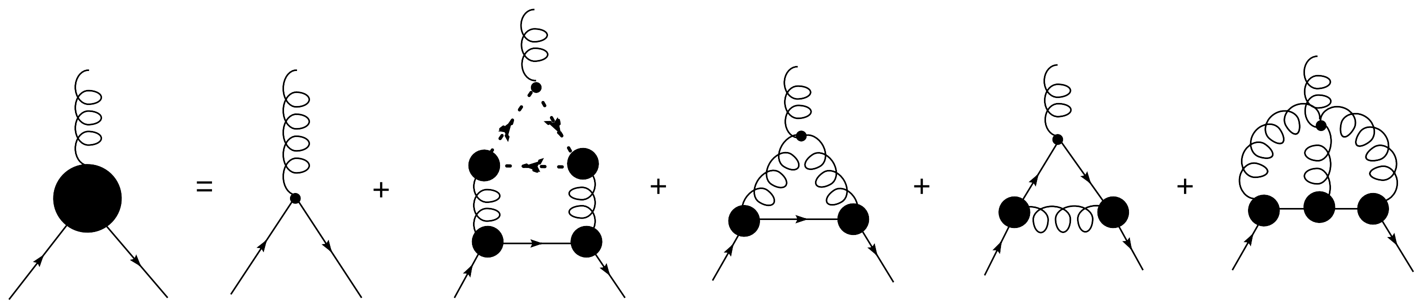


- Quark diagram: Hadronic contributions ('unquenching')

  $\rightarrow$ talk of Dominik Nickel!
- Ghost diagram: Infrared leading

# Infrared Structure of QCD II

- Quark-gluon vertex: **lowest order** in skeleton expansion



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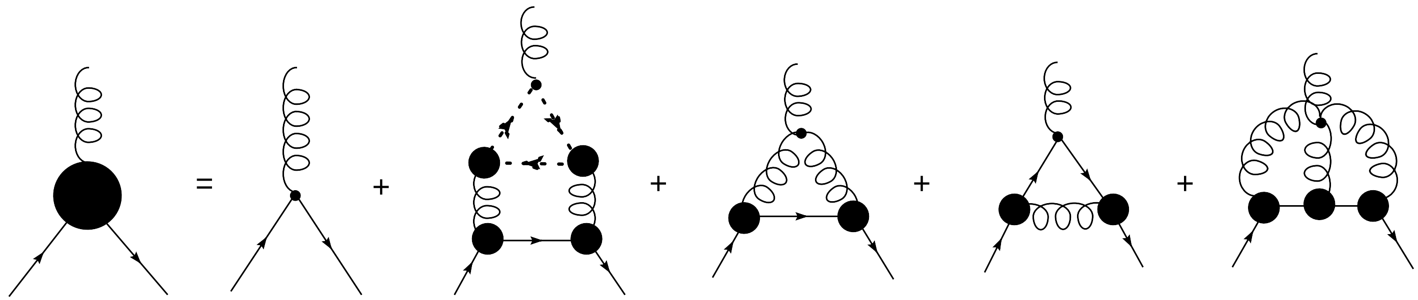


$$S(p) = \frac{\not{p} + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \rightarrow \frac{Z_f \not{p}}{M^2} + \frac{Z_f}{M}$$

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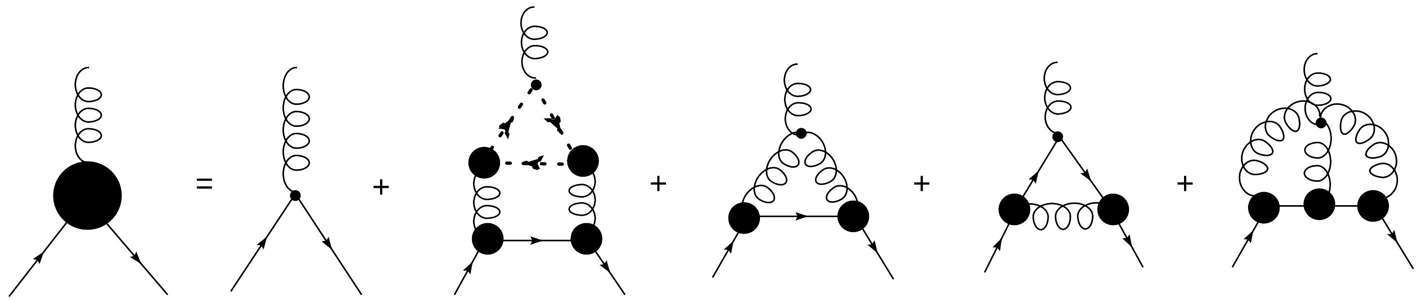


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$$\Gamma_\mu = ig \sum_{i=1}^{4} \lambda_i G^i_\mu\,, \quad G^1_\mu = \gamma_\mu\,,\ G^2_\mu = \hat{p}_\mu\,,\ G^3_\mu = \hat{\not p}\,\hat{p}_\mu\,,\ G^4_\mu = \hat{\not p}\,\gamma_\mu$$

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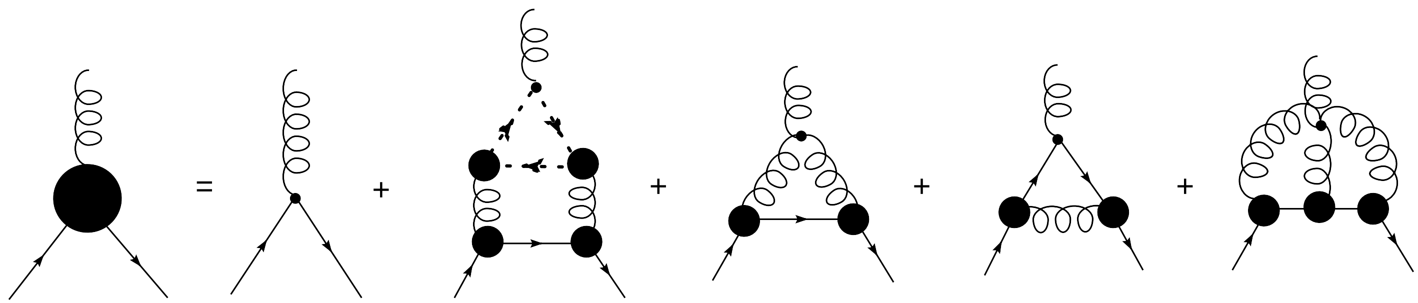


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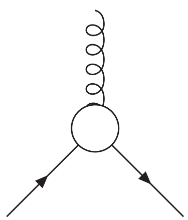
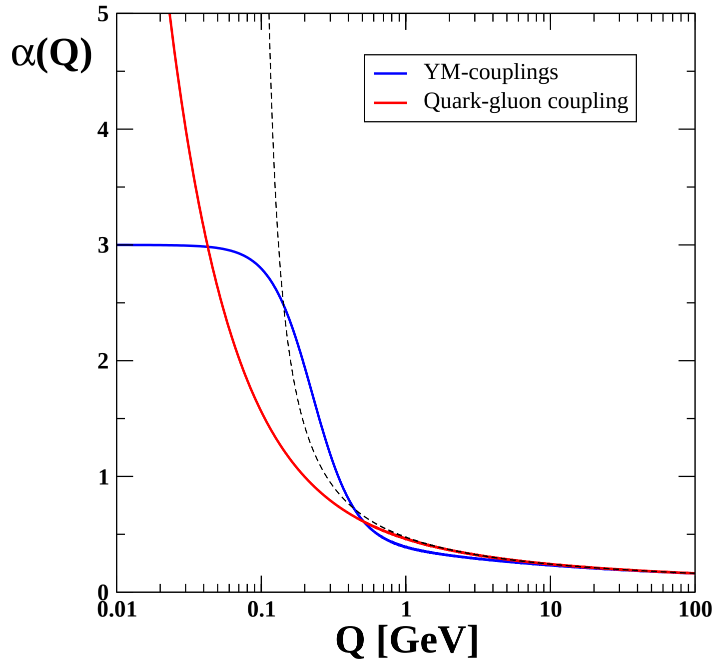
$$\lambda_{1,2,3,4} \sim (p^2)^{-1/2-\kappa}$$

# Running Coupling: IR-slavery

$$\Gamma^{qg}(p^2) \sim (p^2)^{-1/2-\kappa}\,, \quad Z_f(p^2) \sim const\,, \quad Z(p^2) \sim (p^2)^{2\kappa}$$

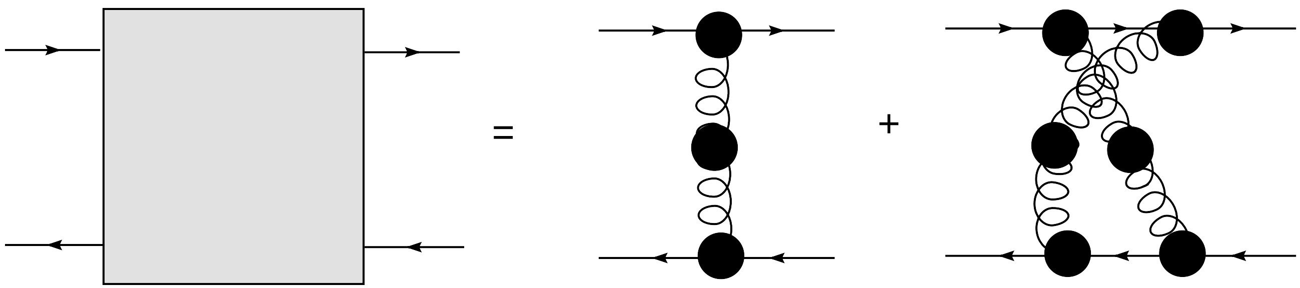


$$\alpha^{qg}(p^2) = \alpha_\mu \left[\Gamma^{qg}(p^2)\right]^2 \left[Z_f(p^2)\right]^2 Z(p^2) \sim \frac{const_{qg}}{N_c}\frac{1}{p^2}$$



# The quark-antiquark potential

- quenched QCD




$$\Gamma^{0,0,2}(p^2) \sim (p^2)^{-1}$$

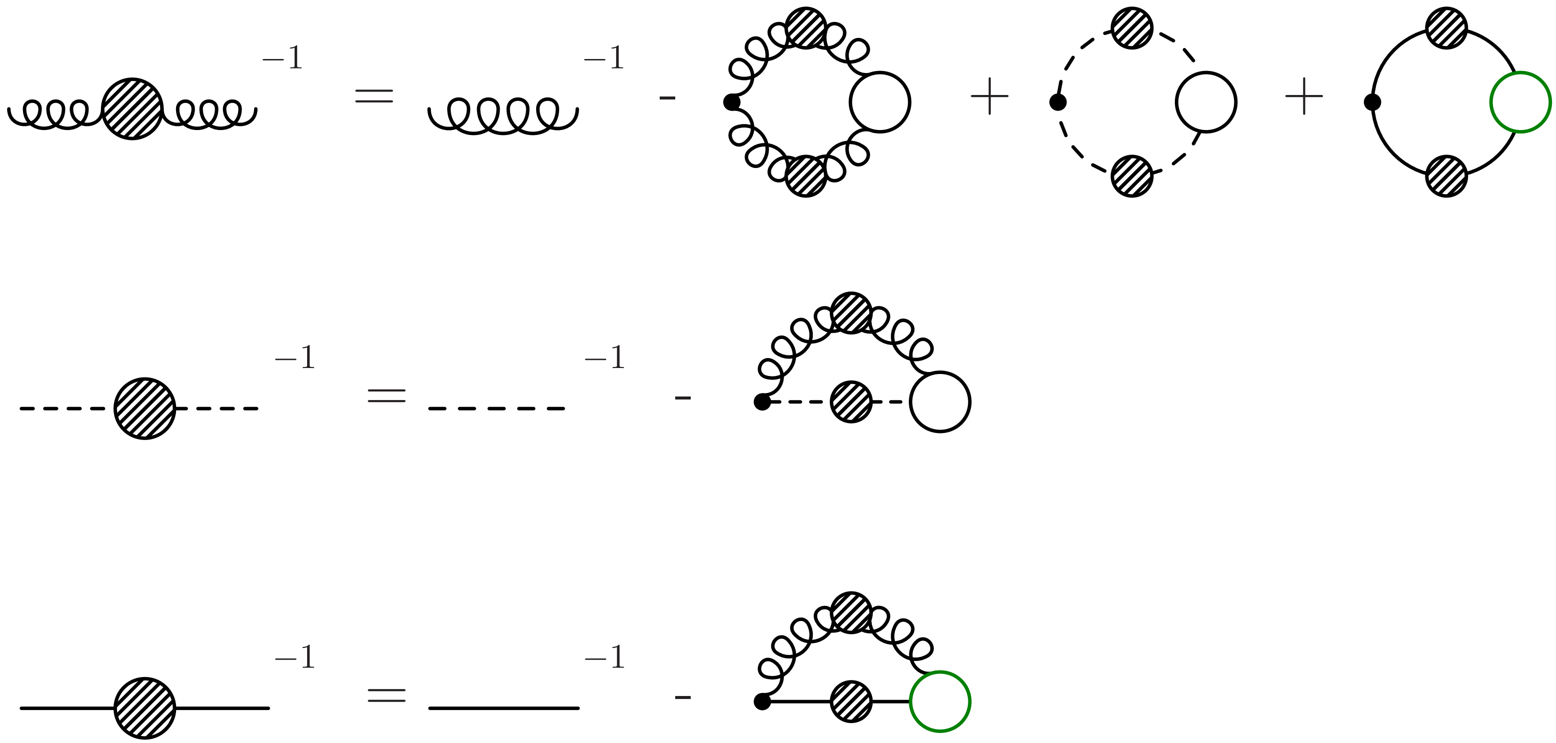
$$V(\mathbf{r}) = \frac{1}{(2\pi)^3} \int d^3p \, e^{i\mathbf{pr}} \, \frac{(p^2)^{-1}}{p^2} \sim |\mathbf{r}|$$

Quark confinement ?!

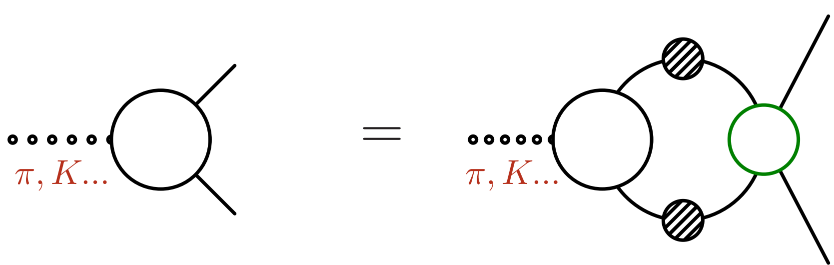
# Outline

- Introduction
- Infrared properties of SU(N) Yang-Mills theory
- Ghost and Glue in a box
- Infrared Slavery in quenched QCD
- **Light mesons: unquenching effects**

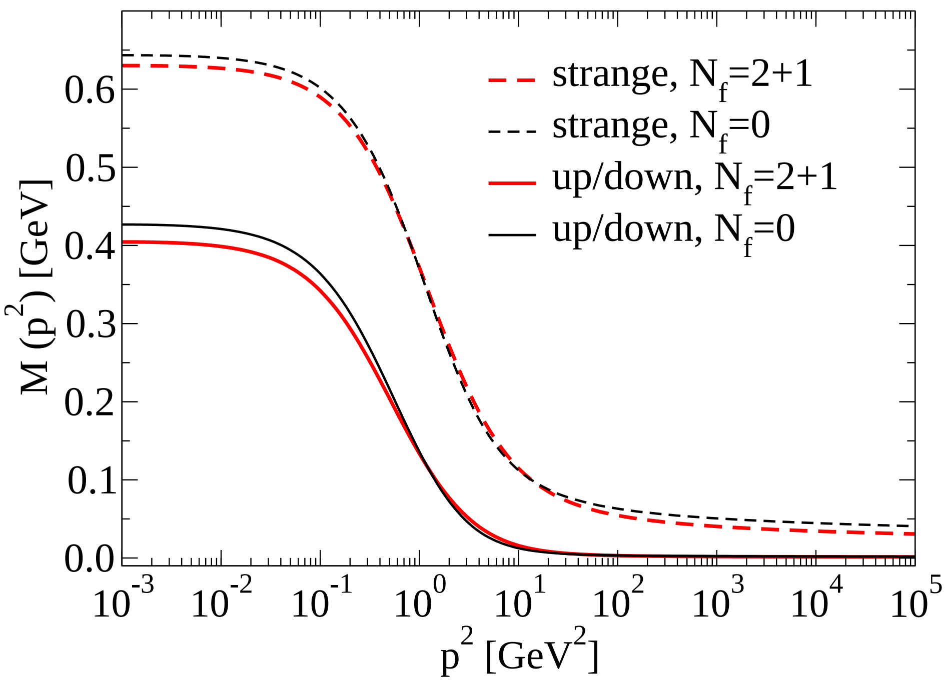
# DSEs and BSE

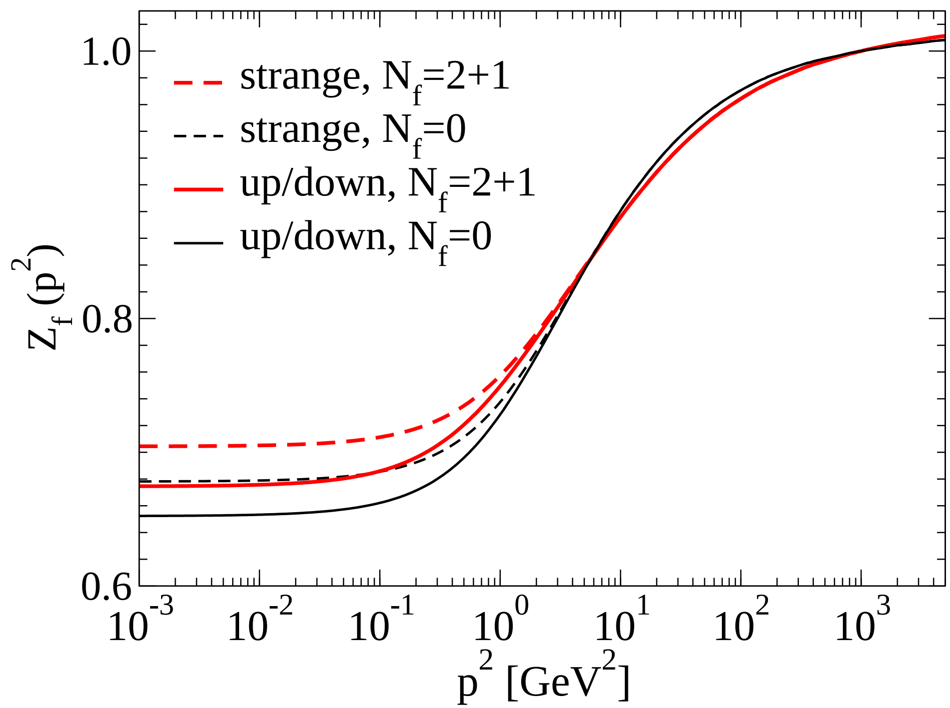



$$\Gamma_\mu(p,k) \;\sim\; \gamma_\mu \; G^2(k)\; A(k)$$



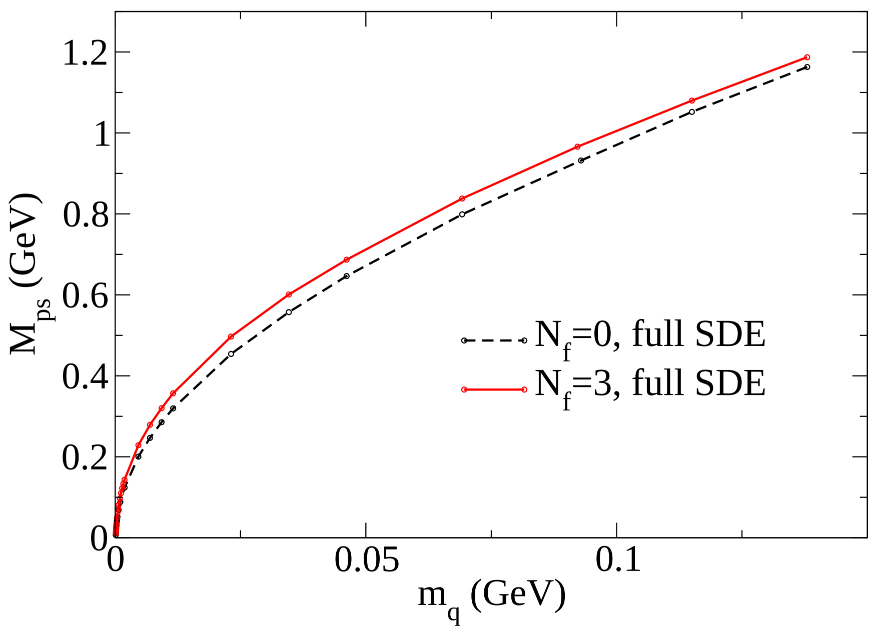
# Partially unquenched quark

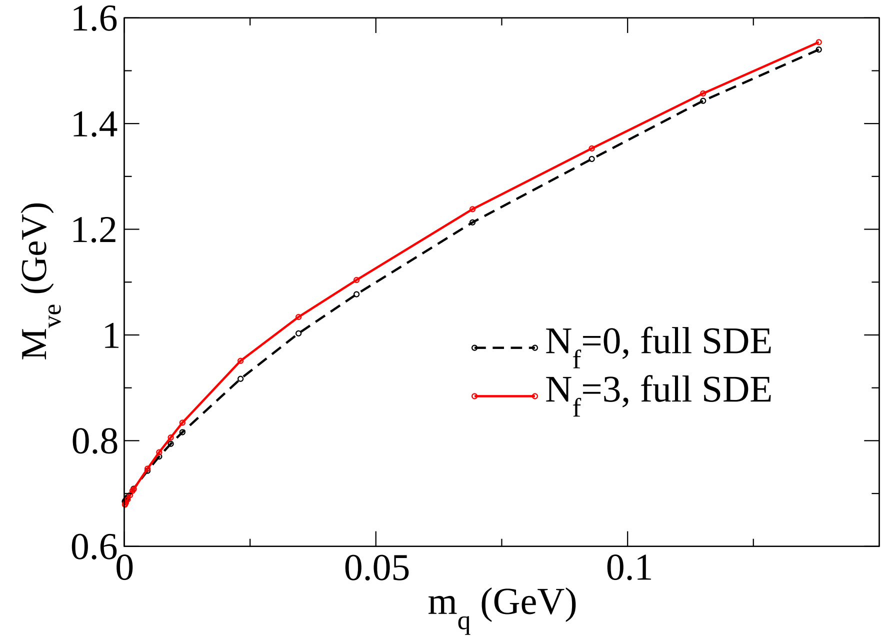



| | $-(<\overline{q}q>^0)^{1/3}$ (MeV) | $M_{ch}(p^2=0)$ (MeV) |
|---|---|---|
| $N_f=0$ | 266 | 416 |
| $N_f=2+1$ | 271 | 388 |

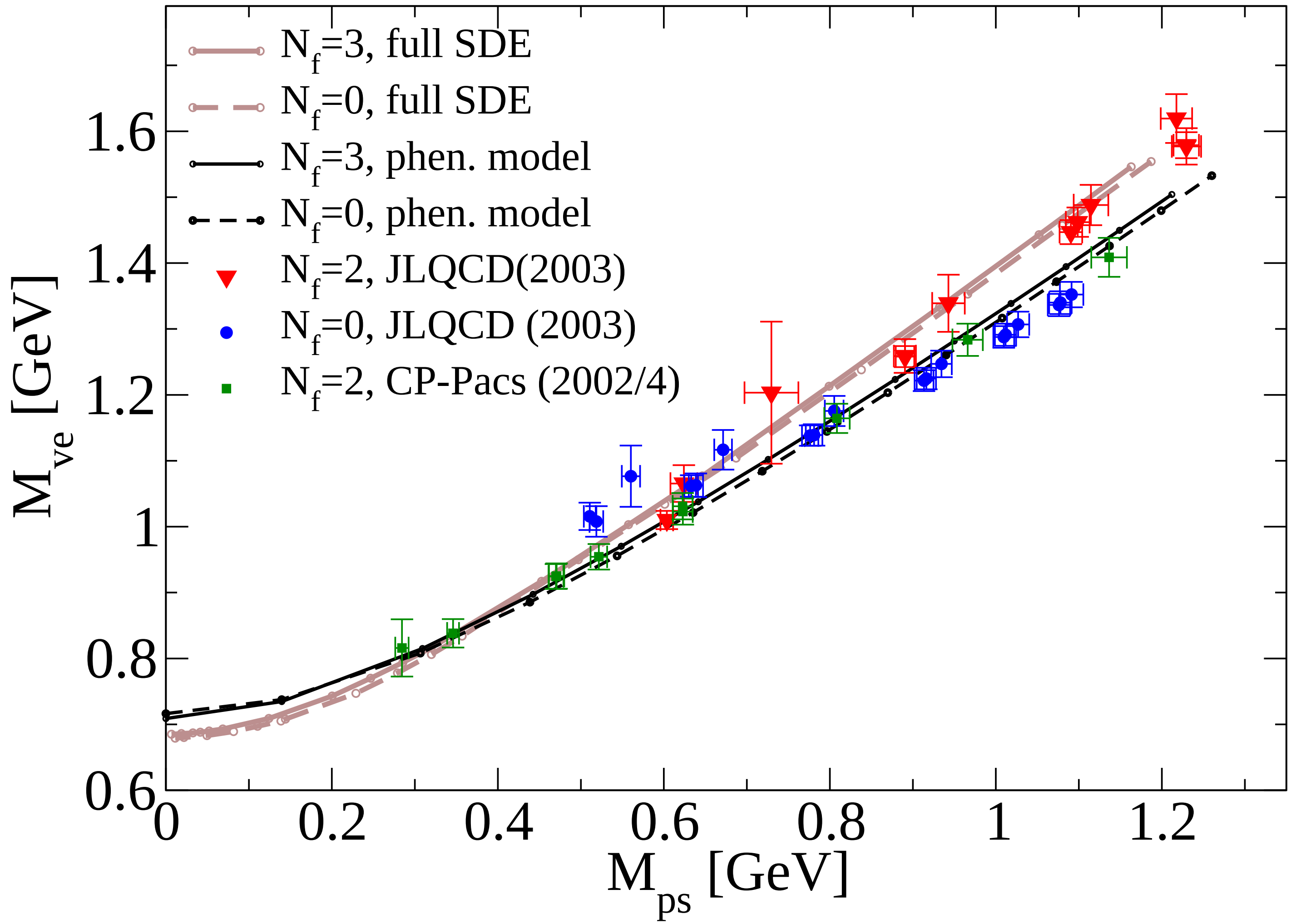
# Partially unquenched light mesons I




| | $m_u$ | $m_s$ | $m_\pi$ | $f_\pi$ | $m_K$ | $f_K$ | $m_\rho$ |
|---|---|---|---|---|---|---|---|
| $N_f = 0$ | 4.17 | 88.2 | 139.7 | 130.9 | 494.5 | 165.6 | 708.0 |
| $N_f = 2 + 1$ | 4.06 | 86.0 | 140.0 | 131.1 | 493.3 | 169.5 | 695.2 |
| $N_f = 3$ | 4.06 | | 139.7 | 130.8 | | | 690.0 |
| PDG | 3.0-5.5 | 80-130 | 139.6 | 130.7 | 493.7 | 160.0 | 770 |

# Partially unquenched light mesons II




C. F., P. Watson and W. Cassing, Phys. Rev. D **72** (2005) 094025

# Summary

**Landau gauge Yang-Mills theory:**

- 1PI-function with $2n$ ghost and $m$ gluon legs:

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

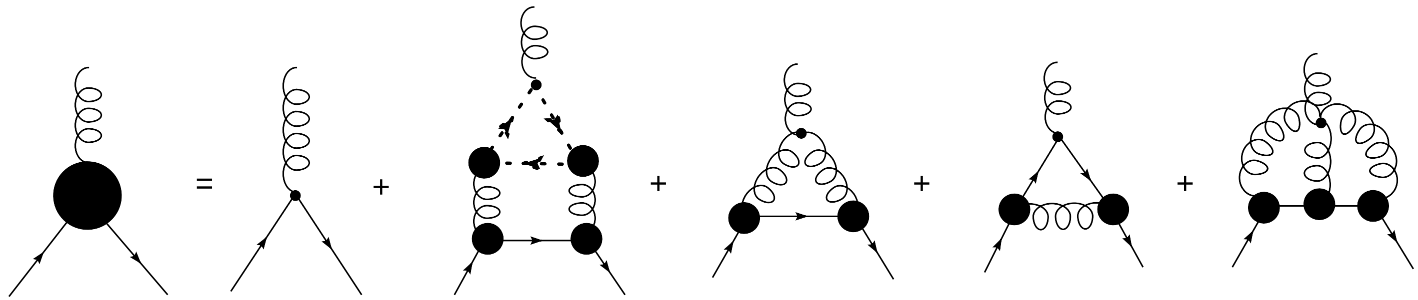
- IR-asymptotic theory dominated by ghosts
- YM-couplings: Universal IR-fixed point

**Landau gauge QCD (quenched):**

- Quark-gluon-coupling: Infrared slavery ?!
- Quark-quark-potential: Linearly rising ?!

# Chiral symmetry restoration I

- Quark-gluon vertex: **lowest order** in skeleton expansion



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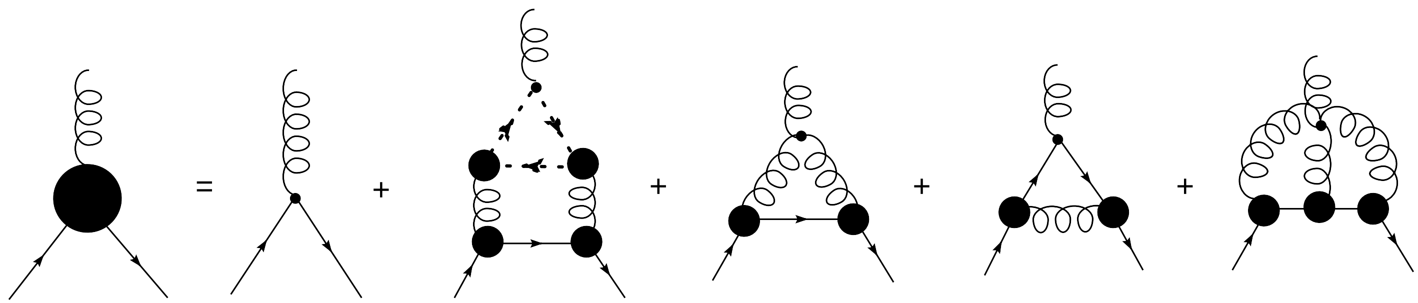


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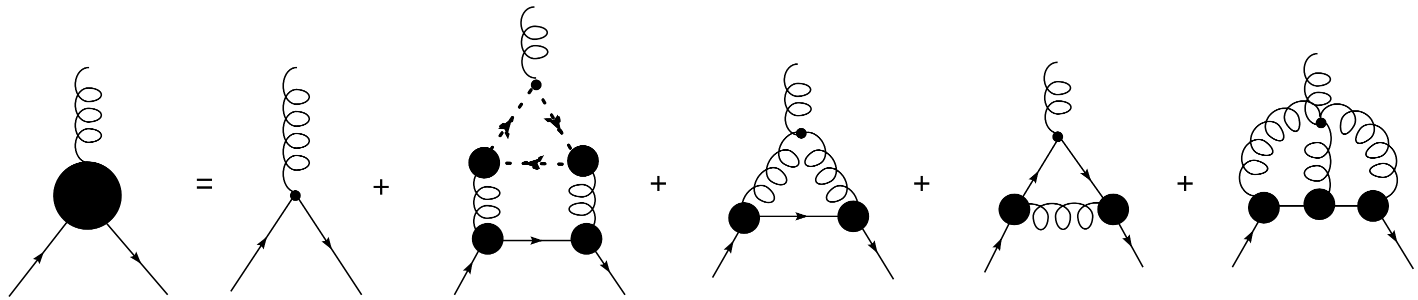


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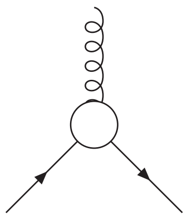
$$\lambda_1 \sim (p^2)^{-\kappa}\,,\ \lambda_2 = 0\,,\ \lambda_3 \sim (p^2)^{-\kappa}\,,\ \lambda_4 = 0$$

# Chiral symmetry restoration II

- Running coupling: Restoration of fixed point



$$\alpha^{qg}(p^2) = \alpha_\mu \, [\Gamma^{qg}(p^2)]^2 \, [Z_f(p^2)]^2 \, Z(p^2) \sim const_{qg}$$

- Quark-antiquark potential: Deconfinement

$$\Gamma^{0,0,2}(p^2) \sim const.$$

$$V(\mathbf{r}) \sim \frac{1}{(2\pi)^3} \int \frac{1}{p^2} e^{i\mathbf{pr}} d^3p \quad \sim \quad \frac{\mathbf{1}}{|\mathbf{r}|}$$

**Chiral symmetry breaking $\leftrightarrow$ Quark confinement**

R. Alkofer, C. F., F. Llanes-Estrada, hep-ph/0607293