The spectrum of static-light mesons from lattice QCD

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Heavy quarks on the lattice

Discretisation of the Dirac operator introduces errors which vary with am_Q.

Require that $am_Q \ll 1$.

With current resources simulations at the b-quark mass are not possible (charm quark?).

Alternatives:

lattice NRQCD

Anisotropic lattice QCD - $a_t \ll 1/m_Q$, $a_s > a_t$.

Static quark approximation (Heavy quark effective theory).

The static limit

- Valid for hadrons containing a single heavy quark.
- $m_Q \gg \Lambda_{QCD}$.

- These hadrons exhibit an approximate heavy quark spin-flavour symmetry.
- In the limit $m_Q \to \infty$ this symmetry becomes exact.

- Corrections to the static limit can be computed order by order in Λ_{QCD}/m_Q (HQET).
- For the b-quark $\Lambda_{QCD}/m_Q \approx 0.1$.

Experimentally little is known about the B meson spectrum.

- Pseudoscalar and Vector channels have been identified.
- $B_j^*(5732)$ and $B_{sj}^*(5850)$ are possible P-wave candidates.

Quark models predict non-trivial structure.

Extraction and identification of excited-state spectrum from lattice simulations is challenging.

Model predictions

Possible inversion of orbitally-excited multiplets with respect to the ‘usual’ atomic ordering.

Quark models - splitting between states in an orbital receives opposing contributions from single gluon exchange and Thomas precession.

For small inter-quark separation single gluon exchange dominates - higher angular momentum states are heavier.

At larger separations Thomas precession comes to dominate.

Isgur (98) predicted the inversion of the P-wave multiplet for static-light mesons.

Signal-to-noise ratios

Traditionally, simulations of static-light systems have been noisy. In a simulation, one generally evaluates the light quark propagator from a single space-time site to all other lattice sites. A static quark propagates only in time which means that static-light interpolating operators are confined to a single spatial lattice site. Signal-to-noise ratio tends to be very poor (Exponential decay with a large exponent).

Solution:

Efficient estimate for all elements of the lattice Dirac propagator. Average static-light correlators over all lattice sites to obtain a dramatic increase in statistics (Liverpool-Helsinki , Regensburg , Dublin).

Estimating all elements of the Dirac propagator

Low-lying eigenmodes of the Dirac operator are important. Write the propagator as a truncated spectral sum and a stochastic contribution.

Evaluate the contribution of the low-lying eigenmodes exactly.

To obtain the stochastic estimate, invert the Dirac operator on a set of Z_4 noise sources which have been 'diluted' in a subset of quarkfield indices.

The static-light spectrum

Heavy-quark symmetry means that there is no hyperfine splitting and $J_l^{P_l}$ are good quantum numbers.

Conventional to use these quantum numbers to label degenerate hyperfine multiplets .

In this notation the S, P and D -wave channels are $\frac{1}{2}^-$, $\frac{1}{2}^+$, $\frac{3}{2}^+$ and $\frac{3}{2}^-$, $\frac{5}{2}^-$

Inversion of natural ordering in the P orbital $\Rightarrow$ $\frac{3}{2}^+$ level lies below the energy of the $\frac{1}{2}^+$ multiplet.

In simulations we exploit degeneracies and average measurements over the channels in a given hyperfine multiplet.

To facilitate the average over the hyperfine multiplet we choose interpolating operators specifically for the light fermionic degrees of freedom.

$J_\ell^{P_\ell}$ labels spinorial irreps of $O(3)$.

Connect source and sink operators with a Wilson line to obtain a gauge-invariant two-point function for the hyperfine multiplet of interest.

$$C(\tau) \propto \langle\Omega|\overline{\mathcal{O}}^{(R)}(t+\tau,\vec{x})\mathcal{W}(\tau+t,t;\vec{x})\mathcal{O}^{(R)}(t,\vec{x})|\Omega\rangle, \quad R \in \{J_l^{P_\ell}\} \qquad (1)$$

In the continuum the interpolating operators consist of quark field spin components + spatial derivatives.

On the lattice, excitations (derivatives) $\longrightarrow$ spatially-extended operators $\Rightarrow$ light quark field components separated from the static-quark along a gauge covariant path.
How does one construct such operators? Will a naive discretisation of continuum operators suffice?

Angular momentum and parity quantum numbers label irreducible representations of O(3), which is a symmetry group of the continuum Hamiltonian.

$$O(3) = SO(3) \otimes C_2, \quad C_2 = \{I, I_s\} \tag{2}$$

Spatially isotropic lattice preserves C_2 but $SO(3)$ is broken to a 24 element subgroup - O.

The relevant lattice symmetry group is $O_h = O \otimes C_2$.

We can't simply discretise the continuum operators.

Resulting operators will have a non-zero overlap with an infinite number of states. Conclusive identification of excited states is impossible.

Instead, construct operators which transform according to the irreps of O_h.

Determine the particle content of the irreps.

Identify the excited state energies by observing patterns in the measured energies across the irreps.

Irreps of the lattice symmetry group

First consider O (subgroup of $SO(3)$).

O has five single-valued (bosonic) irreps labeled A_1, A_2, E, T_1 and T_2.

However, to construct operators for the light degrees of freedom - we require the spinorial representations of O.

Obtain these from the corresponding double cover group O^D.

O^D has 8 single-valued irreps. 5 of these coincide with the single-valued irreps of O. The remaining three are the double-valued irreps of O
Half-integer irreps of $SO(3) \longrightarrow G_1$, G_2, H.

Irreps of the lattice symmetry group

O_h has twice as many irreducible representations as O

$A_1 \longrightarrow A_{1g}, A_{1u}$
$G_1 \longrightarrow G_{1g}, G_{1u}$
where g(u) denote irreps which are even (odd) under parity.

Particle content

- Restrict the continuum irreps (labeled $J_\ell^{P_\ell}$) to the elements of O_h.
- This generates representations for O_h which are in general reducible.
- Decompose these subduced representations into their constiuent irreps.

Lattice irrep	Dimension	J^P
G_{1g}	2	$\frac{1}{2}+$, $\frac{7}{2}+\ldots$
G_{2g}	2	$\frac{5}{2}+$, $\frac{7}{2}+\ldots$
H_g	4	$\frac{3}{2}+$, $\frac{5}{2}+$, $\frac{7}{2}+$..

States with the same J^P but different J_z can be divided between the irreps.

S-wave multiplet ($\frac{1}{2}^-$) appears in the G_{1u} irrep.

P-waves
$\frac{1}{2}^+ \in G_{1g}$.
$\frac{3}{2}^+ \in H_g$.

D-waves
$\frac{3}{2}^- \in H_u$.
$\frac{5}{2}^- \longrightarrow G_{2u} \oplus H_u$.
Should observe near-degenerate energy levels in the G_{2u} and H_u irreps corresponding to $\frac{5}{2}^-$.

Accessing excited states

1. Identify a set of interpolating operators which project onto a particular lattice irrep.
2. Apply a variational analysis to extract the energies of a number of low-lying states

G_{1u} can be accessed using local operators.

G_{1g}, H_g, H_u straight-line displaced operators.

G_{2u} accessed using planar-diagonal displaced operators.

By construction the operators transforming according to different rows of the same representation are orthogonal.

To generate variational bases a number of levels of Jacobi smearing are applied to the light quark spin components.

Variational Analysis

For each irrep compute a matrix of correlation functions.

$$C_{\alpha\beta}(\tau) \propto \sum_{t,\vec{x}} \langle \Omega | \overline{\mathcal{O}}_{\alpha}^{(R)}(t+\tau,\vec{x}) \mathcal{W}(t,t+\tau;\vec{x}) \mathcal{O}_{\beta}^{(R)}(t,\vec{x}) | \Omega \rangle$$

Solve the generalised eigenvalue problem

$$C(t_D)\mathbf{v}_n = \lambda_n(t_D,t_0)C(t_0)\mathbf{v}_n$$

Then

$$\tilde{C}_{nn}(\tau) \equiv \mathbf{v}_n^T C(\tau)\mathbf{v}_n \sim \lambda_n(\tau,\tau_0) \qquad (3)$$
$$\lim_{\tau \gg \tau_0} \lambda_n(\tau,\tau_0) \longrightarrow e^{-E_n(\tau-\tau_0)}$$

Define

$$m_{eff}^{(n)}(\tau,\tau_0) = -\ln\left[\tilde{C}_{nn}(\tau+1)/\tilde{C}_{nn}(\tau)\right], m_{eff}^{(n)}(\tau \gg \tau_0) \longrightarrow E_n$$

Simulation details

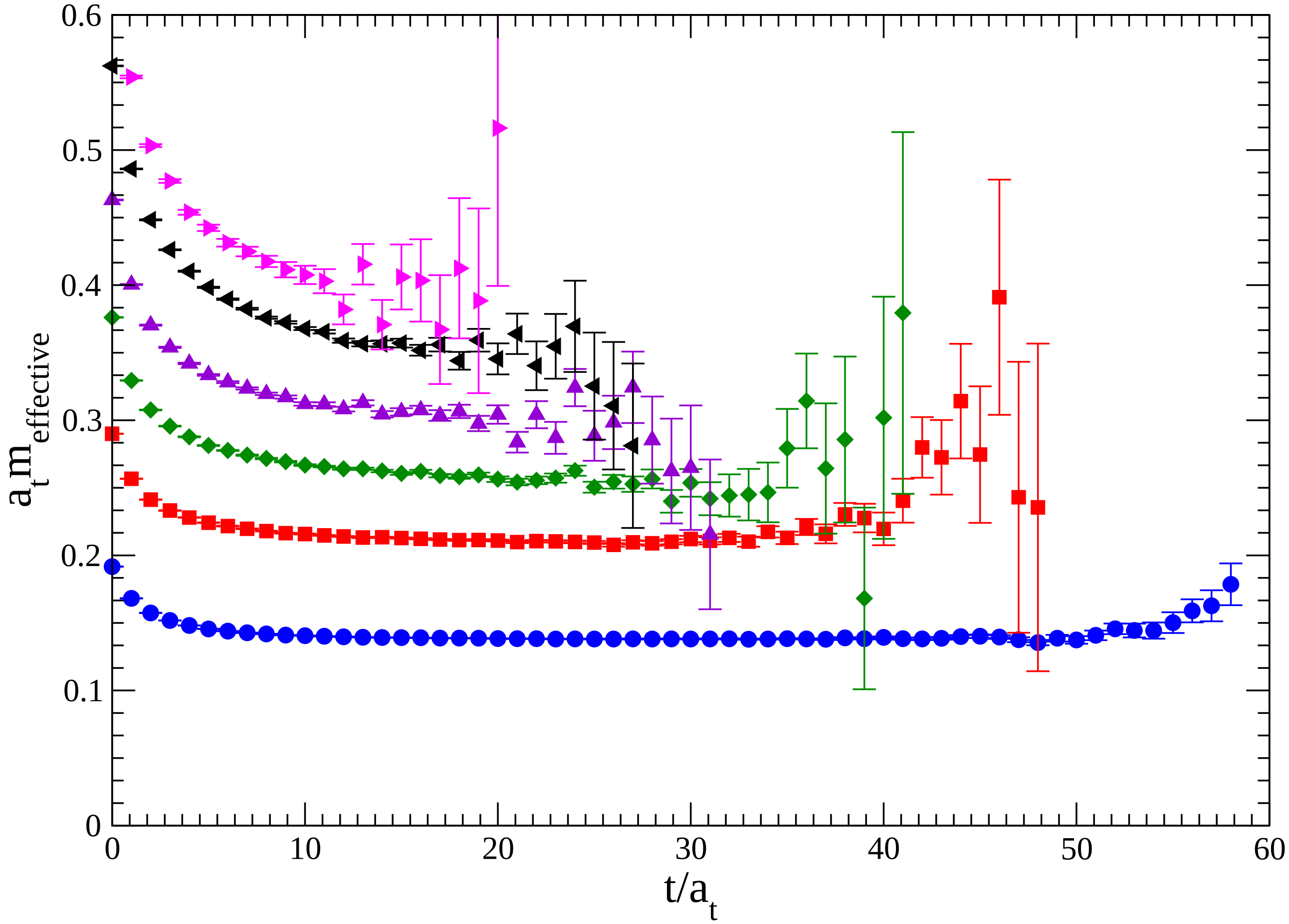
$N_f = 2$ simulations performed on an anisotropic lattice

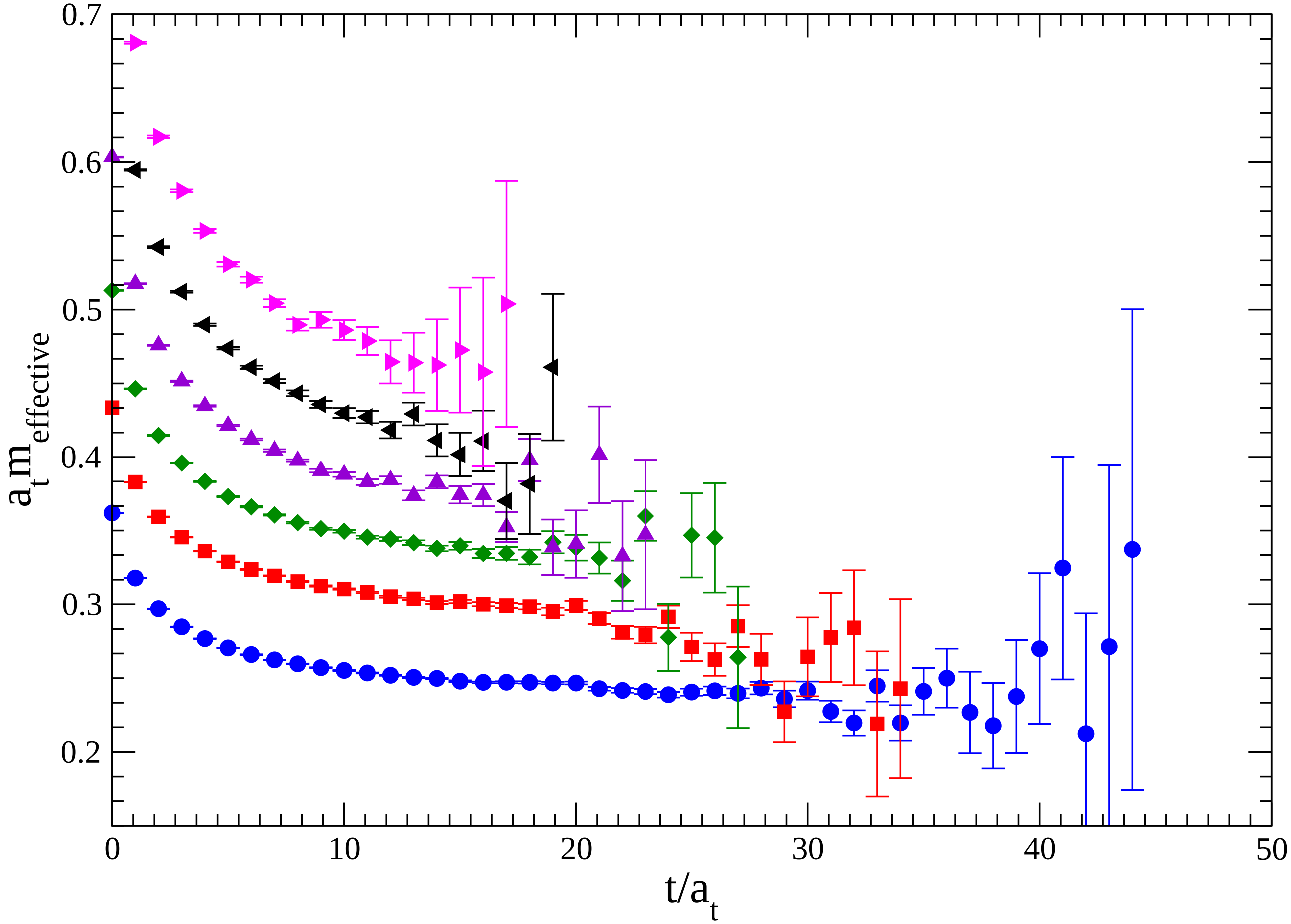
$a_s \approx 0.17$ fm.
$a_t \approx 0.03$ fm.

quark mass close to strange - relevant for the B_s spectrum.

Two spatial volumes $(1.35 \text{ fm})^3$ and (2.03 fm^3).

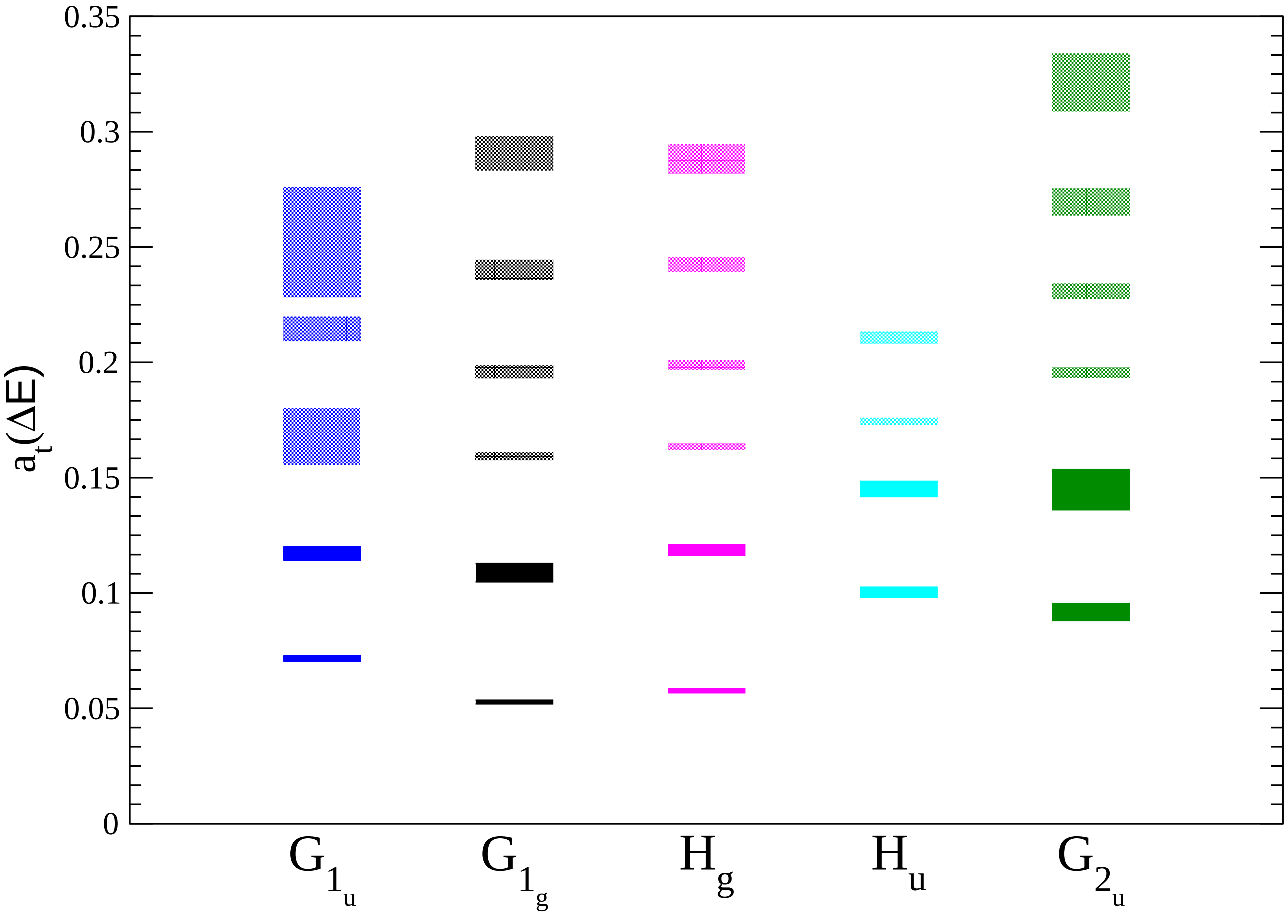
Effective mass plot for the G_{1u} (S-wave) irrep.





Effective mass plot for the G_{2u} irrep which contains the $\frac{5}{2}^-$ D-wave.

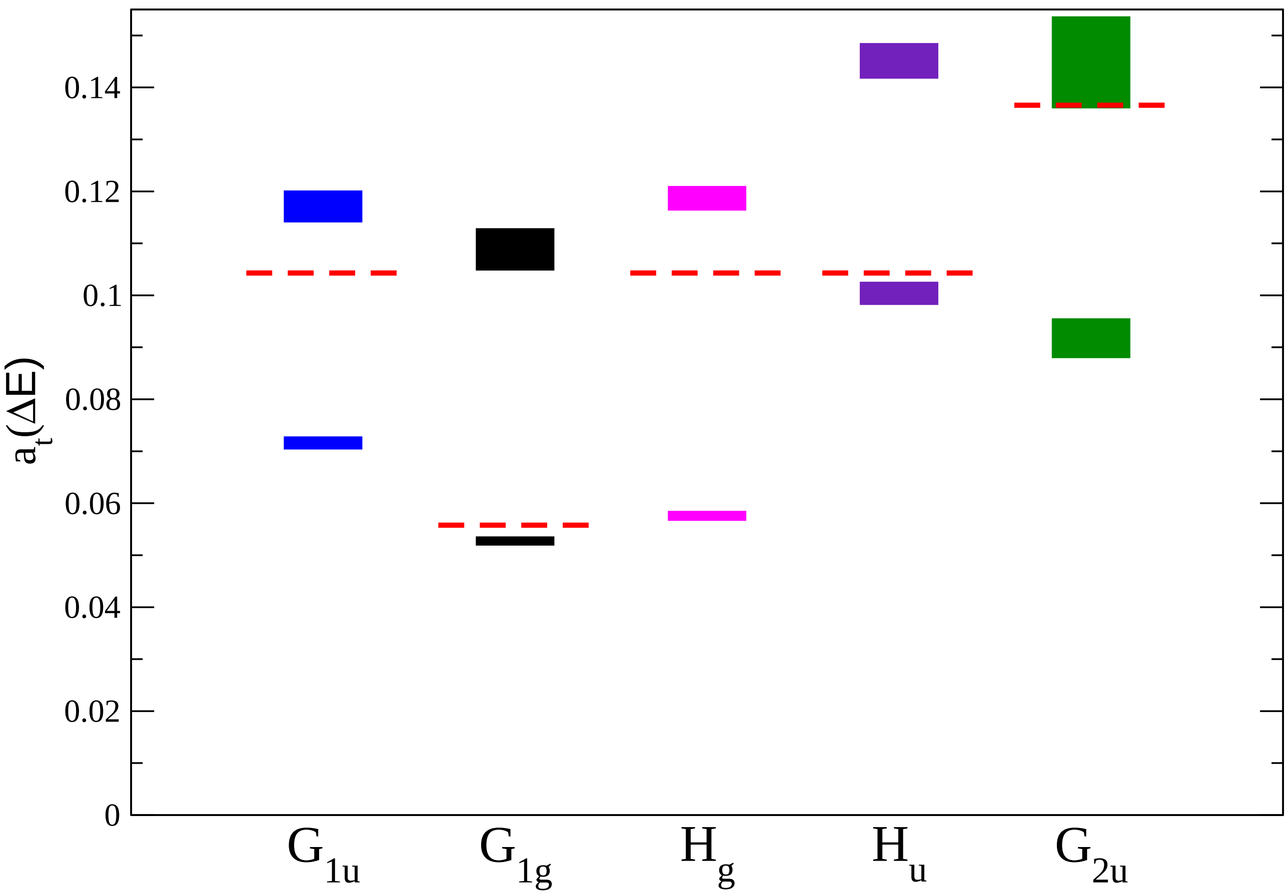
Measured energies contain an unphysical shift. This cancels in energy differences.



Splitting between the ground state in G_{1u} and higher excitations.

- Multiparticle states - spectral weights show a well-defined volume dependence.
- No evidence for these states in the previous figure.
- For sufficiently light quark masses the lowest energy scattering states in each of the irreps consist of a pion plus an S-wave static-light meson.
- A simple qualitative calculation for the threshold energies yields the following result...

Scattering thresholds



Scattering states are certainly permitted in the energy regime under investigation.

Absence of these states from the measured spectrum is a reflection on our choice of interpolating operators.

Interpretation

The lowest-lying state in G_{1u} is the S-wave.
1st and 2nd excited states are radial excitations with the same quantum numbers. Other states appearing in G_{1u} include $\frac{7}{2}^-$ (L=4) states.

Lowest energy level in G_{1g} corresponds to the $\frac{1}{2}^+$ P-wave. 1st excited state is a radial excitation. Also contains the $\frac{7}{2}^-$ F-wave.

H_g ground-state is the $\frac{3}{2}^+$ P-wave. 'Natural' ordering of P-wave multiplets holds but the spin-orbit splitting is very small (34 MeV). 1st excited state is a radial excitation.

Interpretation of the energy levels in the G_{2u} and H_u irreps is much more complicated....

Recall that $\frac{3}{2}^-$ appears in H_u and $\frac{5}{2}^-$ states appear in H_u and G_{2u}.
Assume that we have resolved all the low-lying single-particle energies.

2 possibilities: Usual ordering of states or inversion of the D-wave multiplets.

Neither scenario is compatible with the data!
Most likely explanation is that the data is incomplete. Missing energy-levels in the H_u irrep.

Not surprising given our choices for variational bases. Variational method biased towards radial excitations. Access to the complete spectrum will require a greater variety of interpolating operators.

Summary and outlook

Precision determination of low-energy states in the static-light spectrum (still incomplete).
A precise understanding of the spatial lattice symmetries is crucial.

No inversion of P-wave multiplets.
D-wave ordering still unknown.

Data are (too) compatible with experimental results for the B_s spectrum.

A complete determination will require the use of a greater variety of interpolating operators and ultimately a continuum limit. Can all be achieved in the not-so-distant future.

channel	$\triangle E$ (GeV)
G_{1u}, 1st excitation	0.504(8)
G_{1u}, 2nd excitation	0.82(2)
G_{1g}, ground state	0.371(6)
G_{1g}, 1st excitation	0.76(3)
H_g, ground state	0.405(6)
H_g, 1st excitation	0.84(2)
H_u, ground state	0.706(2)
H_u, 1st excitation	1.02(2)
G_{2u}, ground state	0.65(3)
G_{2u}, 1st excitation	1.02(6)