

The Bethe-Salpeter-equation and mesons

A. Krassnigg (University of Graz, Austria)

Work with:

A. Höll, C. D. Roberts, S. V. Wright (ANL, PHY)

Work performed at/supported by/in collaboration with:

Austrian Research Foundation 

Argonne National Laboratory



University of Graz



- QCD and hadrons
- DSE-BSE
- Solution strategies
- Mesons and their properties
- Symmetries \leftrightarrow exact results
- Example (sophisticated) Ansatz
- Example results
- Conclusions and outlook

- **Dyson Schwinger Equations:**
a modern method in **relativistic QFT**

P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12** (2003) 297

R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281

C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. **45** (2000) S1

A. Holl, C. D. Roberts, S. V. Wright, nucl-th/0601071

C. S. Fischer, J. Phys. G **32** (2006) R253

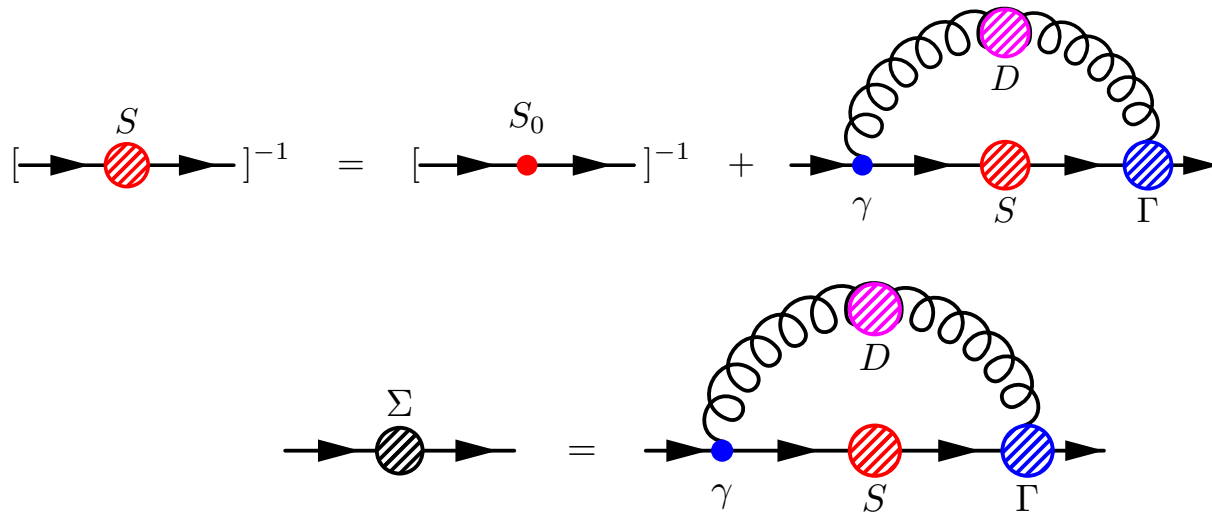
- **Dyson Schwinger Equations:**
a modern method in **relativistic QFT**
P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12** (2003) 297
R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281
C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. **45** (2000) S1
A. Holl, C. D. Roberts, S. V. Wright, nucl-th/0601071
C. S. Fischer, J. Phys. G **32** (2006) R253
- Study hadrons as composites of **quarks** and **gluons** . . .
- . . . including:
 - **Chiral symmetry** and $D_\chi SB$
 - correct perturbative limit (via $\alpha_p(Q^2)$)
 - quark and gluon confinement
 - **Poincaré covariance**
- Propagators and Bethe-Salpeter amplitudes
→ **observables**

- Solutions: **Schwinger functions**
(Euclidean Green functions)
(also calculated on the lattice)

- Solutions: **Schwinger functions**
(Euclidean Green functions)
(also calculated on the lattice)
- Each function satisfies **integral equation** involving **other** functions \Rightarrow
- **Infinite** set of coupled integral equations
- **Truncation scheme** necessary \Rightarrow
- Generating tool for perturbation theory

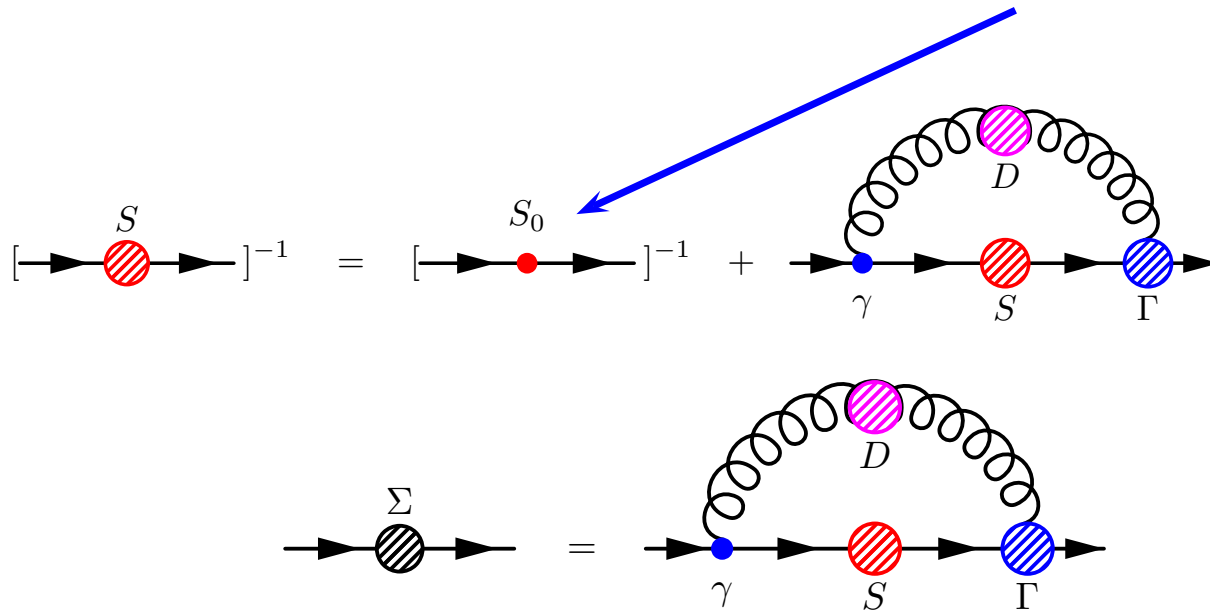
- Solutions: **Schwinger functions**
(Euclidean Green functions)
(also calculated on the lattice)
- Each function satisfies **integral equation** involving **other** functions \Rightarrow
- **Infinite** set of coupled integral equations
- **Truncation scheme** necessary \Rightarrow
- **Nonperturbative** truncation scheme
- Respect **symmetries**
- Prove **exact** (model independent) **results**
- Devise **(sophisticated) models** to illustrate them

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



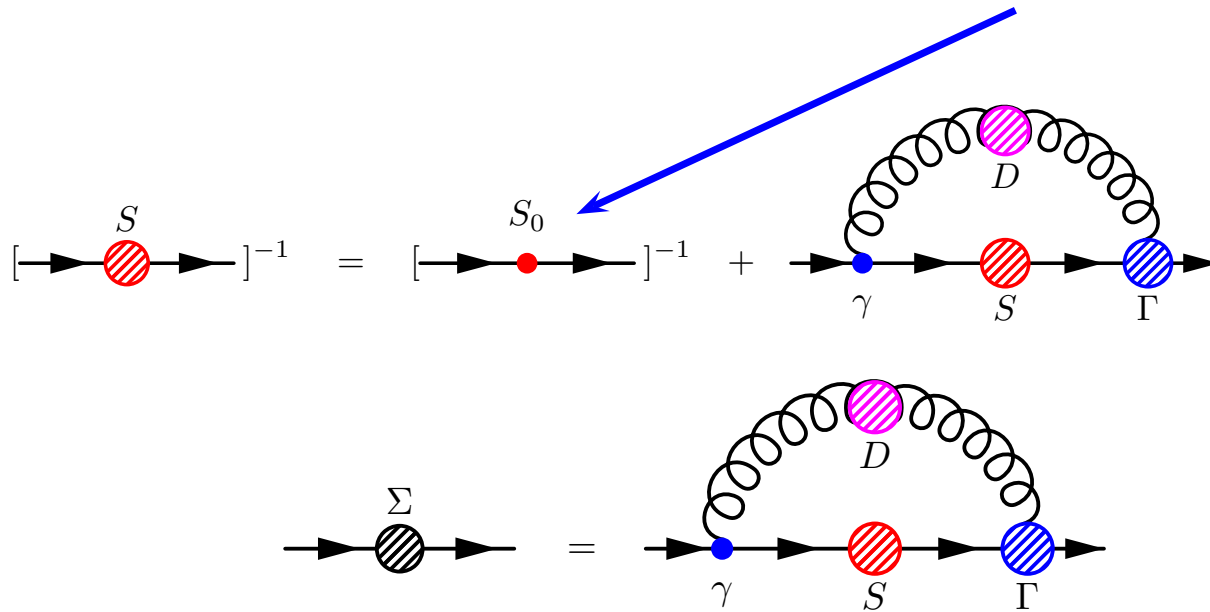
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

current quark mass m_ζ



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

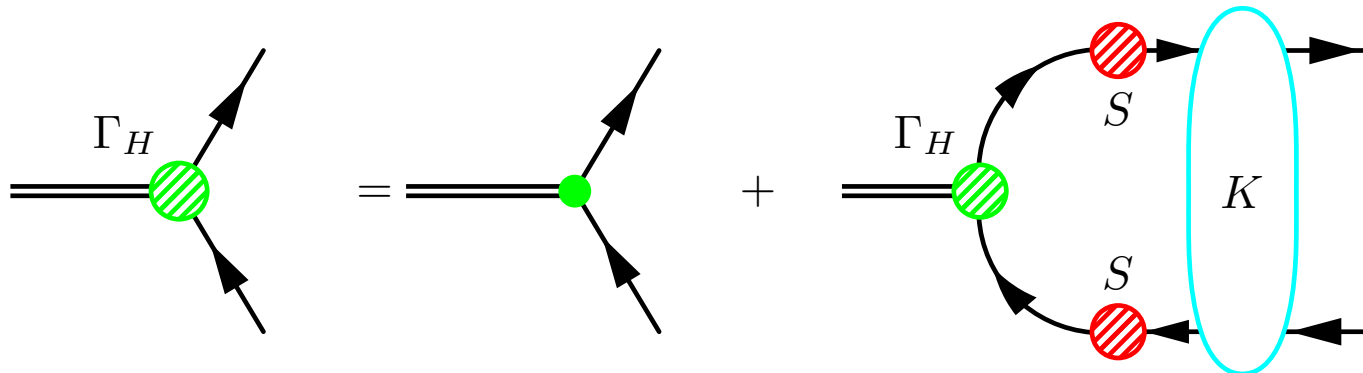
current quark mass m_ζ



- **Weak coupling** expansion reproduces every diagram in **perturbation theory**, but:
- Perturbation theory: $m_\zeta = 0 \Rightarrow M(p^2) \equiv 0$

- BSE for $q\bar{q}$ or qq bound states ($\chi = S\Gamma_H S$)

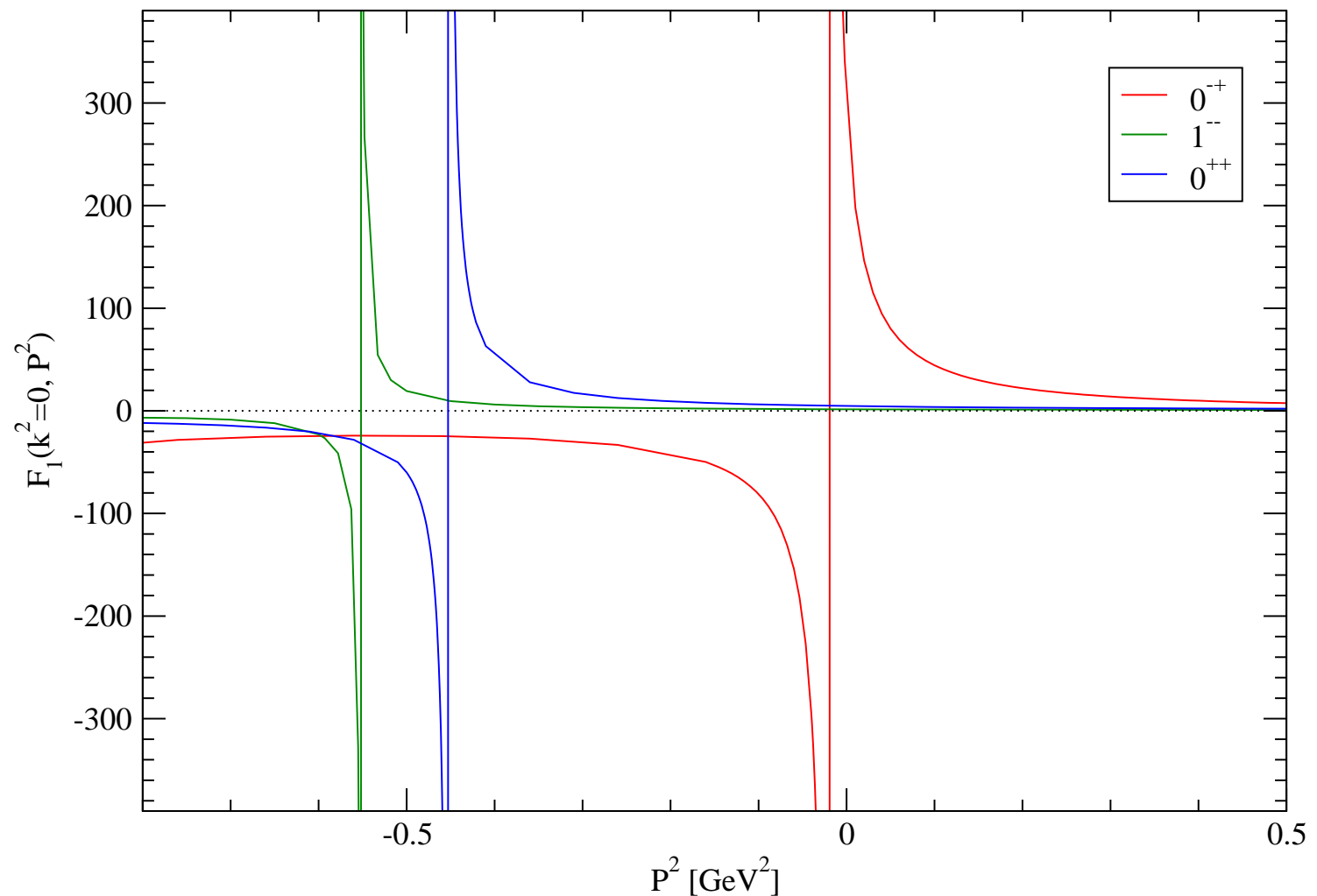
$$\Gamma_H(p; P) = \text{d. t.} + \int d^4q \chi(q; P) K(q, p; P).$$



- Gap eq. **output** \rightarrow BSE **input**
- Bound state at $P^2 = -m_H^2$:

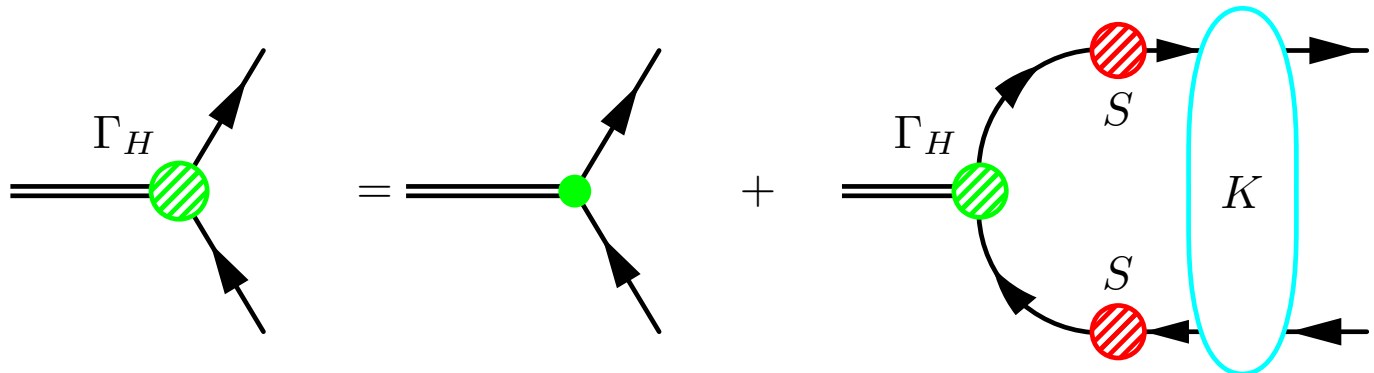
$$\Gamma_H(q; P) = \frac{r_H \Gamma_h(q; P)}{P^2 + m_H^2} + \text{regular terms}$$

- 0^{-+} , 0^{++} , and 1^{--} meson amplitudes



- BSE for $q\bar{q}$ or qq bound states ($\chi = S\Gamma_H S$)

$$\Gamma_H(p; P) = \text{d. t.} + \int d^4q \chi(q; P) K(q, p; P).$$



- Gap eq. **output** \rightarrow BSE **input**
- Bound state at $P^2 = -m_H^2$:

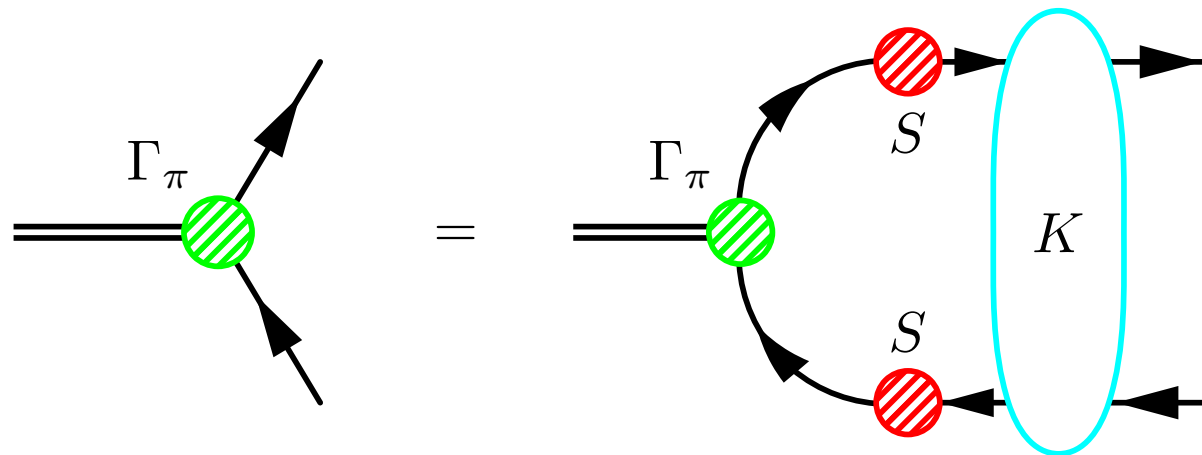
$$\Gamma_H(q; P) = \frac{r_H \Gamma_h(q; P)}{P^2 + m_H^2} + \text{regular terms}$$

Bethe-Salpeter Equation

- BSE for $q\bar{q}$ or qq bound states ($\chi = S\Gamma_h S$)

$$\Gamma_{h\,tu}(p; P) = \int d^4q [\chi(q; P)]_{sr} K_{rs}^{tu}(q, p; P).$$

- homogeneous \rightarrow eigenvalue equation

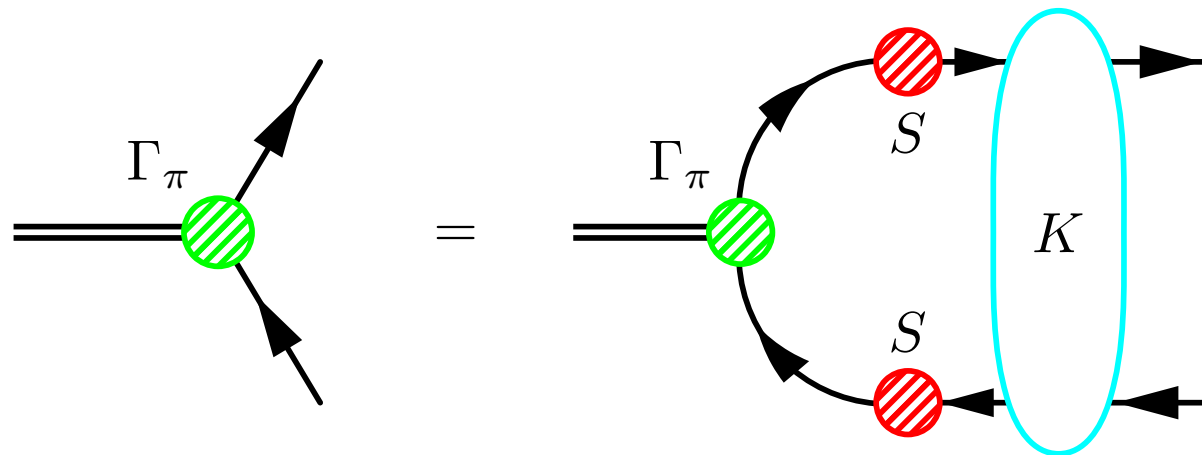


Bethe-Salpeter Equation

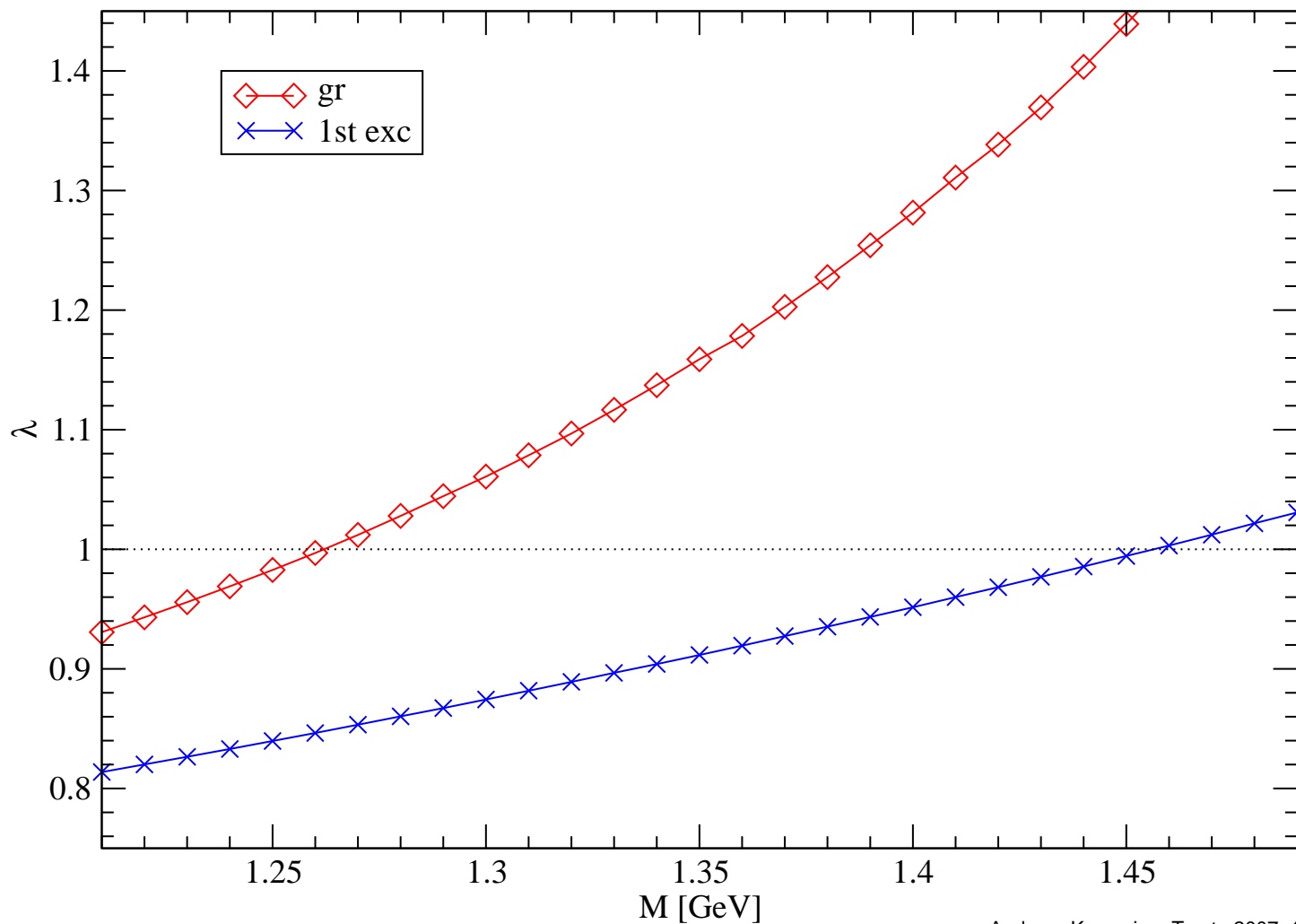
- BSE for $q\bar{q}$ or qq bound states ($\chi = S\Gamma_h S$)

$$\Gamma_{h\,tu}(p; P) = \lambda(P^2) \int d^4q [\chi(q; P)]_{sr} K_{rs}^{tu}(q, p; P).$$

- homogeneous \rightarrow eigenvalue equation



- Solution strategy for homogeneous BSE



- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^j(k; P) = S^{-1}(k_+) i\gamma_5 \frac{\tau^j}{2} + i\gamma_5 \frac{\tau^j}{2} S^{-1}(k_-) - 2i m(\zeta) \Gamma_5^j(k; P),$$

- Axial-vector Ward-Takahashi identity

$$\Gamma_{5\mu}^j(k; P) =$$

$$\Gamma_5^j(k; P)$$

- Consequence (residues):

$$f_{\pi_n} m_{\pi_n}^2 = 2 m(\zeta) \rho_{\pi_n}(\zeta);$$

- Axial-vector Ward-Takahashi identity

$$\Gamma_{5\mu}^j(k; P) = \Gamma_5^j(k; P)$$

- Consequence (residues):

$$f_{\pi_n} m_{\pi_n}^2 = 2 m(\zeta) \rho_{\pi_n}(\zeta);$$

- valid for every pseudoscalar meson
- valid for every current quark mass
- \Rightarrow GMOR, PCAC

P. Maris, C. D. Roberts, Phys. Rev. **C56**, 3369 (1997)

A. Höll, A. K., and C. D. Roberts, Phys. Rev. C **70**, 042203 (2004)

- Investigate the **chiral limit** of

$$f_{\pi_n} m_{\pi_n}^2 = 2 m(\zeta) \rho_{\pi_n}(\zeta);$$

- Investigate the **chiral limit** of

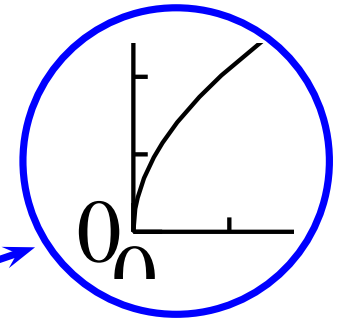
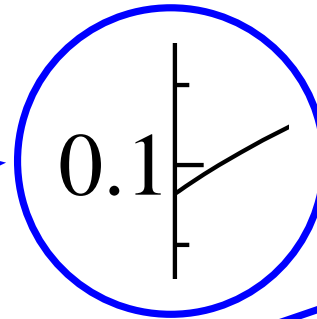
$$f_{\pi_n} m_{\pi_n}^2 = 2 m(\zeta) \rho_{\pi_n}(\zeta);$$

- Ground state pion:**

- $m(\zeta) \rightarrow 0$

- $f_{\pi_{gr}}$ **finite**

- $m_{\pi_{gr}} \rightarrow 0$



- Investigate the **chiral limit** of

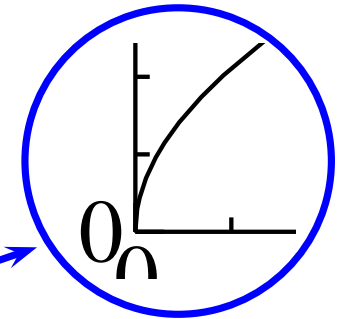
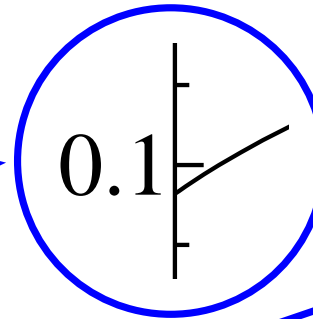
$$f_{\pi_n} m_{\pi_n}^2 = 2 m(\zeta) \rho_{\pi_n}(\zeta);$$

- Ground state pion:**

- $m(\zeta) \rightarrow 0$

- $f_{\pi_{gr}}$ finite

- $m_{\pi_{gr}} \rightarrow 0$

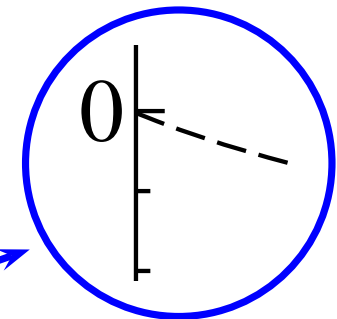
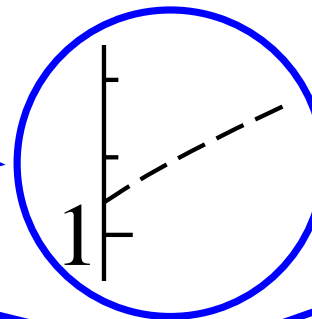


- Excited state pion:**

- $m(\zeta) \rightarrow 0$

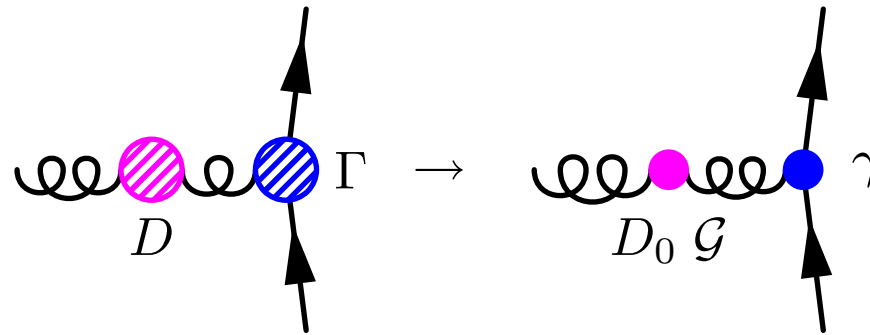
- $m_{\pi_{exc1}}$ finite

- $f_{\pi_{exc1}} \rightarrow 0$



Rainbow-Ladder (RL) Truncation

- Rainbow approximation for gap equation
- Ladder approximation for BSE



- Effective coupling \mathcal{G}
- Bare quark-gluon vertex γ_ν
- Bare gluon propagator $D_{\mu\nu}^{\text{free}}(p - q)$
- How good is this?

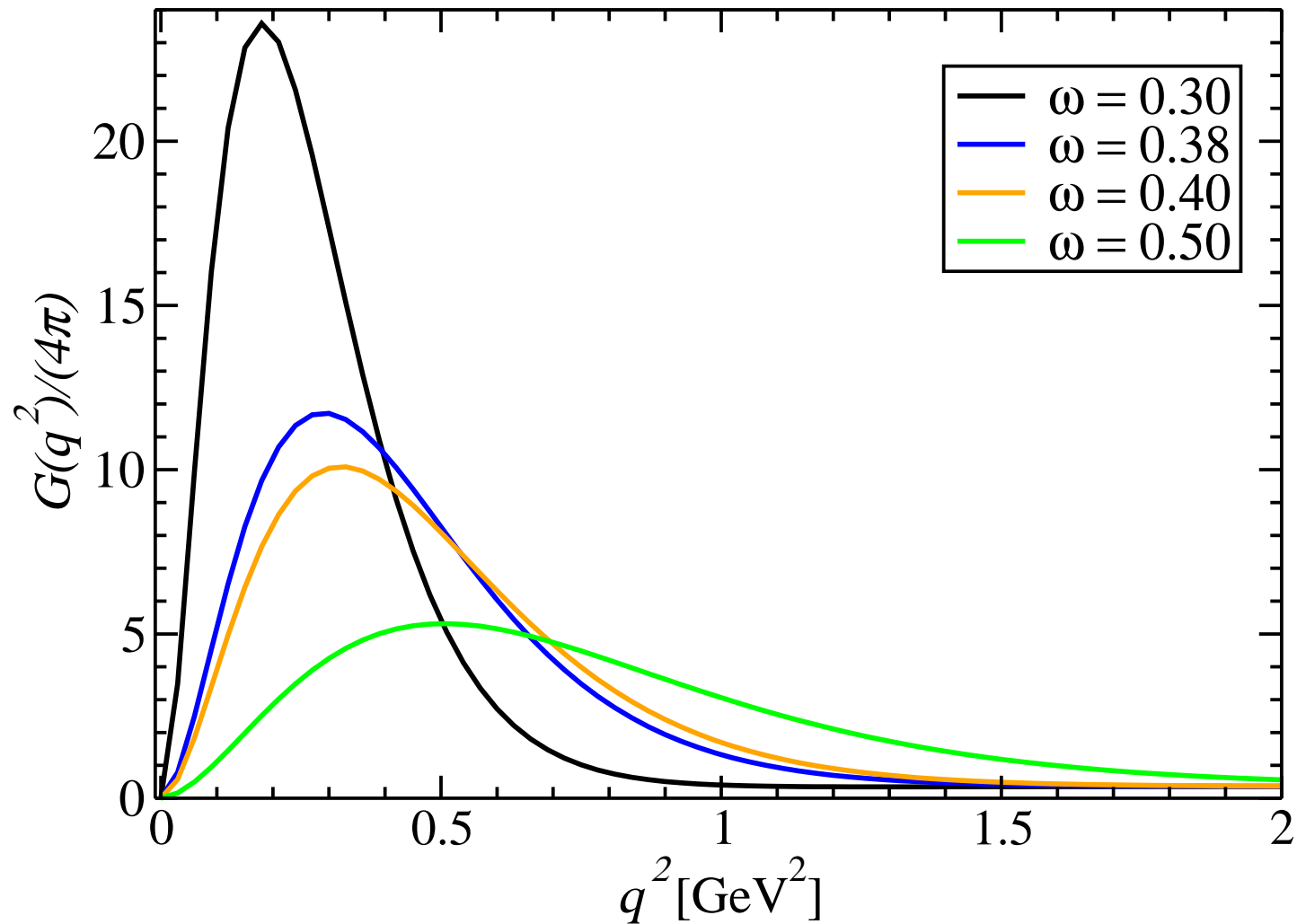
- What do we know?
- Effective running coupling $\mathcal{G}(Q^2)$

- What do we know?
- Effective running coupling $\mathcal{G}(Q^2)$
- Perturbative QCD determines **UV regime**
- **IR unknown** in detail

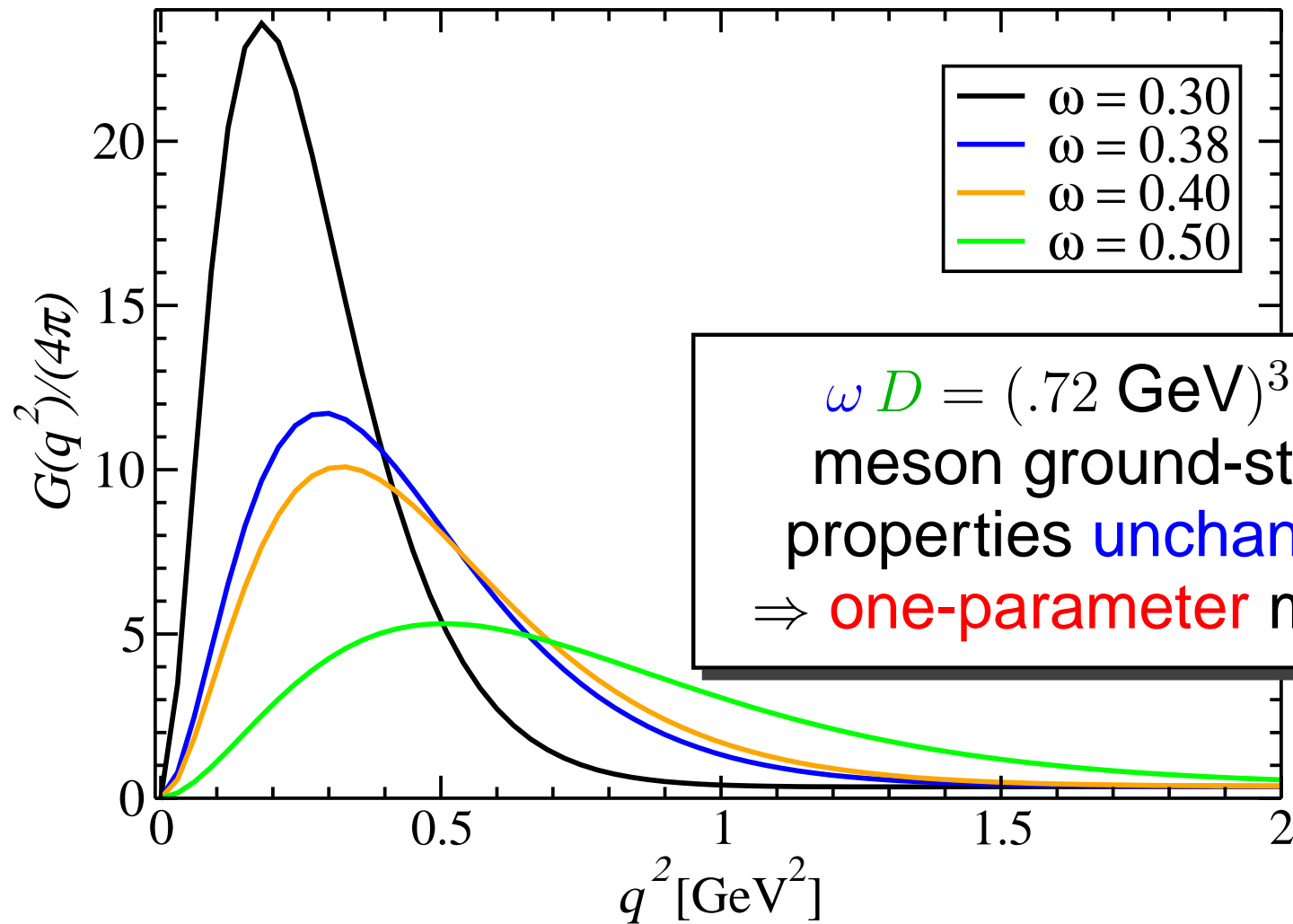
- What do we know?
- Effective running coupling $\mathcal{G}(Q^2)$
- Perturbative QCD determines UV regime
- IR unknown in detail
- IR enhancement necessary for dynamical breaking of chiral symmetry
- Integrated strength is essential
- Precise form at low $Q^2 \rightarrow$ model

- What do we know?
- Effective running coupling $\mathcal{G}(Q^2)$
- Perturbative QCD determines **UV regime**
- **IR unknown** in detail
- IR **enhancement necessary** for dynamical breaking of chiral symmetry
- **Integrated strength** is essential
- Precise form at low $Q^2 \rightarrow$ **model**
- **IR**: two-parameters via Gaussian: strength D and width ω
- **perturbative** α in the **UV** region

- Effective coupling $\mathcal{G}(Q^2)$: $\omega D = \text{const.}$



- Effective coupling $\mathcal{G}(Q^2)$: $\omega D = \text{const.}$

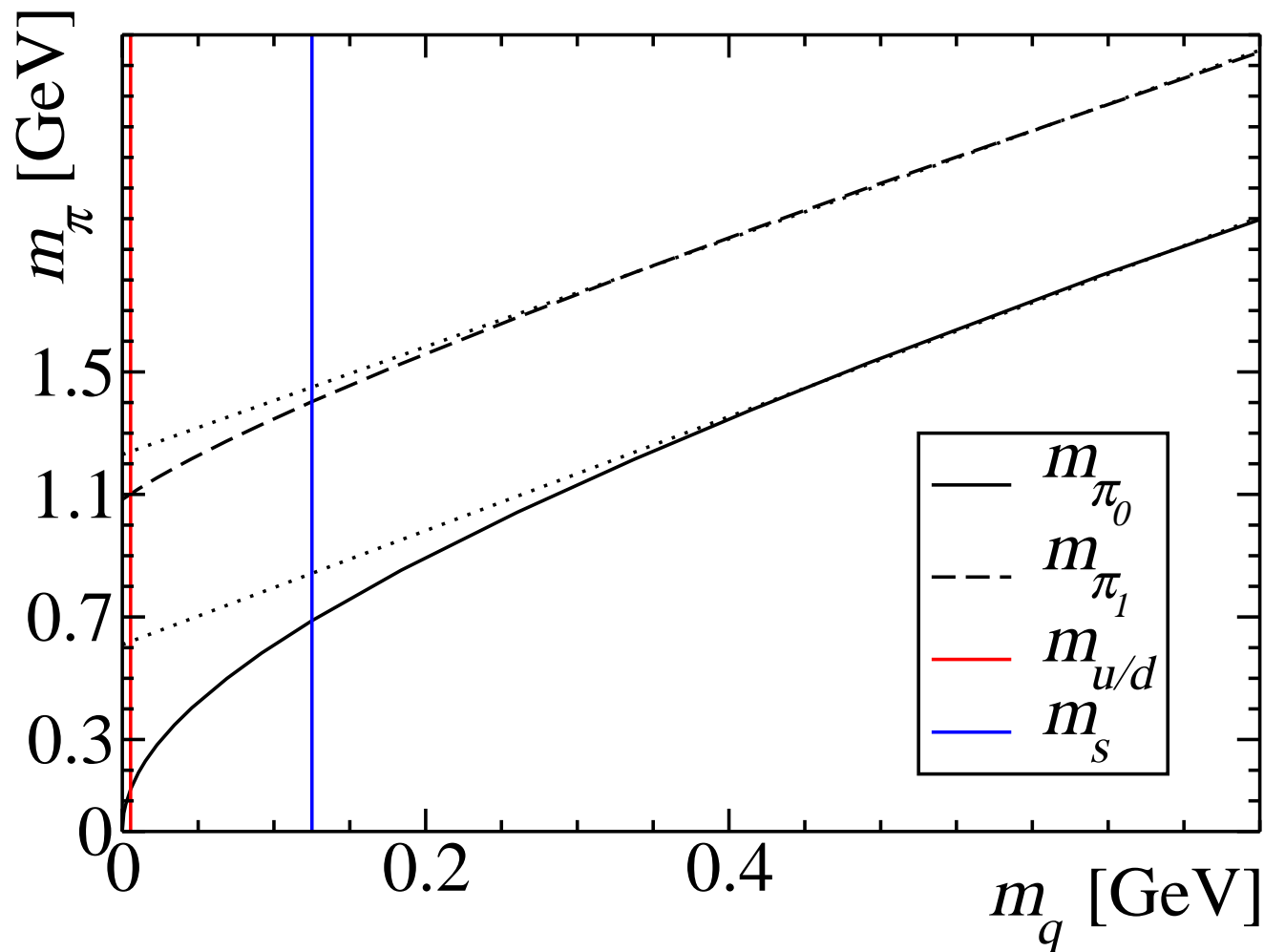


So Far – Ground States

- P. Maris, P. C. Tandy: series of papers following
P. Maris and P. C. Tandy, Phys. Rev. C **60**, 055214 (1999).
- Successful description of light
pseudoscalar and vector mesons

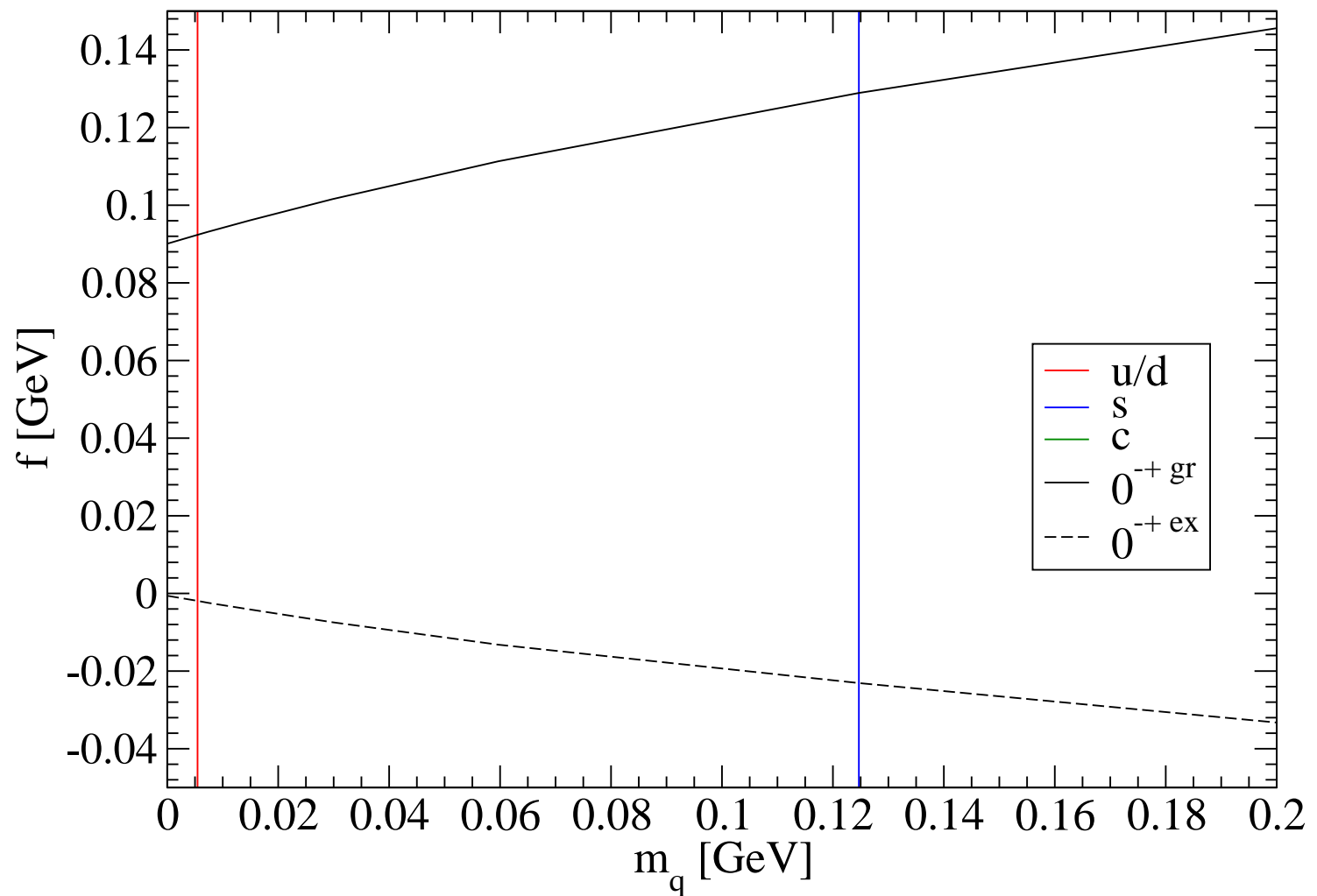
- P. Maris, P. C. Tandy: series of papers following P. Maris and P. C. Tandy, Phys. Rev. C **60**, 055214 (1999).
- Successful description of light **pseudoscalar and vector** mesons
- Now:
 - **Radial** excitations
 - **Scalar** mesons
 - **Axial vector** mesons
- Study **long range part** of the **strong interaction** between **light quarks**

- $m_{0_{gr}^{--+}}$ and $m_{0_{exc1}^{--+}}$ as functions of current quark mass



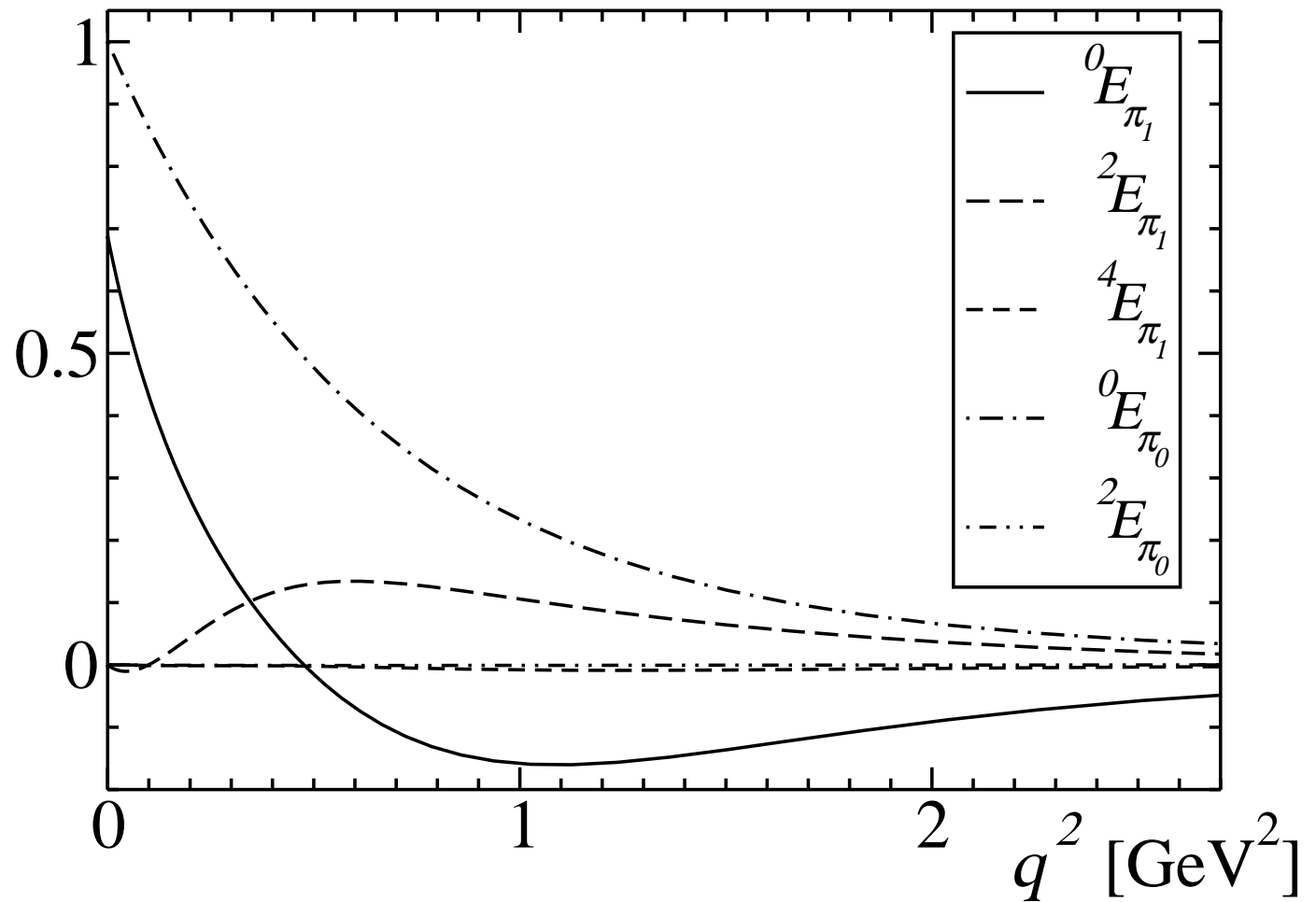
Leptonic Decay Constants

- $f_{0_{gr}^{-+}}$ and $f_{0_{excl}^{-+}}$ as functions of current quark mass



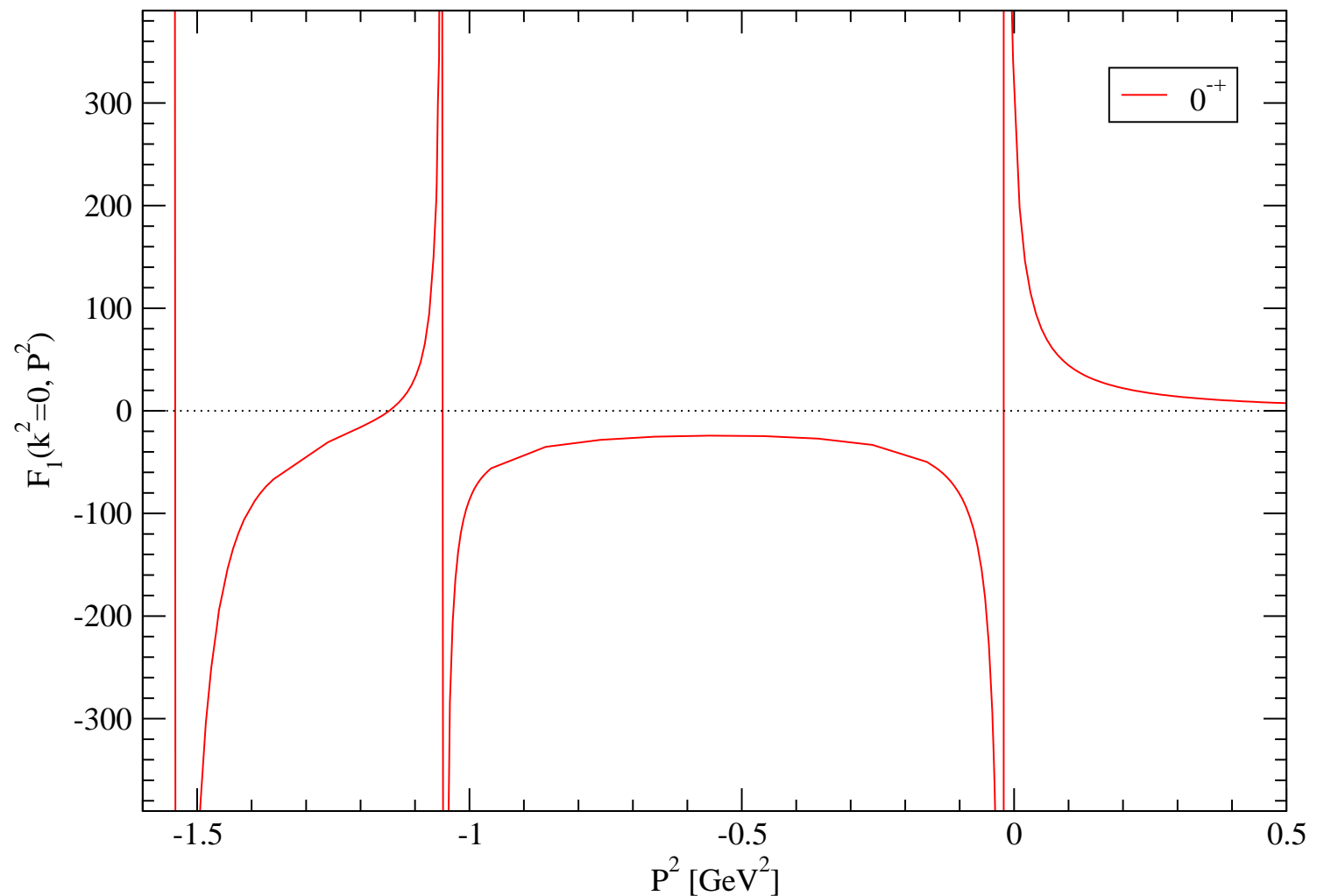
Chebyshev Moments

- Chebyshev moments for π_0 and π_1

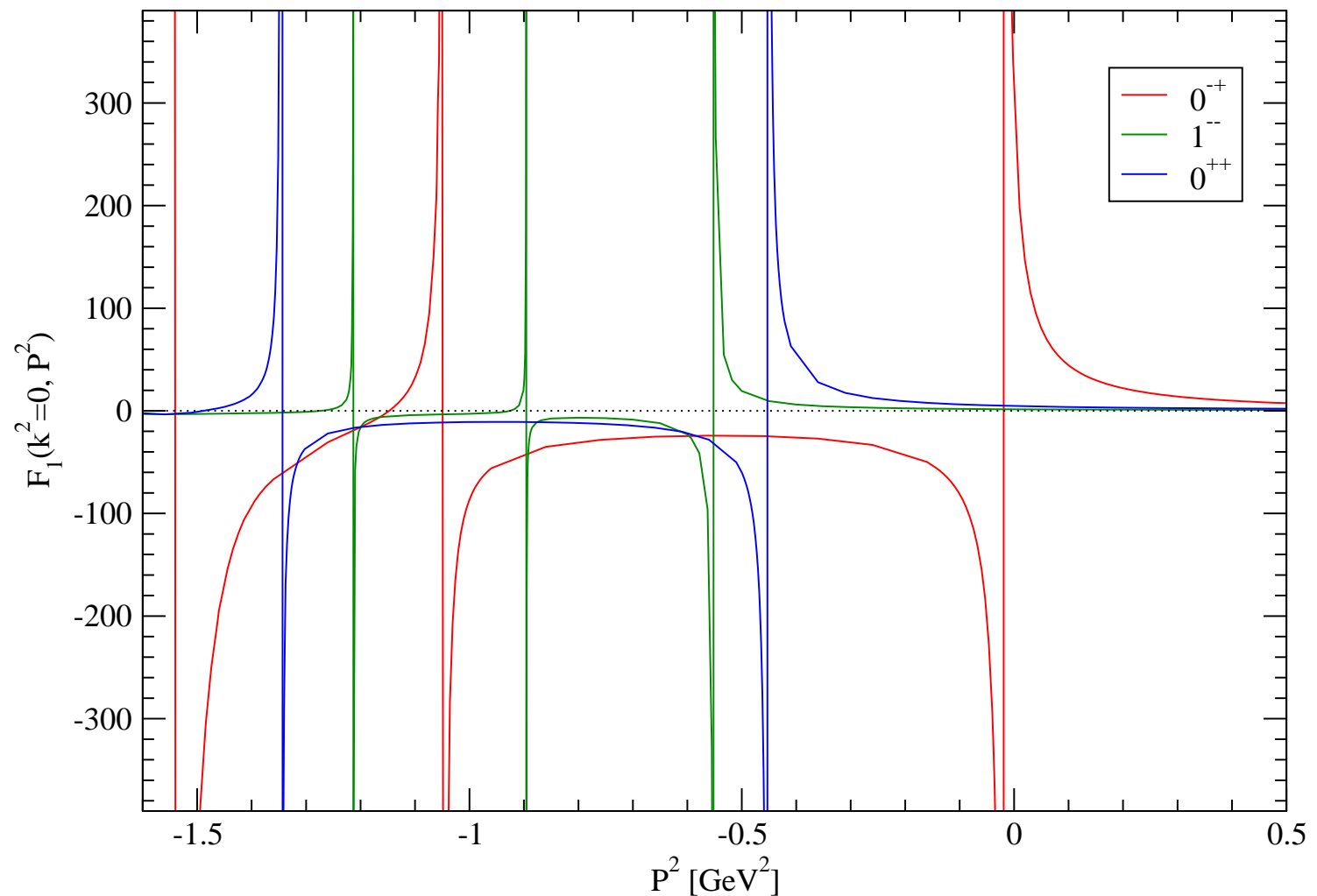


- 0^{-+} meson amplitude

M. Bhagwat, A. Höll, A. K., C. D. Roberts and S. V. Wright, arXiv:nucl-th/0701009

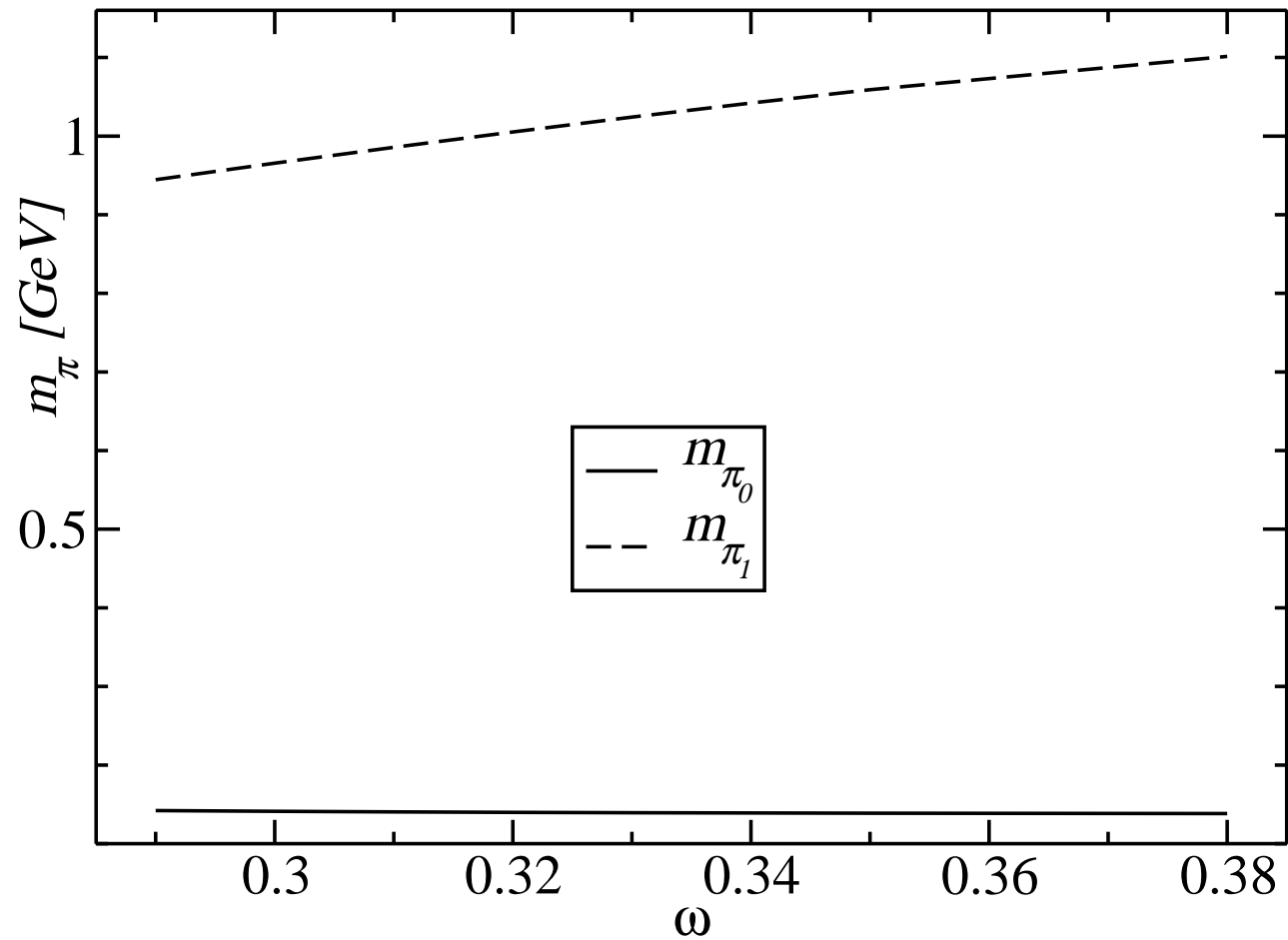


- 0^{-+} , 0^{++} , and 1^{--} meson amplitudes



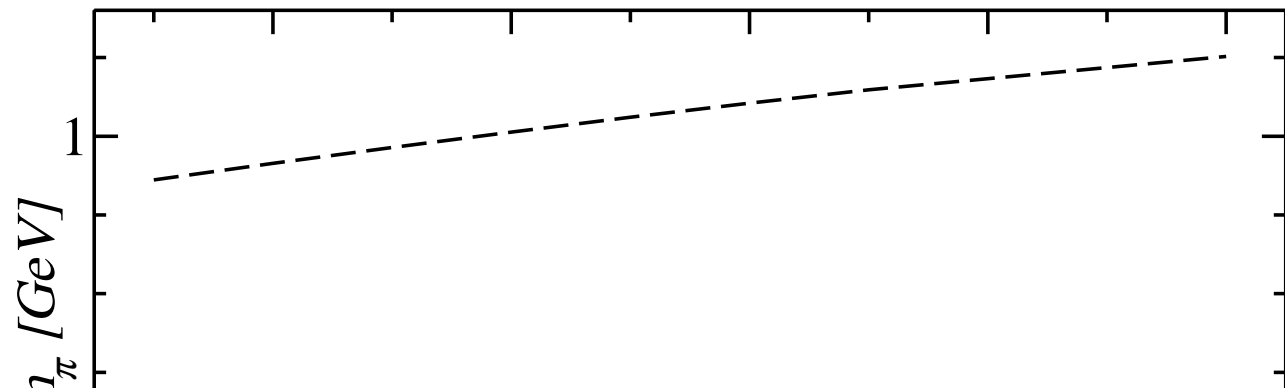
Model parameter dependence

- $m_{\pi_{gr}}$ and $m_{\pi_{exc1}}$ as functions of ω



Model parameter dependence

- $m_{\pi_{gr}}$ and $m_{\pi_{exc1}}$ as functions of ω

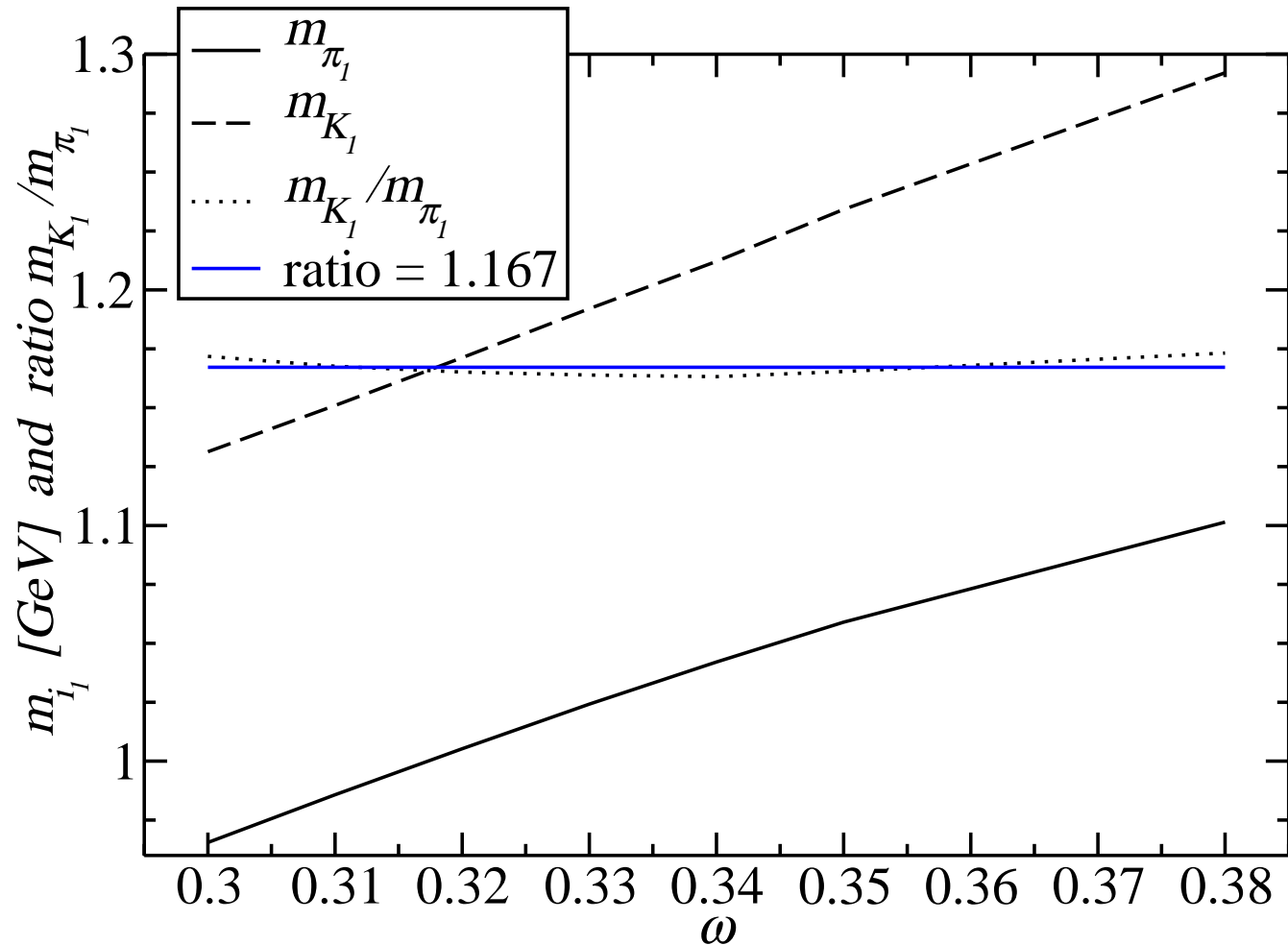


Radial excitations show ω dependence
→ probe long-range part of the
strong interaction

Ratios, e.g. Excited Kaon

- A. Höll *et al.*, Int. J. Mod. Phys. A **20**, 1778 (2005)

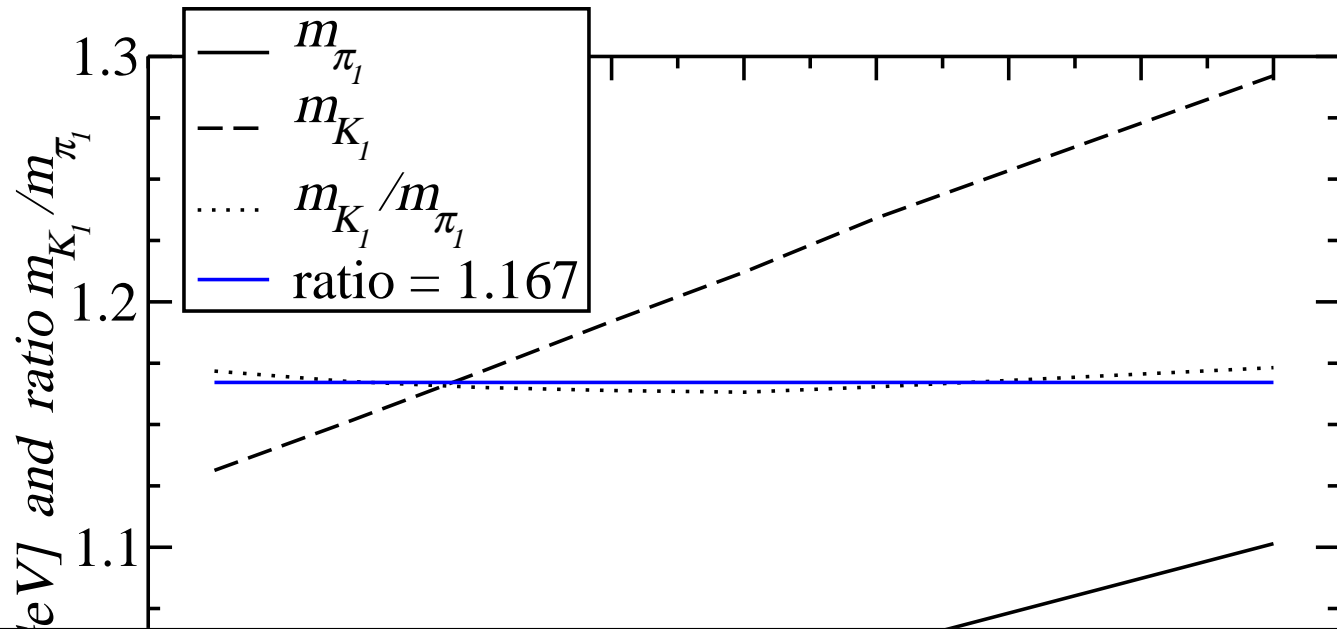
$m_{K_{exc1}}/m_{\pi_{exc1}}$ as a function of ω



Ratios, e.g. Excited Kaon

- A. Höll *et al.*, Int. J. Mod. Phys. A **20**, 1778 (2005)

$m_{K_{exc1}}/m_{\pi_{exc1}}$ as a function of ω



Consider ratio: $m_{K_{exc1}}/m_{\pi_{exc1}} = const \rightarrow$

estimate for $m_{K_{exc1}}$

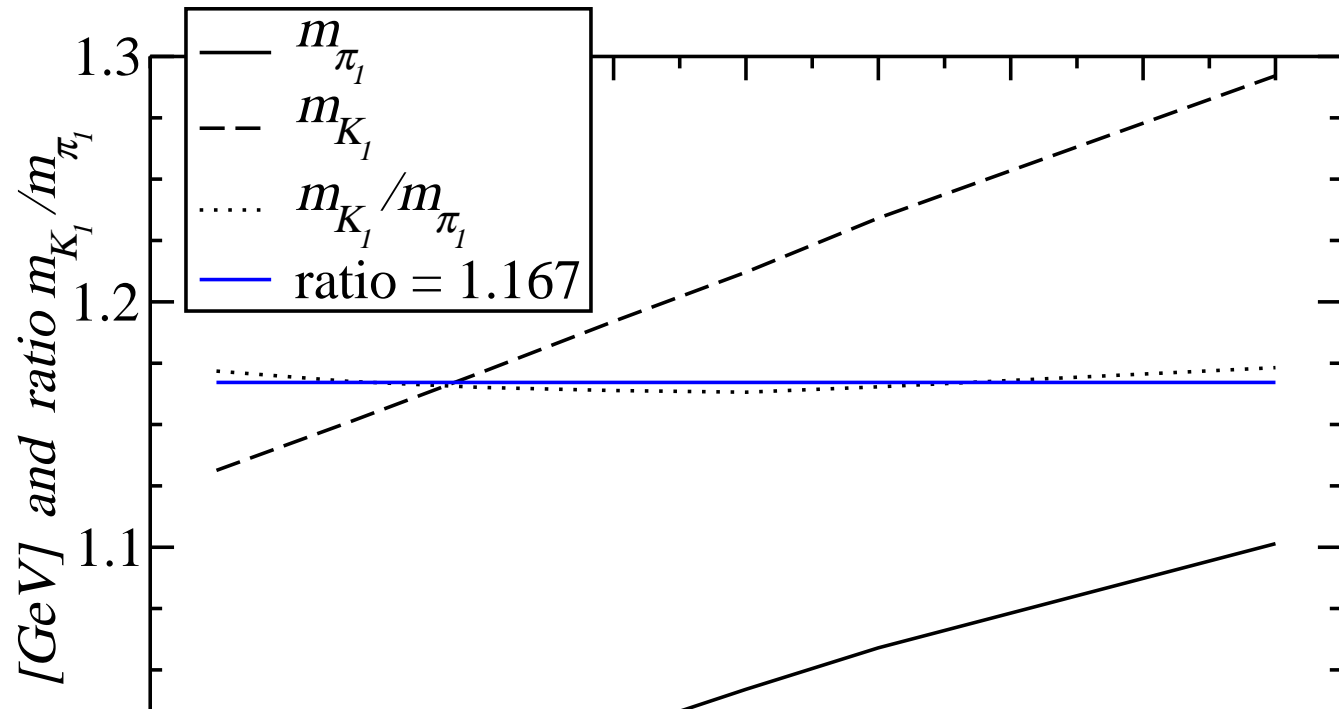
via experimental mass of excited pion:

$$m_{K_{exc1}} \approx 1520 \text{ MeV (Exp: K(1460))}$$

Ratios, e.g. Excited Kaon

- A. Höll *et al.*, Int. J. Mod. Phys. A **20**, 1778 (2005)

$m_{K_{exc1}}/m_{\pi_{exc1}}$ as a function of ω

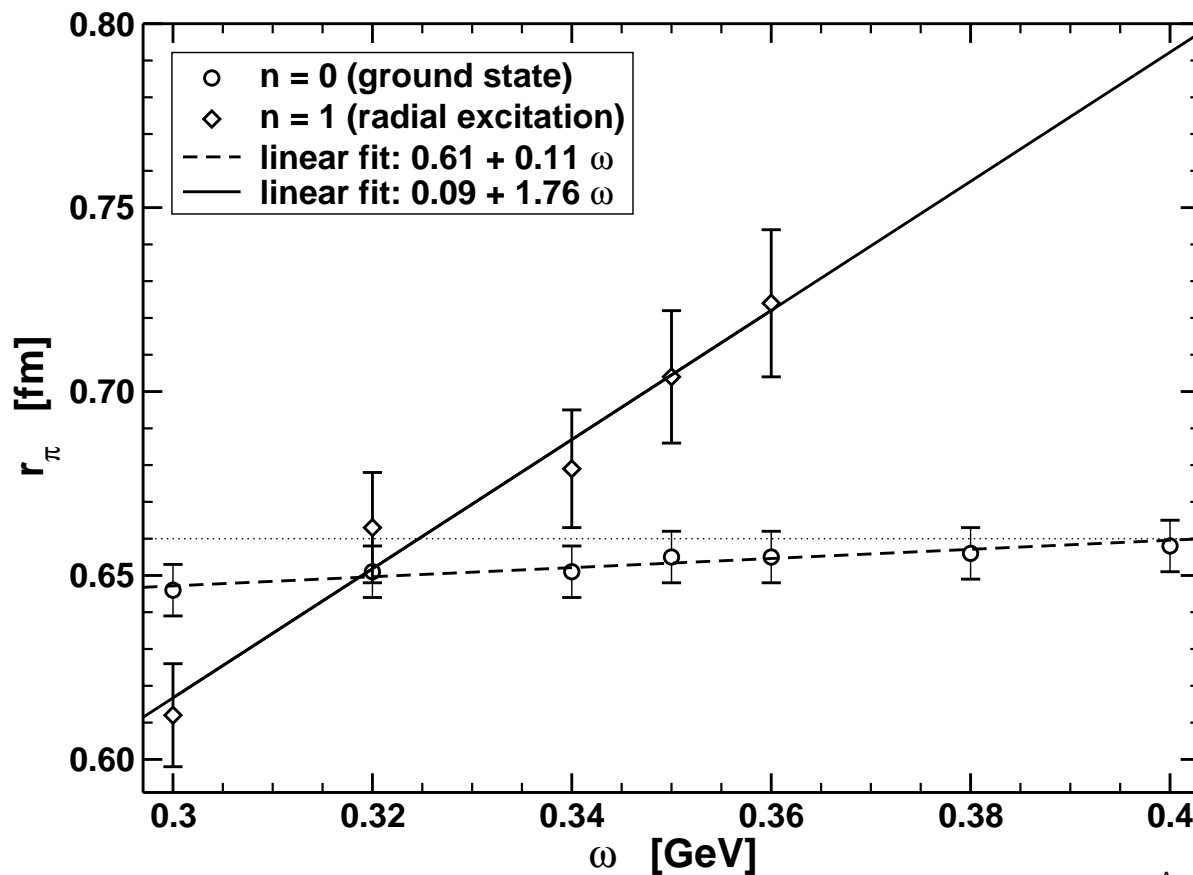


One more ratio: **leptonic decay constants:**

$$f_{K_{exc1}}/f_{\pi_{exc1}} \sim 10$$

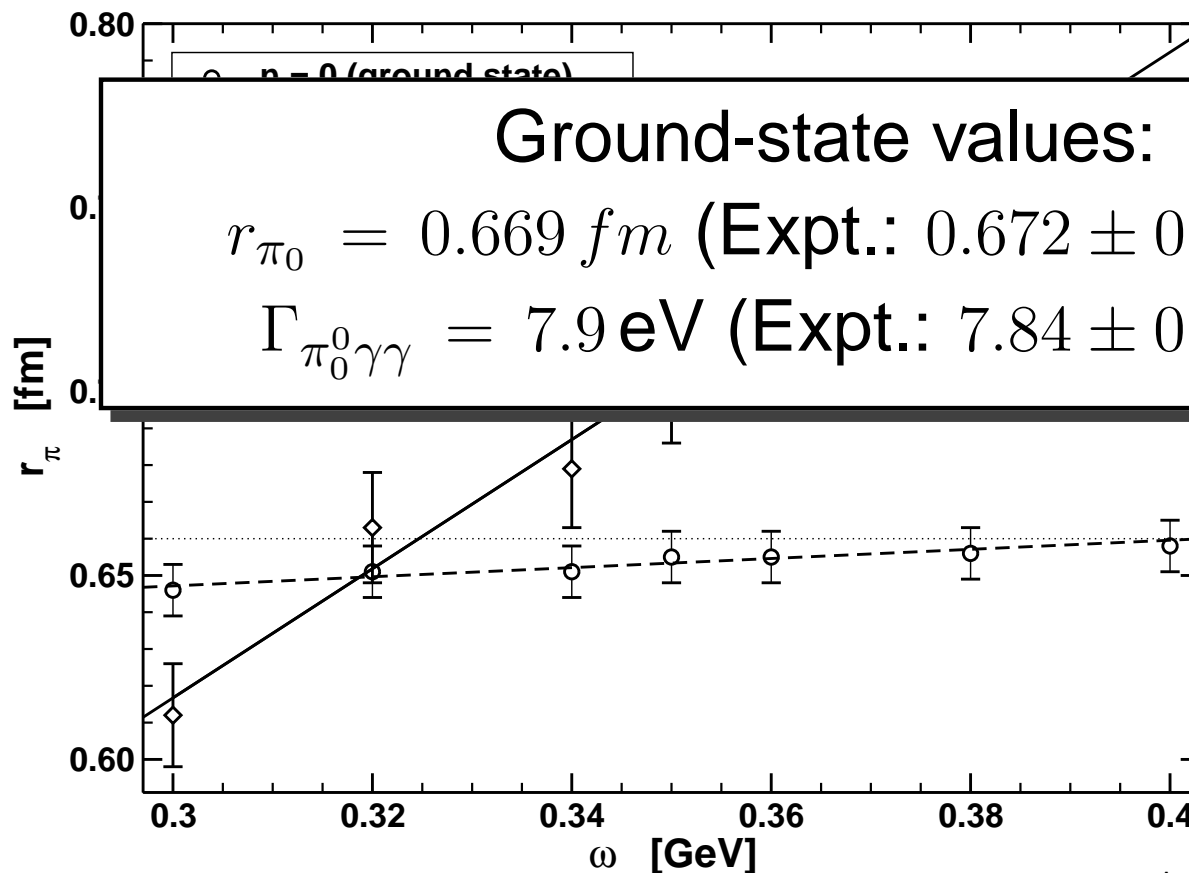
Estimates:

- π_1 charge radius: $r_{\pi_1} \approx 0.93 \text{ fm} \approx 1.4 r_{\pi_0}$
- radiative decay $\pi_1 \rightarrow \gamma\gamma$: $\Gamma_{\pi_1^0 \gamma\gamma} \approx 240 \text{ eV}$



Estimates:

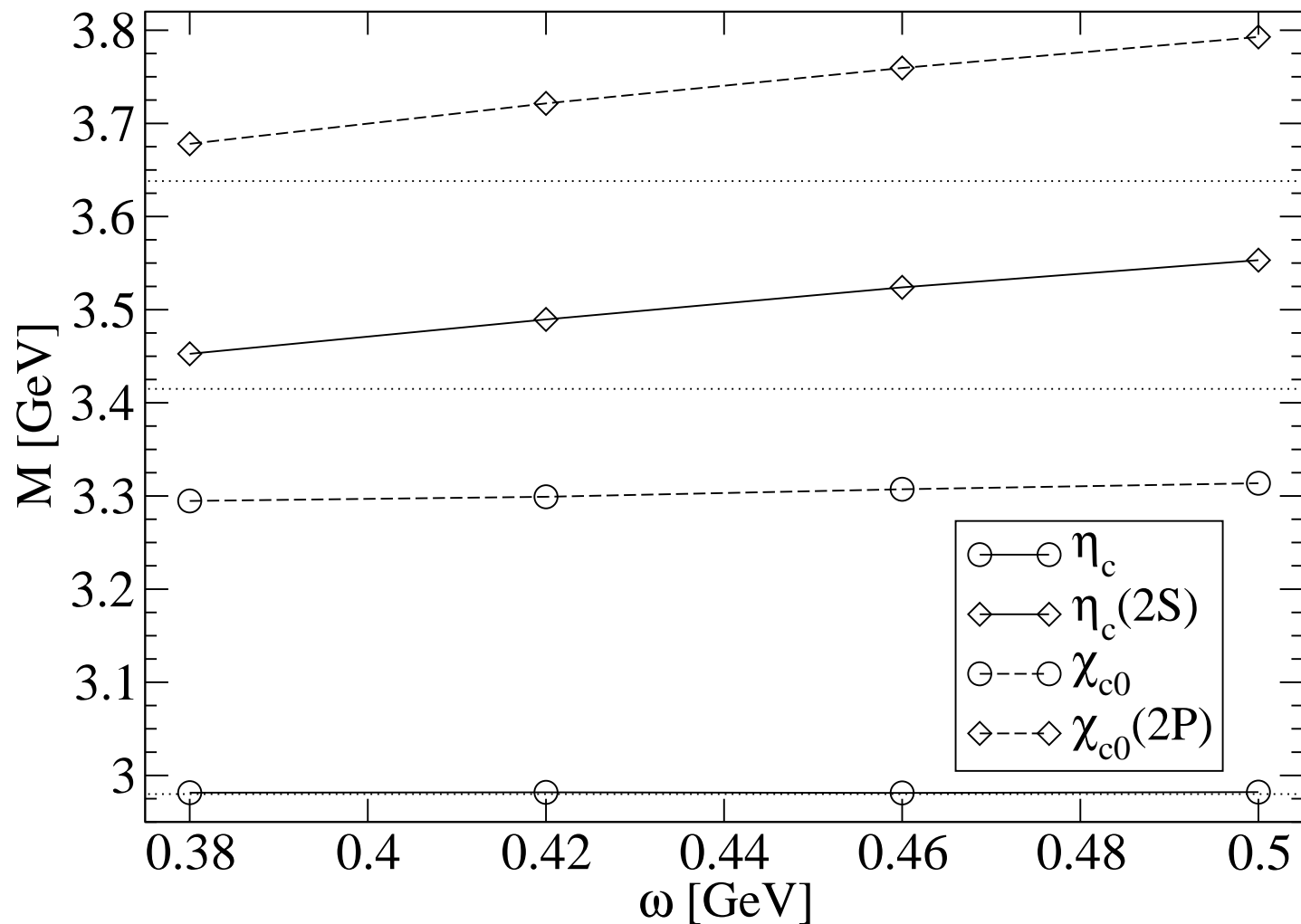
- π_1 charge radius: $r_{\pi_1} \approx 0.93 \text{ fm} \approx 1.4 r_{\pi_0}$
- radiative decay $\pi_1 \rightarrow \gamma\gamma$: $\Gamma_{\pi_1^0\gamma\gamma} \approx 240 \text{ eV}$



Ratios, e.g. Charmonium

- A. K., C. D. Roberts, and S. V. Wright, arXiv:nucl-th/0608039

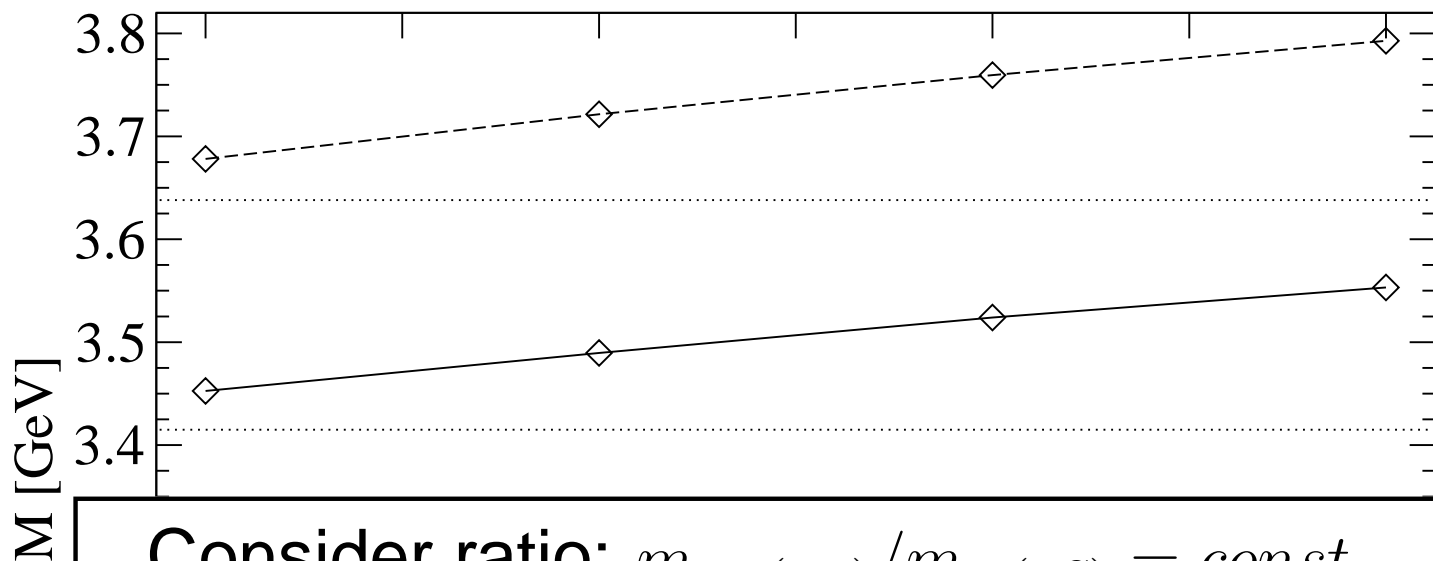
m_{0-+} and m_{0++} as functions of ω



Ratios, e.g. Charmonium

- A. K., C. D. Roberts, and S. V. Wright, arXiv:nucl-th/0608039

m_{0-+} and m_{0++} as functions of ω



Consider ratio: $m_{\chi_{c0}(2P)}/m_{\eta_c(2S)} = \text{const} \rightarrow$
estimate for $m_{\chi_{c0}(2P)}$
 via experimental mass of $\eta_c(2S) = 3.64 \text{ GeV}$:
 $m_{\chi_{c0}(2P)} = \mathbf{3.88 \text{ GeV}}$

0.38 0.4 0.42 0.44 0.46 0.48 0.5
 ω [GeV]

Other Things and Elsewhere

- Estimates for mesons with **exotic** QN, e. g. $\pi_1 (1^{-+})$
- **Heavy-meson** observables
- **Diquark confinement** (model-independent)
- **Gluon propagator** and **quark-gluon vertex**
- Comparison to **lattice** gauge QCD
- **Baryon** studies via quark-diquark Ansatz
- Exploratory steps **beyond RL** with sophisticated interaction

- Work in progress
 - Hadronic decays, e. g. $\pi_{exc1} \rightarrow Q \pi_{gr}$
 - Higher radial excitations
 - Heavy-light mesons and radial excitations
 - Nucleon properties (diquarks)

- Work in progress
 - **Hadronic decays**, e. g. $\pi_{exc1} \rightarrow Q \pi_{gr}$
 - **Higher** radial excitations
 - **Heavy-light** mesons and radial excitations
 - **Nucleon properties** (diquarks)
- Wish list
 - Sophisticated meson model **beyond RLT**
 - Good description of **axial-vector** mesons
 - Study states with **“exotic”** quantum numbers
 - Include **hadronic decay channels** in BSE kernel
 - ...

Summary and Conclusions

- Dyson-Schwinger equations provide a **nonperturbative continuum** approach to QCD
- Bethe-Salpeter equation used to describe bound states in a manifestly **covariant** way
- Symmetry-preserving truncation scheme enables proof of **exact results** and reliable studies of **hadron properties**

Summary and Conclusions

- Dyson-Schwinger equations provide a **nonperturbative continuum** approach to QCD
- Bethe-Salpeter equation used to describe bound states in a manifestly **covariant** way
- Symmetry-preserving truncation scheme enables proof of **exact results** and reliable studies of **hadron properties**
- Step **beyond** Rainbow-Ladder truncation needed to go for axial vectors, scalars, exotics, radially excited states
- These provide means to study the **long-range behavior** of the strong interaction

Thank you!