

Studying pion effects in the quark propagator

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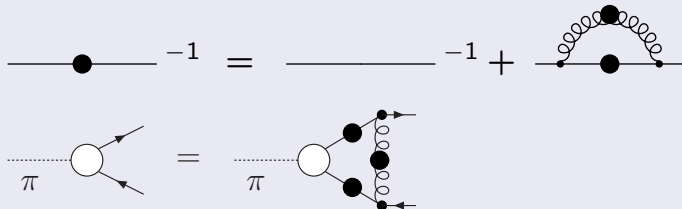
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Confinement: connecting the light- and heavy-quark domains
March 2007, Trento

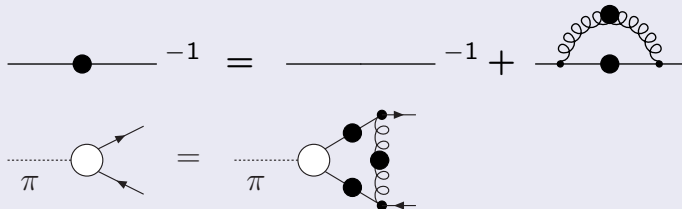


rainbow-ladder approximation



proper description of hadronic observables...

rainbow-ladder approximation

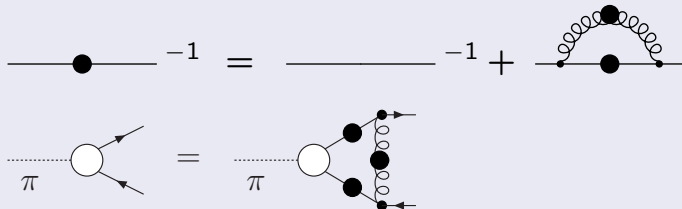


proper description of hadronic observables...

... no inclusion of Goldstone boson dynamics, *i.e.* 'quenched'

→ hadronic decays? width of mesons? parametric dependencies predicted by χ PT? unquenched lattice QCD results?

rainbow-ladder approximation



proper description of hadronic observables...

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→ hadronic decays? width of mesons? parametric dependencies predicted by χ PT? unquenched lattice QCD results?

goal/idea

find viable truncation to study Goldstone boson dynamics

- 1 Inclusion of pion dynamics
- 2 Determination of interaction from unquenched lattice QCD data
- 3 Results
- 4 Open questions and summary

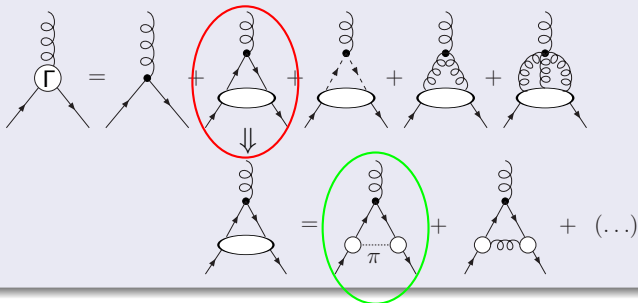
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relevant Dyson-Schwinger equations

DSE of the quark propagator

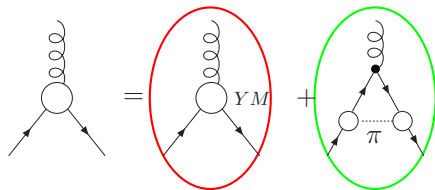
The diagram shows the Dyson-Schwinger equation for the quark propagator. On the left, a horizontal line with a shaded circle (representing a quark) is followed by a minus sign and a superscript -1. This is equal to a horizontal line with a minus sign and a superscript -1, plus a diagram of a quark line with a shaded circle and a vertex labeled Γ connected to a loop of gluons.

- gluon propagator known from DSE studies and lattice calculations
- $q\bar{q}g$ -vertex...



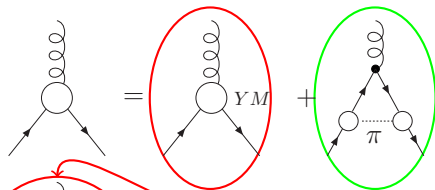
approximation for $q\bar{q}g$ -vertex

this motivates

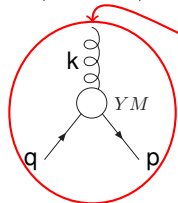


approximation for $q\bar{q}g$ -vertex

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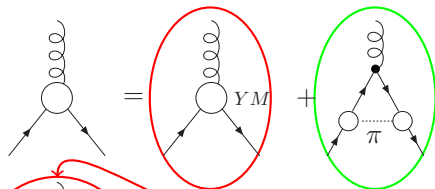
and we assume



$$\Gamma_{\mu}^{YM a}(p, q; k) = ig\Gamma^{YM}(k^2)\gamma_{\mu}\frac{\lambda^a}{2}$$

approximation for $q\bar{q}g$ -vertex

this motivates



and we assume

A diagrammatic equation for the YM vertex. It shows a circle labeled YM with a wavy line entering from the top and two straight lines exiting from the bottom, labeled q and p . The top wavy line is labeled k . This is equated to the expression $\Gamma_{\mu}^{YM a}(p, q; k) = ig\Gamma^{YM}(k^2)\gamma_{\mu}\frac{\lambda^a}{2}$. A red circle highlights the diagram, and a red arrow points from the YM term in the previous diagram to this one.

remark:

A diagrammatic equation for the π vertex. It shows a triangle labeled π with a wavy line entering from the top vertex and two straight lines exiting from the bottom vertices, labeled q and p . The top wavy line is labeled k . This is equated to the expression $\Gamma_{\mu}^{\pi a}(p, q; k) = ig\left(\sum_{i=1}^{12}\lambda_i(p, q; k)L_{\mu}^i\right)\frac{\lambda^a}{2}$. A green circle highlights the diagram, and a green arrow points from the π term in the previous diagram to this one.

truncated DSE of the quark propagator

the DSE of the quark propagator then takes the form

$$\text{quark line} \stackrel{-1}{=} \text{quark line} - \text{rainbow diagram} - \text{pion cloud diagram}$$

with the BSE in rainbow-ladder approximation

$$\text{pion vertex} = \text{pion vertex} + \text{rainbow-ladder diagram}$$

we approximate

$$\text{quark line} \stackrel{-1}{=} \text{quark line} - \text{rainbow diagram} - \text{pion cloud diagram}$$

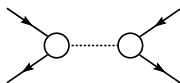
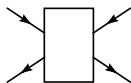
'rainbow'

'pion cloud'

treatment of the pion

consider resonant contribution in 4-point function

$$M_{tu}^{rs}(q, k; P) = \left(\bar{\Gamma}_{\pi}^a(q; -P) \right) \Big|_{rs} \frac{1}{P^2 + m_{\pi}^2} \left(\Gamma_{\pi}^a(q; P) \right) \Big|_{tu} + \dots$$



⇒ need Bethe-Salpeter amplitude (BSA) of pion

chiral limit ($m_q = 0$)

quark propagator

$$S^{-1}(p) = -i\not{p}A(p) + B(p)$$

axial Ward-Takahashi identity (AXWTI) for propagator

$$\begin{aligned} S^{-1}\left(k + \frac{P}{2}\right) i\gamma_5 \frac{\tau^a}{2} + i\gamma_5 \frac{\tau^a}{2} S^{-1}\left(k - \frac{P}{2}\right) &= P_\mu \Gamma_{5\mu}^a(k; P) \\ &= P_\mu \left(\frac{P_\mu f_\pi \Gamma_\pi^a(k; P)}{P^2} + \dots \right) \end{aligned}$$

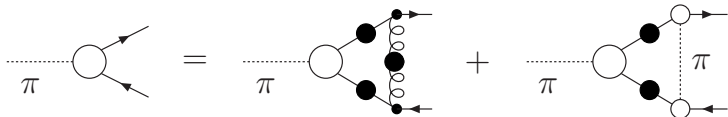
\Rightarrow generalized Goldberger-Treiman relation

$$\Gamma_\pi^a(k; P) = i\gamma_5 \tau^a \frac{B(k)}{f_\pi} + O(P_\mu)$$

\Rightarrow self-consistent scheme on level of quark propagator,
but we want to improve on that...

explicit chiral symmetry breaking ($m_q \neq 0$)

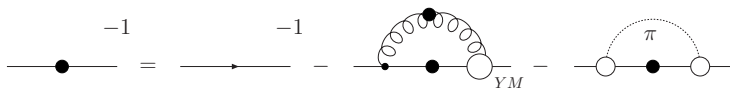
Bethe-Salpeter kernel constrained by AXWTI



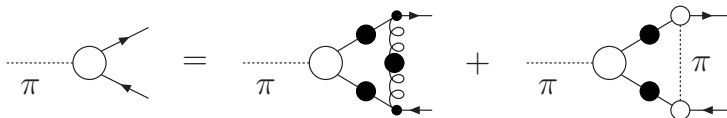
AXWTI fulfilled for $P_\mu \rightarrow 0$

\Rightarrow pion is a Goldstone boson (\rightarrow Gell-Mann–Oakes–Renner relation)

closed truncation scheme



$$S^{-1}(p) = -i\not{p}A(p) + B(p)$$



$$\Gamma_{\pi}^a(p; P) = \gamma_5 \tau^a (iE_{\pi}(p; P) + \not{P} F_{\pi}(p; P) + \not{P} P \cdot p G_{\pi}(p; P) + i\sigma_{\mu\nu} p_{\mu} P_{\nu} H_{\pi}(p; P))$$

only input is $\Gamma_{\mu}^{YM a}(p, q; k) = ig\Gamma^{YM}(k^2)\gamma_{\mu}\frac{\lambda^a}{2}$

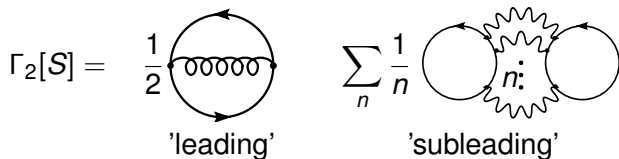
at this stage: further numerical simplifications employed

$1/N_c$ -expansion of 2PI action

qDSE from effective action

$$\Gamma[S] = -\text{Tr} \ln S^{-1} + \text{Tr} \left(1 - Z_2 S_0^{-1} S \right) + \Gamma_2[S]$$

GCM / effective-gluon exchange in $\frac{1}{N_c}$ -expansion

$$\Gamma_2[S] = \frac{1}{2} \text{ (leading) } + \sum_n \frac{1}{n} \text{ (subleading) }$$


resulting in self-energies

$$\Sigma(p) = \text{ (gluon self-energy) } + \text{ (ghost self-energy) }$$


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parametrization of the interaction

factorize vertex ansatz

$$\Gamma^{YM}(k^2) = \underbrace{\Gamma_1(k^2)}_{\text{UV running}} \underbrace{\Gamma_2(k^2)}_{\text{low momentum behavior}} \underbrace{\Gamma_3(k^2)}_{\text{mass dependence}}$$

Bhagwat *et al.* (2003), Fischer *et al.* (2005)

5 parameters in

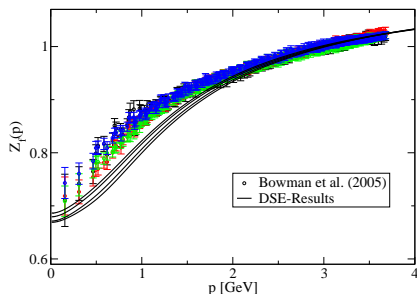
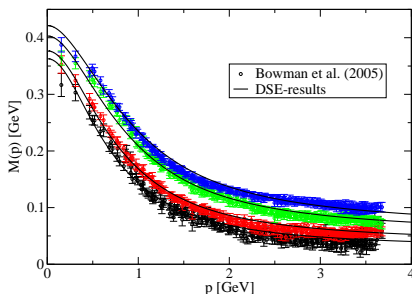
$$\Gamma_1(k^2) = \frac{\pi \gamma m}{\ln(k^2/\Lambda_{QCD}^2 + e - 1)}$$

$$\Gamma_2(k^2) = \tilde{Z}_3 \sqrt{\frac{k^2}{k^2 + \Lambda_{YM}^2}} G(k^2) G(\zeta^2) h [\ln(k^2/\Lambda_{YM}^2 + e - 1)]^{1+\delta}$$

$$\Gamma_3(k^2) = Z_2 \frac{a(M) + k^2/\Lambda_{QCD}^2}{1 + k^2/\Lambda_{QCD}^2}$$

unquenched lattice QCD results for quark propagator

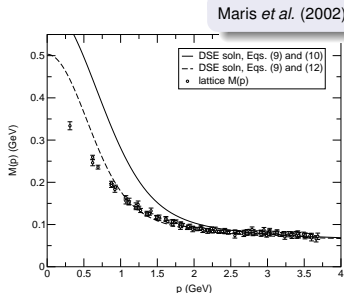
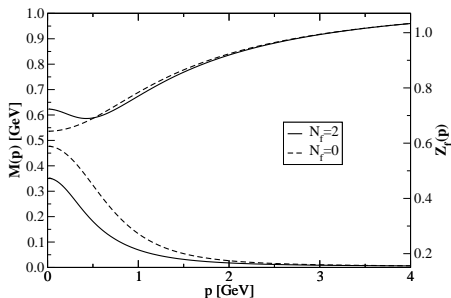
$$S(p) = \frac{1}{-i\not{p}A(p) + B(p)} = \frac{Z_f(p)}{-i\not{p} + M(p)}$$



- can achieve nice agreement with lattice QCD data
- small mass dependence in $Z_f(p)$

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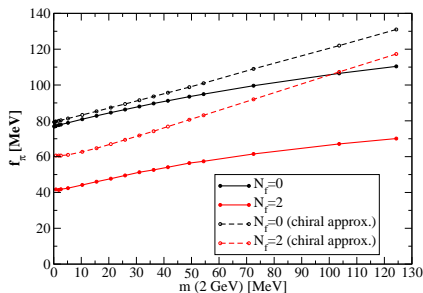
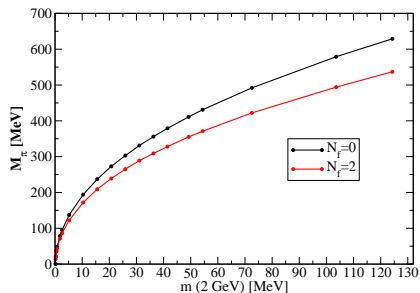
quark propagator at physical point



- unquenching effects sizeable (over-estimated?)
- mass function significantly smaller than in phenomenological models

- $\langle \bar{\psi}\psi \rangle \Big|_{\mu=2\text{GeV}}^{\text{MOM}} = - (191\text{MeV})^3, \quad m^{\text{MOM}}(2\text{GeV}) = 5.2\text{MeV}$

pion mass and decay constant

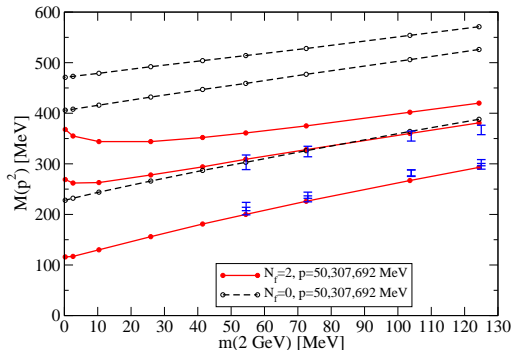


chiral approximation for f_π

$$f_\pi^2 = \frac{3}{4\pi} \int_0^\infty dp^2 \frac{p^2 Z_f(p) M(p)}{(p^2 + M^2)^2} \left(M(p) - \frac{p^2}{2} \frac{dM}{dp^2} \right)$$

- $f_\pi \sim 30\%$ too small
- unquenching effects slightly over-estimated?
- curvature in the m -dependence?

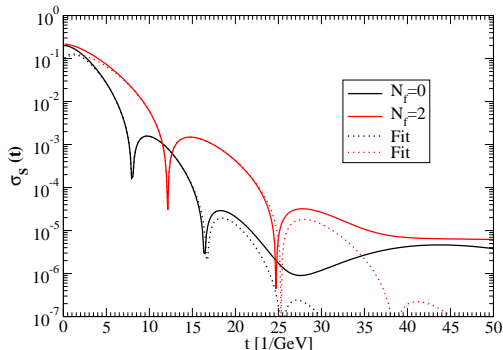
chiral extrapolation of mass function



- curvature in the m -dependence
- linear extrapolation leads to smaller values in chiral limit

analytical properties of the quark propagator

$$\sigma_S(t) = \int d^3x \int \frac{d^4p}{(2\pi)^2} \exp(ip \cdot x) \frac{Z_f(p)M(p)}{p^2 + M^2}$$



- no qualitative change in pole structure

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open questions

- overestimate pion effects due to numerical approximation?
- $q\bar{q}g$ -vertex ansatz for $\Gamma_{\mu}^{YM a}$ suitable?
- conversion of lattice units?

Bowman *et al.* (2005): "the definition of the lattice spacing in quenched calculations is somewhat arbitrary. . . not consistent with that published for the MILC configurations."

summary

- proper description of lattice QCD data
- simple implementation of pion effects!

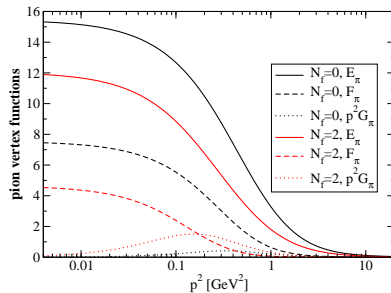
Bethe-Salpeter kernel

$$\begin{aligned} K_{tu}^{rs}(q, p; P) = & \frac{1}{4} \Gamma_{\pi}^a \left(\frac{p+q}{2}; p-q \right) \Big|_{ts} \Gamma_{\pi}^a \left(-\frac{p+q}{2}; p-q \right) \Big|_{ru} D_{\pi}(k) \\ & + \frac{1}{4} \Gamma_{\pi}^a \left(\frac{p+q}{2}; q-p \right) \Big|_{ts} \Gamma_{\pi}^a \left(-\frac{p+q}{2}; q-p \right) \Big|_{ru} D_{\pi}(k) \\ & + \frac{1}{4} \Gamma_{\pi}^a \left(\frac{p+q}{2}; q-p \right) \Big|_{ru} \Gamma_{\pi}^a \left(-\frac{p+q}{2}; q-p \right) \Big|_{ts} D_{\pi}(k) \\ & + \frac{1}{4} \Gamma_{\pi}^a \left(\frac{p+q}{2}; q-p \right) \Big|_{ru} \Gamma_{\pi}^a \left(-\frac{p+q}{2}; q-p \right) \Big|_{ts} D_{\pi}(k) \end{aligned}$$

and

$$D_{\pi}(k) = \frac{1}{(p-q)^2 + m_{\pi}^2}$$

pion vertex functions



- G_π large