Studying pion effects in the quark propagator

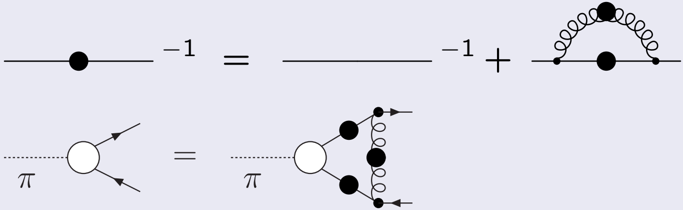
Dominik Nickel[1]

Christian Fischer[1] Jochen Wambach[1,2]

[1]TU Darmstadt [2]GSI Darmstadt

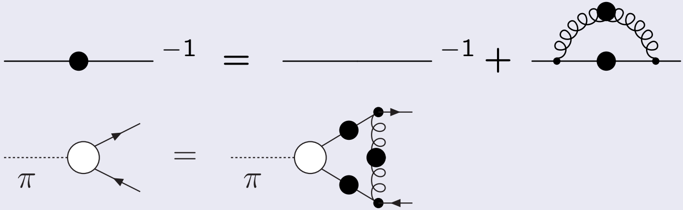
Confinement: connecting the light- and heavy-quark domains
March 2007, Trento

rainbow-ladder approximation



proper description of hadronic observables...

rainbow-ladder approximation



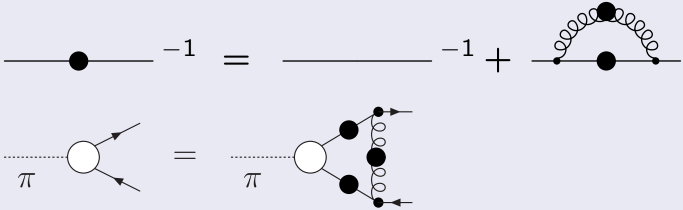
proper description of hadronic observables...

... no inclusion of Goldstone boson dynamics, *i.e.* 'quenched'

$\longrightarrow$ hadronic decays? width of mesons? parametric dependencies predicted by χPT? unquenched lattice QCD results?

introduction

rainbow-ladder approximation



proper description of hadronic observables...

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$\longrightarrow$ hadronic decays? width of mesons? parametric dependencies predicted by χPT? unquenched lattice QCD results?

goal/idea

find viable truncation to study Goldstone boson dynamics

outline

1. Inclusion of pion dynamics
2. Determination of interaction from unquenched lattice QCD data
3. Results
4. Open questions and summary

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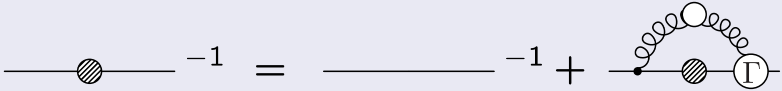
relevant Dyson-Schwinger equations

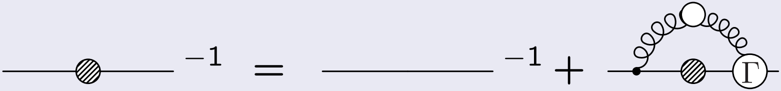
DSE of the quark propagator



- gluon propagator known from DSE studies and lattice calculations
- $q\bar{q}g$-vertex...

relevant Dyson-Schwinger equations

DSE of the quark propagator



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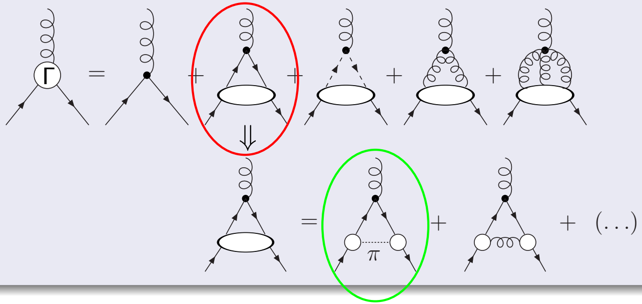
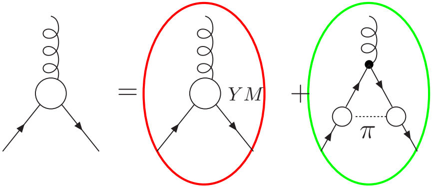


approximation for $q\bar{q}g$-vertex

this motivates



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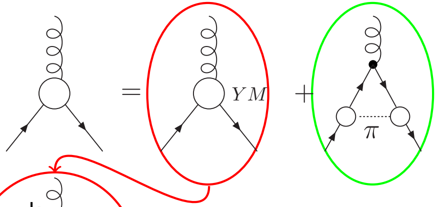
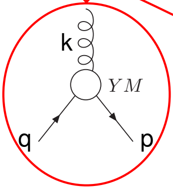
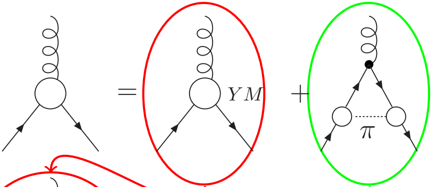


and we assume



$$\Gamma_\mu^{YM\,a}(p,q;k) = ig\Gamma^{YM}(k^2)\gamma_\mu \frac{\lambda^a}{2}$$

this motivates



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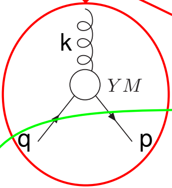
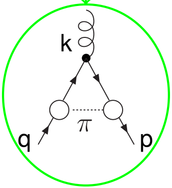


$$\Gamma_\mu^{YM\,a}(p,q;k) = ig\Gamma^{YM}(k^2)\gamma_\mu\frac{\lambda^a}{2}$$

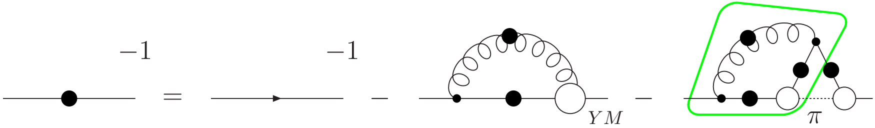
remark:



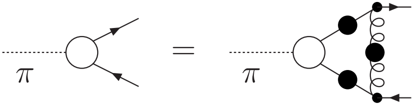
$$\Gamma_\mu^{\pi\,a}(p,q;k) = ig\left(\sum_{i=1}^{12}\lambda_i(p,q;k)L_\mu^i\right)\frac{\lambda^a}{2}$$

truncated DSE of the quark propagator

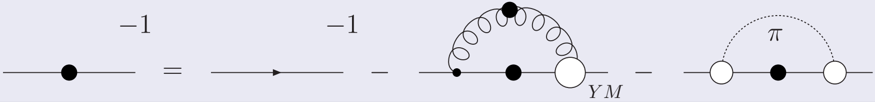
the DSE of the quark propagator then takes the form



with the BSE in rainbow-ladder approximation



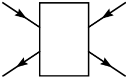
we approximate

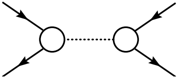


'rainbow' 'pion cloud'

consider resonant contribution in 4-point function

$$M^{rs}_{tu}(q,k;P) \quad = \quad \left(\bar{\Gamma}^a_\pi(q;-P)\right)\Big|_{rs} \frac{1}{P^2+m_\pi^2} \left(\Gamma^a_\pi(q;P)\right)\Big|_{tu} + \dots$$





$\Rightarrow$ need Bethe-Salpeter amplitude (BSA) of pion

chiral limit ($m_q = 0$)

quark propagator

$$S^{-1}(p) = -i\not{p}A(p) + B(p)$$

axial Ward-Takahashi indentity (AXWTI) for propagator

$$\begin{aligned} S^{-1}\left(k+\frac{P}{2}\right) i\gamma_5\frac{\tau^a}{2} + i\gamma_5\frac{\tau^a}{2}S^{-1}\left(k-\frac{P}{2}\right) &= P_\mu\Gamma^a_{5\mu}(k;P) \\ &= P_\mu\left(\frac{P_\mu f_\pi \Gamma^a_\pi(k;P)}{P^2} + \dots\right) \end{aligned}$$

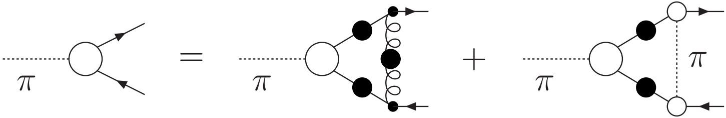
⇒ generalized Goldberger-Treiman relation

$$\Gamma^a_\pi(k;P) = i\gamma_5\tau^a\frac{B(k)}{f_\pi} + O(P_\mu)$$

⇒ self-consistent scheme on level of quark propagator, but we want to improve on that...

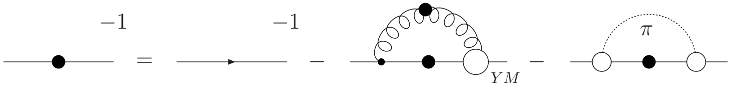
explicite chiral symmetry breaking ($m_q \neq 0$)

Bethe-Salpeter kernel constrained by AXWTI

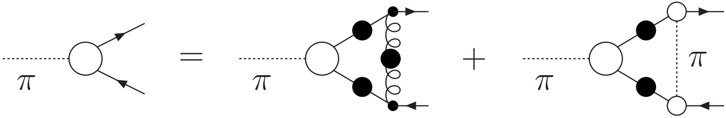


AXWTI fulfilled for $P_\mu \to 0$
$\Rightarrow$ pion is a Goldstone boson ($\to$ Gell-Mann–Oakes–Renner relation)

closed truncation scheme



$S^{-1}(p) = -i\not{p}A(p) + B(p)$



$$\Gamma_\pi^a(p;P) = \gamma_5 \tau^a (iE_\pi(p;P) + \not{P}\, F_\pi(p;P) + \not{p} P\cdot p\, G_\pi(p;P) + i\sigma_{\mu\nu} p_\mu P_\mu\, H_\pi(p;P))$$

only input is $\Gamma_\mu^{YM\,a}(p,q;k) = ig\Gamma^{YM}(k^2)\gamma_\mu \frac{\lambda^a}{2}$

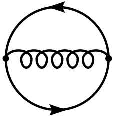
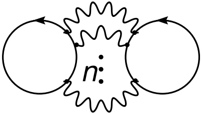
at this stage: further numerical simplifications employed

$1/N_c$-expansion of 2PI action

qDSE from effective action

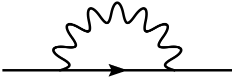
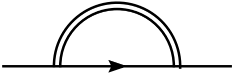
$$\Gamma[\mathcal{S}] \;=\; -\mathrm{TrLn}S^{-1} + \mathrm{Tr}\left(1 - Z_2 S_0^{-1} S\right) + \Gamma_2[S]$$

GCM / effective-gluon exchange in $\dfrac{1}{N_c}$-expansion

$\Gamma_2[S] = \quad \dfrac{1}{2}$  $\quad \sum_n \dfrac{1}{n}$ 

'leading' 'subleading'

resulting in self-energies

$\Sigma(p) \quad =$  $+$ 

outline

parametrization of the interaction

factorize vertex ansatz

$$\Gamma^{YM}(k^2) = \underbrace{\Gamma_1(k^2)}_{\text{UV running}} \quad \underbrace{\Gamma_2(k^2)}_{\text{low momentum behavior}} \quad \underbrace{\Gamma_3(k^2)}_{\text{mass dependence}}$$

Bhagwat *et al.* (2003), Fischer *et al.* (2005)

5 parameters in

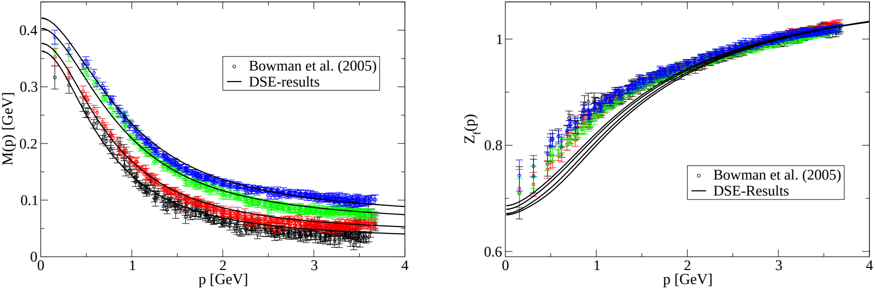
$$\Gamma_1(k^2) = \frac{\pi\gamma_m}{\ln(k^2/\Lambda^2_{QCD} + e - 1)}$$

$$\Gamma_2(k^2) = \widetilde{Z}_3 \sqrt{\frac{k^2}{k^2 + \Lambda^2_{YM}}} G(k^2)\ G(\zeta^2)\ \ h\,[\ln(k^2/\Lambda^2_{YM} + e - 1)]^{1+\delta}$$

$$\Gamma_3(k^2) = Z_2 \frac{a(M) + k^2/\Lambda^2_{QCD}}{1 + k^2/\Lambda^2_{QCD}}$$

unquenched lattice QCD results for quark propagator

$$S(p) = \frac{1}{-i\not{p}A(p) + B(p)} = \frac{Z_f(p)}{-i\not{p} + M(p)}$$



- can achieve nice agreement with lattice QCD data
- small mass dependence in $Z_f(p)$

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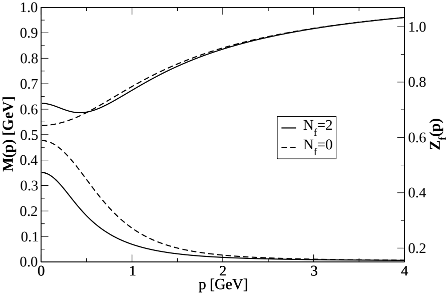
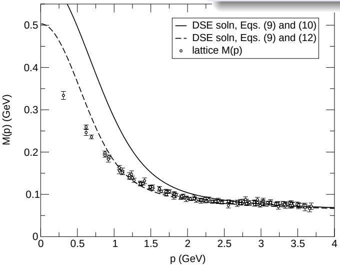
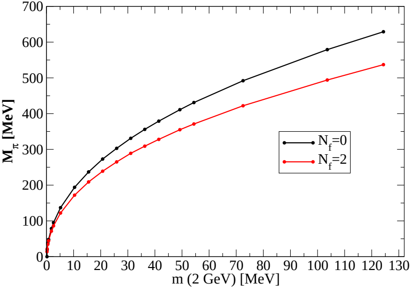
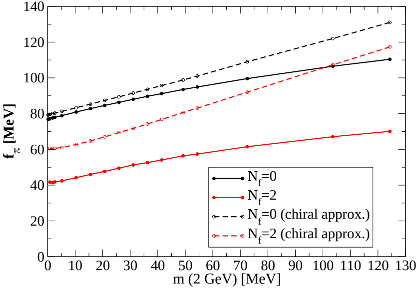
quark propagator at physical point





- unquenching effects sizeable (over-estimated?)
- mass function significantly smaller than in phenomenological models
- $\langle\bar{\psi}\psi\rangle\Big|^{MOM}_{\mu=2\text{GeV}} = -(191\text{MeV})^3, \quad m^{MOM}(2\text{GeV}) = 5.2\text{MeV}$



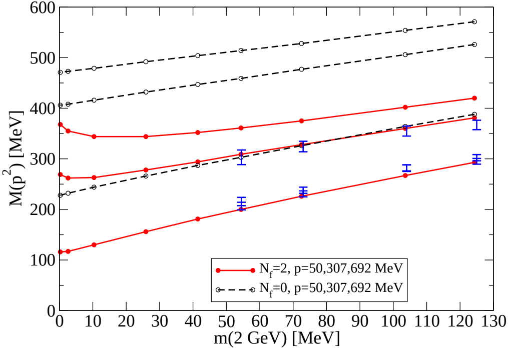


chiral approximation for f_π

$$f_\pi^2 = \frac{3}{4\pi}\int_0^\infty dp^2 \frac{p^2 Z_f(p) M(p)}{\left(p^2 + M^2\right)^2}\left(M(p) - \frac{p^2}{2}\frac{dM}{dp^2}\right)$$

- $f_\pi \sim 30\%$ too small
- unquenching effects slightly over-estimated?
- curvature in the *m*-dependence?

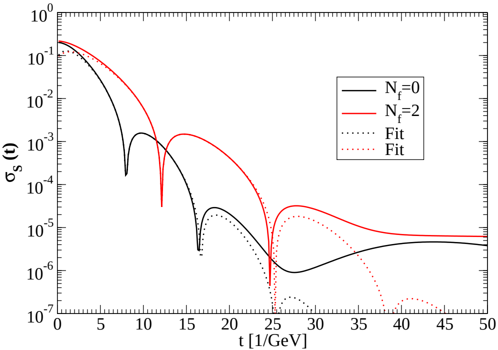
chiral extrapolation of mass function



- curvature in the *m*-dependence
- linear extrapolation leads to smaller values in chiral limit

analytical properties of the quark propagator

$$\sigma_s(t) = \int d^3x \int \frac{d^4p}{(2\pi)^2} \exp(ip \cdot x) \frac{Z_f(p)M(p)}{p^2 + M^2}$$



- no qualitative change in pole structure

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open questions and summary

open questions

- overestimate pion effects due to numerical approximation?
- $q\bar{q}g$-vertex ansatz for $\Gamma_\mu^{YM\,a}$ suitable?
- conversion of lattice units?

> Bowman *et al.* (2005): "the definition of the lattice spacing in quenched calculations is somewhat arbitrary. . . not constistent with that published for the MILC configurations."

summary

- proper description of lattice QCD data
- simple implementation of pion effects!

Bethe-Salpeter kernel

$$
\begin{aligned}
K^{rs}_{tu}(q,p;P) = \; & \frac{1}{4}\,\Gamma^a_\pi\left(\frac{p+q}{2};p-q\right)\Big|_{ts}\,\Gamma^a_\pi\left(-\frac{p+q}{2};p-q\right)\Big|_{ru}D_\pi(k) \\
& +\frac{1}{4}\,\Gamma^a_\pi\left(\frac{p+q}{2};q-p\right)\Big|_{ts}\,\Gamma^a_\pi\left(-\frac{p+q}{2};q-p\right)\Big|_{ru}D_\pi(k) \\
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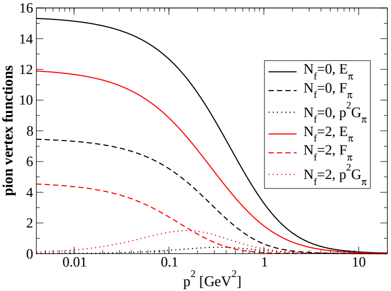
\end{aligned}
$$

and

$$
D_\pi(k) = \frac{1}{(p-q)^2+m_\pi^2}
$$

pion vertex functions



- G_π large