Light baryon magnetic moments and $N \to \Delta\gamma$ transition in a relativistic quark model

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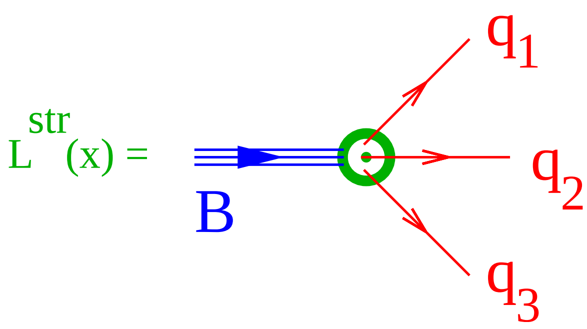
Outline

- ⋆ Main ingredients of the Relativistic Three-Quark Model
- ⋆ Meson cloud corrections
- ⋆ Magnetic moments of light baryons
- ⋆ N-$\Delta\gamma$ transition properties
- ⋆ Summary

The Model

⋆ Covariant description of bound states

⋆ Approach: interaction Lagrangian coupling baryons with constituent quarks



⋆ Meson cloud corrections

- Dressing quarks by pseudoscalar meson clouds: baryon ChPT methods
- Constituent quarks and chiral fields as effective degrees of freedom

⋆ RTQM has been used to compute exclusive semileptonic, nonleptonic, strong and electromagnetic decays of single heavy baryons

⋆ RTQM has been generalized to the study of double and triple heavy baryons

The RTQ Lagrangian

The general Lagrangian $$L^{int} = L^{str} + L^{em} + L^{weak}$$

Non-local strong interaction

$$L^{str}(x) = g_B \bar{B}(x) \int dy_1 \int dy_2 \int dy_3 F(x; y_1, y_2, y_3) J_B(y_1, y_2, y_3) + h.c.$$

The distribution of constituent quarks inside the baryon is given by a phenomenological relativistic vertex function

$$F_B(x; y_1, y_2, y_3) = \delta^{(4)}(x - \sum_{i=1}^{3} \omega_i y_i) \times \Phi_B[\sum_{i<j} (y_i - y_j)^2] \ ,$$

where $\omega_i = \frac{m_i}{\sum_{i=1}^{3} m_i}$ and Φ_B is the correlation function of three constituent quarks.

Low-energy observables are insensitive to the choice of the functional form of the hadron-quark vertex function.

Dubna group, Z.Phys. C 65, 681 (1995)

Choose a universal Gaussian shape:

$$\tilde{\Phi}_B(k_{1E}^2 + k_{2E}^2) = \exp(-\frac{k_{1E}^2 + k_{2E}^2}{\Lambda_B^2})$$

Construct the three-quark baryon currents as products of three-quark fields

$$J_B(y_1, y_2, y_3) = \Gamma_1 q^a(y_1) q^b(y_2) C\Gamma_2 q^c(y_3) \epsilon^{abc}$$

The Γ matrices provide the right spin quantum numbers of a given baryon

$$\Gamma_{1,2} = 1,\ \gamma^5,\ \gamma^\mu,\ \sigma^{\mu\nu},\ \gamma^\mu\gamma^5$$

⋆ Light baryon currents

Nucleon V and T currents: degenerated in the non-relativistic limit

$$J_p^V = \epsilon^{abc}\gamma^\mu\gamma^5 d^a u^b C\gamma_\mu u^c$$
$$J_p^T = \epsilon^{abc}\sigma^{\mu\nu}\gamma^5 d^a u^b C\sigma_{\mu\nu} u^c$$

$$J_p = (1-\beta)J_p^V + \beta J_p^T$$

For the $\Delta^+(1232)$ there exists only a V current

$$J_{\Delta^+}^{V,\mu} = \frac{\epsilon^{abc}}{\sqrt{3}}(d^a u^b C\gamma^\mu u^c + 2u^{Ta} u^b C\gamma^\mu d^c)$$

Very important ingredient: Compositeness Condition

The concept: represent real composite particles by fictitious elementary particles:

$$Z_B = 1 - g_B^2 \Sigma'_B(\not{p})|_{\not{p}=M_B} = 0$$

where $Z_B^{1/2} =< B_0|B >$.

Consequence: the physical baryon is dressed by the interaction with its own constituents, thus a bound state; constituent quarks can exist only as virtual states.

Tool in calculating the coupling constant g_B.
In the calculation of the mass operator we use free quark propagators $S_q(k) = \frac{1}{m_q - \not{k}}$; where m_q are the effective quark masses; $q = u, d, s$

Magnetic moments of light baryons and properties of $N \to \Delta\gamma$ transition

$\star$ The RTQM is used to perform the calculation of the bare matrix elements (valence quark contributions).

$\star$ Additional chiral corrections: nonlinear chiral quark Lagrangian.

$\star$ Magnetic moments: standard exercise
(to fix Λ_B and chiral corrections).

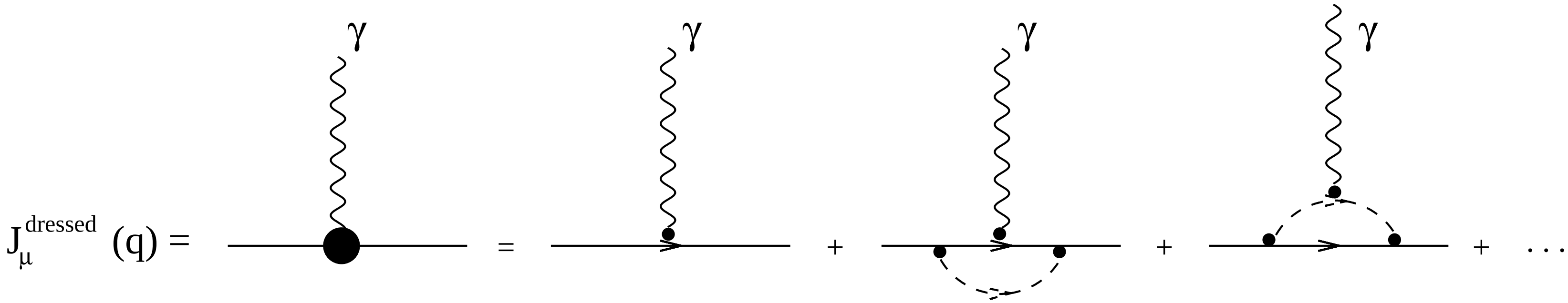
$\star$ $N - \Delta\gamma$ transition: highly nontrivial
(parameter free, prediction; T-V current structure ?)

Dressing quarks by mesonic clouds

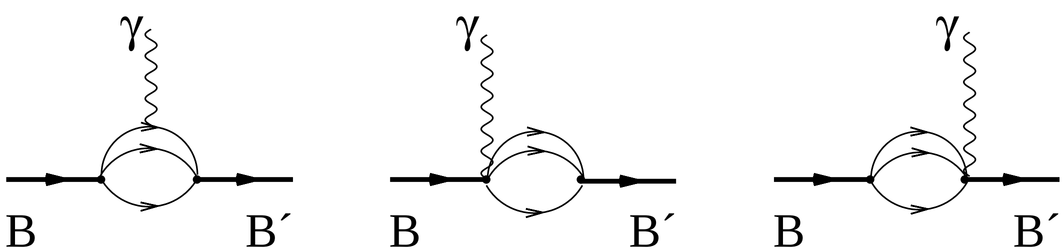
Original concept: Manohar and Georgi NPB 234 (1984) 189

2 main steps :

⋆ Calculate dressed transition quark operators relevant for the interaction of quarks with external fields in the presence of a virtual meson cloud



⋆ Projection on the baryon: calculate baryon matrix elements

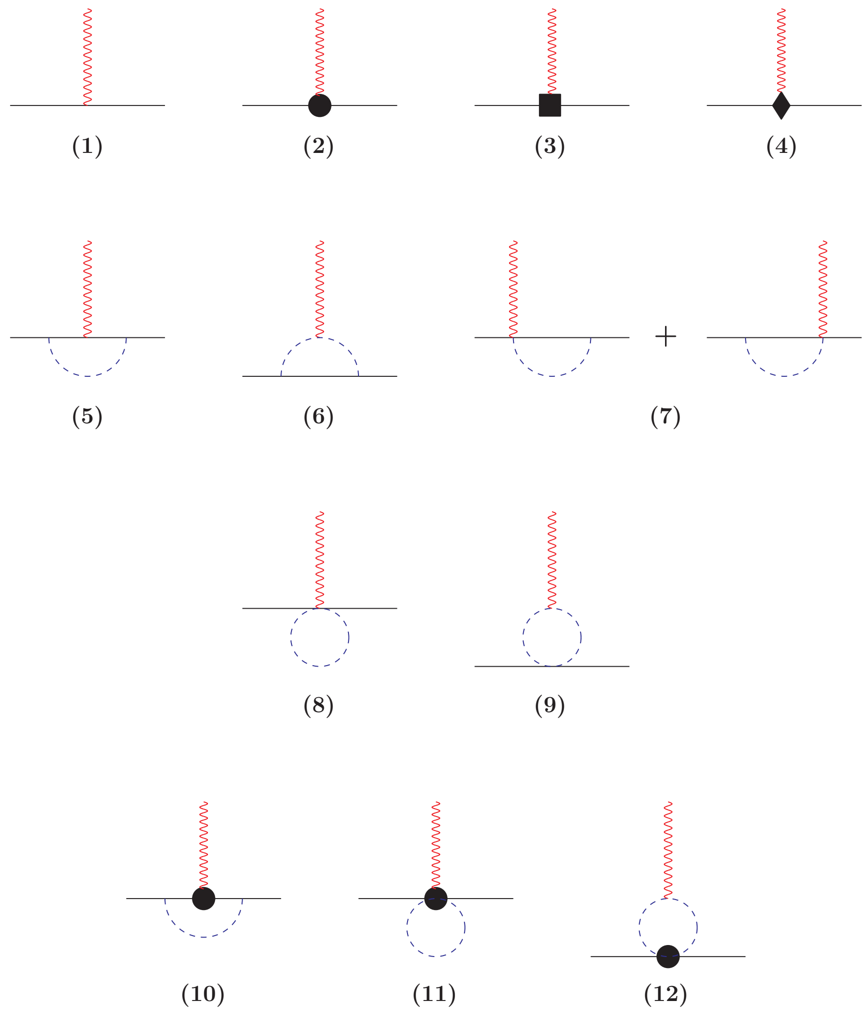


Dressing quarks by mesonic cloud

Becher/Leutwyler, Eur. Phys. J. C9 (1999)
Kubis/Meissner, Eur. Phys. J. C18 (2001)

Dressed quark transition operators: one-body up to 1-loop and to $O(p^4)$

$$\mathcal{L}_{qU} = \mathcal{L}_q + \mathcal{L}_U$$
$$\mathcal{L}_q = \mathcal{L}_q^{(1)} + \mathcal{L}_q^{(2)} + \mathcal{L}_q^{(3)} + \mathcal{L}_q^{(4)} + \ldots$$
$$\mathcal{L}_U = \mathcal{L}_U^{(2)}$$
$$\mathcal{L}_U^{(2)} = < u_\mu u^\mu + \chi_+ >$$
$$\mathcal{L}_q^{(1)} = \bar{q}[i \not{D} - m + \frac{1}{2} g \not{u} \gamma^5] q$$



Projection on baryon level

⋆ Main result : factorization of chiral corrections

$$
\begin{aligned}
& < B(p')|J_\mu^{dress}(q)|B(p) >= \\
= \quad & 2\pi\delta^4(p'-p-q)\bar{u}_B(p')\{\gamma_\mu F_1^B(q^2) + i\frac{\sigma_{\mu\nu}q^\nu}{2m_B}F_2^B(q^2)\}u_B(p) \\
= \quad & 2\pi\delta^4(p'-p-q)\{f_D^q(q^2)V_\mu^B(p',p) + i\frac{q^\nu}{2m}f_P^q(q^2)T_{\mu\nu}^B(p',p)\}
\end{aligned}
$$

$$
\begin{aligned}
V_\mu^B(p',p) &=< B(p')|\bar{q}(0)\gamma_\mu q(0)|B(p) > \\
T_{\mu\nu}^B(p',p) &=< B(p')|\bar{q}(0)\sigma_{\mu\nu} q(0)|B(p) >
\end{aligned}
\quad \Longrightarrow \quad \text{RTQM}
$$

The calculation of the cloud contributions $f_D^q(q^2)$, $f_P^q(q^2)$ and of the bare contributions $V_\mu^B(p',p), T_{\mu\nu}^B(p',p)$ is done independetly !

Meson-cloud form factors $f_P^q(q^2)$, $f_D^q(q^2)$ calculated by Amand Faessler, Th. Gutsche, V.E. Lyubovitskij, K. Pumsa-ard, Phys Rev. D 73, 114021 (2006).

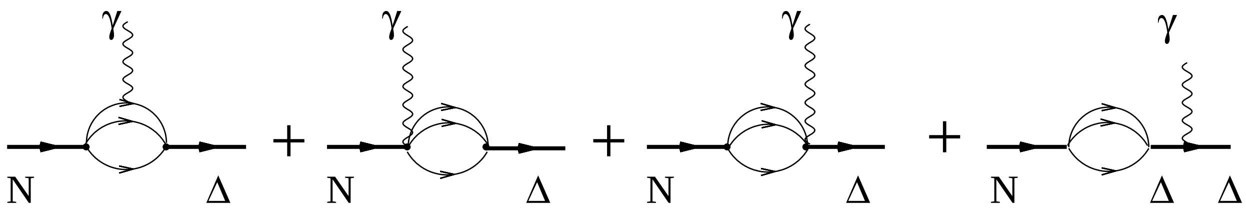
$\star$ $N - \Delta\gamma$ transition matrix elements:

$$< \Delta(p')|J_\mu^{dress}(q)|N(p) >= 2\pi\delta^4(p' - p - q)\bar{u}_\Delta^\nu(p')\Lambda_{\mu\nu}(p, p^{'})u_N(p)$$

$$= 2\pi\delta^4(p' - p - q)\sum_{u,d,s}\{f_D^q(q^2) < \Delta(p')|j_{\mu,q}^{bare}(0)|N(p) >$$

$$+i\frac{q^\nu}{2m}f_P^q(q^2) < \Delta(p')|j_{\mu\nu,q}^{bare}(0)|N(p) >\}$$

$$\Lambda_{\mu\nu}(p, p') = [g_{\mu\nu}b_1(q^2) + p_\mu q_\nu b_2(q^2) + \gamma_\mu q_\nu b_3(q^2) + q_\mu q_\nu b_4(q^2)]\gamma^5$$



$N - \Delta$ *transition observables*
in terms of the relativistic form factors $b_i(q^2)$

- Form Factors

$$G_{M1}(Q^2) = \frac{1}{4}\left\{ b_3(Q^2)\frac{m_+(3m_\Delta + m_N) + Q^2}{m_\Delta} + b_2(Q^2)(m_+m_- + Q^2) - 2b_4(Q^2)Q^2 \right\}$$

$$G_{E2}(Q^2) = \frac{1}{4}\left\{ b_3(Q^2)\frac{m_+m_- - Q^2}{m_\Delta} + b_2(Q^2)(m_+m_- + Q^2) - 2b_4(Q^2)Q^2 \right\}$$

$$G_{C2}(Q^2) = \frac{|\vec{q}|}{2}\left\{ b_3(Q^2) + b_2(Q^2)E_N + b_4(Q^2)\omega \right\}$$

- Helicity amplitudes

$$A_{3/2}(Q^2) = -\sqrt{\frac{\pi\alpha\omega}{2m_N^2}}[G_{M1}(Q^2) + G_{E2}(Q^2)]$$

$$A_{1/2}(Q^2) = -\sqrt{\frac{\pi\alpha\omega}{6m_N^2}}[G_{M1}(Q^2) - 3G_{E2}(Q^2)]$$

- Multipole ratios

$$EMR(Q^2) = \frac{E2}{M1} = -\frac{G_{E2}(Q^2)}{G_{M1}(Q^2)}, \qquad CMR(Q^2) = \frac{C2}{M1} = -\frac{G_{C2}(Q^2)}{G_{M1}(Q^2)}$$

Model Parameters

⋆ The effective quark masses:
$m_u = m_d = m = 0.420$ GeV and $m_s = 0.570$ GeV

⋆ The cut-off parameter $\Lambda_B = 0.75 \div 1.25$ GeV
(a),(b),(c)

⋆ Three low-energy coupling constants enter the mesonic-cloud contributions. $^{(d)}$

[a] M.A. Ivanov, M.P. Locher, V.E Lyubovitskij, Few-Body Syst. 21, 131 (1996)
[b] M.A. Ivanov, J.G. Koerner, V.E Lyubovitskij, A.G. Rusetsky, Phys. Rev. D 60 (1999) 094002
[c] A. Faessler, Th. Gutsche, M.A. Ivanov, J.G. Koerner, V.E Lyubovitskij, Phys. Lett. B 518, 55 (2001)
[d] A. Faessler, T. Gutsche, V.E. Lyubovitskij, K. Pumsa-ard, Phys Rev. D 73, 114021 (2006)

Results

Table 1. Magnetic moments of light baryons (in units of the nuclear magneton μ_N). Results are calculated for the case of a purely vector current.

	$\Lambda_B = 0.75$ GeV			Experiment PDG, LEGS Collab.
	Bare (3q)	Meson cloud	Total	
μ_p(*)	2.621	0.172	2.793	2.793
μ_n(*)	-1.643	-0.270	-1.913	-1.913
μ_Λ(*)	-0.578	-0.035	-0.613	-0.613 $\pm$ 0.004
μ_{Σ^+}	2.430	0.130	2.560	2.458 $\pm$ 0.010
μ_{Σ^-}	-0.962	-0.235	-1.197	-1.160 $\pm$ 0.025
μ_{Ξ^0}	-1.310	-0.076	-1.386	-1.250 $\pm$ 0.014
μ_{Ξ^-}	-0.562	0.014	-0.548	-0.6507 $\pm$ 0.003
$\mu_{N\Delta}$	3.102	0.356	3.458	3.642 $\pm$ 0.019 $\pm$ 0.085

(*) fitted values

Results

Sensitivity of the bare contributions to the light baryon magnetic moments (μ_N) on the choice of the octet baryon $3q$-current. The scale parameter is chosen to be $\Lambda_B = 0.8$ GeV

	Vector current	Tensor current	PDG Collab.,LEGS Collab.
μ_p	2.614	2.804	2.793
μ_n	-1.634	-1.814	-1.913
μ_Λ	-0.579	-0.594	-0.613 $\pm$ 0.004
μ_{Σ^+}	2.423	2.509	2.458 $\pm$ 0.010
μ_{Σ^-}	-0.960	-0.973	-1.160 $\pm$ 0.025
μ_{Ξ^0}	-1.303	-1.385	-1.250 $\pm$ 0.014
μ_{Ξ^-}	-0.567	-0.560	-0.6507 $\pm$ 0.003
$\mu_{N\Delta}$	2.984	2.740	3.642 $\pm$ 0.019 $\pm$ 0.085

Results

N $\rightarrow \Delta\gamma$ transition (Set III: $\Lambda_B = 0.75$ GeV)

	Bare (3q)	Meson cloud	Total	Experiment PDG, LEGS, A1 Collab.
EMR (%) at $Q^2 = 0$	-3.43	0.30	-3.13	-2.5 $\pm$ 0.5; -3.07 $\pm$ 0.26 $\pm$ 0.24
EMR (%) at $Q^2 = 0.06$ GeV^2	-3.35	0.30	-3.05	-2.28 $\pm$ 0.29 $\pm$ 0.20
CMR (%) at $Q^2 = 0$	-3.98	0.25	-3.73	
CMR (%) at $Q^2 = 0.06$ GeV^2	-5.17	0.33	-4.84	-4.81 $\pm$ 0.27 $\pm$ 0.26
$A_{1/2}(0)$ in (10^{-3} GeV$^{-1/2}$)	-114.3	-14.3	-128.6	-135 $\pm$ 6
$A_{3/2}(0)$ in (10^{-3} GeV$^{-1/2}$)	-228.1	-25.4	-253.5	-250 $\pm$ 8
$G_{E2}(0)$	0.130	0.002	0.132	0.137 $\pm$ 0.012 $\pm$ 0.043
$G_{M1}(0)$	3.800	0.435	4.235	4.460 $\pm$ 0.023 $\pm$ 0.104
$G_{C2}(0)$	0.151	0.007	0.158	
$Q_{N\Delta}$ (fm^2)	-0.102	-0.002	-0.104	-0.108 $\pm$ 0.009 $\pm$ 0.034
$\mu_{N\Delta}$	3.102	0.356	3.458	3.642 $\pm$ 0.019 $\pm$ 0.085
$\Gamma_{\Delta \rightarrow N\gamma}$ (MeV)	0.53	0.13	0.66	0.58 - 0.67

Results

Sensitivity of low-energy observables on the choice of the cut-off parameter.
Results are calculated for the case of a purely vector current.

	Set I ($\Lambda_B = 1.25$ GeV)			Set II ($\Lambda_B = 0.8$ GeV)			Set III ($\Lambda_B = 0.75$ GeV)		
	Bare (3q)	Meson cloud	Total	Bare (3q)	Meson cloud	Total	Bare (3q)	Meson cloud	Total
$\mu_{\Sigma^+}(\mu_N)$	2.336	0.196	2.532	2.423	0.148	2.571	2.430	0.130	2.560
$\mu_{N\Delta}(\mu_N)$	2.357	0.439	2.796	2.984	0.354	3.338	3.102	0.356	3.458
$EMR_{Q^2=0\,GeV^2}$ (%)	-3.22	0.29	-2.93	-3.41	0.31	-3.10	-3.43	0.30	-3.13
$CMR_{Q^2=0\,GeV^2}$ (%)	-3.69	0.34	-3.35	-3.95	0.26	-3.69	-3.98	0.25	-3.73

Results of other approaches

Values of the $N \to \Delta\gamma$ helicity amplitudes $A_{1/2}$, $A_{3/2}$ and the ration E2/M1 in different model
All the values are given at $Q^2 = 0$.

	$A_{1/2}$ in 10^{-3} GeV$^{-1/2}$	$A_{3/2}$ in 10^{-3} GeV$^{-1/2}$	E2/M1 (%)
non-relativistic quark models $^{(a)}$	-103	-179	-2 to 0
MIT bag models $^{(b)}$	-102	-176	0
quark models + π, σ exchange $^{(c)}$	-91	-182	-3.5
our model ($\Lambda_B = 0.8$ GeV) $^{(d)}$	-124.3	-244.7	-3.1
Experiment $^{(e)}$	-135 $\pm$ 6	-250 $\pm$ 8	-2.5 $\pm$ 0.5

[a] N. Isgur, G. Karl, R. Koniuk, PRD 25, 2394 (1982)
[b] J.F. Donoghue, E. Golowich, B.R. Holstein, PRD 12, 2875 (1975)
[c] A.J. Buchman, E. Hernandez, Amand Faessler, PRC 55, 448 (1997)
[d] Amand Faessler, Th. Gutsche, B.R. Holstein, V.E. Lyubovitskij, D.Nicmorus, K. Pumsa-ard
[e] Y.M. at al. [PDG], J. Phys.G 33, 1 (2006)

The importance of the vector current

1

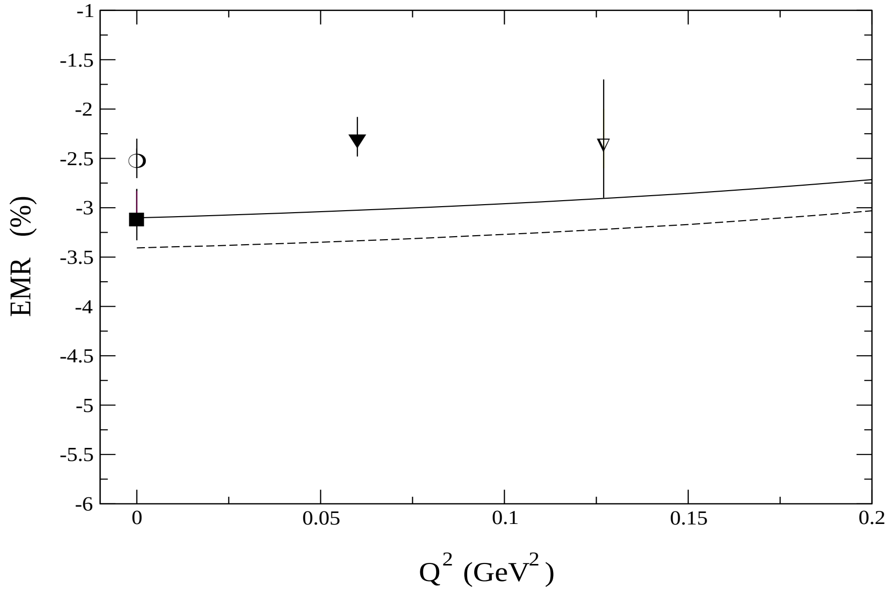
Sensitivity of the EMR and CMR ratios to the choice of the proton $3q$-current

	Mixing parameter β													
	0	0.025	0.05	0.075	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.5	0.75	1
Set III ($\Lambda_B = 0.75$ GeV)														
EMR (%)	-3.13	-2.84	-2.58	-2.33	-2.07	-1.62	-1.19	-0.79	-0.41	-0.07	0.26	0.85	2.10	3.00
CMR (%)	-3.73	-3.39	-3.06	-2.75	-2.44	-1.87	-1.34	-0.85	-0.40	0.03	0.43	1.16	2.65	3.80

$$J_p = (1-\beta)J_p^V + \beta J_p^T$$

A correct negative sign for the two ratios is achieved with the choice of a pure vector current for the nucleon. The occurence is based on the dependence of the two ratios on the relativistic form factors.

EMR and CMR



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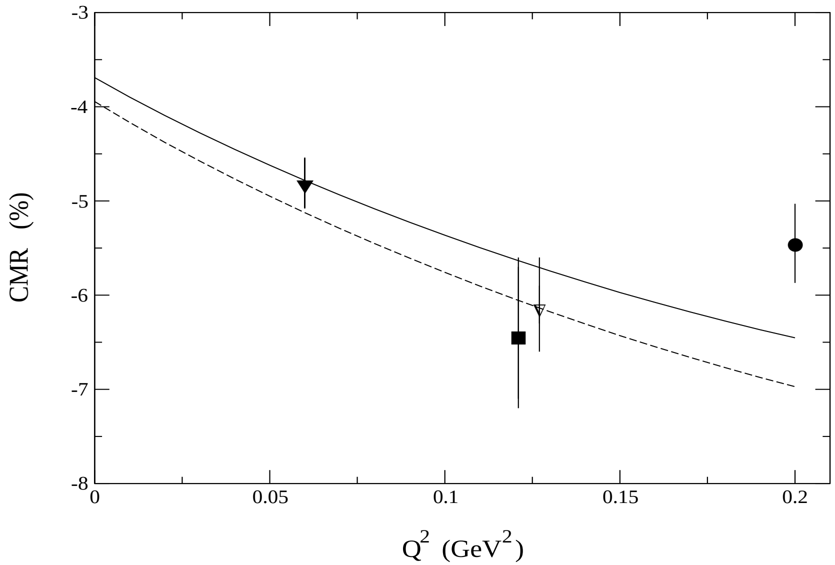
filled box - LEGS Collab.
opened circle - MAMI
opened triangle - OOPS Collab.

filled triangle - A1 Collab.
opened triangle - OOPS Collab.
filled box - MAMI
filled circle - MAMI



Summary and Conclusions

Investigation of the electromagnetic properties of the light baryons.

Advantages of the RTQM:
- ⋆ Lorentz and gauge invariance
- ⋆ self-consistent calculational technique
- ⋆ few parameters

Lorentz covariant chiral quark approach: meson-cloud corrections.

Test the sensitivity of the observables with the scale parameter.
Test the sensitivity of the EMR and CMR ratios with the choice of the baryonic currents.

Light baryon observables and $N - \Delta$ transition observables are in good agreement with experiment.

The decomposition of the vertex function $\Lambda_{\mu\nu}(p, p^{'})$ is not unique

M.M. Giannini, Rep. Prog. Phys. 54, 453 (1990)

$$\Lambda_{\mu\nu}(p, p') = [g_{\mu\nu} b_1(q^2) + p_\mu q_\nu b_2(q^2) + \gamma_\mu q_\nu b_3(q^2) + q_\mu q_\nu b_4(q^2)]\gamma^5$$

Due to the factorization, the relativistic form factors can be written as:

$$b_i(q^2) = b_i^{bare}(q^2) + b_i^{cloud}(q^2)$$

$$< \Delta(p')|j_{\mu,q}^{bare}(0)|N(p) > \quad \Longrightarrow \quad b_i^V(q^2)$$

$$< \Delta(p')|j_{\mu\nu,q}^{bare}(0)|N(p) > \quad \Longrightarrow \quad b_i^T(q^2)$$

$$< \Delta(p')|J_\mu^{dress}(q)|N(p) > \quad \Longrightarrow \quad f_D^q(q^2) \cdot b_i^V(q^2) + f_P^q(q^2) \cdot b_i^T(q^2)$$

Definition of baryon quantities

⋆ Magnetic moments of the baryon octet: $\mu_B = [F_1^B(0) + F_2^B(0)]\frac{m_p}{2m_B}$.
Can be split into the valence quark contribution and the meson cloud contribution

$$\begin{aligned}
\mu_B &= \mu_B^{bare} + \mu_B^{cloud} \\
\mu_B^{bare} &= \sum_{q=u,s,d} f_D^q(0)[F_1^{Bq}(0) + F_2^{Bq}(0)] \\
\mu_B^{cloud} &= \sum_{q=u,s,d} f_P^q(0) G_2^{Bq}(0)
\end{aligned}$$

where $F_{1(2)}^{Bq}(q^2)$ and $G_{1(2)}^{Bq}(q^2)$ are the Pauli and Dirac form factors describing the distribution of quarks q in the baryon B.

Meson-cloud form factors $f_P^q(q^2)$, $f_D^q(q^2)$ calculated by Amand Faessler, Th. Gutsche, V.E. Lyubovitskij, K. Pumsa-ard, Phys Rev. D 73, 114021 (2006).