Baryon properties in the Poincaré-covariant Faddeev approach of Landau gauge QCD

From dynamical scalar quark confinement to nucleon form factors

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ECT*, Trento, May 14, 2008

Outline

cf. the talks by Craig Roberts on Monday and by Peter Tandy today

No free quarks and gluons detected: but a
plethora of corresponding bound states, the **hadrons**!

CONFINEMENT

QCD correlation functions

Motivation: Why Functional Approaches to QCD?

No free quarks and gluons detected: but a plethora of corresponding bound states, the **hadrons**!

CONFINEMENT

QCD correlation functions

★ are connected to confinement of
- gluons,
- quarks, and
- colored composites.

★ provide input into hadron phenomenology:
- DχSB and chiral phase transition
- Bethe-Salpeter equations for Mesons
  form factors, decays, reactions, ...
- Faddeev equations for Baryons
  nucleon form factors, meson production, ...

Motivation: Why Functional Approaches to QCD?

No free quarks and gluons detected: but a plethora of corresponding bound states, the **hadrons**!

CONFINEMENT

QCD correlation functions

describe **hadrons** as bound states
in terms of underlying substructure.

★ Tested by experiments!

★ Future facilites @ GSI, JLab, . . .

Motivation: Why Functional Approaches to QCD?

CONFINEMENT

implies

- a non–perturbative RG invariant confinement scale

$$\Lambda = \mu \exp\left(-\int^{g} \frac{dg'}{\beta(g')}\right) \stackrel{g\to 0}{\to} \mu \exp\left(-\frac{1}{2\beta_0 g^2}\right)$$

- infrared singularities $\Longleftrightarrow$ continuum approach

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Motivation: Why Functional Approaches to QCD?

Some Selected Approaches to Confinement:

see *e.g.* R.A. and J. Greensite, *Quark Confinement: The Hard Problem of Hadron Physics* , J. Phys. **G34** (Special focus issue on Hadron Physics) (2007) S3.

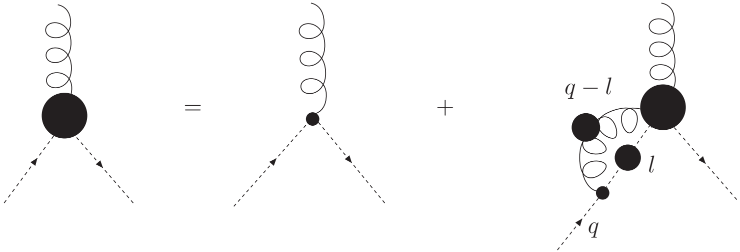
- chromomagnetic **monopoles**
  'tHooft, diGiacomo, ...
- center **vortices**
  Greensite, Olejnik, ...
- **AdS$_5$** / QCD correspondence
  Maldacena, Brodsky, ...
- **Coulomb** confinement
  Gribov, Zwanziger, ...
- Landau gauge **Green Functions**
  Smekal, Fischer, ...

!!! Infrared behaviour of Green functions;

e.g. in linear covariant gauges:
7 primitively divergent Green functions in QCD,
5 primitively divergent Green functions in Yang-Mills theory.

- gluon, ghost, and quark propagators as well as
- 3-gluon, 4-gluon, gluon-ghost, and quark-gluon vertices.

- Starting point in gauges with transverse gluon propagator: Ghost-Gluon-Vertex fulfills Dyson-Schwinger eq.

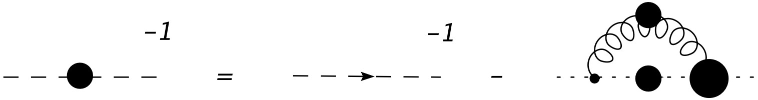


- $l_\mu D_{\mu\nu}(l-q) = q_\mu D_{\mu\nu}(l-q) \Rightarrow$ Bare Vertex for $q_\mu \to 0$
- No anomalous dimensions in the IR

J. C. Taylor, Nucl. Phys. B **33** (1971) 436.
C. Lerche, L. v. Smekal, PRD **65** (2002) 125006.
A. Cucchieri, T. Mendes and A. Mihara, JHEP 0412:012 (2004).
W. Schleifenbaum, A. Maas, J. Wambach and R. A., Phys.Rev.D72 (2005) 014017.

Infrared Exponents for Gluons and Ghosts

- Dyson-Schwinger eq. for the ghost-propagator:



Ansatz for Gluon, $Z(p^2) \sim (p^2)^\alpha$,
and Ghost Ren. Fct., $G(p^2) \sim (p^2)^\beta$.

L. v. Smekal, A. Hauck, R. A., Phys. Rev. Lett. **79** (1997) 3591

- Selfconsistency $\Rightarrow \quad -\beta = \alpha + \beta =: \kappa$ i.e.

$$Z(p^2) \sim (p^2)^{2\kappa}\,, \quad G(p^2) \sim (p^2)^{-\kappa}$$

- IR enhanced ghost propagator: $0.5 \leq \kappa < 1$

Kugo–Ojima confinement criterion
and Gribov–Zwanziger horizon condition fulfilled!

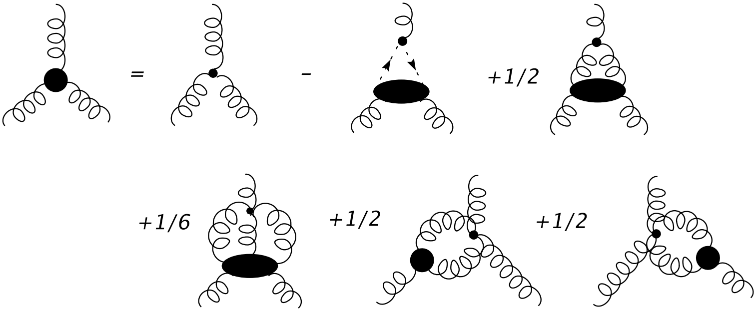
P. Watson and R. A., Phys. Rev. Lett. **86** (2001) 5239

Infrared Exponents for Gluons and Ghosts

R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Example: DSE for 3-gluon-vertex



Use DSEs and ERGEs:

→ Two different towers of equations for Green functions

E.g. ghost propagator

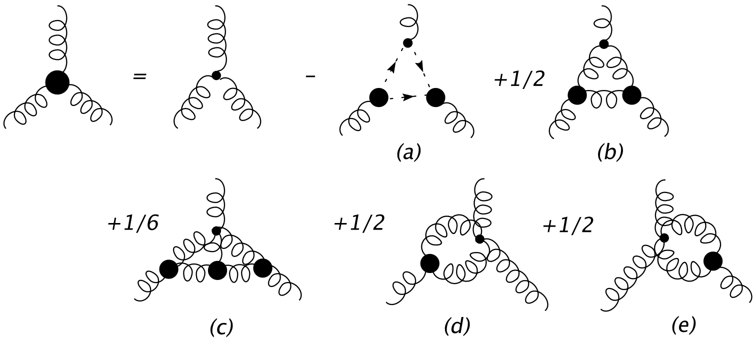
Infrared Exponents for Gluons and Ghosts

R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Skeleton expansion &

generalized formulas (neg. dim.) for Feynman integrals:

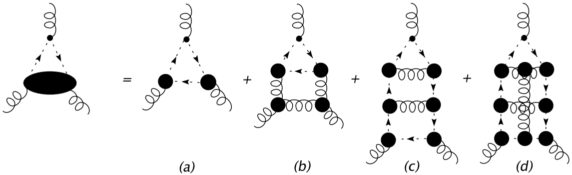


Use DSEs and ERGEs:

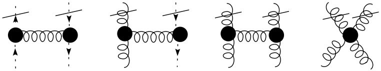
R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Three-gluon vertex: **higher order** in skeleton expansion



built from insertions



insertions have zero IR anomalous dimensions ⇒

IR-analysis valid to all orders in skeleton expansion

Use DSEs and ERGEs:

Infrared Exponents for Gluons and Ghosts

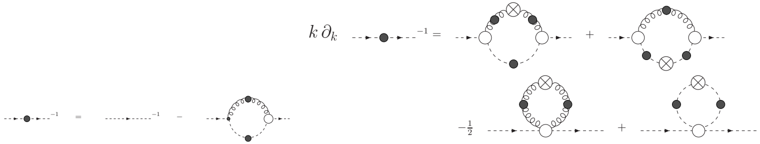
R. A., C. S. Fischer, F. Llanes-Estrada, Phys. Lett. **B611** (2005) 279.

Apply asymptotic expansion to all primitively divergent Green functions:

Use DSEs and ERGEs:

$\longrightarrow$ Two different towers of equations for Green functions

E.g. ghost propagator



IR-Analysis of whole tower of equations $\Rightarrow$

Solution unique! C.S. Fischer and J.M. Pawlowski, PRD **75** (2007) 025012.

Infrared Exponents for Gluons and Ghosts

$2n$ external ghost legs and m external gluon legs
(one external scale p^2; **solves DSEs and STIs**):

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa}$$

- Ghost propagator IR divergent
- Gluon propagator IR suppressed
- Ghost-Gluon vertex IR finite if all external momenta vanish
- 3- & 4- Gluon vertex IR divergent if external momenta vanish
- IR fixed point for the coupling from each vertex
- Conformal nature of Infrared Yang-Mills theory!
- Ghost sector of YM-theory dominates IR!

  D. Zwanziger, Phys. Rev. D **69** (2004) 016002

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YM Running Coupling: IR fixed point

$$G(p^2) \sim (p^2)^{-\kappa}\,, \quad Z(p^2) \sim (p^2)^{2\kappa}$$
$$\Gamma^{3g}(p^2) \sim (p^2)^{-3\kappa}\,, \quad \Gamma^{4g}(p^2) \sim (p^2)^{-4\kappa}$$

$$\alpha^{gh-gl}(p^2) = \alpha_\mu \, G^2(p^2) \, Z(p^2) \sim \frac{const_{gh-gl}}{N_c}$$

$$\alpha^{3g}(p^2) = \alpha_\mu \, [\Gamma^{3g}(p^2)]^2 \, Z^3(p^2) \sim \frac{const_{3g}}{N_c}$$

$$\alpha^{4g}(p^2) = \alpha_\mu \, [\Gamma^{4g}(p^2)]^2 \, Z^4(p^2) \sim \frac{const_{4g}}{N_c}$$

Running Coupling

Ghost-Gluon-Vertex UV finite:

$$\alpha_S(\mu^2) = \frac{g^2(\mu^2)}{4\pi} = \frac{1}{4\pi\beta_0} g_0^2 Z(\mu^2) G^2(\mu^2)$$

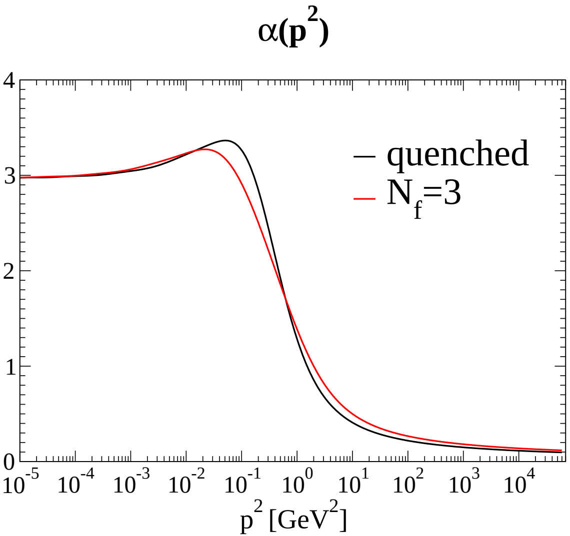
With known IR behavior of gluon (Z) and ghost (G) renormalization function:

IR fix point

$$\alpha_c = \alpha_S(k^2 \to 0) \simeq 2.972^*$$

$^*\alpha_S(0) = \frac{4\pi}{6N_c} \frac{\Gamma(3-2\kappa)\Gamma(3+\kappa)\Gamma(1+\kappa)}{\Gamma^2(2-\kappa)\Gamma(2\kappa)}$

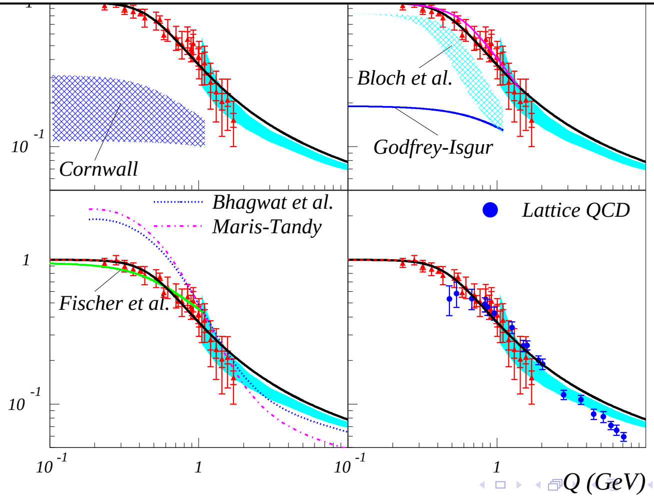
Running Coupling



$\alpha_S(M_Z^2) \stackrel{!}{=} 0.118$

Running Coupling

A. Deur, V. Burkert, J.P. Chen, and W. Korsch, " Determination of the effective strong coupling constant $\alpha_{s,g_1}(Q^2)$ from CLAS spin structure function data´´, arXiv:0803.4119



Positivity violation and gluon confinement

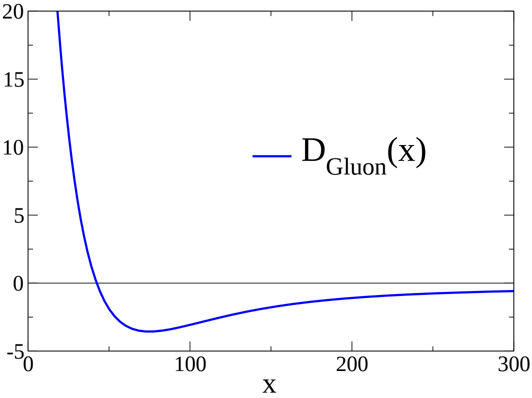
Simple argument [Zwanziger]:
IR vanishing gluon propagator implies

$$0 = D_{gluon}(k^2 = 0) = \int d^4x D_{gluon}(x)$$

$\Longrightarrow D_{gluon}(x)$ has to be negative for some values of x.

Positivity violation and gluon confinement

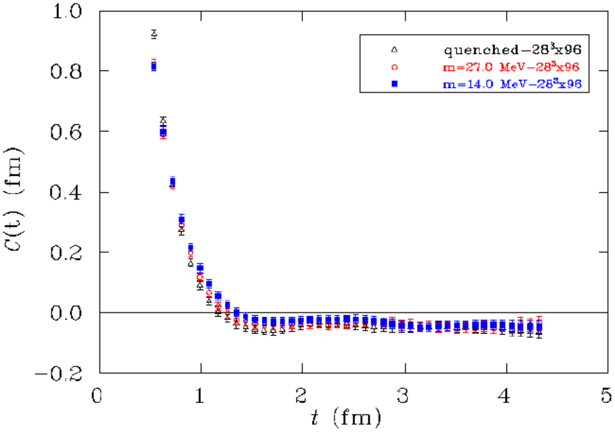
Fourier transform of DSE result:



Gluons unobservable $\Longrightarrow$ **Gluon Confinement!**

Positivity violation and gluon confinement

P. Bowman et al., Phys.Rev.D76 (2007) 094505



Positivity violation and gluon confinement

R.A., W. Detmold, C.S. Fischer and P. Maris, PR**D70** (2004) 014014

$$D^{\text{fit}}_{gluon}(p^2) = w \frac{1}{p^2} \left(\frac{p^2}{\Lambda^2_{\mathtt{QCD}} + p^2} \right)^{2\kappa} \left(\alpha_{\text{fit}}(p^2) \right)^{-\gamma}$$

- IR part: cut for $-\Lambda^2_{\mathtt{QCD}} < p^2 < 0$
- D^{fit}_{gluon}: cut along negative, i.e. timelike, half-axis!

Wick rotation possible!

- w arbitrary normalization parameter
- $\kappa = \frac{93-\sqrt{1201}}{98}$ fixed from IR analysis
- $\gamma = \frac{-13N_c+4N_f}{22N_c-4N_f}$ from perturbation theory
- **Effectively one parameter**†**:** $\Lambda_{\mathtt{QCD}}$**=520 MeV!**

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† from fits to lattice data: $\Lambda_{\mathtt{QCD}} \approx 380$ MeV

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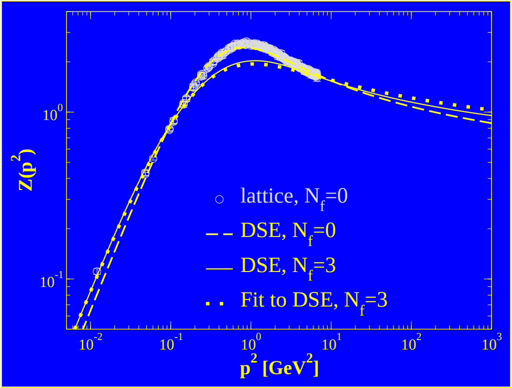

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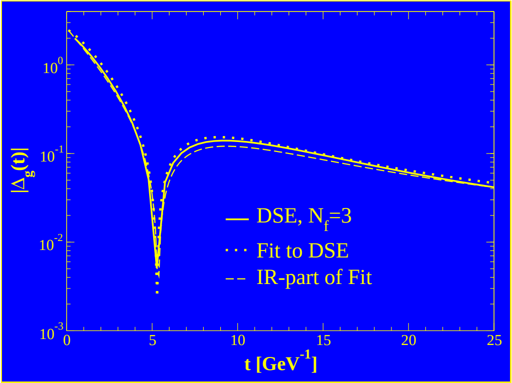

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from fits to lattice data: $\Lambda_{\tt QCD} \approx 380$ MeV

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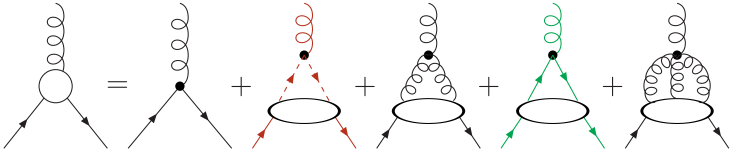
Dynamically induced scalar quark confinement

R.A., C.S. Fischer, F. Llanes-Estrada, K. Schwenzer, arXiv0804.3042[hep-ph].

Quark-gluon vertex:

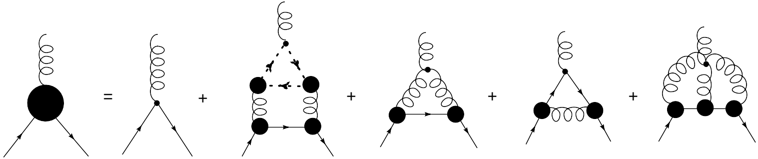


Quark diagram: Hadronic contributions ('unquenching')

Ghost diagram: Infrared leading!

Dynamically induced scalar quark confinement

Quark-gluon vertex: **lowest order** in skeleton expansion



chiral symmetry dynamically or explicitely broken:

$$S(p) = \frac{\not p + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2) \to \frac{Z_f \not p}{M^2} + \frac{Z_f}{M}$$

AND

$$\Gamma_\mu = ig \sum_{i=1}^{4} \lambda_i G^i_\mu\,, \quad G^1_\mu = \gamma_\mu\,, \quad G^2_\mu = \hat{p}_\mu\,, \quad G^3_\mu = \hat{\not p}\,\hat{p}_\mu\,, \quad G^4_\mu = \hat{\not p}\,\gamma_\mu$$

WITH $\lambda_{1,2,3,4} \sim (p^2)^{-1/2-\kappa}$

Dynamically induced scalar quark confinement

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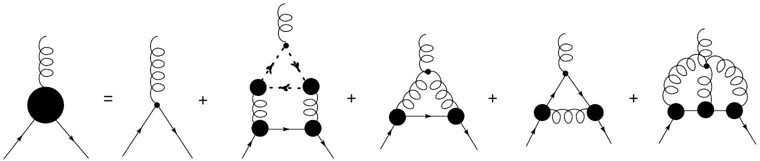

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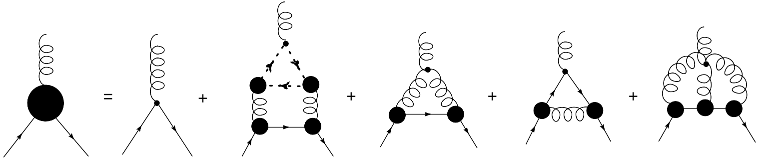

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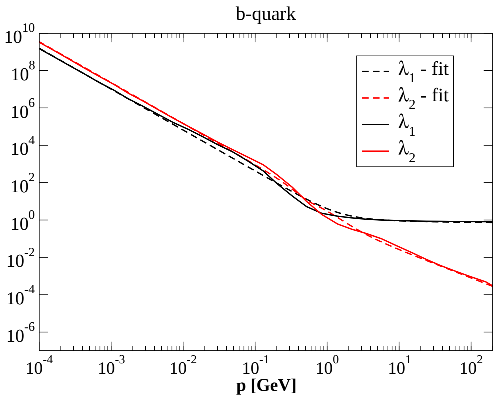
Dynamically induced scalar quark confinement

chiral symmetry dynamically or explicitely broken:

$$\lambda_{1,2,3,4} \sim (p^2)^{-1/2-\kappa} \text{ i.e.}$$

Quark-Gluon vertex IR divergent!

Scalar component λ_2 in IR even **larger** than vector component λ_1!



Dynamically induced scalar quark confinement

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Scalar component λ_2 in IR even **larger** than vector component λ_1!

As

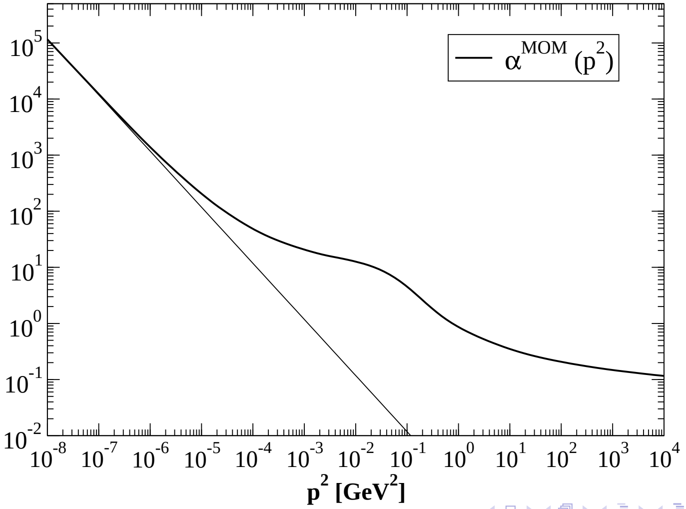
$$\Gamma^{qg}(p^2) \sim (p^2)^{-1/2-\kappa}\,, \quad Z_f(p^2) \sim const\,, \quad Z(p^2) \sim (p^2)^{2\kappa}$$

running coupling from quark-gluon is IR divergent:

$$\alpha^{qg}(p^2) = \alpha_\mu\, [\Gamma^{qg}(p^2)]^2\, [Z_f(p^2)]^2\, Z(p^2) \sim \frac{const_{qg}}{N_c}\frac{1}{p^2}$$

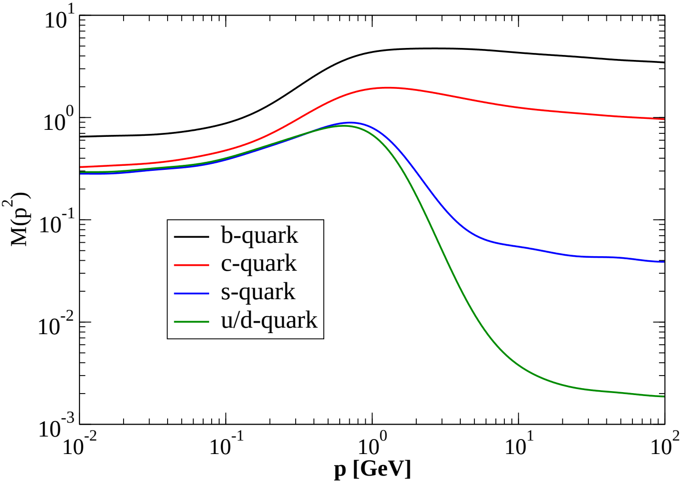
Dynamically induced scalar quark confinement

Running coupling from quark-gluon vertex:



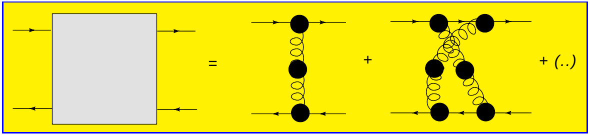
Dynamically induced scalar quark confinement

Dynamically generated quark mass function:



Dynamically induced scalar quark confinement

"Quenched´´ quark-antiquark potential



infrared divergent such that

$$V(\mathbf{r}) = \int \frac{d^3p}{(2\pi)^3} H(p^0 = 0, \mathbf{p}) e^{i\mathbf{pr}} \quad \sim \quad |\mathbf{r}|$$

i.e. linear, dominantly scalar, quark confinement!

$U_A(1)$ anomaly and η' mass from quark confinement

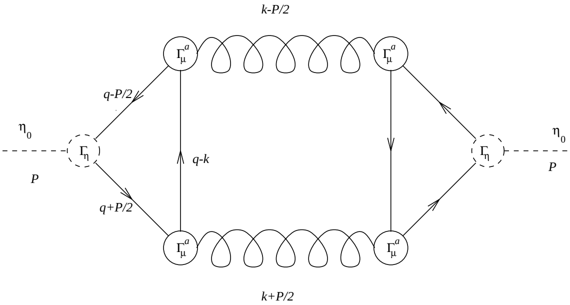
R.A., C. S. Fischer, R. Williams, arXiv:0804.3478[hep-ph]

$U_A(1)$ symmetry anomalous $\Rightarrow \eta'$ mass $\gg \pi$ mass

Where is this encoded in the Green functions?

J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.

E.g. in:



$$\Gamma_\mu D^{\mu\nu}\Gamma_\nu \propto 1/k^4$$

R.A., C. S. Fischer, R. Williams, arXiv:0804.3478[hep-ph]

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QCD vacuum: winding number spots as, *e.g.*, instantons, couple to chiral quark zero modes $\Rightarrow U_A(1)$ symmetry broken!

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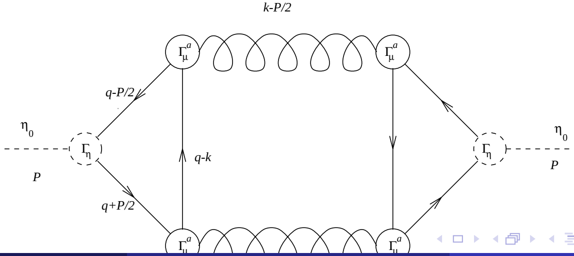

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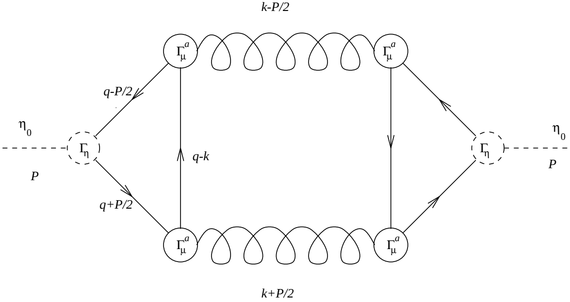


$$\Gamma_\mu D^{\mu\nu} \Gamma_\nu \propto 1/k^4$$

η' mass from quark confinement

However: Infinitely many diagrams (n-gluon exchange) contribute!

Nevertheless:
Calculate contribution from **diamond diagram only** employing DSE results for the gluon and quark propagators and quark-gluon vertex (provides correct pseudoscalar and vector meson masses):

$\chi^2 \approx (160\mathrm{MeV})^4$ vs. phenomenological value $(180\mathrm{MeV})^4$

results in: $m_\eta = 479$MeV, $m_{\eta'} = 906$MeV, $\theta = -23^0$.

A Poincaré-covariant Faddeev approach

G. Eichmann, A. Krassnigg, M. Schwinzerl, R.A., Annals of Physics, in press (available online) [arXiv:0712.2666[hep-ph]].

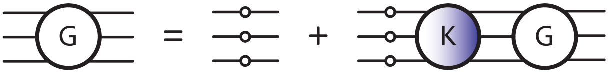
- Aim:
  ab initio calculation of nucleon properties
  from (continuum) QCD
- 1st step:
  determine the nucleons' quark core

- Status:
  - Covariant description of nucleon in terms of a
    nonperturbative quark core
    in a Dyson-Schwinger/Bethe-Salpeter approach
  - Building blocks are propagators from DSE / lattice calculations
  - (still) approximative treatment of quark-quark-T matrix
  - Meson (Pion) cloud effects missing

Poincaré-covariant Faddeev equation

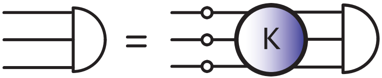
Starting point: Dyson's equation for quark 6-point function

$$G = G_0 + G_0 \, K \, G \quad \Longleftrightarrow \quad G^{-1} = G_0^{-1} - K$$

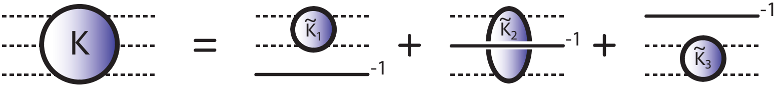


Pole approximation: bound state equation for the baryon

$$\Psi = G_0 \, K \, \Psi$$

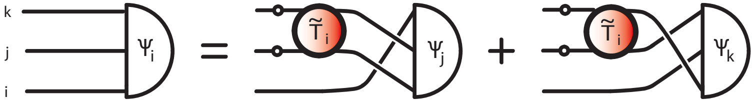


Neglecting all irreducible three-particle interactions



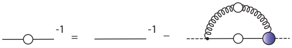
leads to the Faddeev equations

$$\Psi_i = S_j\, S_k\, \widetilde{T}_i\, (\Psi_j + \Psi_k)$$



Quark Propagator

Dressed quark propagator as solution of a model quark DSE



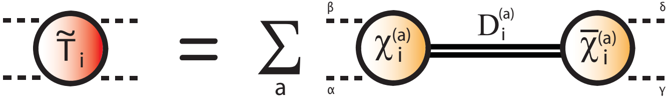
$$S(p) = Z_f(p^2)\frac{i\not{p} - M(p^2)}{p^2 + M^2(p^2)}$$

with model parameters adjusted such that solutions of coupled DSEs and/or corresponding lattice data are reproduced.

“Diquarks”

Expanding the 2-quark correlation function by employing effective diquarks:

$$\widetilde{T}_i = \sum_a \chi_i^a \, D_i^a \, \bar{\chi}_i^a$$



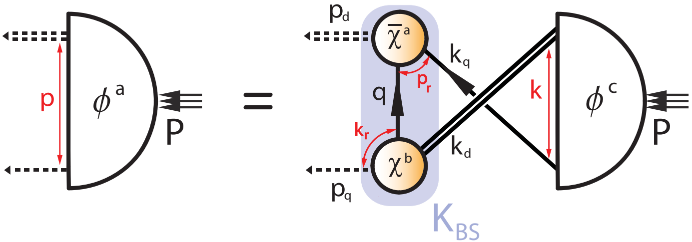
D^a	... diquark propagator
$\chi^a, \bar{\chi}^a$	... diquark amplitudes
	from solutions of model Bethe-Salpeter eqs.

Sum over scalar, axial-vector, ... correlations

Quark-diquark Bethe-Salpeter eqs

This leads to a set of coupled quark-“diquark" Bethe-Salpeter equations:

$$\phi^a(p,P) = \sum_{b,c} \int \frac{d^4k}{(2\pi)^4} \underbrace{\chi^b \, S^T(q) \;\, \bar{\chi}^{a\,T}}_{K_{BS}(p,k,P)} S(k_q) \, D^{bc}(k_d) \, \phi^c(k,P)$$



$\Rightarrow$ Interaction via quark exchange *as required by Pauli principle*

Spin $\frac{1}{2}$ amplitudes

Nucleon (Spin $\frac{1}{2}$): $\Phi^N_{\alpha\beta\gamma} = \Phi^5_\alpha \, (\chi^5)_{\beta\gamma} + \Phi^\mu_\alpha \, (\chi^\mu)_{\beta\gamma}$

$$\Phi^5 = \sum_{i=1}^{2} S_i(p;P) \, \Gamma_i(\gamma^\mu, p, P) \, u$$

$$\Phi^\mu = \sum_{i=1}^{6} A_i(p;P) \, \Gamma_i^\mu(\gamma^\mu, p, P) \, u$$

with constraints on the Dirac matrices Γ_i such that

- nucleon has positive parity and positive energy.
- the two independent momenta

  P baryon momentum and

  p quark-diquark relative momentum

  are basis vecors.

Spin $\frac{1}{2}$ amplitudes

N wave fcts in the rest frame	eigenvalue $l(l+1)$ of $\mathbf{L}^2$	eigenvalue $s(s+1)$ of $\mathbf{S}^2$
$\mathcal{S}_1 u(\gamma_5 C) = \binom{\chi}{0}(\gamma_5 C)$	0 s	$\frac{3}{4}$
scalar		
$\mathcal{S}_2 u(\gamma_5 C) = \binom{0}{\frac{1}{p}(\vec{\sigma}\vec{p})\chi}(\gamma_5 C)$	2 p	$\frac{3}{4}$
$\mathcal{A}_1^\mu u(\gamma^\mu C) = \hat{P}^0 \binom{\frac{1}{p}(\vec{\sigma}\vec{p})\chi}{0}(\gamma^4 C)$	2 p	$\frac{3}{4}$
$\mathcal{A}_2^\mu u(\gamma^\mu C) = \hat{P}^0 \binom{0}{\chi}(\gamma^4 C)$	0 s	$\frac{3}{4}$
$\mathcal{B}_1^\mu u(\gamma^\mu C) = \binom{i\sigma^i\chi}{0}(\gamma^i C)$	0 s	$\frac{3}{4}$
axialvector		
$\mathcal{B}_2^\mu u(\gamma^\mu C) = \binom{0}{\frac{i}{p}\sigma^i(\vec{\sigma}\vec{p})\chi}(\gamma^i C)$	2 p	$\frac{3}{4}$
$\mathcal{C}_1^\mu u(\gamma^\mu C) = \binom{i\left(\hat{p}^i(\vec{\sigma}\hat{\vec{p}}) - \frac{1}{3}\sigma^i\right)\chi}{0}(\gamma^i C)$	6 d	$\frac{15}{4}$
$\mathcal{C}_2^\mu u(\gamma^\mu C) = \binom{0}{i(\ldots\ \frac{1}{\ldots}\ \ldots)}(\gamma^i C)$ [illegible]	2 p	$\frac{15}{[illegible]}$

Spin $\frac{3}{2}$ amplitudes

Delta (Spin $\frac{3}{2}$):

$$\Phi^{\Delta}_{\alpha\beta\gamma} = \Phi^{\mu}_{\alpha} \, (\chi^{\mu})_{\beta\gamma}$$

$$\Phi^{\mu} = \sum_{i=1}^{8} D_i(p;P) \, \Gamma_i^{\mu\nu}(\gamma^{\mu}, p, P) \, u^{\nu}$$

- Note: 8 scalar functions, respectively, describe N and Δ baryon.

Partial waves

J	$=$	**L**	$+$	**S**
total angular momentum good quantum number		orbital		spin
		angular momentum frame dependent!		

Choose rest frame and decompose Pauli-Lubanski vector into orbital and spin part:

$$W_i = L_i + S_i = \frac{1}{2}\epsilon_{ijk}\left(\mathbb{L}^{jk} + \mathbb{S}^{jk}\right)$$

$$\mathbb{L}^{jk} = \left(p^j \frac{\partial}{\partial p^k} - p^k \frac{\partial}{\partial p^j}\right)$$

$$\mathbb{S}^{jk} = \frac{1}{2}\sigma^{jk} \otimes \mathbf{1} \otimes \mathbf{1} + \text{permutations}$$

Partial waves

$$L^i L^i \Phi^{8,10}_{\alpha\beta\gamma} = l(l+1)\Phi^{8,10}_{\alpha\beta\gamma}$$
$$S^i S^i \Phi^{8,10}_{\alpha\beta\gamma} = s(s+1)\Phi^{8,10}_{\alpha\beta\gamma}$$

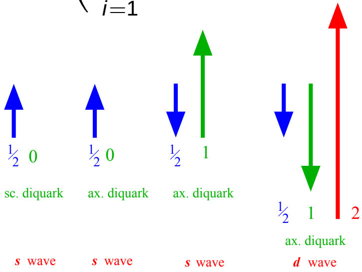
Nucleon (Spin $\frac{1}{2}$):
Upper components: 3 s–waves, 1 d–wave
Lower components: 4 p–waves

Delta (Spin $\frac{3}{2}$):
Upper components: 1 s–wave, 3 d–waves
Lower components: 3 p–waves, 1 f–wave

Decomposition of the quark-diquark amplitudes

... in Dirac space with Lorentz-invariant coefficients:
(restriction of positive energy ⇒ 8 components)

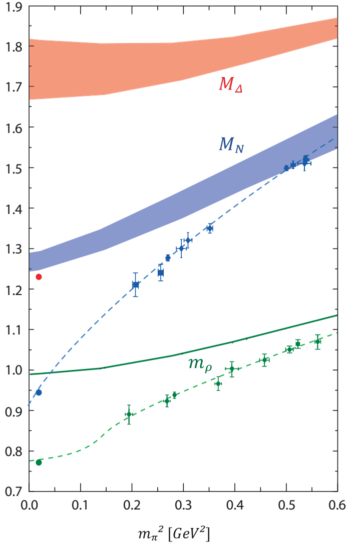
$$\begin{pmatrix} \Phi^5(p,P) \\ \Phi^\mu(p,P) \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{2} Y_i(p^2, p\cdot P)\, \mathcal{S}_i(p,P) \\ \sum_{i=1}^{6} Y_{i+2}(p^2, p\cdot P)\, \gamma^5 \mathcal{A}_i^\mu(p,P) \end{pmatrix}$$



(R.A. and M. Oettel, Schladming Proceedings 2005 [arXiv:nucl-th/0507003])

NB: NUCLEON IS NOT SPHERICALLY SYMMETRIC!

Quark core masses



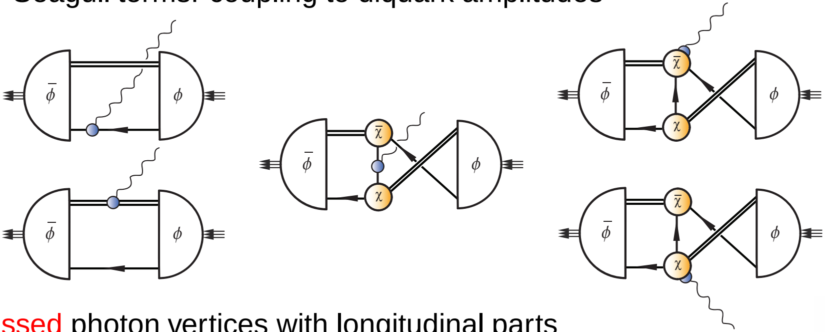
lattice data: Adelaide, Graz, JLQCD, QCDSF, and CP-PACS groups

Electromagnetic Current

Current conservation requires the following diagrams:

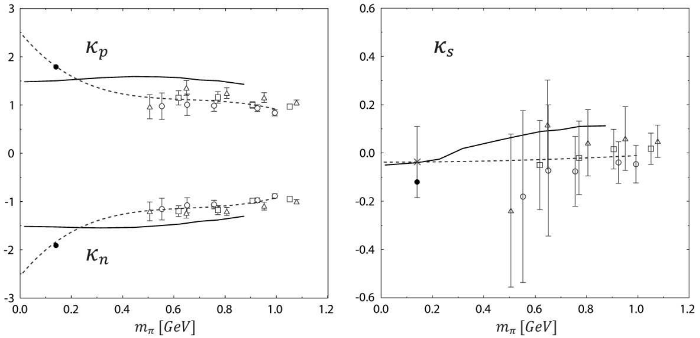
(M. Oettel, M.A. Pichowsky and L. von Smekal, Eur.Phys.J.A 8, (2000) 251-281)

- Photon-quark coupling
- Photon-diquark coupling
- Coupling to exchange quark
- Seagull terms: coupling to diquark amplitudes



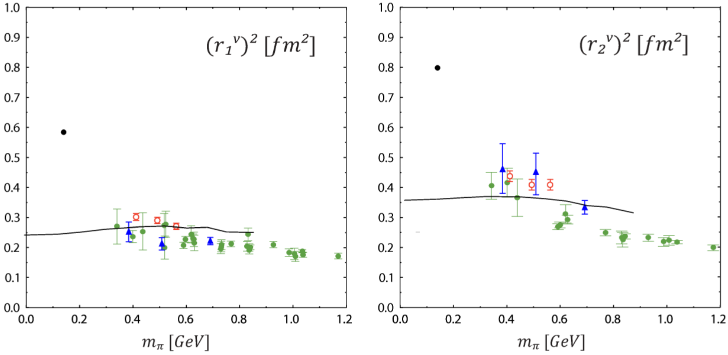
Dressed photon vertices with longitudinal parts constrained by Ward-Takahashi identities

Magnetic moments



lattice data: QCDSF
chiral extrapolation: M. Göckeler et al.

E.m. isovector radii



lattice data: C. Alexandrou et al., M. Göckeler et al.

Electromagnetic Form Factors

Many more results in Gernot Eichmann's talk next week!

Summary

Landau gauge IR QCD Green functions:

- Gluon confinement by positivity violation
- Infrared-finite strong running coupling in Yang-Mills theory
- Chiral symmetry dynamically broken! In 2- and **3**-point function!
- Quark confinement: In IR dominantly scalar!
- η' mass generated ($U_A(1)$ anomaly)
- First step towards baryon observables in a functional 'first-principle' approach!
- Electromagnetic form factors as function of current quark masses

- Improve on T-matrix in covariant Faddeev equation
- Include pion cloud
- Transition form factors
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