Strong formfactors of the nucleon in relativistic CQM

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Outline

1. Introduction
2. PFSM construction of strong decay operators and ff
3. Comparison with other approaches (Phen, Lat)
4. Summary
5. Incorporating field-theoretic aspects “ex post facto”

Quantum Chromodynamics

- Fundamental strong-interaction (field) theory with
  d.o.f.: current quarks and gluons
- Constituent Quark Models
  Effective description for low energy hadrons
  d.o.f.: constituent quarks with interaction
  Low-e QCD $\rightarrow$ Spontaneous-Breaking of χ-symmetry
  $\rightarrow$ Massive constituent quarks and Golstone-B-Ex hyperfine interactions
  (alternatives)

Pleople that worked around the Graz CQM (for more than a decade)

Plessas, Wagenbrunn, Glozman, Klink, Schweiger, Krassnigg, Melde, Berger, Sengl, Boffi, Radici, ...

CQM model with GBE hyperfine interaction

Pion-quark (ps) coupling related to the pion-nucleon (ps) coupling using the Golberger-Treiman relations (and $g_Q^A = 0.6 \times g_N^A$):

$$\frac{g_{\pi Q}^2}{4\pi} = \left(\frac{g_Q^A}{g_N^A}\right)^2 \left(\frac{m_Q}{m_N}\right)^2 \frac{g_{\pi N}^2}{4\pi} = 0.67\,.$$

THIS FIXES AT ONCE THE $\pi \cdot Q$ COUPLING.

Interaction left with 5 parameters to fit low-lying baryon spectra

Other model hf interactions are possible (color-magnetic OGE) but "parity inversion $N(1440)^+ - N(1535)^-$ together with correct Δ-N mass splitting is obtained only with large GBE fraction"

(Glozman, Papp, Plessas, Varga, Wagenbrunn, PRC **57** (1998) 3406)

NO parameters in EM ff, meson decays, strong ff

The Hamiltonian

$$H_{free} = \sum_{i=1}^{3} \sqrt{m_i^2 + \vec{k}_i^2}$$

$$V_{ij}^{conf} = V_0 + Cr_{ij}$$

The quark-quark interaction is an istantaneous pseudoscalar-meson exch. potential

$$V_{ij}^{hf} = \left[\sum_{a=1}^{3} V_{ij}^{\pi} \lambda_i^a \lambda_j^a + \sum_{a=4}^{7} V_{ij}^{K} \lambda_i^a \lambda_j^a + V_{ij}^{\eta} \lambda_i^8 \lambda_j^8 + \frac{2}{3} V_{ij}^{\eta'} \right] \vec{\sigma}(i) \cdot \vec{\sigma}(j).$$

... but other variants (i.e. One-Gluon-Exchange potential models) have been tested...

ROADMAP (WISH MAP)

- CQM → spectra worked out and appear OK
- PFSM → EM ff analyzed and do quite well (caveat...)
- PFSM → Meson Decays analyzed and do not so well
- PFSM → Strong ff we are here!
- With the strong ff a new door opens:

.... incorporation of π-field theoretic aspects retroactively
keywords: Bare & dressed hadrons, pion loops, including FSI in meson decays, including CC dynamics, pion dynamics. .... but
¿ are we doing double counting ?

Why Relativistic Quantum Mechanics?

- Requirements of special relativity are satisfied
- Finite number of degrees of freedom
- Description of composite particles
- Large class of admissible interactions
- Few-body-type calculations are tractable

But ... it is not a field theory!

- it requires definition of suitable spectator-models operators and currents
- these are effective many-body operators that cannot be uniquely defined and are *ansätze*

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Poincaré invariance for a system of free particles is easy!
... but for interacting systems is "really difficult" (Dirac, 1949)

- One way out: the Bakamjian-Thomas construction
- Linearize the procedure with an intermediate step:
  introduce the interaction in "auxiliary" operators which include the mass operator

Point Form Spectator Model construction

Special methods of adding interactions exist

in particular, instant, front and point form

Properties of point form

- Kinematic subgroup is Lorentz group
- So-called velocity states represent suitable basis:
  - Separation of the center-of-momentum motion
  - Spin coupling identical to non-relativistic ones
- CQM interactions can be used to define invariant mass operator
- $\longrightarrow$ CQM wave functions can be used as input
- $\longrightarrow$ Poincaré invariant transition amplitudes

Relativistic Reduced Transition Amplitude for mesonic decay (once the overall factor $2MV_0\delta^3\left(M\vec{V}-M'\vec{V}'-\vec{Q}\right)$ has been sorted out)

$$F_{i\to f}\equiv\langle V',M',J',\Sigma'|\,\hat{D}^m_{rd}\,|V,M,J,\Sigma\rangle=\frac{2}{MM'}\sum_{\sigma_i\sigma_i'}\sum_{\mu_i\mu_i'}\int d^3\vec{k}_2 d^3\vec{k}_3 d^3\vec{k}_2' d^3\vec{k}_3'$$

$$\sqrt{\frac{(\omega_1+\omega_2+\omega_3)^3}{2\omega_1 2\omega_2 2\omega_3}}\sqrt{\frac{\left(\omega_1'+\omega_2'+\omega_3'\right)^3}{2\omega_1' 2\omega_2' 2\omega_3'}}\Psi^{\star}_{M'J'\Sigma'}\left(\vec{k}_i';\mu_i'\right)\prod_{\sigma_i'}D^{\star\frac{1}{2}}_{\sigma_i'\mu_i'}\left\{R_W\left[k_i';B\left(V'\right)\right]\right\}$$

$$\langle p_1',p_2',p_3';\sigma_1',\sigma_2',\sigma_3'|\,\hat{D}^m_{rd}\,|p_1,p_2,p_3;\sigma_1,\sigma_2,\sigma_3\rangle\prod_{\sigma_i}D^{\frac{1}{2}}_{\sigma_i\mu_i}\left\{R_W\left[k_i;B\left(V\right)\right]\right\}\Psi_{MJ\Sigma}\left(\vec{k}_i;\mu_i\right)$$

Point-Form Spectator Model

$$\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' | \hat{D}_{rd}^{pv,m} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle$$

$$= -3\mathcal{N} \frac{i\, g_{qqm}}{2m_1} \frac{1}{\sqrt{2\pi}} \bar{u}(p_1', \sigma_1') \gamma_5 \gamma^\mu \mathcal{F}^m u(p_1, \sigma_1) \tilde{q}_\mu$$

$$\times 2p_{20} \delta^3 \left(\mathbf{p}_2 - \mathbf{p}_2'\right) \delta_{\sigma_2 \sigma_2'} 2p_{30} \delta^3 \left(\mathbf{p}_3 - \mathbf{p}_3'\right) \delta_{\sigma_3 \sigma_3'}$$

We currently use the pseudovector coupling

- Meson couples to quark 1, quarks 2 and 3 are spectators
- Spectator conditions plus total momentum conservation determine the active quark momentum transfer $\tilde{q}^\mu = p_1^\mu - p_1'^\mu \neq q^\mu$
- Normalization $\mathcal{N}$ not completely constrained by Poincaré invariance alone
- PFSM is an effective many-body operator!

Meet ON-shell conditions $\longrightarrow$ Mesonic Decays

Transition Amplitude

$$F_{i\to f} = \langle M', \mathbf{V}', \Sigma', M_{\Sigma'}, T', M_{T'} | \hat{D}^{m}_{rd} | M, \mathbf{V}, \Sigma, M_{\Sigma}, T, M_T \rangle$$

$\uparrow$

mesonic decay operator

Decay Width

$\downarrow$

e.g.: $\Delta(1232) \to \pi$N 33-35 MeV (theory PFSM) 118$\pm$2 (exp)

$\Delta(1232) \to \pi$N 62 MeV (II theory Bonn)
Metsch et al. AIP Conf. Proc. 717, 646 (2004)

In search of off-shell extension

The off-shell extension of the decay amplitudes leads to the strong ff

- All quarks and baryons are on mass-shell
- 3-momentum is conserved at the hadron level: $\vec{q_\pi} = \vec{P}_i - \vec{P}_f$
- Total Energy is not conserved: $\omega_\pi \neq E_i - E_f$
- $Q^2 \equiv -q^2 = -(E_i - E_f)^2 + (\vec{P}_i - \vec{P}_f)^2$

The PFSM strong form factors in the rest frame of the emitting baryon are introduced according to

$$\frac{1}{\sqrt{2\pi}}\frac{f_{NN\pi}G_{NN\pi}\left(Q^2\right)}{m_\pi}\bar{u}(p',\sigma')\gamma_5\gamma^\mu Q_\mu u(p,\sigma)=F^{\mathrm{CQM}}_{i\to f},$$

$$\frac{1}{\sqrt{2\pi}}\sqrt{\frac{2}{3}}\frac{f_{N\Delta\pi}G_{N\Delta\pi}\left(Q^2\right)}{m_\pi}\bar{u}\left(p',\sigma'\right)u_\mu\left(p,\sigma_\Delta\right)Q^\mu=F^{\mathrm{CQM}}_{i\to f}$$

for the πNN and πNΔ vertex, respectively

A warm-up: strong ff in terms of 4-Dim Q^2

PFSM CQM Formfactors are analyzed in terms of Q^2 by fitting over analytical expressions:

$$f_m G_\mathrm{m} \quad = \quad A_\mathrm{m} \left(\frac{\Lambda_m^2 - m_\pi^2}{\Lambda_m^2 + Q^2} \right)^m \tag{1}$$

Table: Fit parameters obtained for the πNN and πNΔ formfactors. Cut-offs Λ are referred to monopole expressions!

	Nucleon			Delta		
m-pole	Λ GeV	A	f(0)	Λ GeV	A	f(0)
1	0.448	1.032	0.933	0.707	1.345	1.293
1.5	$\times\sqrt{1.5}$	1.031	0.933	-	-	-
2	$\times\sqrt{2}$	1.030	0.933	$\times\sqrt{2}$	1.344	1.293
3	–	-	-	$\times\sqrt{3}$	1.344	1.293

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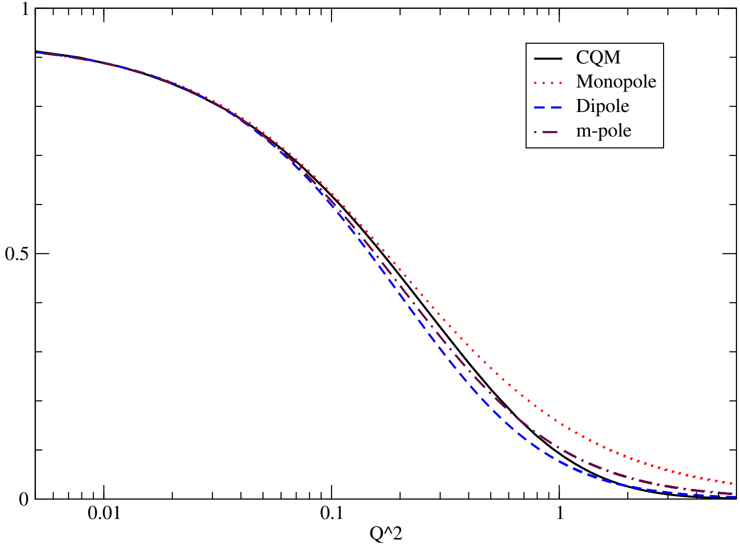
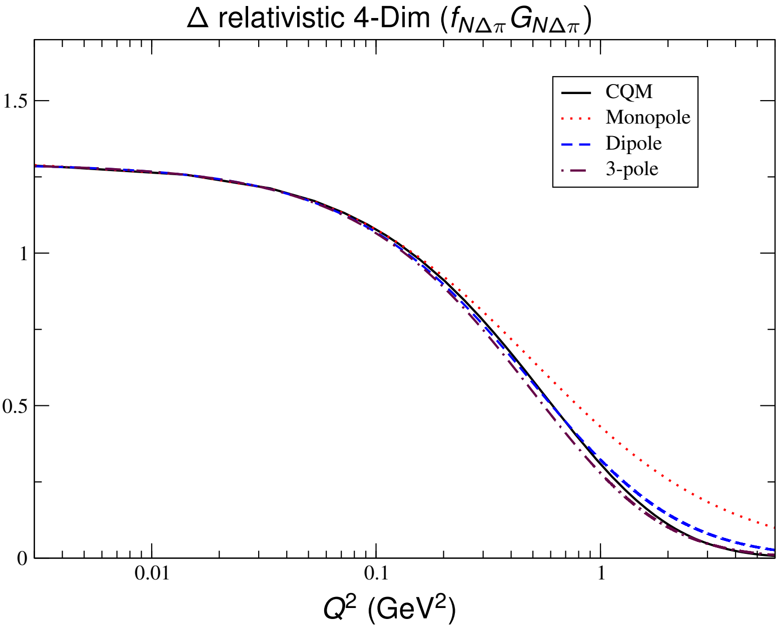
NUCLEON relativistic 4-Dim ($f_{NN\pi} G_{NN\pi}$)



Q^2 (GeV^2)


Δ relativistic 4-Dim ($f_{N\Delta\pi} G_{N\Delta\pi}$)
CQM
Monopole
Dipole
3-pole
1.5
1
0.5
0
0.01
0.1
1
Q^2 (GeV^2)

strong FF

First results: RELATIVISTIC 4-dim Q^2 PFSM CQM

- N: Rather soft cut-offs (Λ= 0.448 GeV) and in between a Dipole/Monopole structure (m=1.5) sesqui-pole
- Δ: Mild soft cut-offs (Λ= 0.707 GeV). Beyond a Dipole structure ?

Form-factors in terms of 3-dim $\vec{q}^2$

Monopole, or Dipole, is that the question?

- Standard 3-DIM $\vec{q}$ monopole form

$$G_{\rm Mon} = A_{\rm Mon} \frac{1}{1 + \left(\frac{\vec{q}}{\Lambda_{\rm Mon}}\right)^2}$$

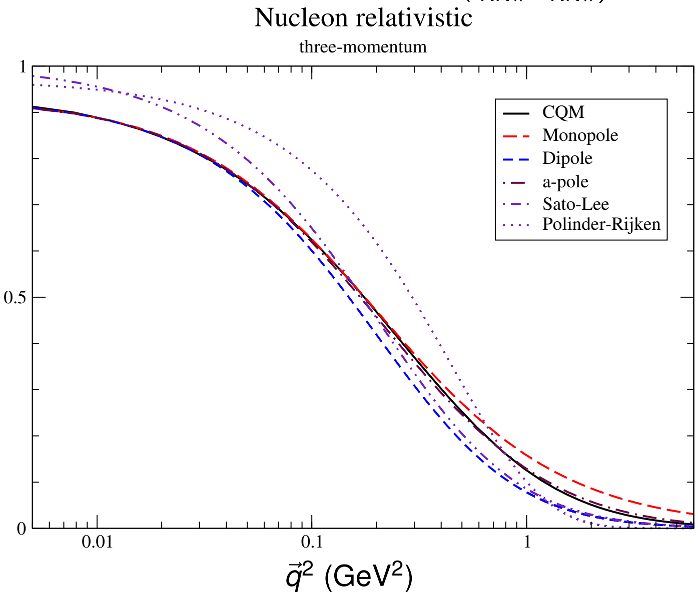
- Dipole form

$$G_{\rm Dip} = A_{\rm Dip} \left(\frac{1}{1 + \left(\frac{\vec{q}}{\Lambda_{\rm Dip}}\right)^2} \right)^2$$

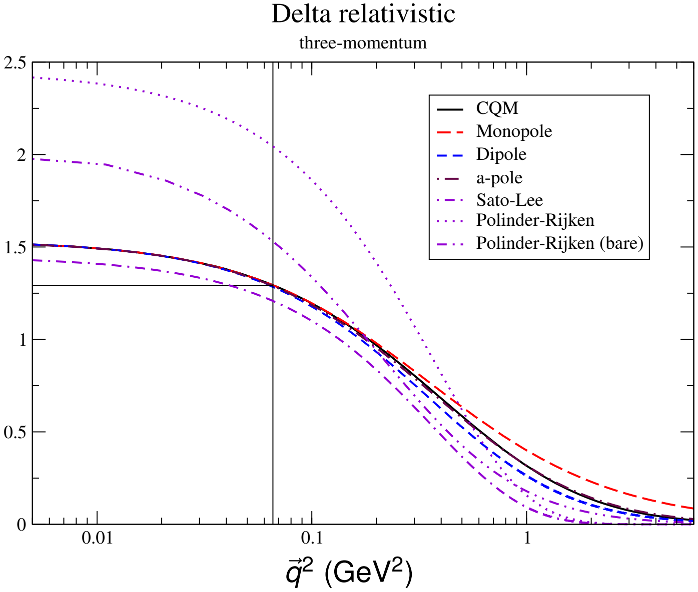
- “Mixed” form of the type

$$G_{\rm a-pole} = A_{\rm a-pole} \frac{1}{1 + \left(\frac{\vec{q}}{\Lambda_{\rm a-pole}}\right)^2 + \left(\frac{\vec{q}}{\Lambda'_{\rm a-pole}}\right)^4}$$

NUCLEON relativistic 3-Dim ($f_{NN\pi} G_{NN\pi}$)



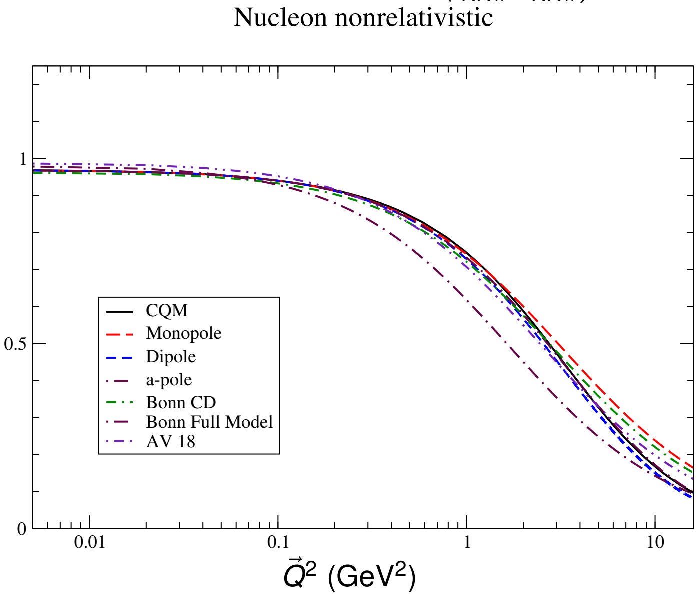
Δ relativistic 3-Dim ($f_{N\Delta\pi}$ $G_{N\Delta\pi}$)



Non Relativistic Reduced Transition Amplitude for mesonic decay

$$F^{NR}_{i\to f} = \sqrt{2E}\sqrt{2E'}\sum_{\mu_i\mu_i'}\int d^3\vec{k}_2 d^3\vec{k}_3 \Psi^{\star}_{M'J'\Sigma'}\left(\vec{k}_i'; \mu_i'\right)$$
$$\times\frac{-3ig_{qqm}}{2m_1}\frac{\mathcal{F}^m}{\sqrt{2\pi}}\left\{\left[\vec{\sigma}_1\cdot\vec{q} - \frac{\omega_m}{2m_1}\vec{\sigma_1}\cdot(\vec{p}_1+\vec{p_1'})\right]\delta_{\mu_2\mu_2'}\delta_{\mu_3\mu_3'}\right\}_{\mu_1\mu_1'}$$
$$\times\Psi_{MJ\Sigma}\left(\vec{k}_i; \mu_i\right)$$

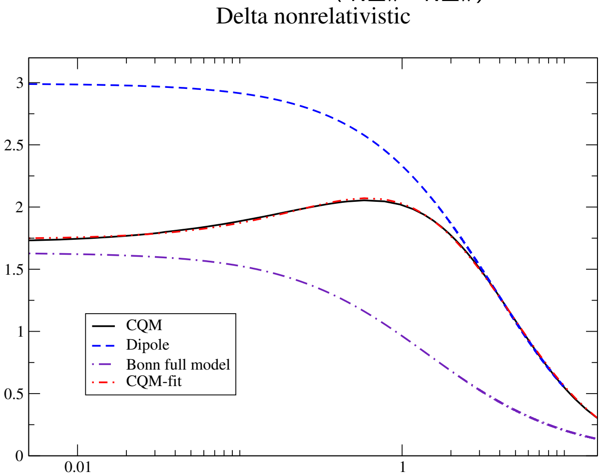
NUCLEON nonrelativistic ($f_{NN\pi} G_{NN\pi}$)



The nonrelativistic reduction of the $\Delta N\pi$ transition shows an unusual low-momentum behaviour, which requires a fit of the type

$$G_{\mathrm{a}}^{\Delta} = \frac{A}{\left[1+\left(\frac{\vec{q}}{\Lambda}\right)^2\right]^2} - \frac{B}{1+\left(\frac{\vec{q}}{\Lambda'}\right)^2}\,. \qquad (2)$$

Δ nonrelativistic ($f_{N\Delta\pi} G_{N\Delta\pi}$)



$\vec{q}^2$ (GeV^2)

Fitted with the new expression (Dipole - Monopole)

Extracted coupling constant and cut-offs

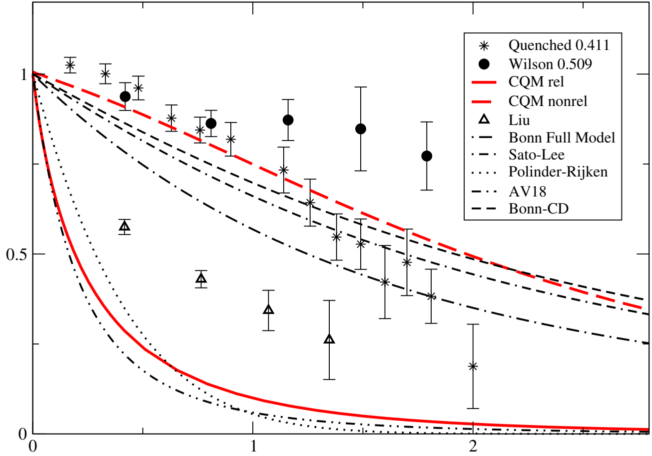
Table: Fit parameters for the form factors in dependence of three-momentum $\vec{q}$. The CQM fits follow Eq.2, except the one denoted by the *. In this case the fit follows the Dipole-Monopole form.

Nucleon	CQM-rel	CQM-nr	Sato-Lee	Bonn	Bonn CD	AV18
$f^2_{NN\pi}/4\pi$	0.0691	0.0747	0.08	0.077	0.074	0.078
Λ [GeV]	0.451	1.804	0.454	1.3	1.72	1.58
Λ' [GeV]	0.931	2.829	0.64218	x	x	x

Delta	CQM-rel	CQM-nr*	Sato-Lee	Bonn	P-R
$f^2_{\Delta N\pi}/4\pi$	0.188	0.241	0.334	0.218	0.167
Λ [GeV]	0.594	1.978	0.458	1.2	exp
Λ' [GeV]	0.998	0.842	0.64818	x	exp

Comparison with Lattice data (2007, 1995) πNN

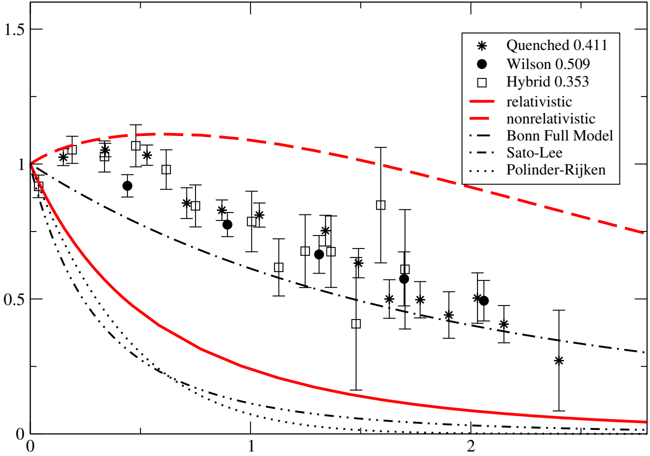
Q^2-dependence for the normalized πNN form factor. Lattice data by Alexandrou et al. PRD (2007) and by Liu et al PRL (1995)



Q^2 (GeV^2).

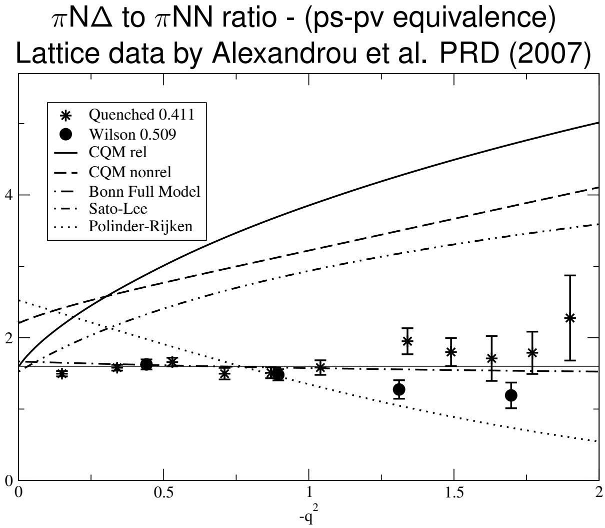
Comparison with Lattice data (2007) $\pi N\Delta$

Q^2-dependence for the normalized $\pi N\Delta$ form factor. Lattice data by Alexandrou et al. PRD (2007)



Q^2 (GeV^2).

Comparison with Lattice data (2007) N-Δ ratio



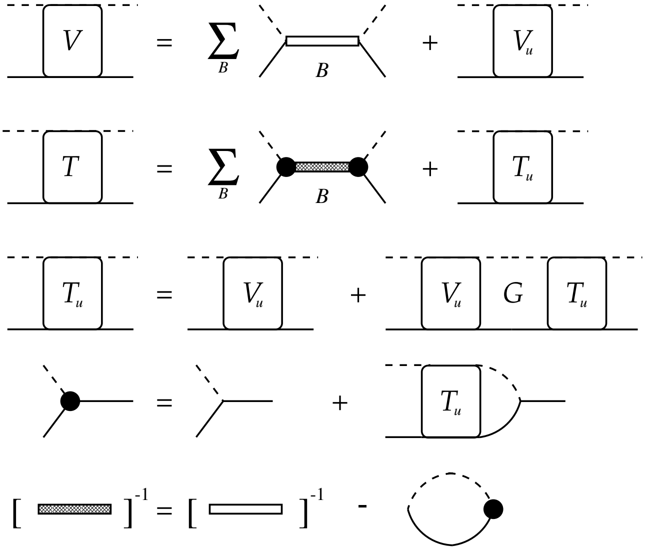
Q^2 (GeV^2).

- In the literature there is a "dicotomy" of πNN & πNΔ vertex structure in πN (usually relativistic) and NN (usually nonrelativistic) calculations.
- Our PFSM results are in line with soft cut-offs, typical of CC πN dynamics
- The nonrelativistic form factors have larger coupling constants than the relativistic ones and are in line with phenomenological values
- Our nonrelativistic πNN results reproduce extremely well the hard cut-offs used in modern phenomenological NN interactions.
- The nonrelativistic πNΔ results exhibit a new and anomalous trend, with a maximum centered around 1 GeV^2, and a new analytical expression was needed
- Recent Lattice results ('07) exhibit a trend similar to our nonrelativistic results, both N and Δ (see talk by Tina Alexandrou). Older lattice data ('95) are remarkably different and exhibit a softer vertex structure, somewhat closer to our relativistic results.

Outlook: The Pion Dressing of CQM baryon resonances, retroactively

It appears evident that standard CQM spectra describe “bare” nucleons and “bare” resonances.
Current step: 1) We have in the light sector calculated microscopically strong “bare” CQM vertices.
Next: Plug it into a CC integral equation (4-Dim type BS-eq) or most likely a 3-Dim reduction -a so called quasi-potential equation. Need also u-channel and t-channel “background” mechanisms. (see talk by Harry Lee, but also Pascalutsa-Tjon, Polinder-Rijken, Gross-Surya and many others...)
Results: (A) Vertices get dressed, (with FSI), this will likely modify baryon masses and widths. In addition there will be mixing (CC) effects, too.

Diagrammatic representation of renormalization eq.



The GRAZ GBE CQM is well suited for this program: it is an optimal choice for incorporating dressing via CC equations

This is because one dresses with the pion the “bare” baryon and not the quarks. Costituent quarks already have their physical mass (whatever that means).

Lowest order dressing: Not possible to represent this as single quark dressing. For classification of diagrams in the pion-3f system see: L. Canton, Phys. Rev. C **58** (1998), L. Canton, T. Melde, J. P. Svenne, Phys. Rev. C **63** (2001)