The Pion Form Factor: Theory

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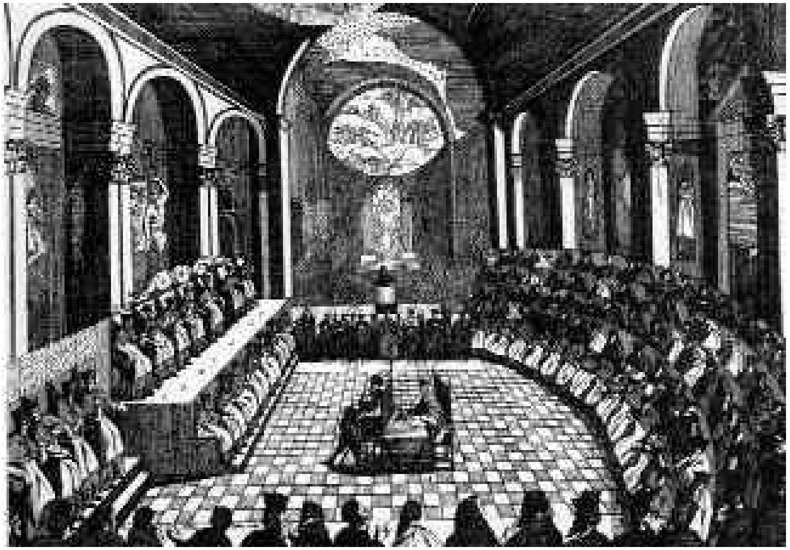

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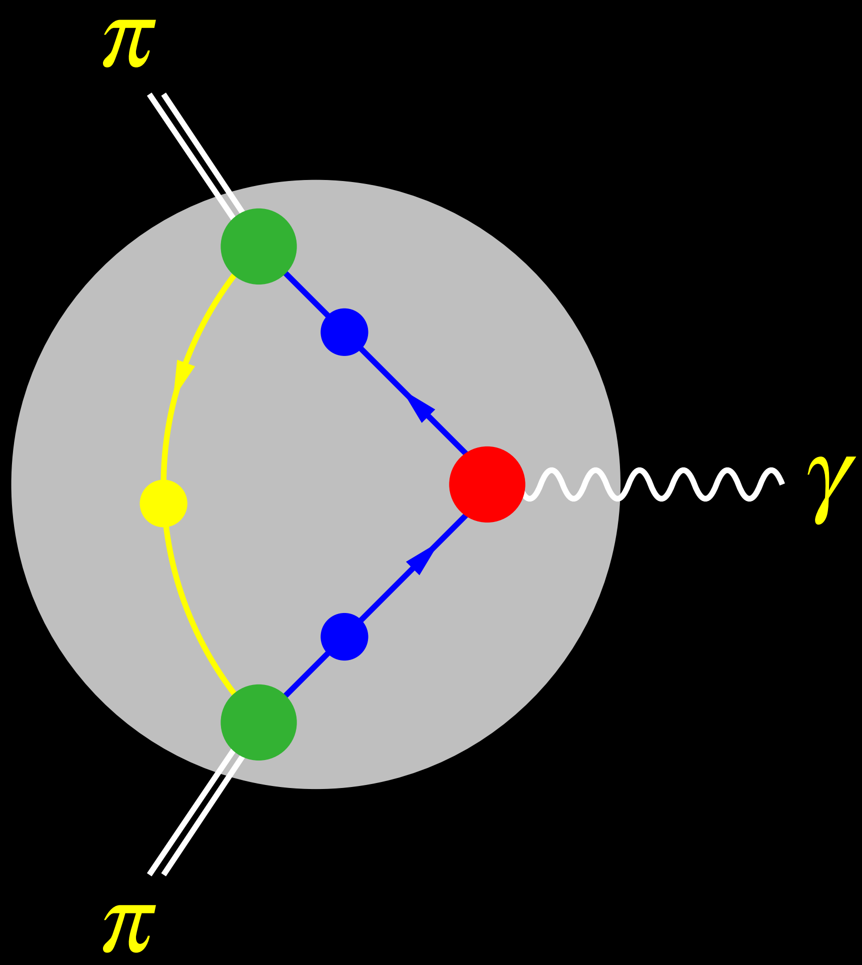
Pion electromagnetic form factor

$$\Lambda_\mu \;=\; (P' + P)_\mu \, F_\pi(Q^2) \quad = \quad N_c \int \frac{d^4q}{(2\pi)^4} \, \mathrm{Tr}\big[\bar{\Gamma}^\pi \, S \, i\Gamma_\mu \, S \, \Gamma^\pi \, S\big]$$



Point Mesons and Vector Meson Dominance

- $\Lambda^\mu = \langle \pi(P') | \hat{J}^\mu_{\rm em}(0) | \pi(P) \rangle = q_\pi (P' + P)^\mu \, F_\pi(Q^2)$
- 1960s phenomenology, VMD: $\hat{J}^\mu_{\rm em}(x) \sim \rho^\mu(x), \omega^\mu(x)$
- $\mathcal{L}^{\rm VMD}_{\rm int}(x) = -\frac{1}{4}\, \rho^{\mu\nu}(x)\, F_{\mu\nu}(x) + J^\mu_{\pi\pi}(x)\, A_\mu(x)$
- $F^{\rm VMD}_\pi(q^2) = F^{\rm VMD}_{\rm core}(q^2) - g_{\rho\pi\pi}\, \left[\frac{1}{q^2 - m^2_\rho + i\, m_\rho\, \Gamma_\rho(q^2)}\right] \frac{q^2}{g_\rho}$
- $F^{\rm VMD}_{\rm core}(q^2) = 1$, (non-$\rho$ degrees of freedom)
- Universal vector coupling: $g_{\rho\pi\pi} \sim g_\rho$
- $r^2_\pi = -6 F'_\pi(0) \sim \frac{6}{m^2_\rho} \frac{g_{\rho\pi\pi}}{g_\rho} \to \frac{6}{m^2_\rho} = 0.44~{\rm GeV}^2$
- It works, but $\cdots$

VMD and role of meson fields

- $F_\pi^{\rm VMD}(q^2) = [F_{\rm core}^{\rm VMD}(q^2) = 1] - g_{\rho\pi\pi}\ [\frac{1}{q^2 - m_\rho^2 + i\, m_\rho\, \Gamma_\rho(q^2)}]\ \frac{q^2}{g_\rho}$
- Should be only part of the story
- Away from the resonance region $q^2 \neq m_\rho^2$: $g_{\rho\pi\pi} \to ?$ $f_\rho \equiv \frac{m_\rho^2}{g_\rho} \to ?$
- Propagator of a bound state (ρ) away from its mass-shell ?

QCD Modeling:

- In general: $F_{\rm H}(q^2) = F_{\rm core}(q^2) + F_{\rm res}(q^2)$
- For IR: both relevant; for UV: $F_{\rm core}(q^2)$ should dominate
- For $F_\pi(Q^2)$, the π/resonant degree of freedom plays a dual role.
- Quark-gluon dynamics underlies both core and non-core; DSE-based approach
- Model dependent separation: DSE approach $\Rightarrow$ quark-gluon core, VMD(ρ), π loop $\sim$ 45%, 40%, 15% of r_π^2

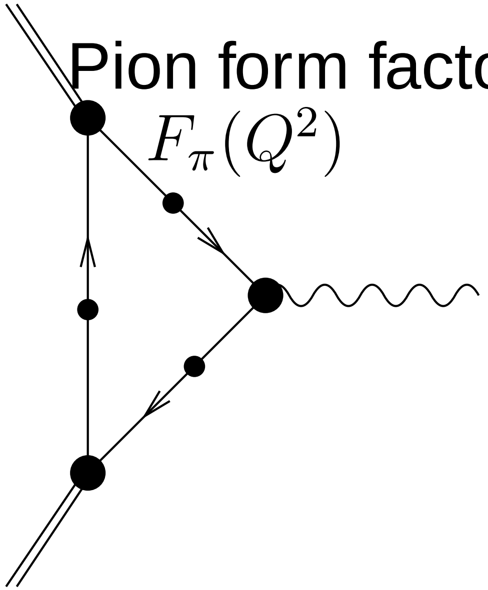
Theory Menu for $F_\pi(Q^2)$

- Constituent quark models
- Dispersion relations
- QCD Sum Rules: $\langle 0|T J_{\rm H}(x) J_{\rm em}(0) J_{\rm H}(y)|0\rangle$
- Chiral Perturbation Theory (low powers of Q^2)
- Light front quark models
- Quark-hadron duality
- AdS/CFT correspondence
- Covariant models based on QCD's Dyson-Schwinger Eqns of Motion
- Lattice-QCD

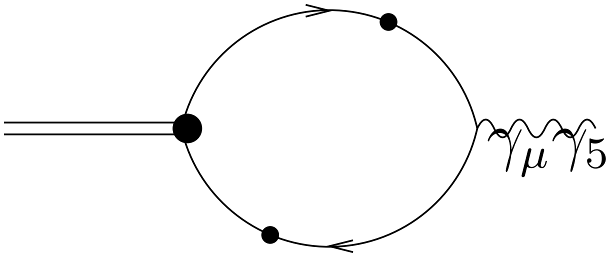
To Calculate Meson Observables

Pion form factor

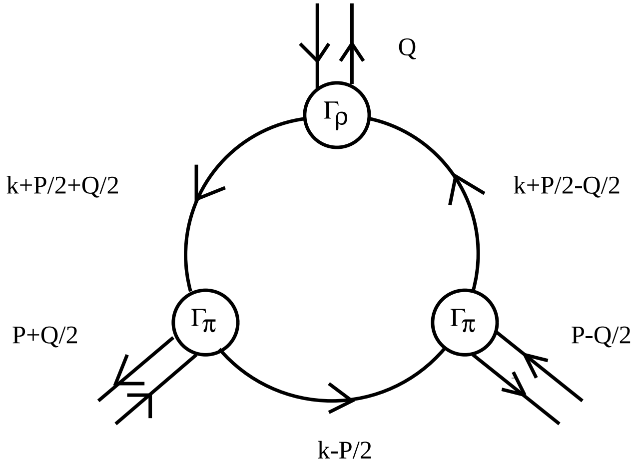
$F_\pi(Q^2)$



decay constant f_π



Strong decay $\rho \rightarrow \pi\pi$



Lattice-QCD and DSE-based modeling

- Lattice: $\langle \mathcal{O} \rangle = \int \; D\bar{q}qG \;\; \mathcal{O}(\bar{q}, q, G) \;\; e^{-\mathcal{S}[\bar{q},q,G]}$
  - Euclidean metric, x-space, Monte-Carlo
  - Issues: lattice spacing and vol, sea and valence m_q, fermion Det
  - Large time limit $\Rightarrow$ nearest hadronic mass pole

- EOMs (DSEs): $0 = \int \; D\bar{q}qG \;\; \frac{\delta}{\delta q(x)} \; e^{-\mathcal{S}[\bar{q},q,G]+(\bar{\eta},q)+(\bar{q},\eta)+(J,G)}$
  - Euclidean metric, p-space, continuum integral eqns
  - Issues: truncation and phenomenology
  - Analtyic contin. $\Rightarrow$ nearest hadronic mass pole

Guidelines for DSE-based modeling

- Soft physics: truncate DSEs to min: 2-pt, 3-pt fns
- Should be relativistically covariant—-convenient for decays, Form Factors, etc
  - No boosts needed on wavefns of recoiling bound st.
  - ∞ d.o.f. $\rightarrow$ few quasi-particle effective d.o.f.
- Do not make a 3-dimensional reduction
- Preserve 1-loop QCD renorm group behavior in UV
- Preserve global symmetries, conserved currents, etc
- Especially preserve chiral symm/Goldstone's Thm
- Can't preserve local color gauge covariance—-just choose Landau gauge
- Parameterize the deep infrared (large distance) QCD coupling

Conserved Vector Current

- Vector: $\Delta\mathcal{L}(x) \propto \partial_\mu J^\mu(x) = \partial_\mu \bar{q}(x)\gamma^\mu q(x)$
  - $\langle 0|T\{\partial_\mu J^\mu(z) q(x)\bar{q}(y) + \delta\, q(x)\bar{q}(y)\}|0\rangle = 0$
  - $iP_\mu \Gamma_\mu(k;P) = S^{-1}(k_+) - S^{-1}(k_-)$
- $\Rightarrow$ kernel of DSE for $S^{-1}(k)$ and $K_{\rm BSE}$ for $\Gamma_\mu(k;P)$ are related
- Quark dressing (not bare) will require a consistently dressed photon-quark vertex
- Any deficiency in electromagnetic structure of hadrons will then not be due to individual quarks!
- $\Rightarrow F_\pi(Q^2 = 0) = Q_\pi = 1$, independent of model details

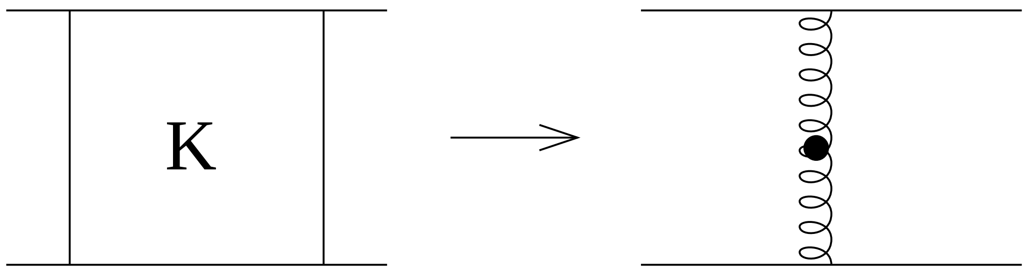
Partially Conserved Axial-vector Current

- Axial vector (flav non-singlet): $J_5^\mu(x) = \bar{q}(x)\gamma^\mu\gamma_5\tau q(x)$
  - $\Delta\mathcal{L}(x) \propto \partial_\mu J_5^\mu(z) - 2m_q J_5(z)$
  - $-iP_\mu\Gamma_{5\mu}(k;P) = S^{-1}(k_+)\gamma_5\frac{\tau}{2} + \gamma_5\frac{\tau}{2}S^{-1}(k_-)$ $-2\,m_q(\mu)\,\Gamma_5(k;P)$
- Independent of model details:
  - DCSB: $\mathcal{L}(m_q = 0)$, but $M_q(p^2) \neq 0$ from dynamics
  - Goldstone Theorem preserved: $m_\pi(m_q = 0) = 0$
  - At physical m_q: ps octet masses good if $\langle\bar{q}q\rangle$ is good
- DCSB $\Rightarrow \pi$: $\Gamma_\pi^0(p^2) = \frac{i\gamma_5}{f_\pi^0}\ \left[\frac{1}{4}\,\mathrm{tr}\,S_0^{-1}(p^2)\right] + \cdots$

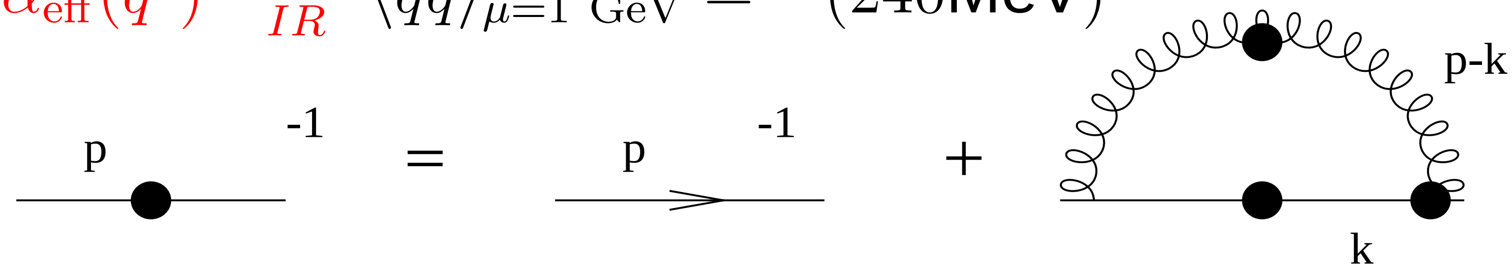
Application to Form Factors

- Manifest covariance—convenient for form factors
- Rainbow DSE, ladder BSE, and IA for $F_\pi(Q^2)$ are symm-matched set
- $\Rightarrow F_\pi(Q^2 = 0) = Q_\pi = 1$, model indep
- $\Rightarrow$ leading asymptotic $F_\pi(Q^2)$ phys content present
- A systematic symm-preserving correction scheme is available

Ladder-Rainbow Model



- $K_{\rm BSE} \to -\gamma_\mu \frac{\lambda^a}{2} \; 4\pi\alpha_{\rm eff}(q^2) \; D_{\mu\nu}^{\rm free}(q) \; \gamma_\nu \frac{\lambda^a}{2}$
- $\alpha_{\rm eff}(q^2) \;\; \underset{UV}{\longrightarrow} \;\; \alpha_s^{1-{\rm loop}}(q^2)$
- $\alpha_{\rm eff}(q^2) \;\; \underset{IR}{\longrightarrow} \;\; \langle \bar{q}q \rangle_{\mu=1\ {\rm GeV}} = -(240\text{MeV})^3$



- P. Maris & P.C. Tandy, PRC60, 055214 (1999)
  $M_\rho, M_\phi, M_{K^\star}$ good to 5%, $f_\rho, f_\phi, f_{K^\star}$ good to 10%

Flavor nonsinglet axial-vector Ward-Takahashi Id

$$-iP_\mu \Gamma_{5\mu}(k;P) = S^{-1}(k_+)\gamma_5 \frac{\tau}{2} + \gamma_5 \frac{\tau}{2} S^{-1}(k_-) - 2\, m_q(\mu)\, \Gamma_5(k;P)$$

$$\nwarrow \; \frac{f_\pi\, P_\mu}{P^2+m_\pi{}^2}\Gamma_\pi \qquad\qquad \frac{-ir_P}{P^2+m_\pi{}^2}\Gamma_\pi \; \nearrow$$

$$S^{-1}(p) = i\not{p} A(p^2) + B(p^2), \qquad \Gamma_\pi(q;P) = \tau\gamma_5\; iE_\pi(q,P) + \cdots$$

- Chiral limit PS pole analysis ($m_\pi = 0$)

  $f_\pi^0\, E_\pi(k;P=0) = B_0(k^2)$, DCSB, Goldstone Thm

- $m_q(\mu) \neq 0$ pole analysis

  $m_\pi^2\; f_\pi = 2\, m_q(\mu)\; r_P(m_q)$

- $m_q \to 0 \Rightarrow$ Gell-Mann–Oakes–Renner:

  $r_P(m_q \to 0) \to -\frac{\langle \bar{q}q\rangle_\mu^0}{f_\pi^0} = \frac{N_c\, Z_4}{f_\pi^0}\, \mathrm{tr_s} \int_q^\Lambda\; S_0(q)$

(Maris, Roberts, Tandy, PLB420 (1998) 267)

Meson Bethe-Salpeter Amplitudes

- General form for pseudoscalar mesons: 4 inv ampls

$$\Gamma(q_+, q_-) = \gamma_5 \left[iE + \not{P}\, F + \not{k}\, G + \sigma_{\mu\nu}\, P_\mu q_\nu\, H \right]$$

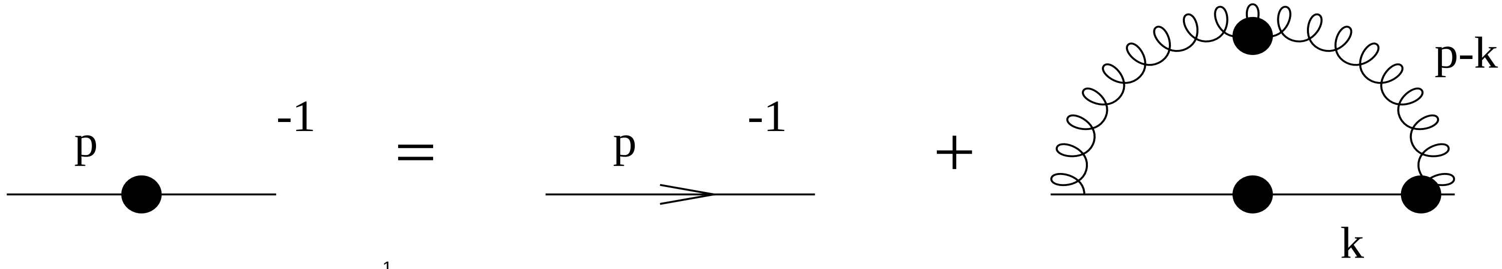
- General form for vector mesons: 8 inv ampls

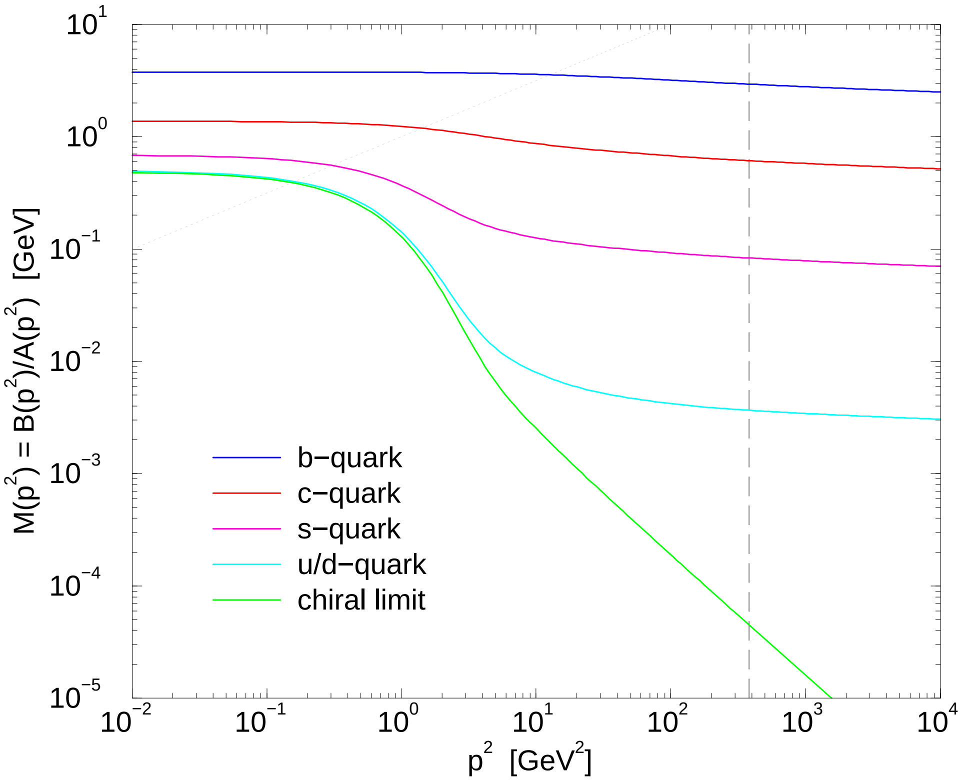
$$\Gamma_\nu(q_+, q_-) = T_{\nu\mu}(P) \big\{ \gamma_\mu\, V_1 + q_\mu\, \not{q}\, V_2 + q_\mu\, \not{P}\, V_3 +$$
$$\gamma_5 \epsilon_{\mu\alpha\nu\beta} \gamma_\alpha q_\nu P_\beta\, V_4 + q_\mu\, V_5 + \sigma_{\mu\nu} q_\nu\, V_6 + \sigma_{\mu\nu} P_\nu\, V_7 + q_\mu \sigma_{\alpha\beta} q_\alpha P_\beta\, V_8 \big\}$$

summary

Quark mass functions from DSE solutions

- Interaction adds mass to both quarks as well as providing binding





Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle^0_\mu$	(0.236 GeV)^3	(0.241†)^3
m_π	0.1385 GeV	0.138†
f_π	0.0924 GeV	0.093†
m_K	0.496 GeV	0.497†
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	7.6 $\cdot 10^6$ s^{-1}	7.38
$\Gamma(K_{\mu 3})$	5.2 $\cdot 10^6$ s^{-1}	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
$m_{K^\star}$	0.892 GeV	0.936
$f_{K^\star}$	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^\star K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^\star K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^\star K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

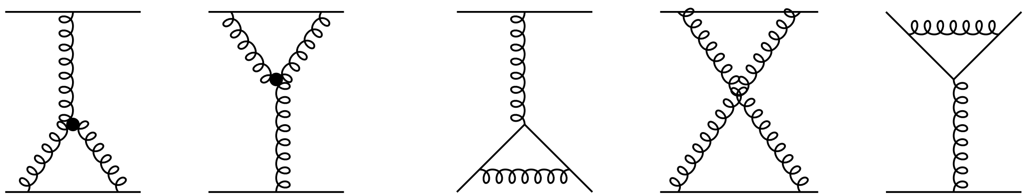
a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

bsampl

From Gluon vertex to BSE Kernel

- $K_{\mathrm{BSE}}(x',y';x,y) = -\frac{\delta}{\delta S(x,y)}\Sigma(x',y')$
- Vertex $\Gamma_\mu(p,q) = \sum$ diagrams $\Rightarrow K_{\mathrm{BSE}} = \sum$ diagrams
- If Σ contains:

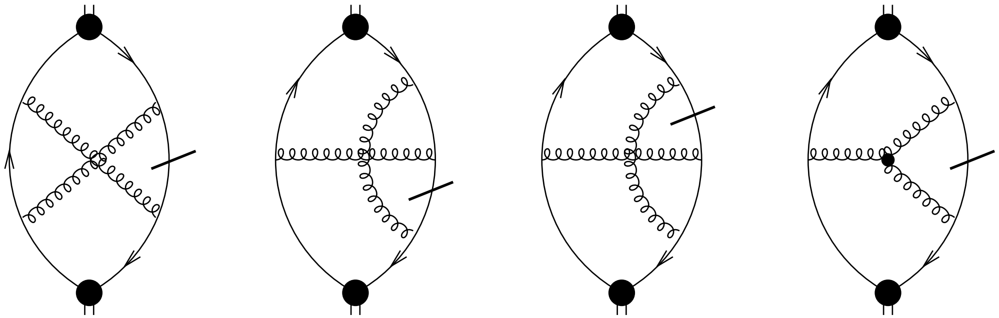
- K_{BSE} contains:



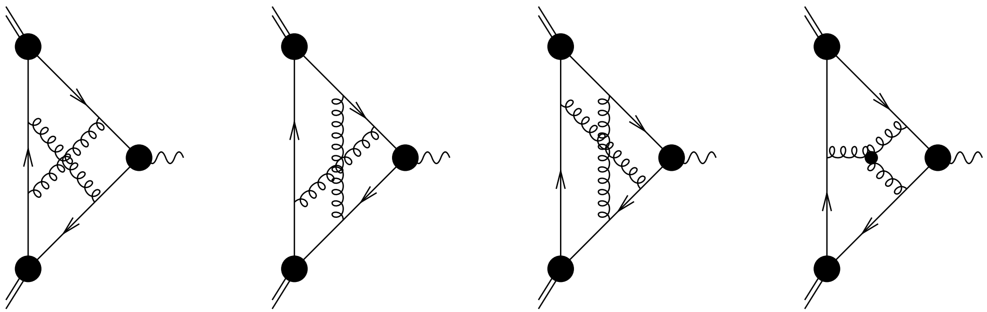
- Axial vector and vector WTIs (color singlet) preserved, if all q props are self-consistently dressed (Munczek 95); Goldstone Thm preserved
- Independent of model parameters. Model does not fight chiral symmetry, use light vector mesons to fix parameters

Beyond ladder-rainbow ⇒ beyond IA

Corrections to the ladd-rainb truncation ⇒ corresp corrections to the BSE norm condition

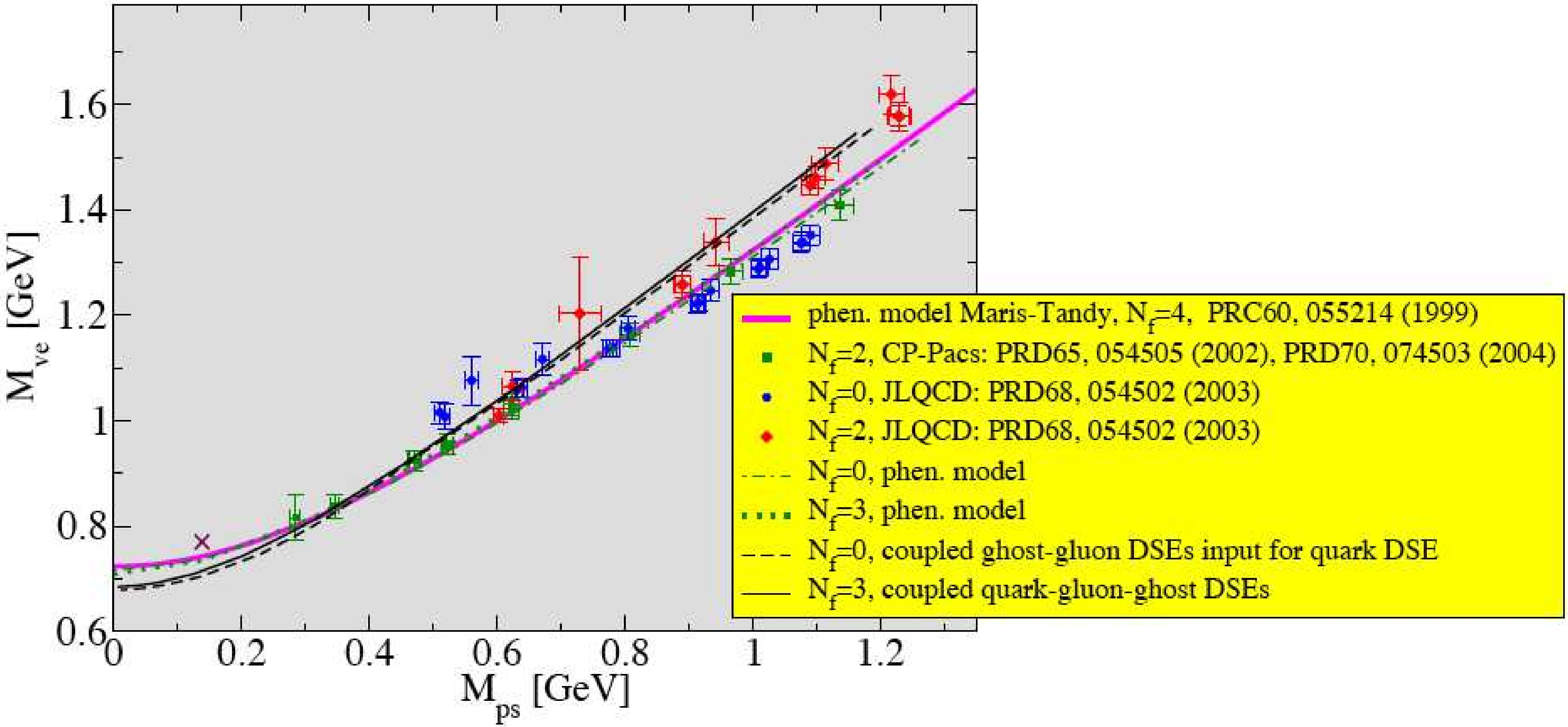


⇒ (via vector Ward Id) corresp corrections to IA for $F_\pi(Q^2 = 0)$:



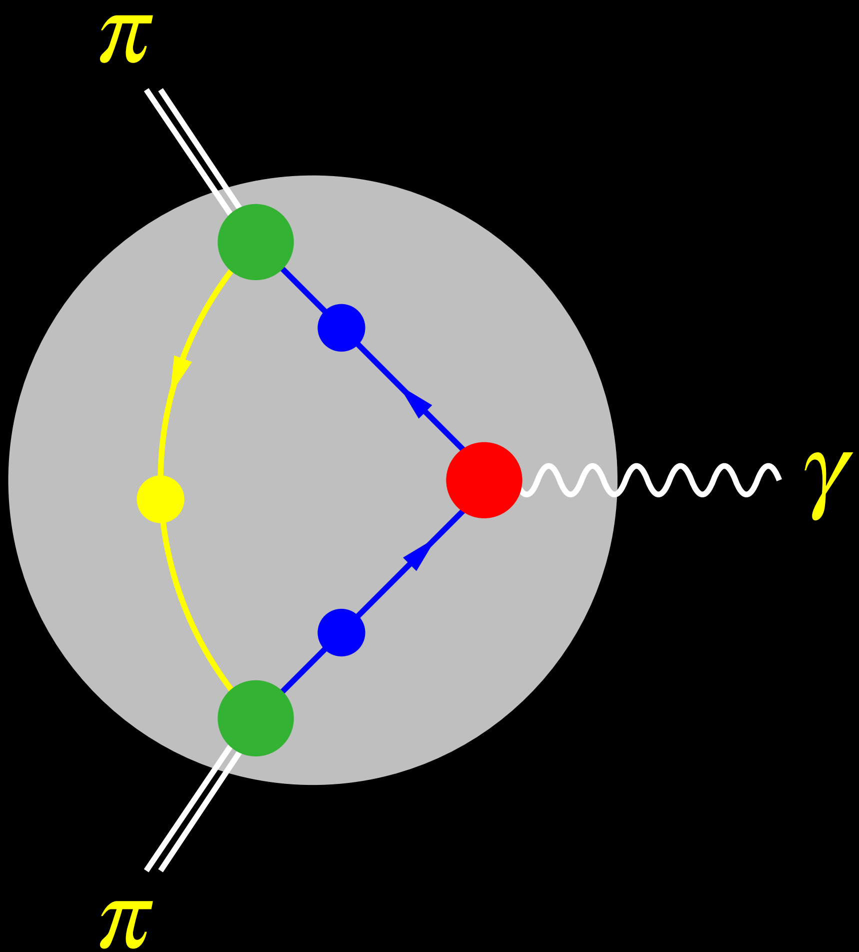
and so on $\cdots$. Different organizations may absorb some into a wavefn.

DSE and Lattice results for M_V and M_{ps}



Pion electromagnetic form factor

$$\Lambda_\mu \;=\; (P' + P)_\mu \, F_\pi(Q^2) \quad = \quad N_c \int \frac{d^4q}{(2\pi)^4} \, \mathrm{Tr}\big[\bar{\Gamma}^\pi \, S \, i\Gamma_\mu \, S \, \Gamma^\pi \, S\big]$$

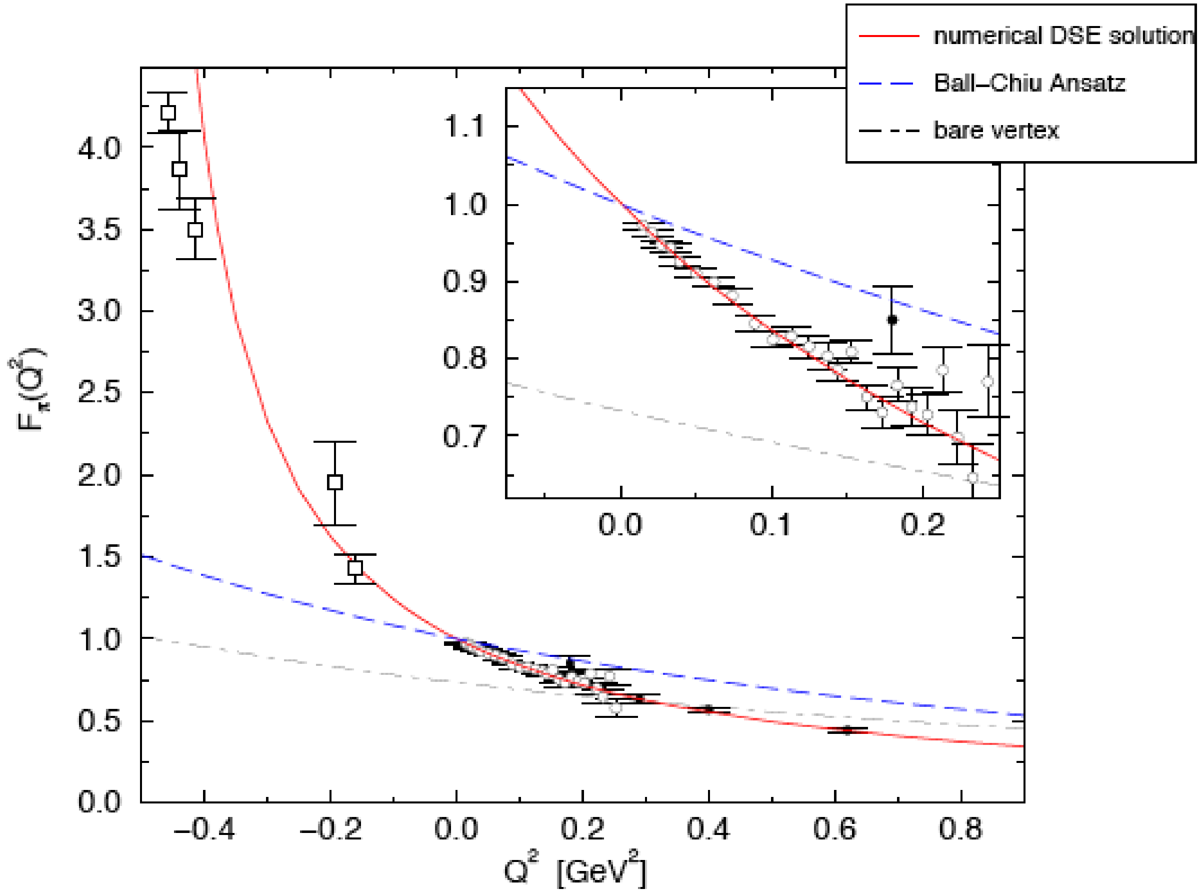


Pion $F(Q^2)$: Low Q^2

(P Maris & PCT, PRC 61, 045202 (2000)

(P. Maris & PCT, PRC 62, 0555204 (2000)

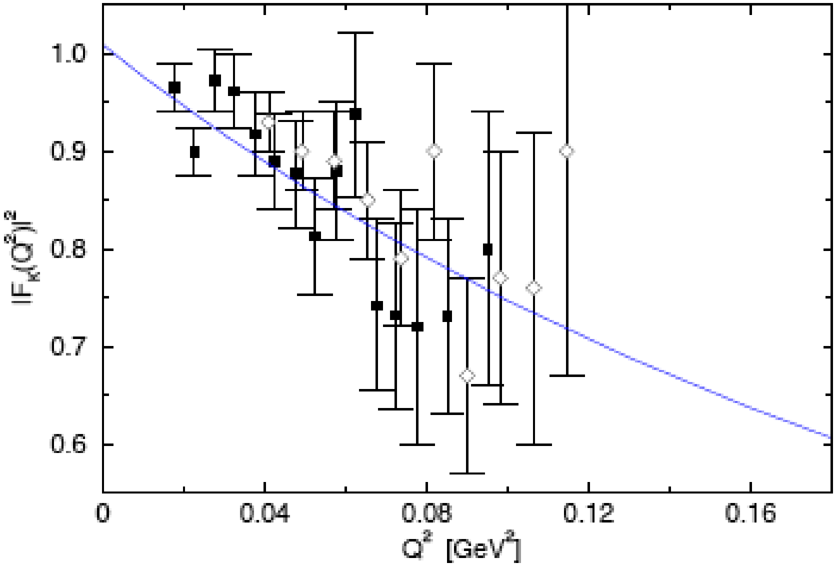
$r_\pi^{\rm DSE} = 0.68$ fm $\qquad r_\pi^{\rm expt} = 0.663 \pm .006$ fm



Kaon $F(Q^2)$: Low Q^2

- Impulse approx + rainbow/ladder $\Rightarrow$

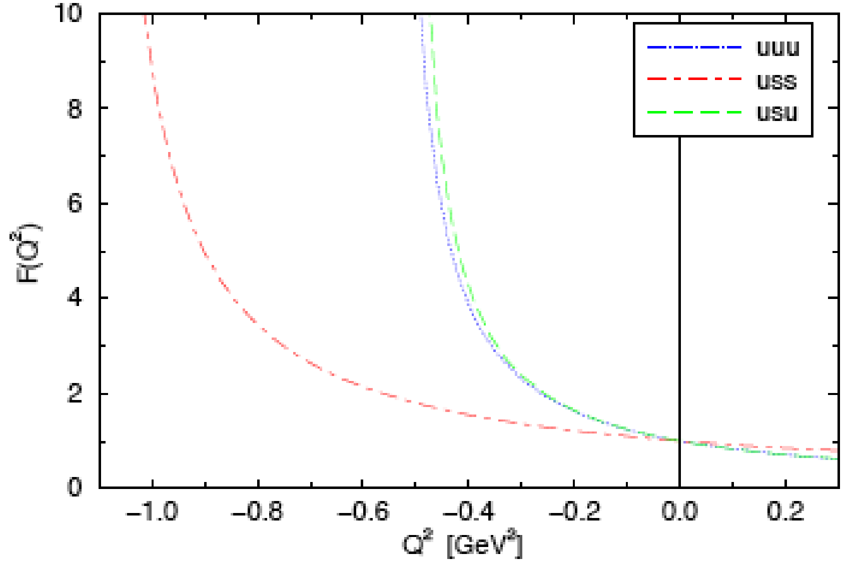
  conserved em current, correct charge of K^+ and K^0



charge radii	experiment	DSE calc
r^2_π	0.44 ± 0.01 fm^2	0.45 fm^2
r^2_{K+}	0.34 ± 0.05 fm^2	0.38 fm^2
$r^2_{K^0}$	-0.054 ± 0.026 fm^2	-0.086 fm^2

Pion and Kaon $F(Q^2)$: Timelike

Behavior in time-like region



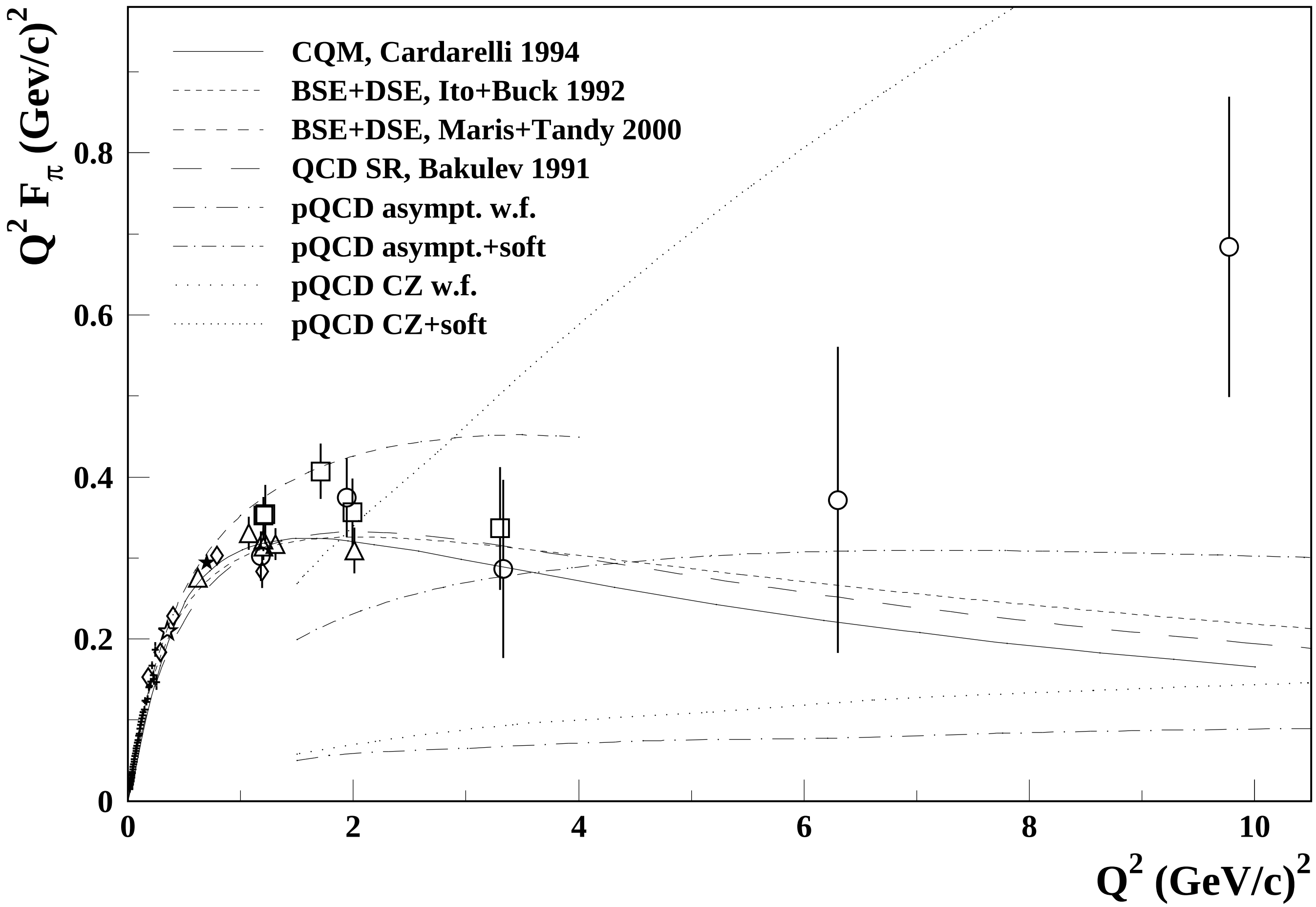
- Can be used to extract strong decay coupling constants

$$F(Q^2) \sim \frac{m_\rho^2}{g_\rho} \frac{g_{\rho\pi\pi}}{m_\rho^2 + Q^2}$$

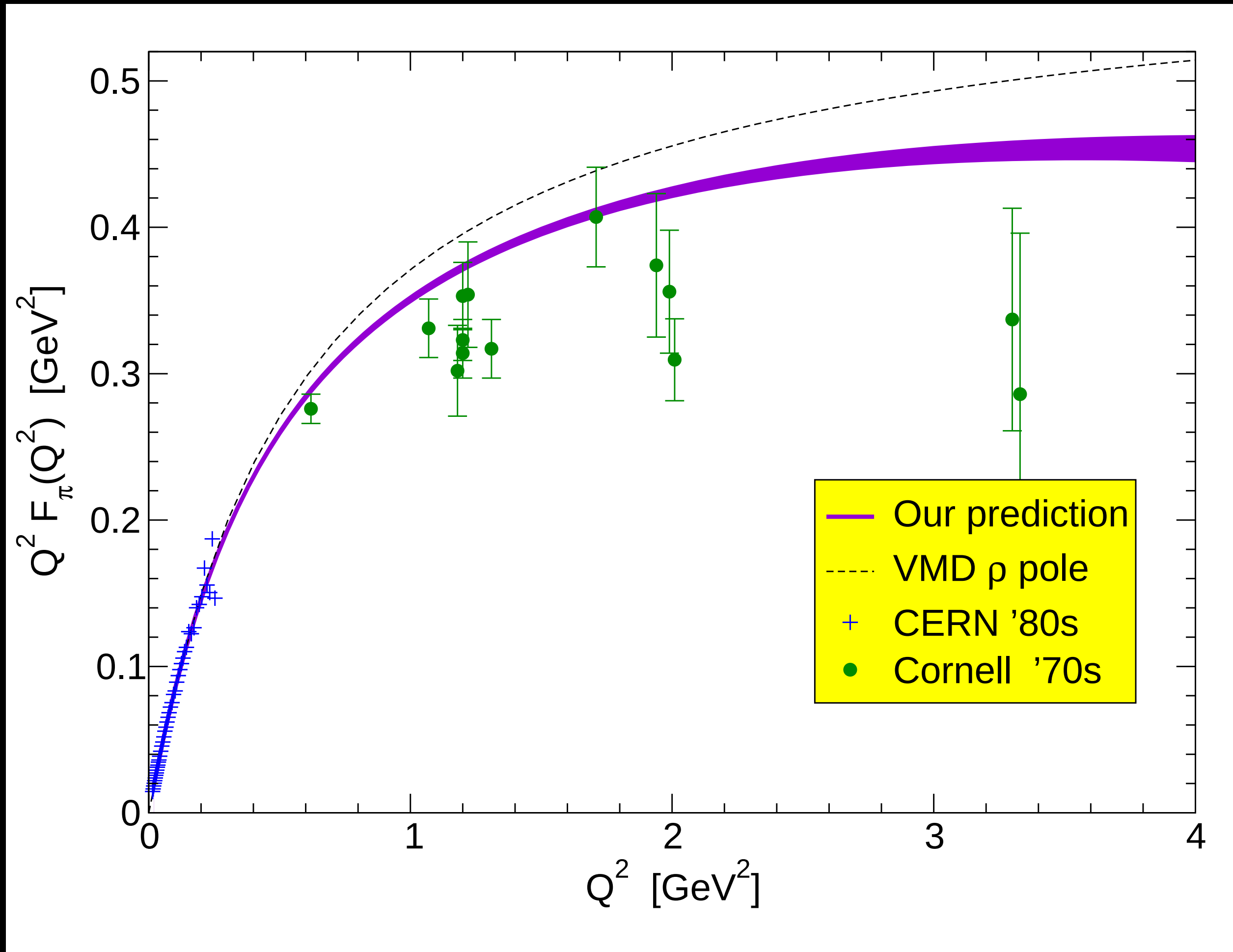
	experiment	calc.	dir calc.
$g_{\rho\pi\pi}$	$6.02 \pm .05$	5.2	5.14
$g_{\phi KK}$	$4.64 \pm .14$	4.3	4.25

Pion Charge Form Factor Pre-2000

- Jochen Volmer, PhD thesis, Oct 3, 2000, Vrije U, Amsterdam.

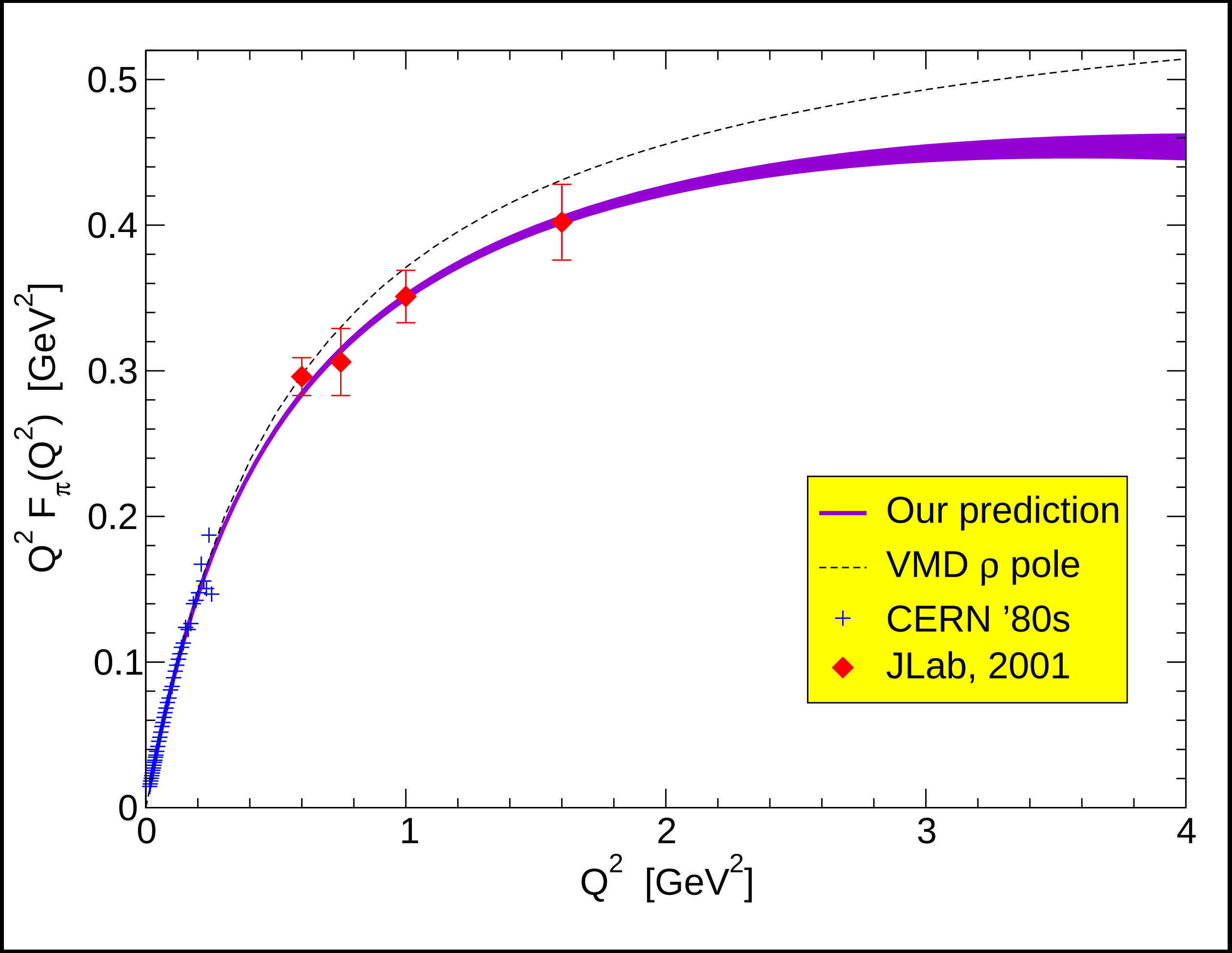


Pion electromagnetic form factor



PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

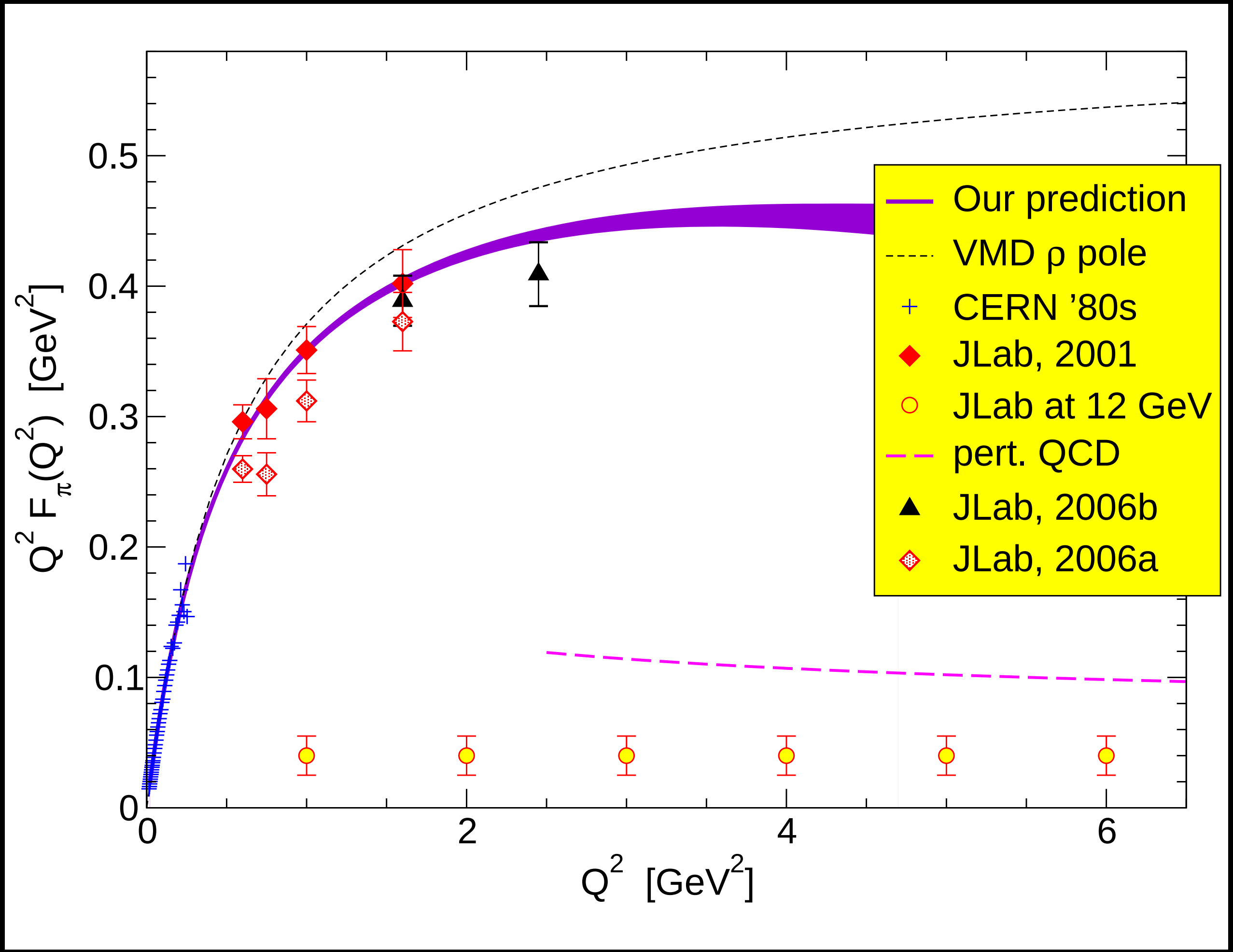
Pion electromagnetic form factor



JLab data from Volmer *et al*, PRL86, 1713 (2001) [nucl-ex/0010009]

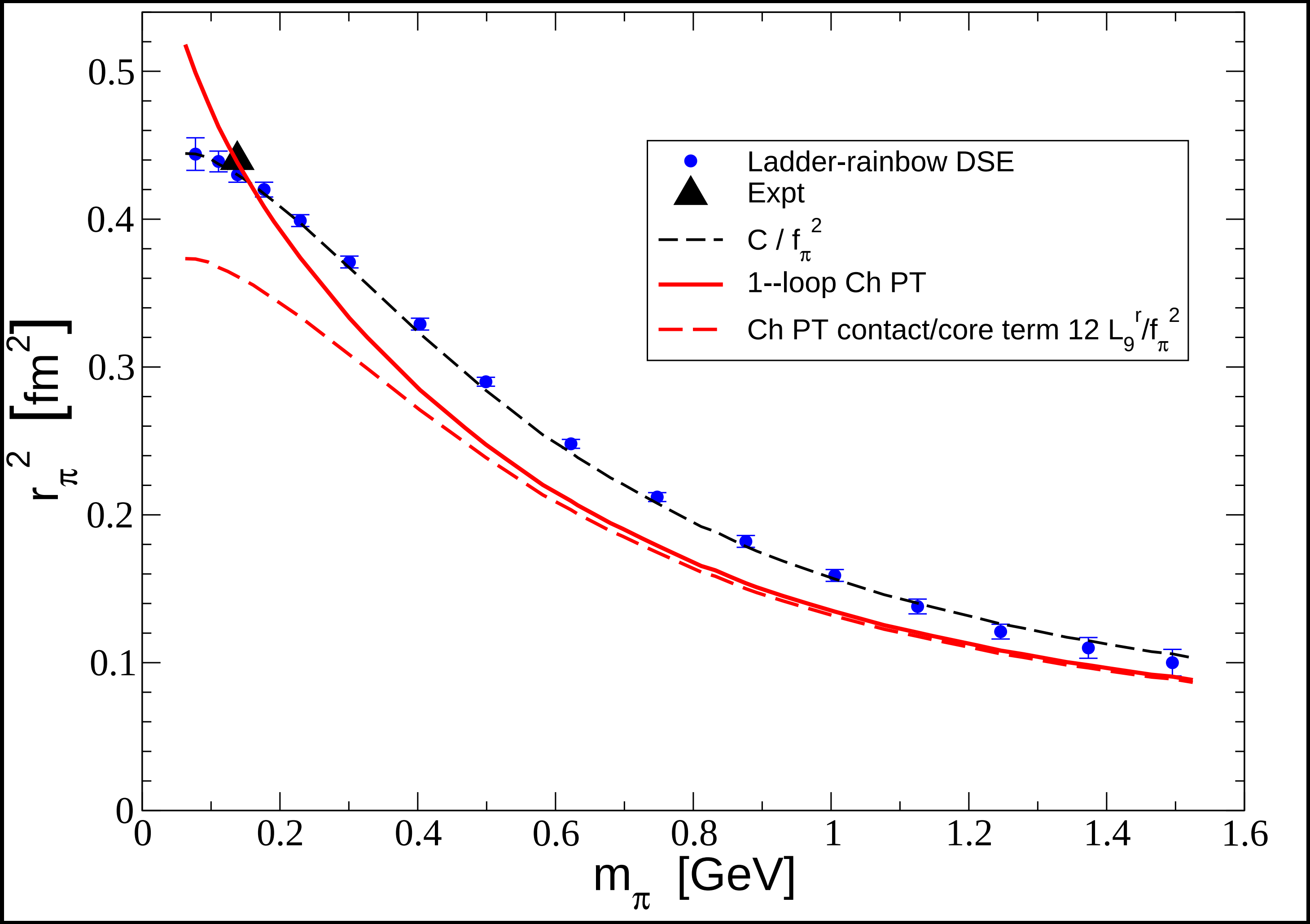
PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]

Pion electromagnetic form factor



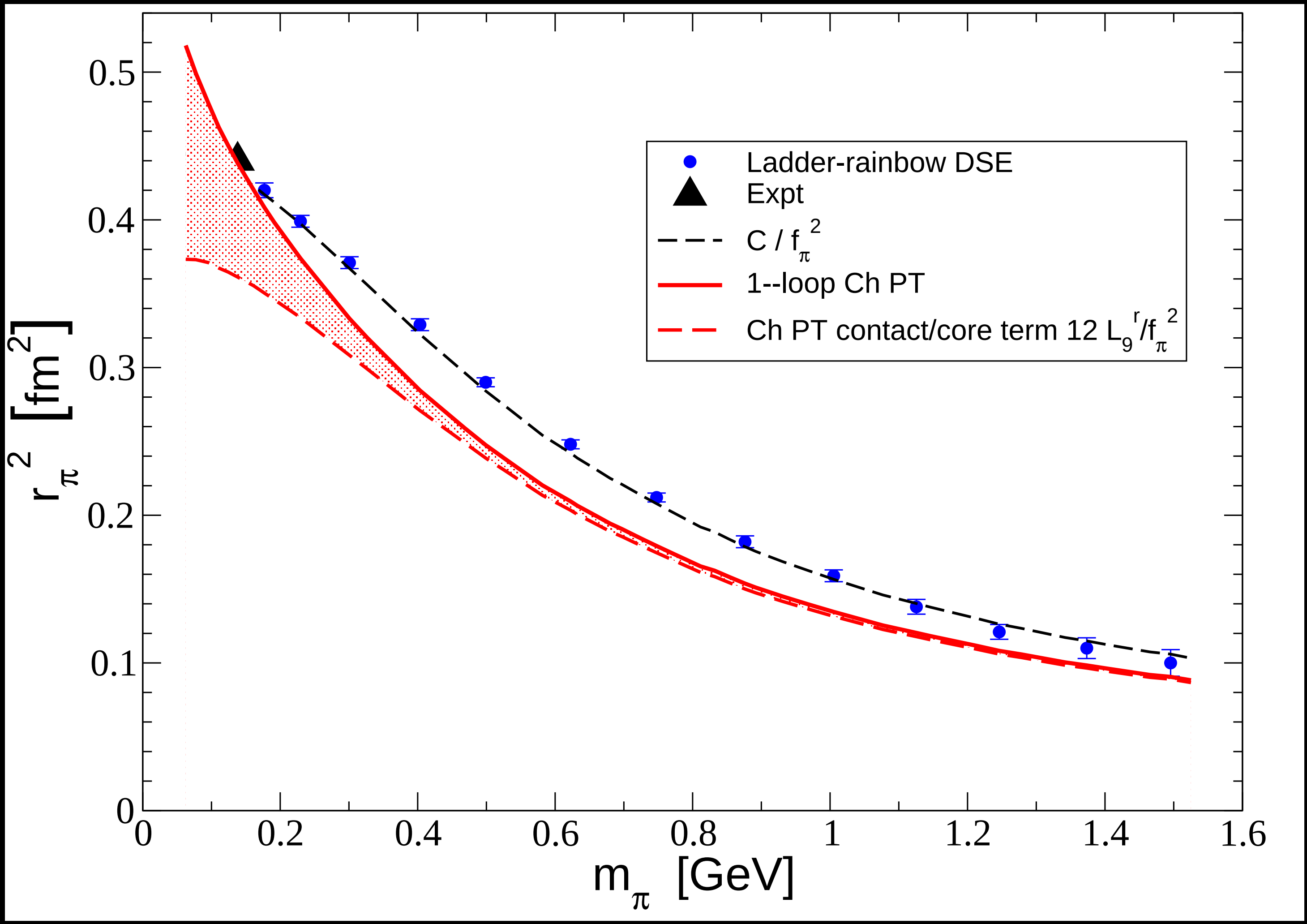
PM and Tandy, PRC62,055204 (2000) [nucl-th/0005015]
2006a: V. Tadevosyan *et al*, [nucl-ex/0607007], 2006b: T. Horn *et al*, [nucl-ex/0607005]

1-loop chiral correction to r_π vs m_π



P. Maris and PCT, in preparation

1-loop chiral correction to r_π vs m_π

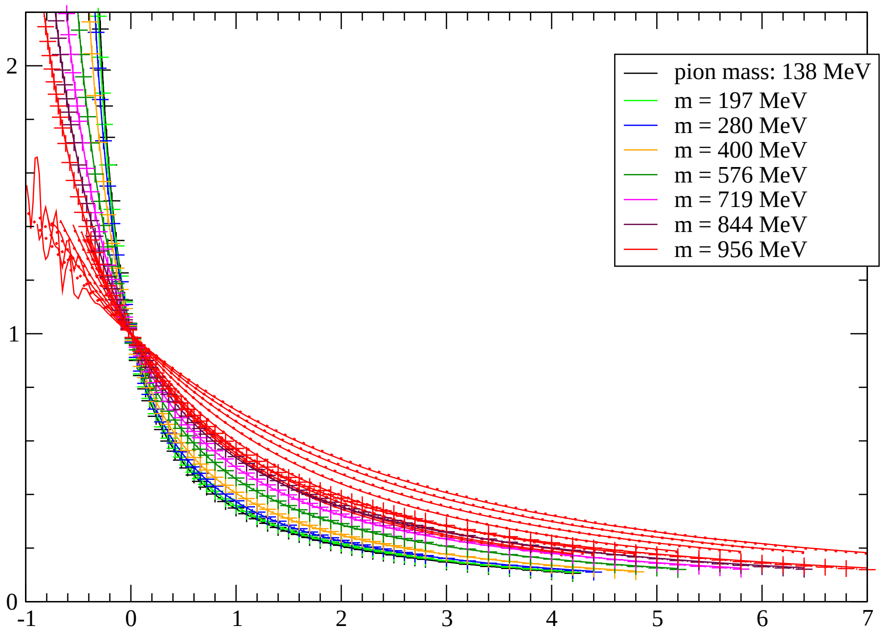


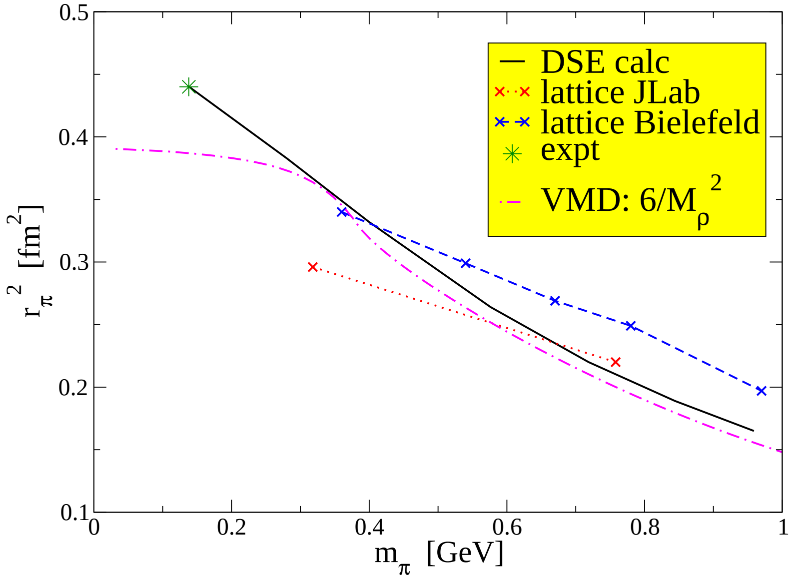
P. Maris and PCT, in preparation

m_q dependence of pion EM form factor and radius

- Maris and PCT, Light-Cone 2005, nucl-th/0511017
- Ladder-rainbow—no chiral loops, yet

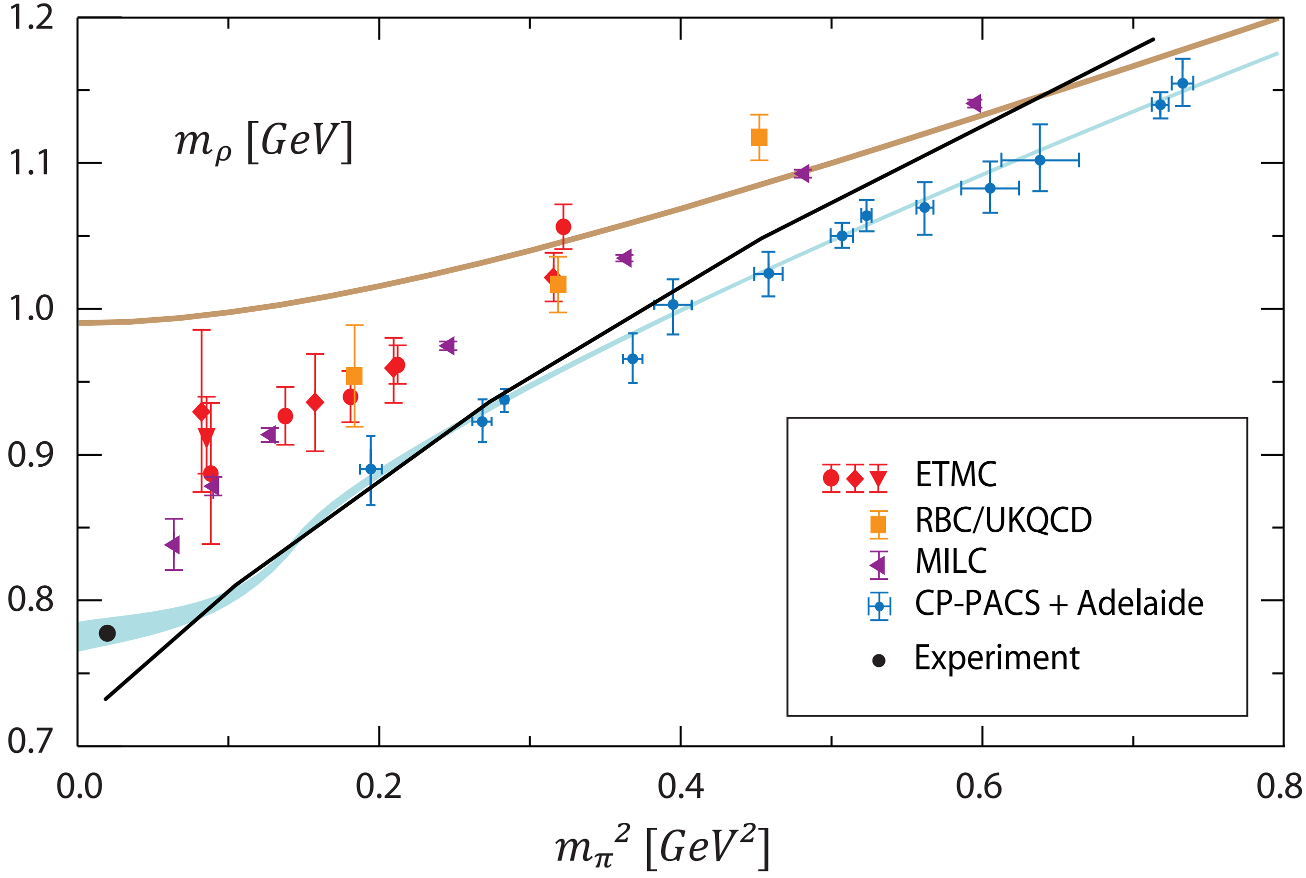
$F(Q^2)$: colored: best calc. ; red: using BC Ansatz





- Also have $M_{\rm ps}(m_q), f_{\rm ps}(m_q)$ for qq, uq, sq mesons.

Leave Room for Corrections to LR DSE



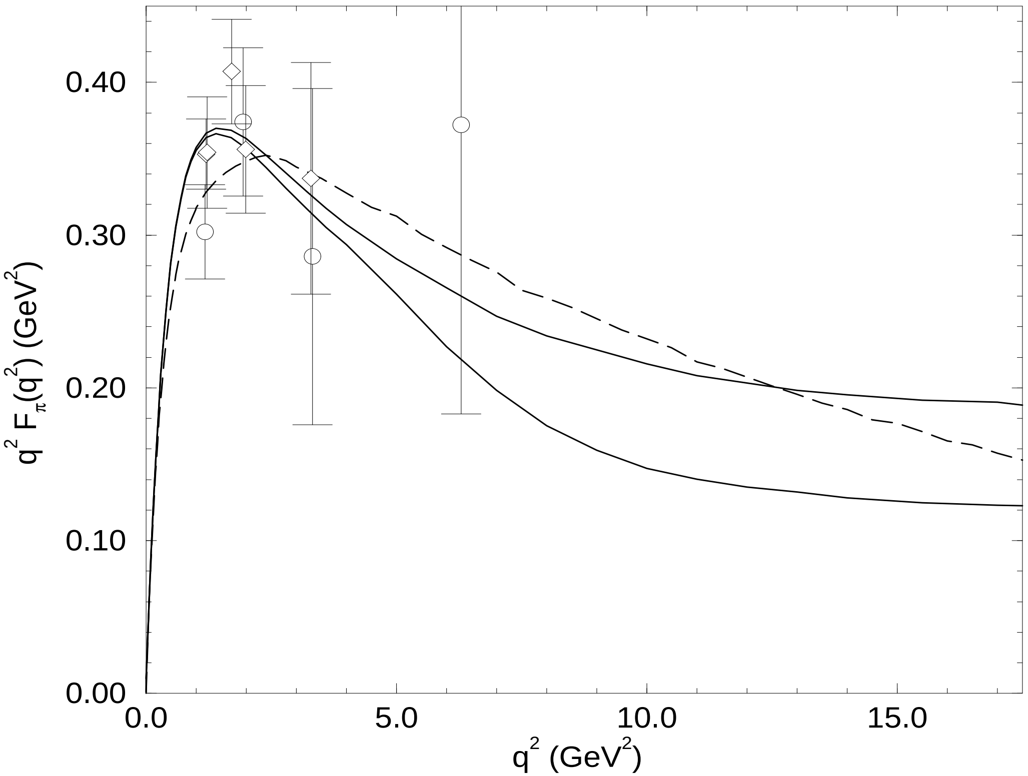
Asymptotic Limit of $F_\pi(Q^2)$

$$Q^2\ F_\pi(Q^2) \to 8\pi\alpha_s(Q^2) f_\pi^2 \ \ ; \qquad f_\pi = 131\ \mathrm{MeV}$$

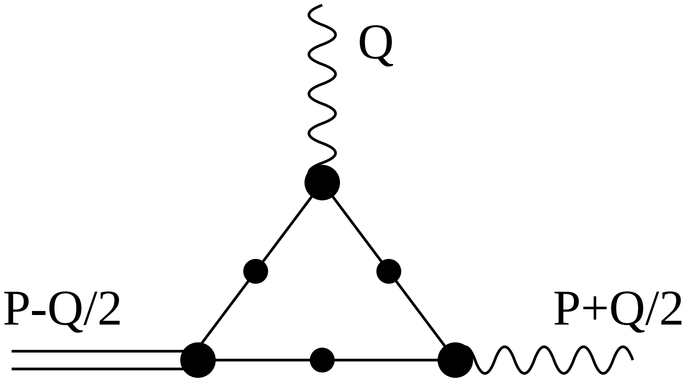
- Limit $\sim$ 0.15 at 10 GeV^2
- Farrar and Jackson, PRL (1979); LePage and Brodsky, PLB (1979)
- Infinite momentum frame, factorization of hard and soft components
- Hard to connect to this via a numerical approach having dynamical quark propagator ampls

Indication of Asymptotic Scale for $F_\pi(Q^2)$

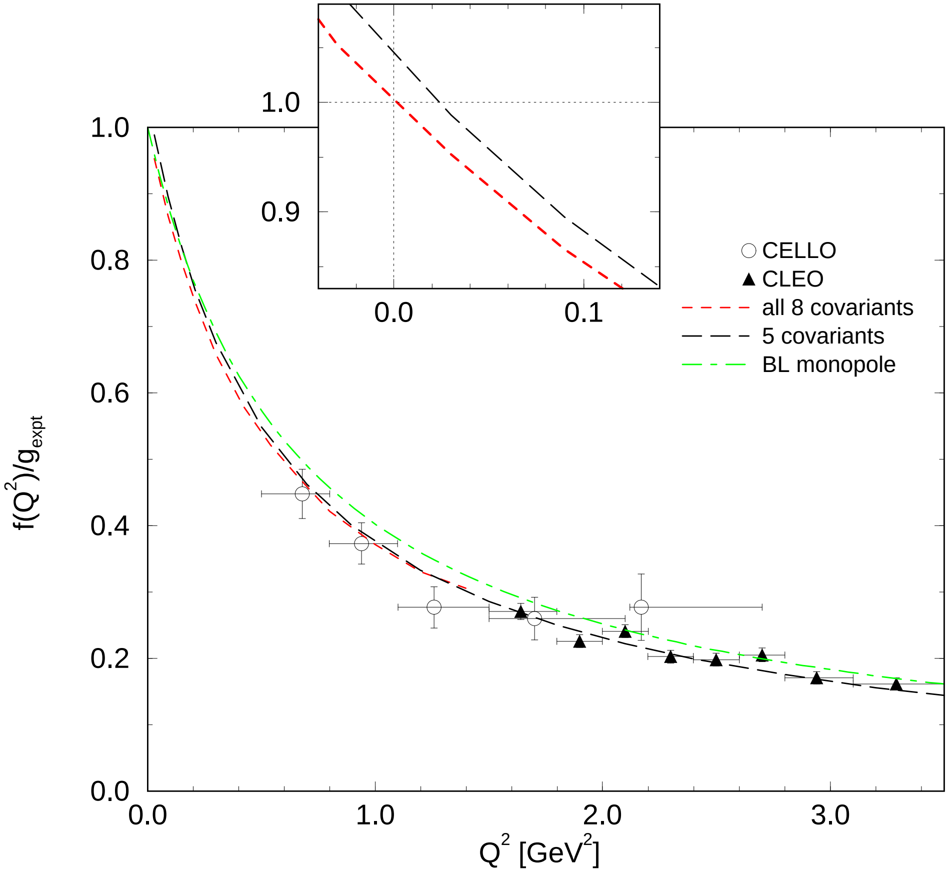
- Semi-analytic DSE approach. $\Gamma_\pi(k,P) = \gamma_5 \left[iE + \not{P}\, F + \not{k}\, G + \sigma_{\mu\nu}\, P_\mu q_\nu\, H \right]$
- P. Maris, C.D. Roberts, Phys. Rev. C58, 3659 (1998)
- Amplitudes $F(k,P)$ and $G(k,P)$ dominate $F_\pi(Q^2)$ at large Q^2



$\gamma^{\star}\pi^{0} \rightarrow \gamma$ *Transition Form Factor*

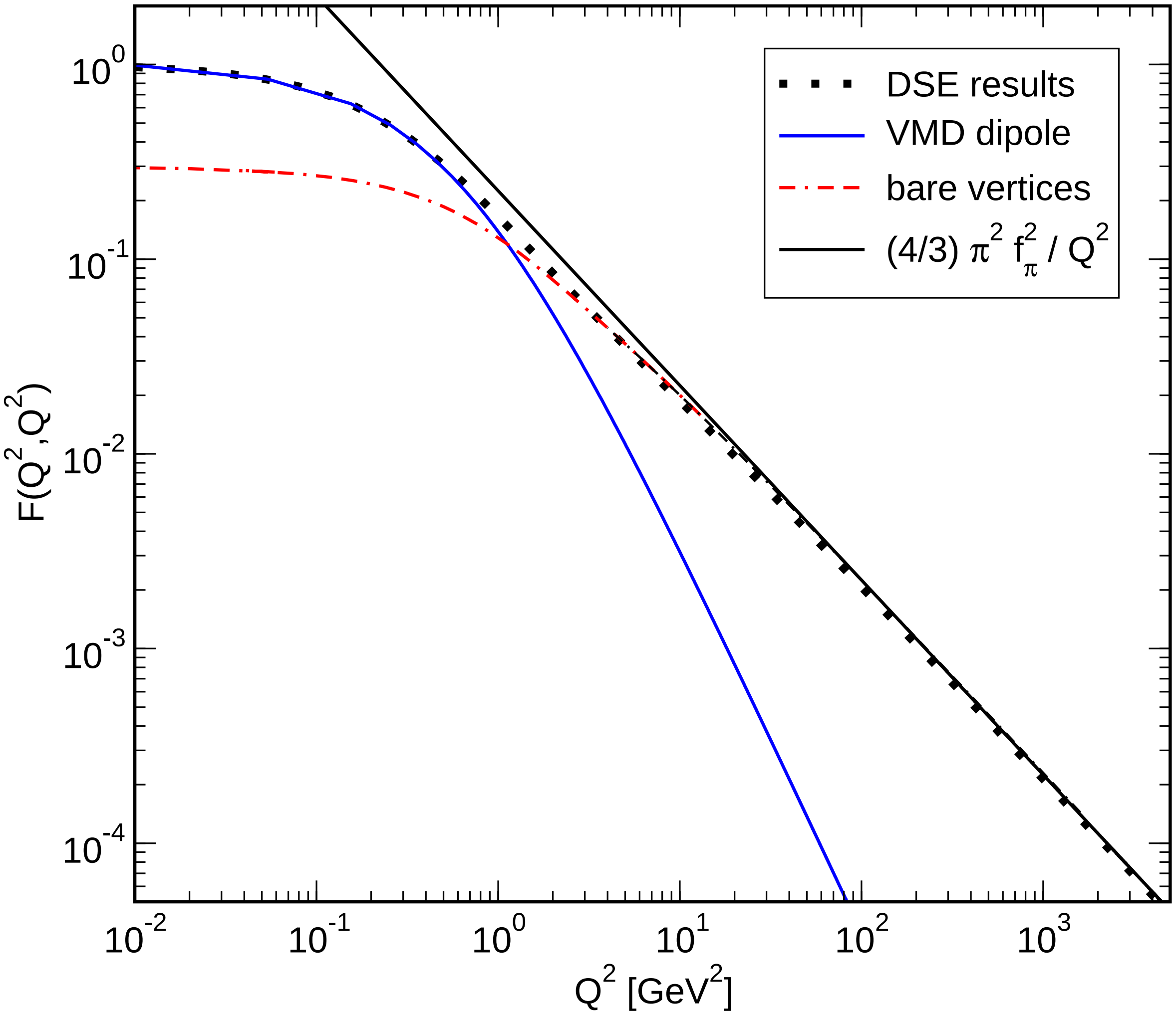


- Abelian axial anomaly $+\ \pi$ pole in $\Gamma_{5\mu} \Rightarrow G(0,0)$
- Chiral limit $G(0,0) = \dfrac{1}{2}$ $\Rightarrow \Gamma_{\pi\gamma\gamma}$ to 2%



$\gamma^\star\pi\gamma^\star$ *Asymptotic Limit*

Lepage and Brodsky, PRD22, 2157 (1980): LC-QCD/OPE $\Rightarrow$



Collaborators

- Craig Roberts, Argonne National Lab
- Pieter Maris, Iowa State University

Thankyou!