Skyrme model investigation of an anomalous two−photon effect in electron nucleon scattering

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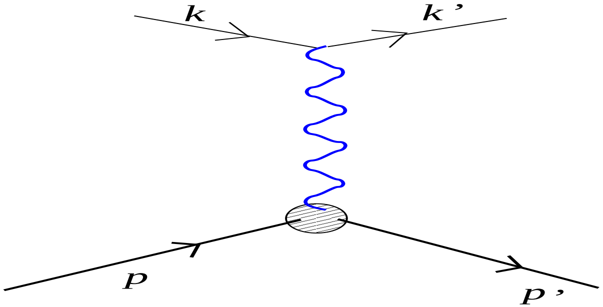
Hadron Electromagnetic Form Factors

Based on: M. Kuhn, HW, arxiv:0804.3334 [nucl–th]

I) Introduction

⋆ kinematics of electron–nucleon scattering



$$q = k - k' = p' - p$$

$$\tau = -\frac{q^2}{4M^2} = \frac{Q^2}{4M^2}$$

$$\nu = = \tfrac{1}{4}(k + k') \cdot (p + p')$$

⋆ photon polarization parameter:

$$\epsilon = \frac{\nu^2 - M^4\tau(1+\tau)}{\nu^2 + M^4\tau(1+\tau)}$$

⋆ photon nucleon coupling (form factors):

$$\langle N(\vec{p}\,')|J_\mu|N(\vec{p})\rangle = \overline{U}(\vec{p}\,')\left[\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M}F_2(Q^2)\right]U(\vec{p})\,,$$

⋆ elastic unpolarized scattering:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} \frac{\epsilon\, G_E^2(Q^2) + \tau\, G_M^2(Q^2)}{\epsilon(1+\tau)}$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)\ , \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

⋆ Rosenbluth method: plot numerator as function of $\epsilon \in [0,1]$
extract $G_E(Q^2)$ and $G_M(Q^2)$
from slope and intercept

⋆ result consistent with

$$R(Q^2) = \frac{\mu_p G_E(Q^2)}{G_M(Q^2)} \approx 1 \qquad \mu_p = G_M(0)$$

reproduced with new data (super Rosenbluth)

⋆ polarization method

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}}\frac{G_E(Q^2)}{G_M(Q^2)}$$

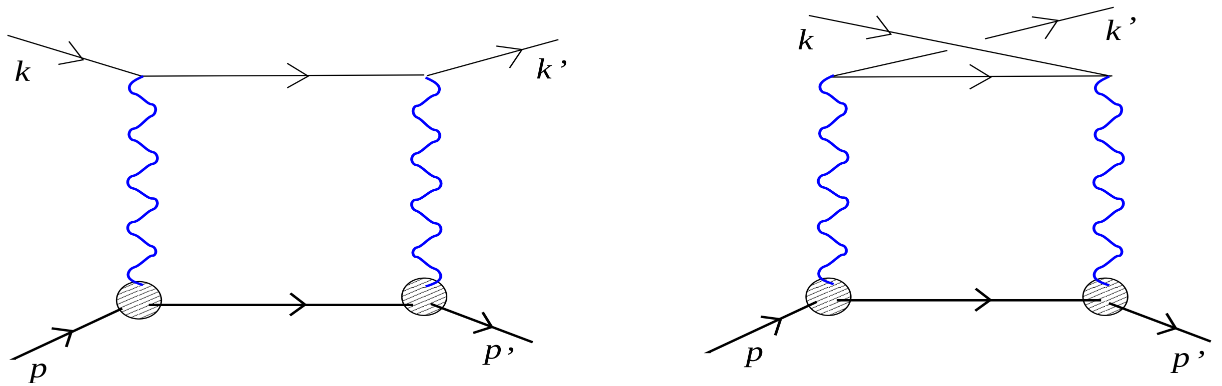
yields

$$R(Q^2) \approx 1 - 0.13\left(Q^2[\mathrm{GeV}]^2 - 0.04\right)$$

possible zero at $Q^2 \approx 8\mathrm{GeV}^2$

⋆ different methods yield different results for a fundamental quantity **?**

⋆ possible two–photon contaminations



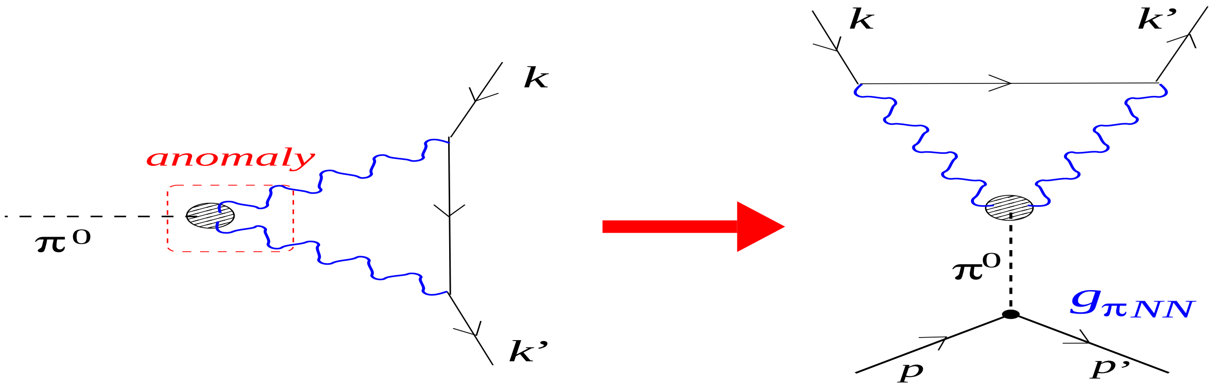
Afanasev, Brodsky, Blunden, Carlson, Chen, Jain, Melnichouk, Tomasi–Gustafsson, Vanderhaeghen, ...

- requires off–shell form factors
- many intermediate states
- some constraints from GDPs and/or dispersion relations
- deviation from linear ϵ dependence in unpolarized cross section

⋆ helps up to about 3GeV^2

⋆ possible additional anomalous contribution: $\pi^0 \to \gamma\gamma \to e^+e^-$

(Savage, Luke & Wise; Dorokhov & Ivanov, ...)



plus cross terms and η's in SU(3)

⋆ $\mathcal{L}_{\pi\gamma\gamma} \sim \epsilon^{\mu\nu\rho\sigma}\partial_\mu\pi^0 A_\nu \partial_\rho A_\sigma$ and axial type

$\Longrightarrow$ $M_{fi} \propto m_e$ *i. e.* potentially small

⋆ anomaly also induces multiple (odd) pion exchanges
$\vec{\tau}\cdot\partial_\mu\vec{\pi} \to U^\dagger\partial_\mu U \neq$ total derivative couplings, may be sizable

II) The model (recent review: HW Springer Lecture Notes **743**)

- ⋆ Skyrme model as a simple tool to study anomalous nucleon coupling to two photons: contained in the (gauged) Wess–Zumino term
- ⋆ chiral field as non–linear representation of the pion field

$$U = \exp\left[i\frac{\vec{\pi}\cdot\vec{\tau}}{f_\pi}\right]$$

- ⋆ most simple extension of the non–linear σ–model

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{1}{32e_{\mathrm{Sk}}^2}\mathrm{tr}\left(\left[\partial_\mu U, \partial_\nu U^\dagger\right]\left[\partial^\mu U, \partial^\nu U^\dagger\right]\right) + \frac{f_\pi^2 m_\pi^2}{4}\mathrm{tr}\left(U + U^\dagger - 2\right)$$

$$f_\pi = 93\mathrm{MeV} \qquad m_\pi = 138\mathrm{MeV} \qquad e_{\mathrm{Sk}} \approx 4$$

- ⋆ fourth order Skyrme term allows (hedgehog) soliton solution

$$U_0(\vec{r}) = \exp\left[i\vec{\tau}\cdot\hat{r}F(r)\right]$$

⋆ topological structure induces baryon number

$$F(0)=\pi \quad \text{and} \quad F(\infty)=0 \qquad \Longrightarrow \qquad B=1$$

solve classical equation of motion for $F(r)$

⋆ collective coordinate quantization to generate nucleon states

$$U(\vec{r},t)=A(t)U_0(\vec{r})A^\dagger(t)$$

$$\langle A|J=I,t,s\rangle=\left[\frac{2J+1}{8\pi^2}\right]^{1/2} D^{J=I}_{t,s}(A)$$

⋆ spin operator $\quad \vec{J}=\alpha^2[U_0]\,\mathrm{tr}\left[(-i)\vec{\tau}A^\dagger \frac{dA}{dt}\right]$

⋆ isospin operator $\quad I_a=-D_{ab}J_b \qquad D_{ab}=\frac{1}{2}\,\mathrm{tr}\left[\tau_a A\tau_b A^\dagger\right]$

⋆ nucleon matrix elements

$$\langle s,t|D_{ab}|s',t'\rangle \;=\; -\tfrac{4}{3}\langle s,t|I_aJ_b|s',t'\rangle$$

$$\langle s,t|D_{3a}D_{3b}|s',t'\rangle \;=\; \tfrac{1}{3}\langle s,t|\delta_{ab}|s',t'\rangle$$

⋆ photon interaction via covariant derivative

$$\partial_\mu U \longrightarrow D_\mu U = \partial_\mu U - ieA_\mu \left[\hat{Q}, U\right] \qquad \hat{Q} = \tau_3/2 + \mathbb{1}$$

$$\mathcal{L}_{\mathrm{nl}\sigma}^{(\mathrm{gauged})} = \frac{f_\pi^2}{4}\mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) - i\frac{f_\pi^2 e}{2}A_\mu \mathrm{tr}\left(\hat{Q}\left[U^\dagger \partial^\mu U + U\partial^\mu U^\dagger\right]\right)$$

$$-\frac{f_\pi^2 e^2}{4}A_\mu A^\mu\, \mathrm{tr}\left(\left[\hat{Q}, U\right]\left[\hat{Q}, U^\dagger\right]\right)$$

⋆ induces local two–photon interactions

⋆ Wess–Zumino term has no classical contribution (in $SU(2)$) requires trial and error procedure to achieve gauge invariance (Witten, Schechter *et al.*)

$$\mathcal{L}_{\mathrm{WZ}}^{\mathrm{gauged}} = \frac{e}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\Big\{A_\mu \mathrm{tr}\left(\hat{Q}\left[U^\dagger\partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U - U\partial_\nu U^\dagger U \partial_\rho U^\dagger U \partial_\sigma U^\dagger\right]\right)$$

$$+ieA_\mu \partial_\nu A_\rho \mathrm{tr}\left(2\hat{Q}^2\left[U^\dagger\partial_\sigma U - U\partial_\sigma U^\dagger\right] + \hat{Q}\partial_\sigma U \hat{Q} U^\dagger - \hat{Q} U \hat{Q}\partial_\sigma U^\dagger\right)\Big\}$$

⋆ nucleon form factors

$$\mathcal{L}^{\text{gauged}} \sim \mathcal{L}^{(0)} + eA_\mu J^\mu + e^2 A_\mu A^\mu S + e^2 \epsilon^{\mu\nu\rho\sigma} A_\mu \partial_\nu A_\rho W_\sigma$$

$$J^\mu \quad \longrightarrow \quad G_E(Q^2) \quad \text{and} \quad G_M(Q^2)$$

$$\langle N(\vec{p}\,')|S|N(\vec{p})\rangle = \overline{U}(\vec{p}\,')S(Q^2)U(\vec{p})$$

$$\langle N(\vec{p}\,')|W_\mu|N(\vec{p})\rangle = \overline{U}(\vec{p}\,')\left[\gamma_\mu F_A(Q^2) + q_\mu F_p(Q^2) + i\sigma_{\mu\nu}q^\nu F_E(Q^2)\right]\gamma_5 U(\vec{p})$$

⋆ model calculation in the Breit frame

$$E = p_0 = p_0' = \sqrt{M^2 + Q^2/4} \qquad \vec{p} = -\vec{p}\,' = \vec{q}/2$$

$$\langle N(\vec{p}\,')|J^0(0)|N(\vec{p})\rangle = 2MG_E(Q^2)\langle s_3'|s_3\rangle \quad \langle N(\vec{p}\,')|J^i(0)|N(\vec{p})\rangle = -2iG_M(Q^2)\epsilon^{ijk}q^j\langle s_3'|S_k|s_3\rangle$$

$$\langle N(\vec{p}\,')|W_i(0)|N(\vec{p})\rangle = \pm\frac{2}{9\pi M}\langle s_3'|H_0(Q^2)S_i + H_2(Q^2)\left(S_i - 3\hat{q}_i\hat{q}\cdot\vec{S}\right)|s_3\rangle$$

$$\text{e. g.} \qquad H_0(Q^2) = M^2\int_0^\infty dr r^2\left[\frac{dF}{dr} + \frac{2}{r}\sin F\cos F\right]j_0(Qr)$$

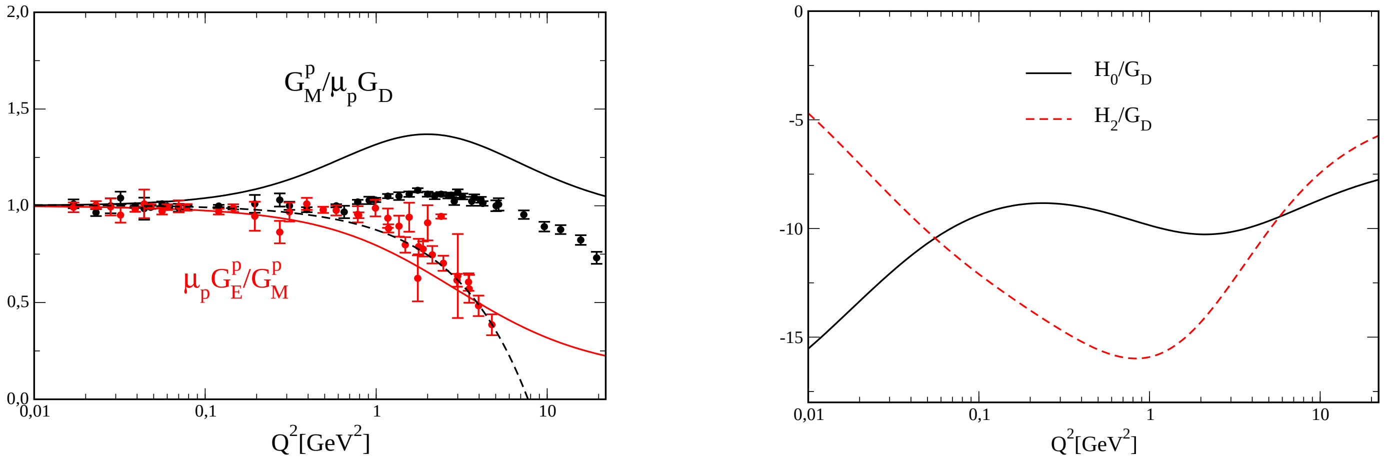
$$F_A(Q^2) = \pm\frac{1}{18\pi ME}\left[H_0(Q^2) + H_2(Q^2)\right]$$

⋆ relativizing by Lorentz boost (Ji, Holzwarth)
extension to larger momenta, generically

$$G(Q^2) \longrightarrow \gamma^{-2n} G\left(\frac{Q^2}{\gamma^2}\right) \qquad \gamma = \sqrt{1+\tau}$$

maps $[0, 4M^2] \mapsto [0, \infty]$ $\left(n = 2 \;\sim\; \text{cluster of three particles}\right)$

⋆ form factor results $\left(G_D(Q^2) = 1/\left(1 + Q^2/0.71\right)^2\right)$



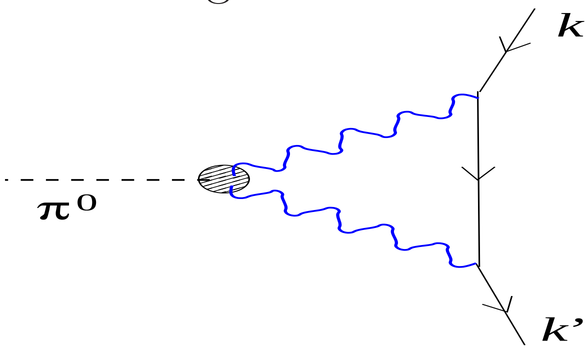
⋆ fine–tuning of the model (*e. g.* vector mesons) improves agreement

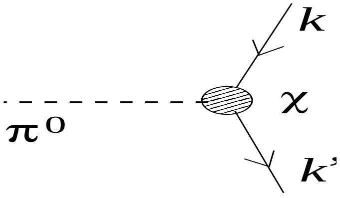
III) UV divergence and $\pi^0 \to e^+e^-$ decay

⋆ coefficient of WZ–term completely fixed

$\longrightarrow$ no tuneable parameter ?

⋆ Feynman diagram





is ultra–violet divergent and induces the

⋆ counterterm (Savage, Luke & Wise)

$$\mathcal{L}_{\rm c.t.} = \frac{i\alpha^2}{32\pi^2}\chi(\Lambda)\overline{\Psi}_e\gamma^\mu\gamma_5\Psi_e {\rm tr}\left(2\hat{Q}^2\left[U^\dagger\partial_\sigma U - U\partial_\sigma U^\dagger\right] + \hat{Q}\partial_\sigma U\hat{Q}U^\dagger - \hat{Q}U\hat{Q}\partial_\sigma U^\dagger\right)$$

(effective form factor for off–shell photon in the loop)

⋆ cancellation of divergence

$$\chi_{\rm fin}(\Lambda) = 6\left(\frac{4}{4-D} - \gamma + \ln 4\pi\right) - \chi(\Lambda)$$

⋆ renormalization condition $(\xi = m_\pi^2/4m_e^2)$

$$\frac{\Gamma(\pi^0 \to e^+e^-)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{\alpha^2}{2\pi^2}\frac{\sqrt{\xi^2-1}}{\xi^3}|A(\xi)|^2 = (6.3 \pm 0.5) \times 10^{-8}$$

$$\mathsf{Re}A(\xi) = \chi_{\rm fin}(\Lambda) - 6\ln\frac{m_\pi^2}{\Lambda^2} + \widetilde{A}(\xi) \qquad \mathsf{Im}A(\xi) = \frac{4\pi\xi}{\sqrt{\xi^2-1}}\ln\left(\xi + \sqrt{\xi^2-1}\right)$$

⋆ renormalization scale

$$\Lambda = 1\text{GeV} \quad \Longrightarrow \quad -24 < \chi_{\rm fin}(\Lambda) < -10$$

IV) Interference in unpolarized cross section

⋆ one photon exchange

$$M_\gamma = i\frac{4\pi\alpha}{q^2}\overline{u}(k')\gamma^\mu u(k)\overline{U}(\vec{p}\,')\left[\gamma_\mu G_M(Q^2) - \frac{1}{2M}\left(p_\mu + p'_\mu\right)F_2(Q^2)\right]U(\vec{p})$$

⋆ two photon WZ–term transition matrix element (axial type)

$$\begin{aligned} iM_{2\gamma}^{\mathrm{WZ}} &= -i\alpha^2\overline{u}(k')\left[w_1(Q^2)k'^\mu + w_2(Q^2)q^\mu + w_3(Q^2)\gamma^\mu\right]\gamma_5 u(k) \\ &\quad \times\overline{U}(\vec{p}\,')\left[\gamma_\mu F_A(Q^2) + q_\mu F_p(Q^2) + \left(p_\mu + p'_\mu\right)F_E(Q^2)\right]\gamma_5 U(\vec{p}) \end{aligned}$$

⋆ photon loop and counterterm contained in *electron* form factors w_i

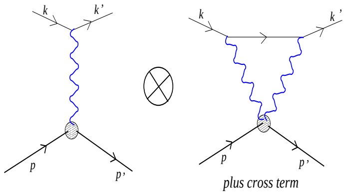
(computed as Feynman parameter integrals)

e. g. $$w_3(Q^2) = \frac{1}{2}\chi_{\mathrm{fin}}(\Lambda) - 3\ln\left(\frac{Q^2}{\Lambda^2}\right) + \widetilde{w}_3(Q^2)$$

scale dependence cancels when χ_{fin} replaced by $\Gamma(\pi^0 \to \mathrm{e}^+\mathrm{e}^-)$

infra–red divergence $\ln(m_e)$ remains in $\widetilde{w}_3$

⋆ interference



$$= \sum_{\text{spins}} M_\gamma^* M_{2\gamma}^{\mathrm{WZ}} = \frac{128\pi\alpha^3}{q^2} w_3 F_A G_M \left[(k\cdot p')^2 - (k\cdot p)^2 \right]$$

$$= 128\pi\alpha^3\, w_3 F_A G_M M^2 \sqrt{\tau\,(1+\tau)\,\frac{1+\epsilon}{1-\epsilon}}$$

if W_μ were a total derivative, $H_0 = -H_2$ and $F_A \equiv 0$ (only $F_p \neq 0$)

$\longrightarrow$ no interference in one–pion–exchange approx.

⋆ non–linear σ term $(S(Q^2))$ contributes only $\mathcal{O}(m_e)$

⋆ no $\mathcal{O}(1/N_C)$ contribution

⋆ reduced unpolarized cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \frac{\epsilon}{\tau}(1+\tau)\frac{d\sigma}{d\Omega} \Big/ \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}}$$

$$= \frac{G_M^2}{\tau}\left[\tau + \epsilon\frac{G_E^2}{G_M^2} + \frac{2\alpha}{\pi} w_3 M^2 \frac{F_A}{G_M}\sqrt{\tau^3\,(1+\tau)\,(1-\epsilon^2)}\right]$$

V) Numerical results

⋆ 'ratio' of cross–sections

$$r_{\text{mod}}(\epsilon) = \frac{1 + \frac{\epsilon}{\tau}\frac{G_E^2}{G_M^2} + \frac{2\alpha}{\pi} w_3 M^2 \frac{F_A}{G_M}\sqrt{\tau(1+\tau)(1-\epsilon^2)}}{1 + \frac{2\alpha}{\pi} w_3 M^2 \frac{F_A}{G_M}\sqrt{\tau(1+\tau)}}$$

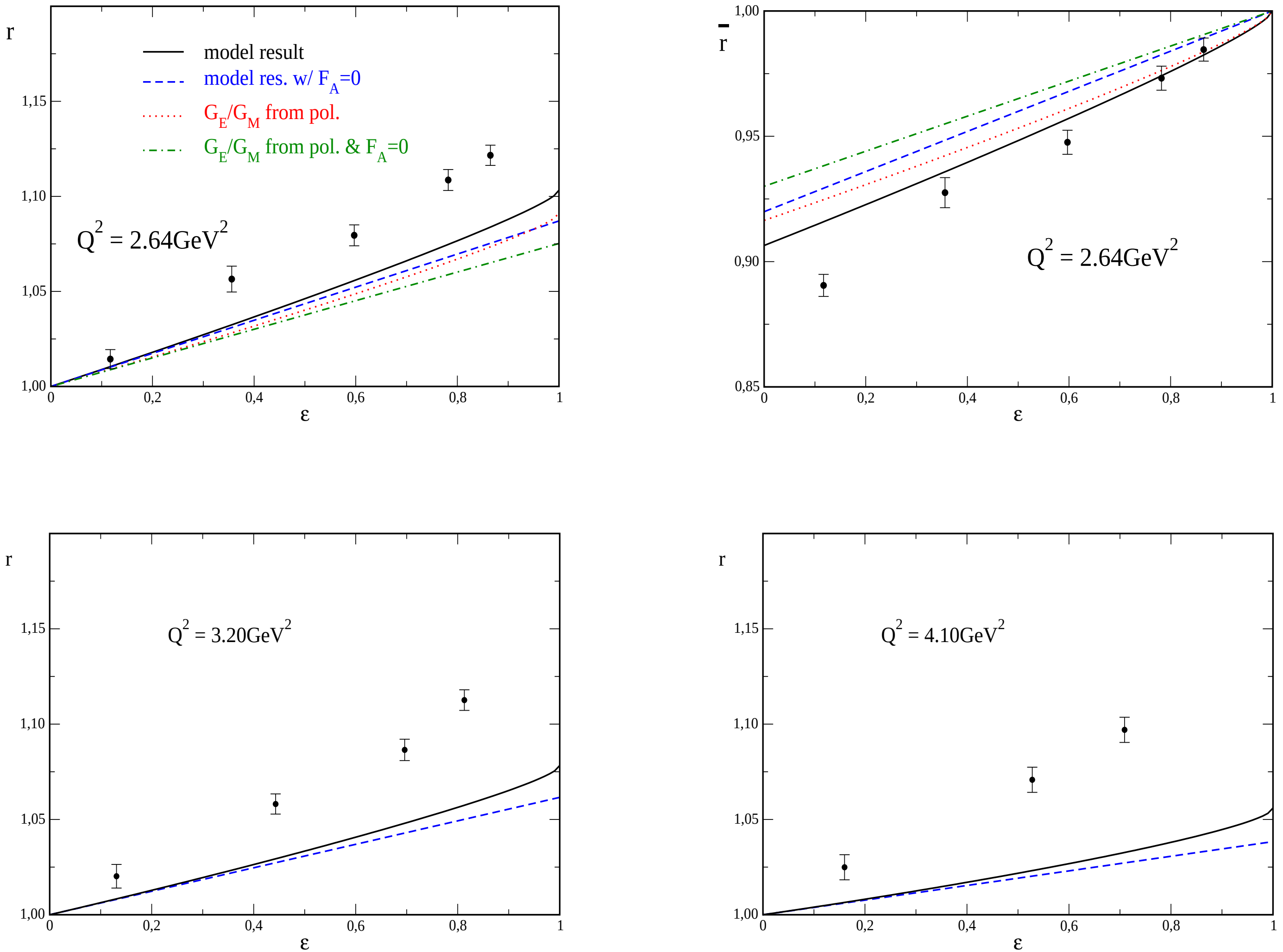
⋆ experimental analog

$$r_{\text{exp}}(\epsilon) = \frac{(d\sigma(\tau,\epsilon)/d\Omega)_R}{(d\sigma(\tau,0)/d\Omega)_R}$$

mag. ff. from Rosenbluth extraction

⋆ alternatively

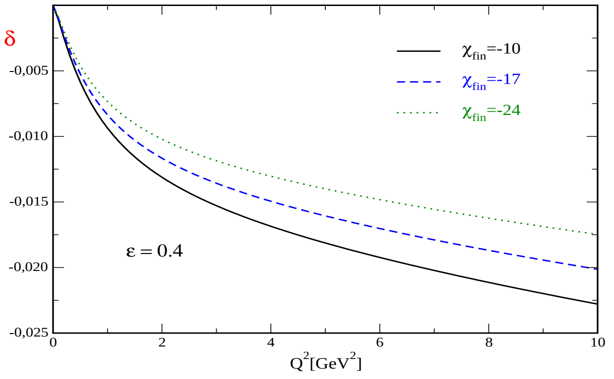
$$\overline{r}_{\text{mod}}(\epsilon) = \frac{1 + \frac{\epsilon}{\tau}\frac{G_E^2}{G_M^2} + \frac{2\alpha}{\pi} w_3 M^2 \frac{F_A}{G_M}\sqrt{\tau(1+\tau)(1-\epsilon^2)}}{1 + \frac{1}{\tau}\frac{G_E^2}{G_M^2}}$$

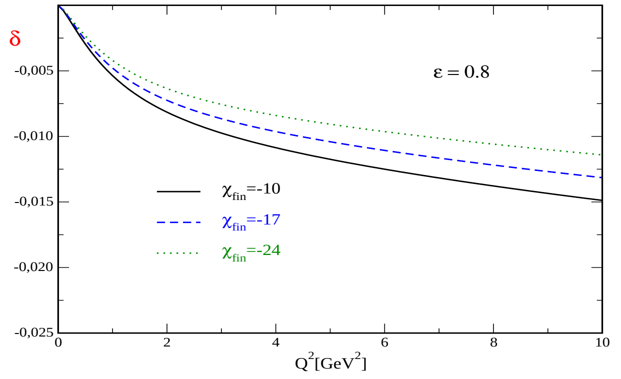


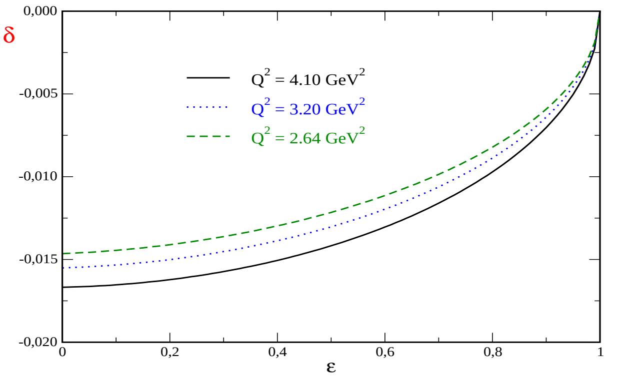
r
model result
model res. w/ F_A=0
G_E/G_M from pol.
G_E/G_M from pol. & F_A=0
Q^2 = 2.64GeV^2
1,00
1,05
1,10
1,15
0
0,2
0,4
0,6
0,8
1
ε
r
Q^2 = 2.64GeV^2
0,85
0,90
0,95
1,00
ε
r
Q^2 = 3.20GeV^2
ε
r
Q^2 = 4.10GeV^2
ε

★ separation of two photon contribution

$$\left(\frac{d\sigma}{d\Omega}\right)_R = \left[G_M^2 + \frac{\epsilon}{\tau}G_E^2\right](1+\delta)$$

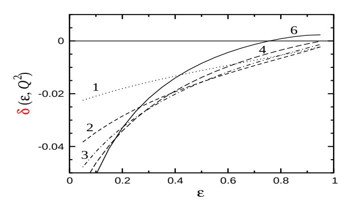






- δ has large slope at small momenta
- at fixed Q^2, δ has biggest slope at $\epsilon \approx 1$
- supplementary to box–graph contributions

(Blunden *et al.*)



V) Conclusions

- ⋆ anomaly may contribute an additional two–photon piece to electron nucleon scattering
- ⋆ interference in unpolarized scattering vanishes in the one–pion–exchange approximation
- ⋆ Skyrme soliton model simplest tool to investigate higher order contributions (anomaly enters via WZ term)
- ⋆ corrections tend in proper direction but are too small to explain the full discrepancy
  no contradiction because they supplement the box–graph contributions
- ⋆ effect on polarization process to be studied next