Nucleon Form Factors & Quark Distributions

(in covariant quark-diquark theories)

Ian Cloët

(Argonne National Laboratory)

Collaborators

Craig Roberts (ANL)

Wolfgang Bentz (Tokai Uni) & Anthony Thomas (JLab)

ECT workshop on Hadron Electromagnetic Form Factors*

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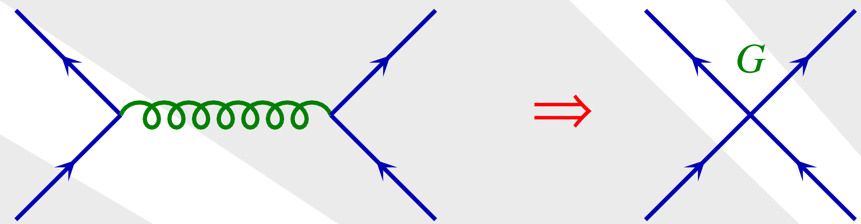
Theme

- Demonstrate utility of Nambu–Jona-Lasinio model
  - nucleon form factors and quark distributions
  - higher twist, GPDs, SDD, ...
- Are nucleon properties modified by the nuclear medium?
  - Of fundamental importance
  - Remains an open question
- Areas where medium modifications seem important:
  - in-medium nucleon form factors: ^{4}He(e,e'p)^{3}H
  - nuclear structure functions: EMC & Polarized EMC effects

Nambu–Jona-Lasinio Model

- Low energy chiral effective theory of QCD

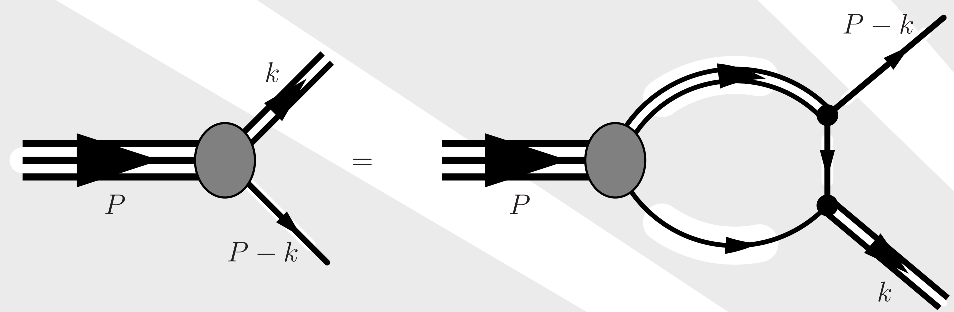


- Investigate the role of quark degrees of freedom.
- Much in common with DSE
- Lagrangian has same symmetries as QCD:
  - ✦ Importantly chiral symmetry and CSB,
    - $\longrightarrow$ Dynamically generated quark masses,
    - $\longrightarrow$ Non-zero chiral condensate.
- Lagrangian ($\Gamma =$ Dirac, colour, isospin matrices)

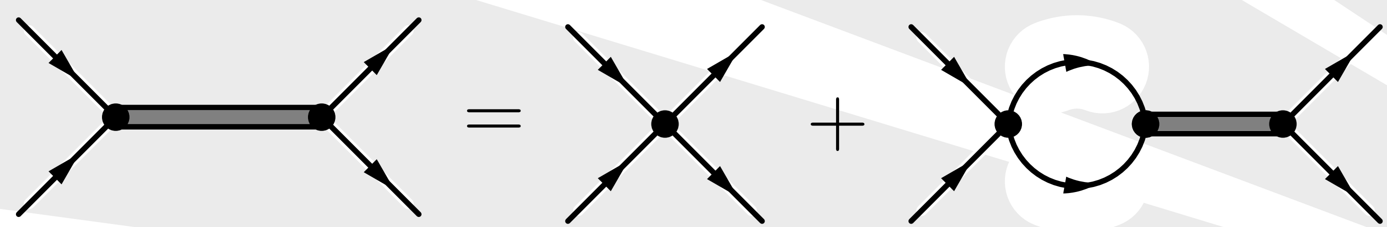
$$\mathcal{L}_{NJL} = \overline{\psi}\left(i\partial\!\!\!/ - m\right)\psi + G\left(\overline{\psi}\Gamma\psi\right)^2$$

Nucleon in the NJL model

- Nucleon approximated as quark-diquark bound state.
- Use relativistic Faddeev approach:



- Diquark - bound state of two quarks:
- Solve Bethe-Salpeter equation for diquark.



- We include scalar and axial-vector diquarks.

Regularization

- Proper-time regularization

$$\frac{1}{X^n} = \frac{1}{(n-1)!} \int_0^\infty d\tau\, \tau^{n-1}\, e^{-\tau X}$$

$$\longrightarrow \quad \frac{1}{(n-1)!} \int_{1/(\Lambda_{UV})^2}^{1/(\Lambda_{IR})^2} d\tau\, \tau^{n-1}\, e^{-\tau X}.$$

- Λ_{IR} eliminates unphysical thresholds for the nucleon to decay into quarks: $\rightarrow$ simulates confinement.
  - G. Hellstern, R. Alkofer and H. Reinhardt, Nucl. Phys. A **625**, 697 (1997).
- Needed for: nuclear matter saturation, Δ baryon.
  - W. Bentz, A.W. Thomas, Nucl. Phys. A **696**, 138 (2001)

Model Parameters

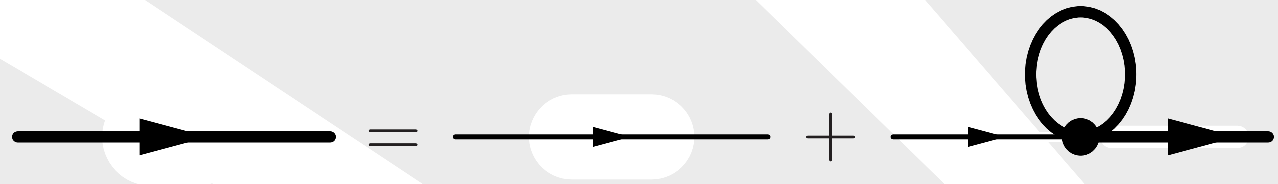
- Free Parameters:

  $\Lambda_{IR},\ \Lambda_{UV},\ M_0,\ G_\pi,\ G_s,\ G_a,\ G_\omega$ and G_ρ

- Constraints:
  - $f_\pi = 93$ MeV, $m_\pi = 140$ MeV & $M_N = 940$ MeV
  - $\int_0^1 dx\ (\Delta u_v(x) - \Delta d_v(x)) = g_A = 1.267$
  - $(\rho, E_B/A) = (0.16\,\text{fm}^{-3}, -15.7\text{ MeV})$
  - $a_4 = 32$ MeV
  - $\Lambda_{IR} = 240$ MeV

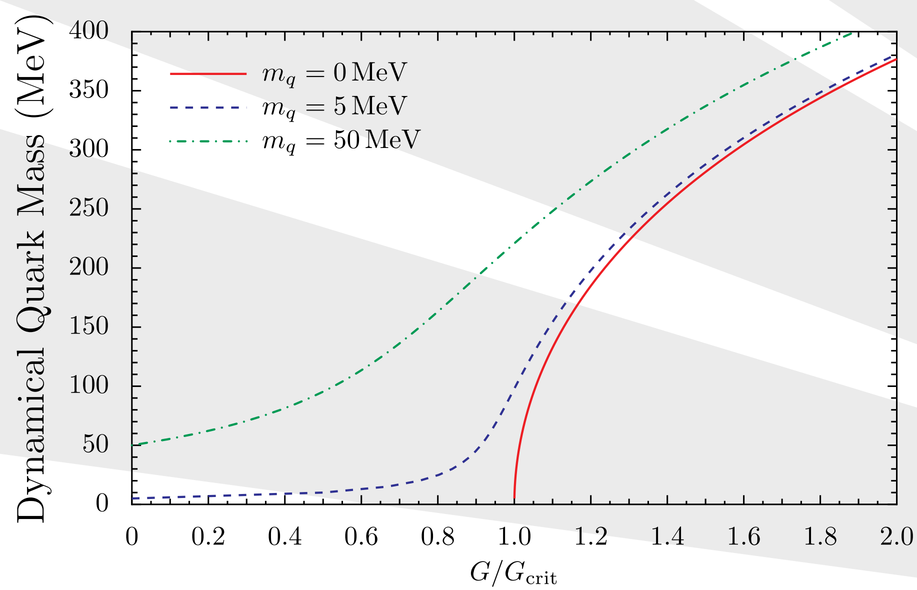
- We obtain [MeV]:
  - $\Lambda_{UV} = 644$
  - $M_0 = 400,\quad M_s = 690,\quad M_a = 990,\quad \ldots$

Gap Equation



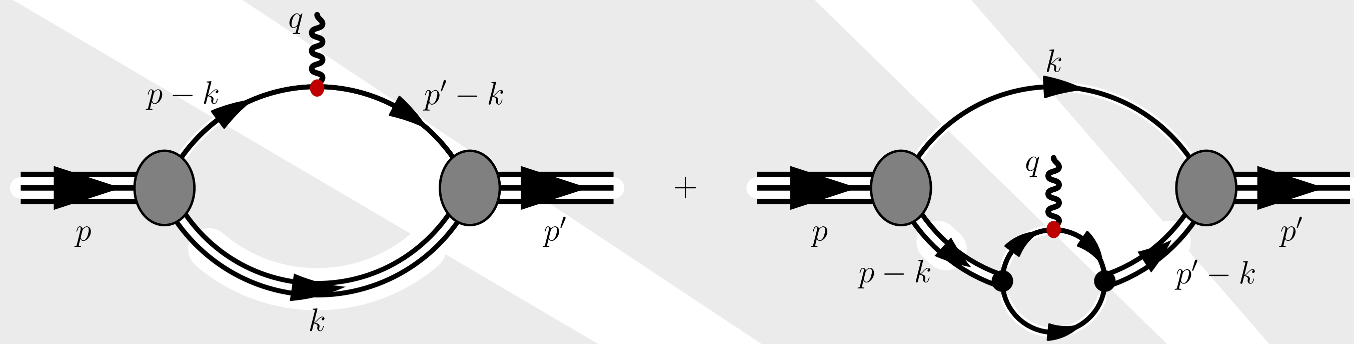
- Self-consistent solution – gives Quark Propagator

$$\frac{1}{\not{p} - m + i\varepsilon} \longrightarrow \frac{1}{\not{p} - M + i\varepsilon}$$



Nucleon Form Factors

- Form Factor Feynman diagrams



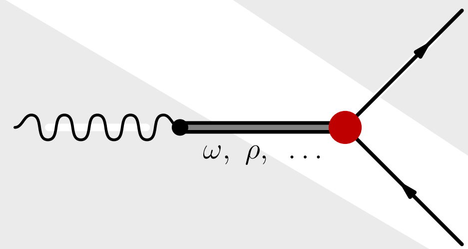
✦ $\bullet = \gamma^\mu F_{1q}(Q^2) + \frac{i\,\sigma^{\mu\nu} q_\nu}{2M} F_{2q}(Q^2)$

- Approach is completely covariant
- No frame is ever assumed
- Charge is conserved automatically
- Diagrams are expressed in form:

$$\langle J^\mu \rangle = \overline{u}_N(p') \left[\gamma^\mu F_{1N}(Q^2) + \frac{i\,\sigma^{\mu\nu} q_\nu}{2M_N} F_{2N}(Q^2) \right] u_N(p)$$

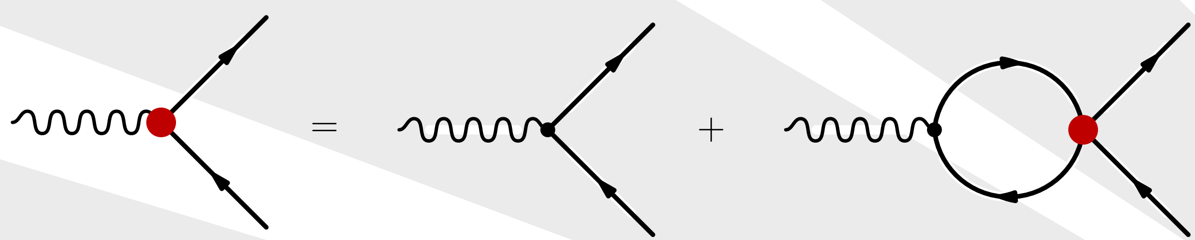
Constituent Quark Form Factors

- Vector Meson Dominance – traditional view



$$\propto \quad \frac{M_\rho^2}{M_\rho^2 + Q^2}$$

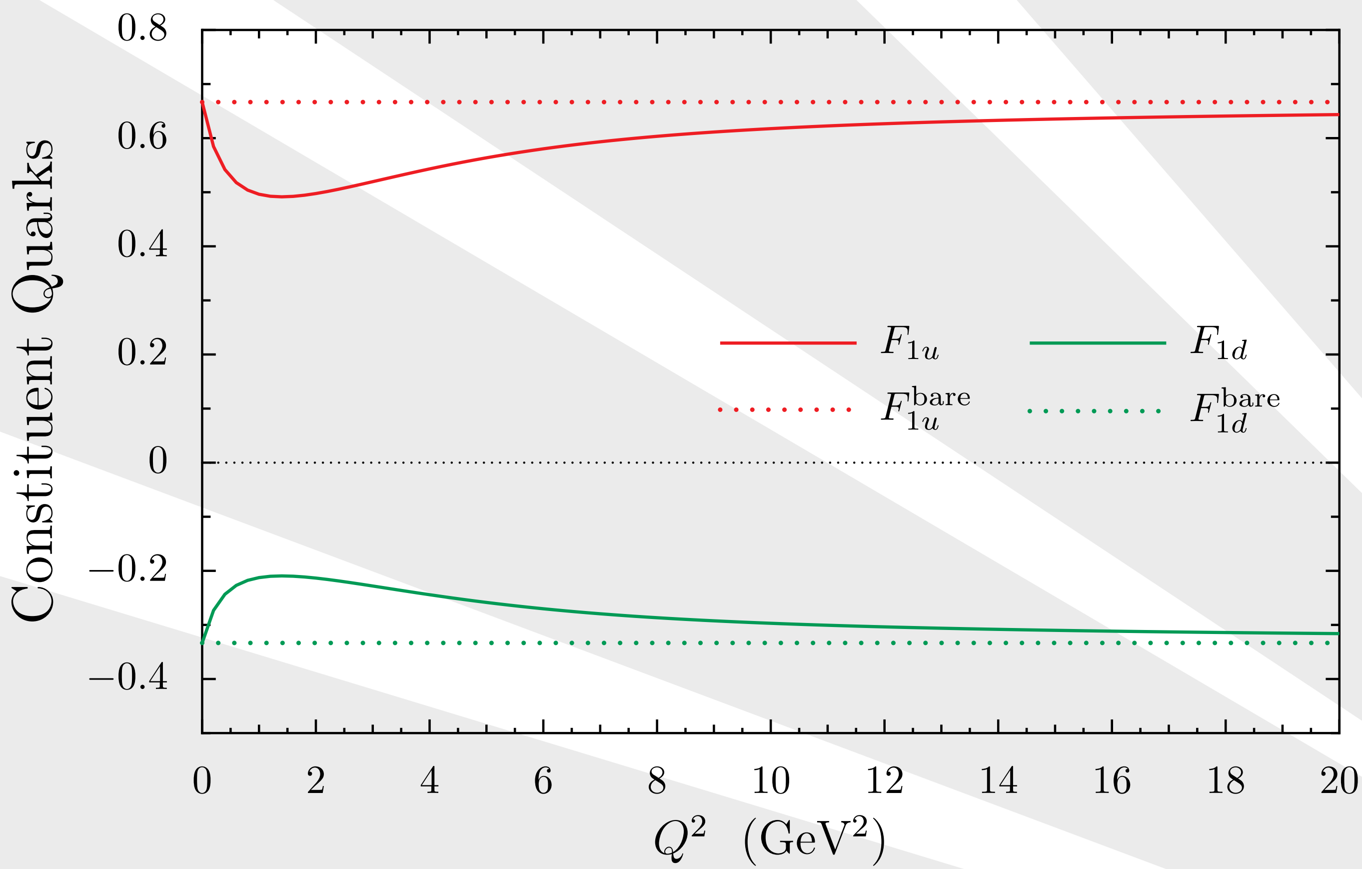
- We solve integral equation for vertex



- Vertex becomes

$$\left(\tfrac{1}{6}+\tfrac{\tau_3}{2}\right)\gamma^\mu \;\to\; \frac{1}{6}\left[1-\frac{2\,G_\omega\,\Pi(q^2)}{1+2\,G_\omega\,\Pi(q^2)}\right]\gamma^\mu+\frac{\tau_3}{2}\left[1-\frac{2\,G_\rho\,\Pi(q^2)}{1+2\,G_\rho\,\Pi(q^2)}\right]\gamma^\mu=\left[\tfrac{1}{6}\,F_\omega+\tfrac{\tau_3}{2}\,F_\rho\right]\gamma^\mu$$

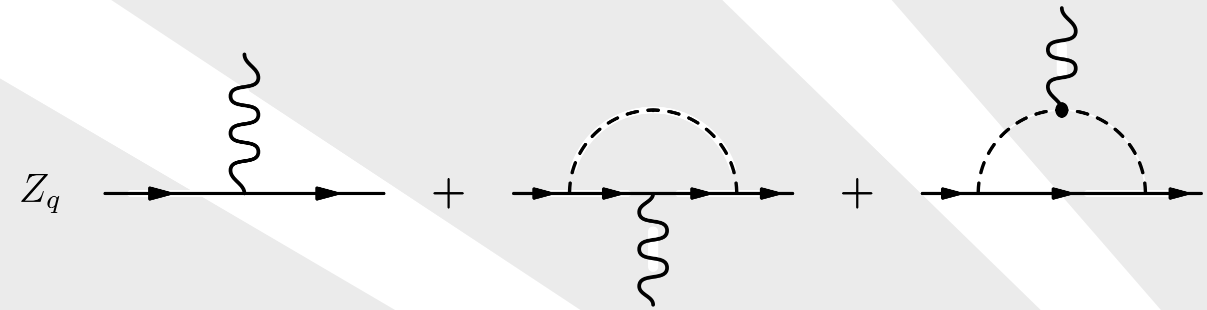
VMD Results



- Expanding about $Q^2 = 0$ gives

$$\left[\tfrac{1}{6}\,F_\omega + \tfrac{\tau_3}{2}\,F_\rho\right]\gamma^\mu \quad\sim\quad \left[\frac{1}{6}\,\frac{M_\omega^2}{M_\omega^2+Q^2} + \frac{\tau_3}{2}\,\frac{M_\rho^2}{M_\rho^2+Q^2}\right]\gamma^\mu$$

Constituent Quarks – Pion



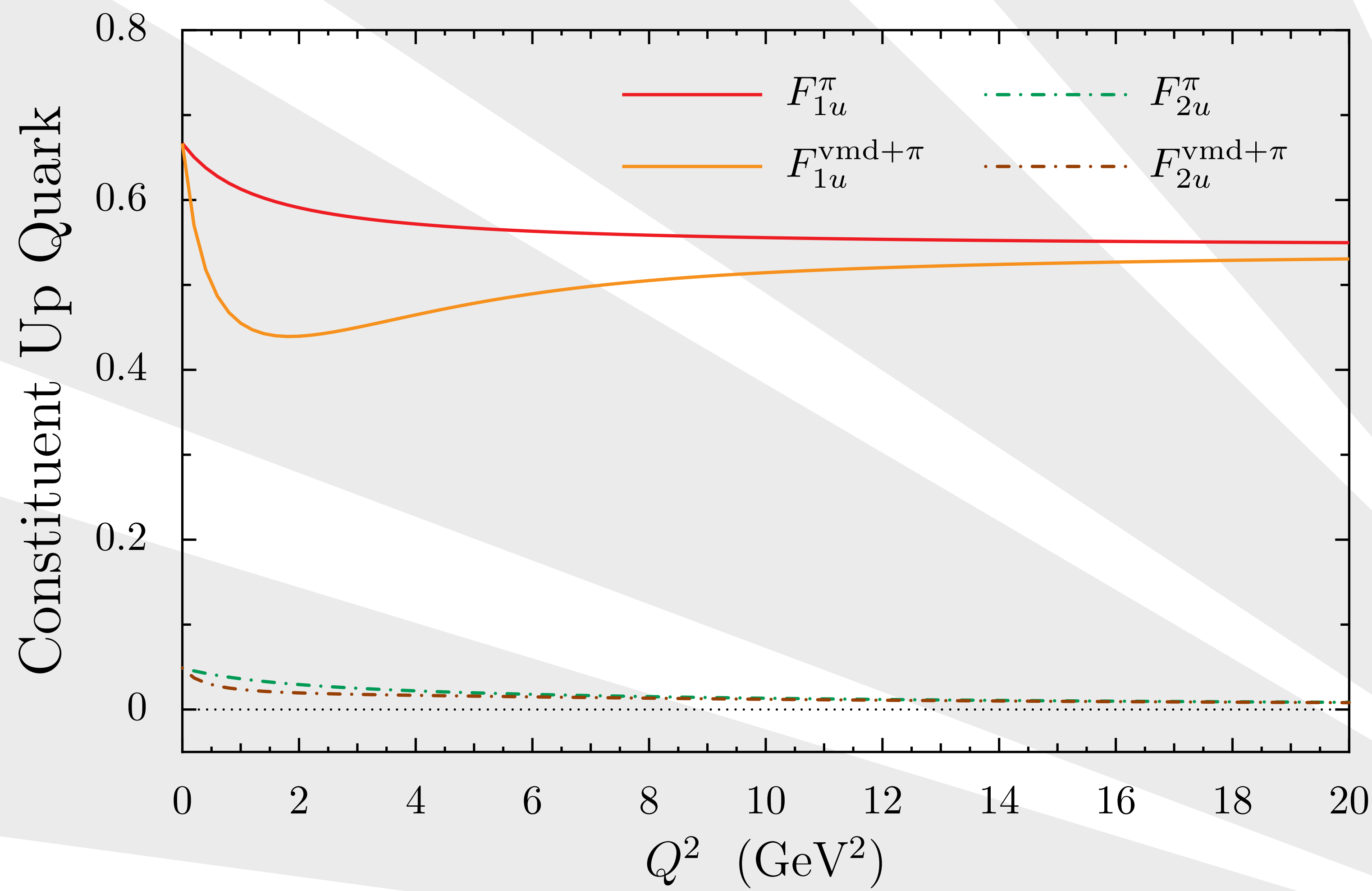
- Probability for find bare quark: $Z_q = 1 + \frac{\partial \Sigma_q}{\partial \not{p}}$
- Pion cloud → anomalous m.m for constituent quarks.

$$F_{1q}(Q^2) = Z_q \left(\frac{1}{6} F_\omega + \frac{1}{2} \tau_3 F_\rho\right) + (F_\omega - \tau_3 F_\rho)\ F_{1q}^{(q)} + \tau_3 F_\rho\ F_{1q}^{(\pi)}$$

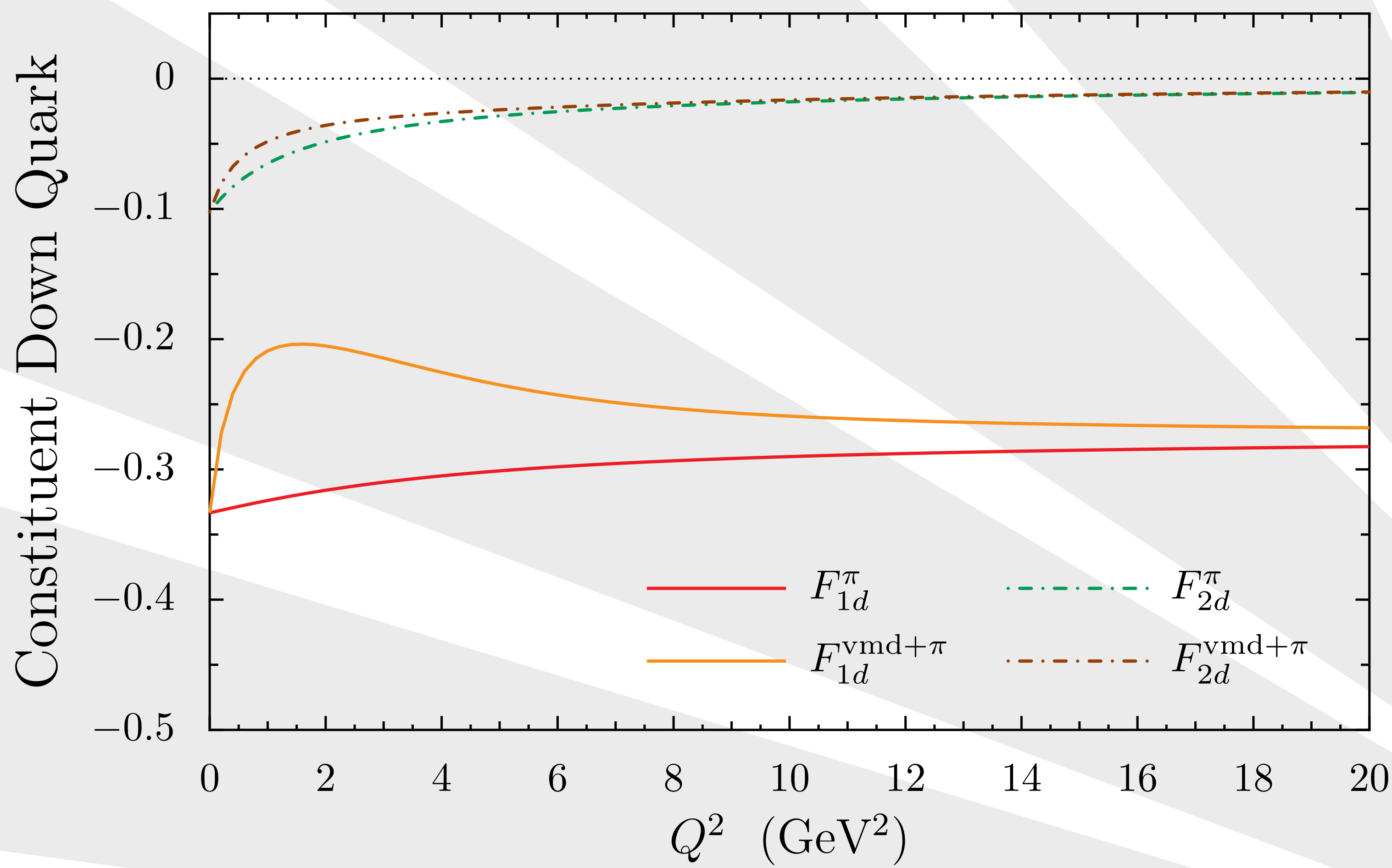
$$F_{2q}(Q^2) = F_{2q}(Q^2)(F_\omega - \tau_3 F_\rho)\ F_{2q}^{(q)} + \tau_3 F_\rho\ F_{2q}^{(\pi)}$$

- No pion exchange between quarks
- Better to add pion at nucleon level

Constituent Up Quark Results

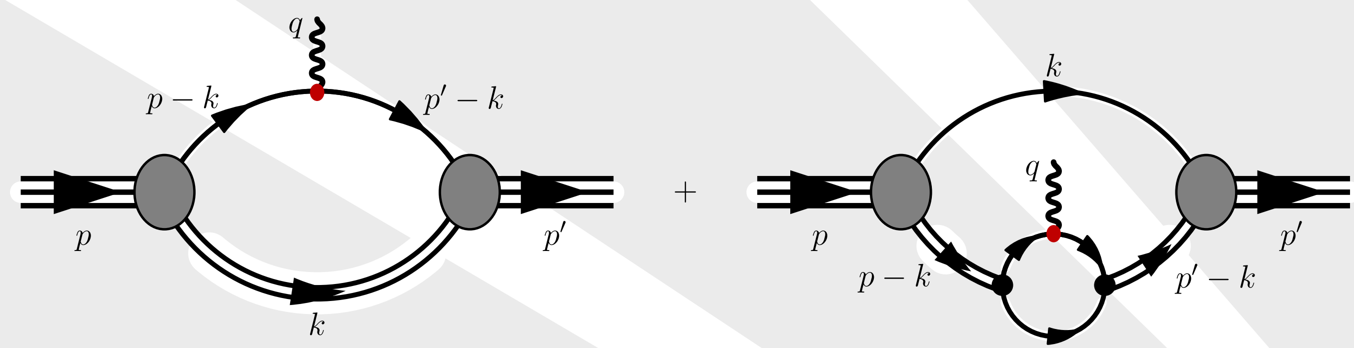


Constituent Down Quark Results



Nucleon Form Factors – Recall

- Form Factor Feynman diagrams

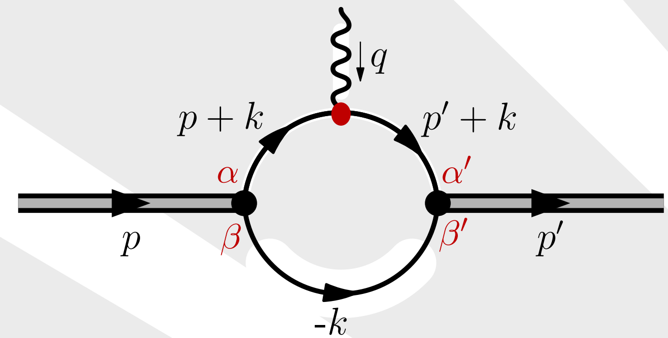


✦ $\bullet = \gamma^\mu F_{1q}(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M} F_{2q}(Q^2)$

- Diagrams are expressed in form:

$$\langle J^\mu \rangle = \overline{u}_N(p') \left[\gamma^\mu F_{1N}(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2M_N} F_{2N}(Q^2) \right] u_N(p)$$

Scalar Diquark & Pion Form Factors

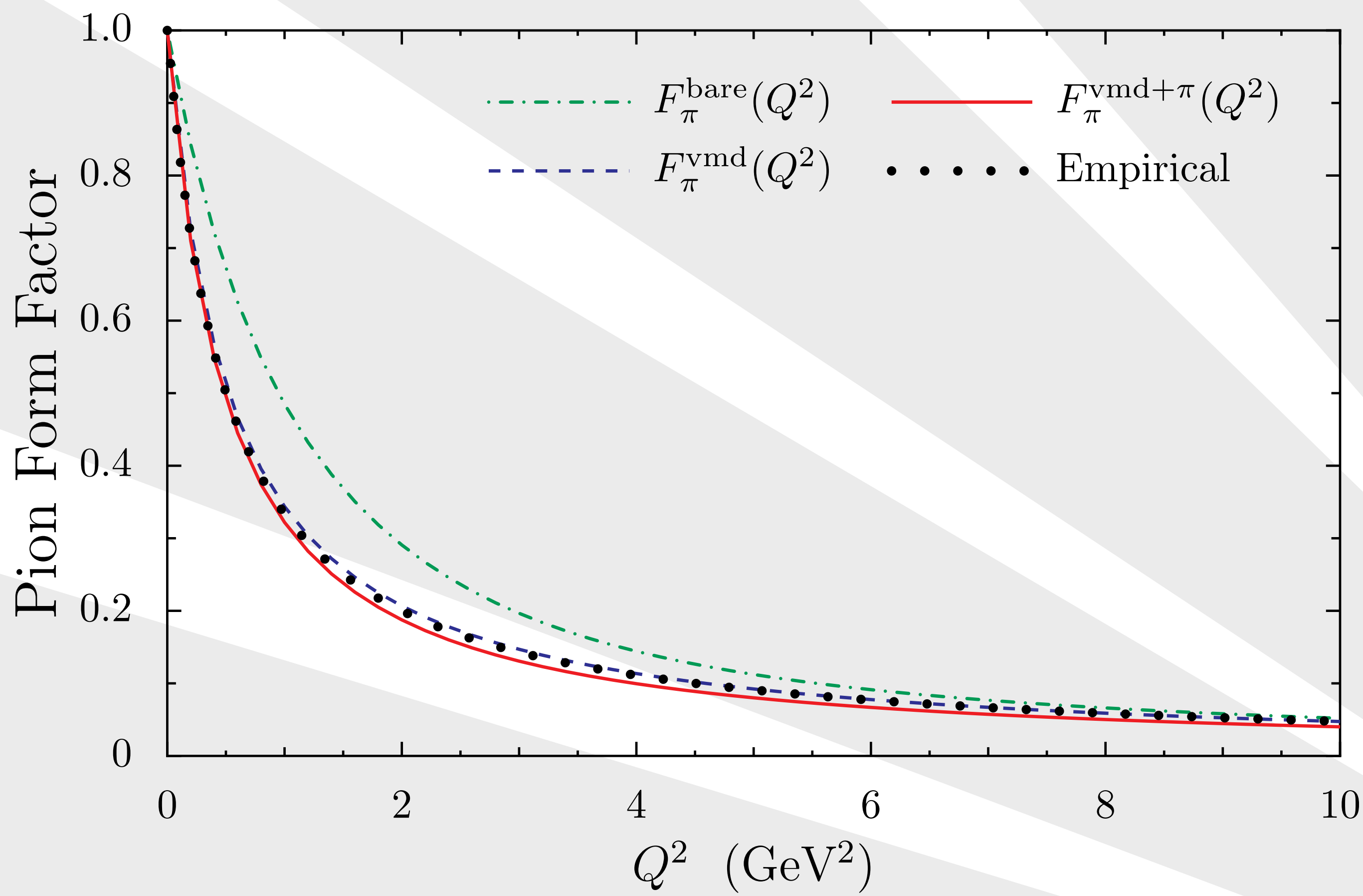


- Scalar diquark: $(\gamma_5\, C\, \tau_2\, \beta^{A'})(C^{-1}\gamma_5\, \tau_2\, \beta^{A})$, Pion: $(\tau\, \gamma_5)$
- Form Factor expressions same: $g_\pi \leftrightarrow g_s, \;\; m_\pi \leftrightarrow M_s$
- Two form factors in general

$$\langle J^\mu_\pi \rangle = (p' + p)^\mu\, F_\pi(Q^2) + \underbrace{(p' - p)^\mu\, F^{\text{os}}_\pi(Q^2)}_{\to\, 0}$$

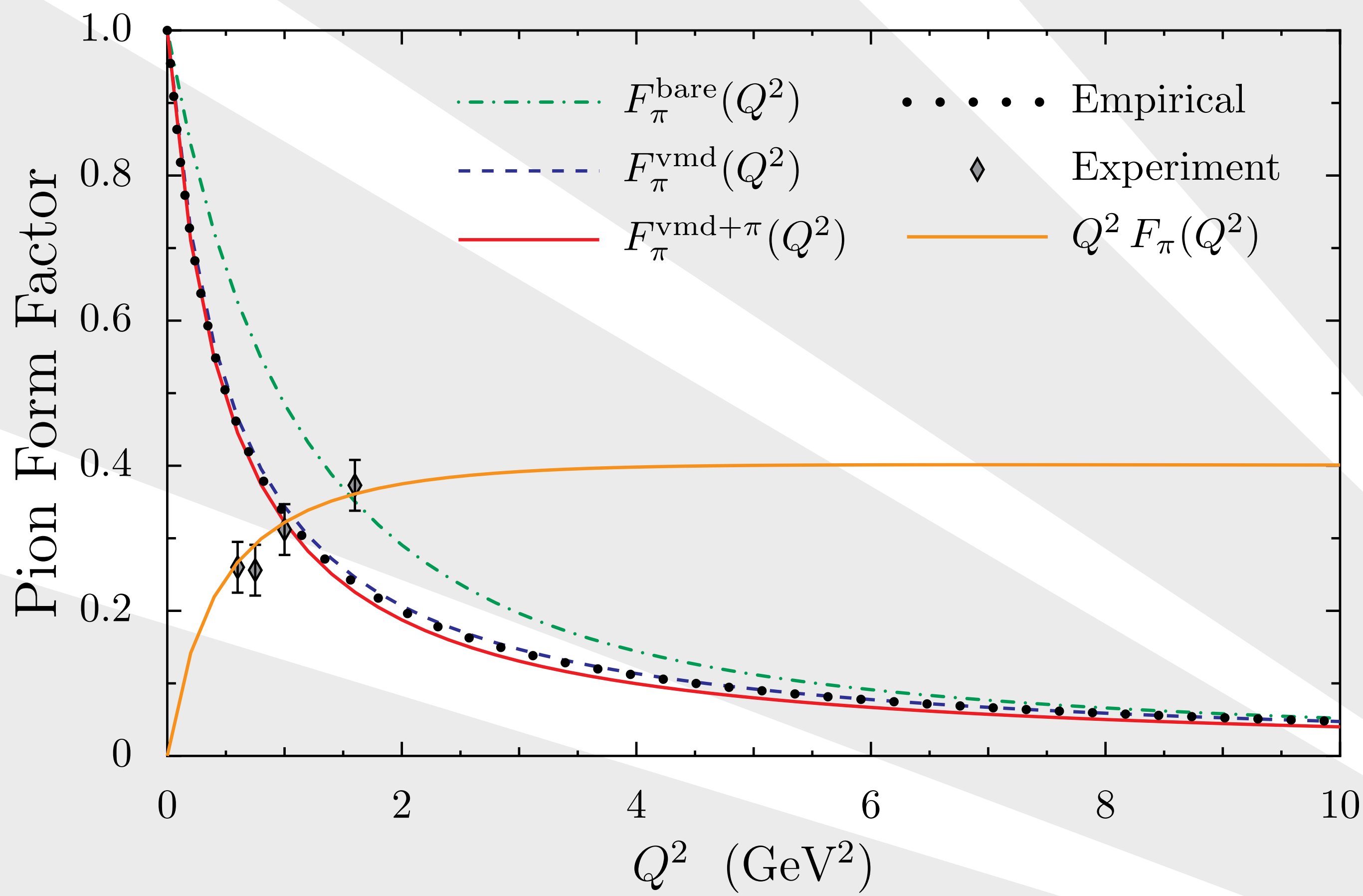
- Result charge radius: $\langle r^2_E \rangle_\pi = 0.46\,\text{fm}^2$
- Experiment: $\langle r^2_E \rangle_\pi = 0.45\, \pm 0.01\text{fm}^2$

Pion Form Factor



- $F_\pi(Q^2) = \left[1 + Q^2/\Lambda^2\right]^{-1}$ $\quad \Lambda^2 = 0.5\,\mathrm{GeV}^2$

Pion Form Factor



- V. Tadevosyan *et al.* [Jefferson Lab F(pi) Collaboration], Phys. Rev. C **75**, 055205 (2007)
- $Q^2\,F_\pi(Q^2) \to 16\pi\, f_\pi^2\, \alpha_s(Q^2) \implies \alpha_{NJL}=0.94 \implies Q^2 \sim 0.46$ GeV^2

Axial-Vector Diquark & Rho Form Factors

- AV diquark: $(\gamma^\beta\, C\, \tau_i \tau_2\, \beta^{A'})(C^{-1}\gamma^\alpha\, \tau_2\tau_j\, \beta^A)$, Rho: $(\tau_j\, \gamma^\mu)$
- Form Factor expressions same: $g_\rho \leftrightarrow g_a,\;\; m_\rho \leftrightarrow M_a$
- 3 on shell form factors

$$\langle J^\mu_\rho \rangle = g^{\alpha\beta}\,(p+p')^\mu\, F_1(Q^2) - \frac{q^\alpha q^\beta}{2M_a^2}\,(p+p')^\mu\, F_2(Q^2) - \left(q^\alpha\, g^{\mu\beta} - q^\beta\, g^{\mu\alpha}\right) F_3(Q^2)$$

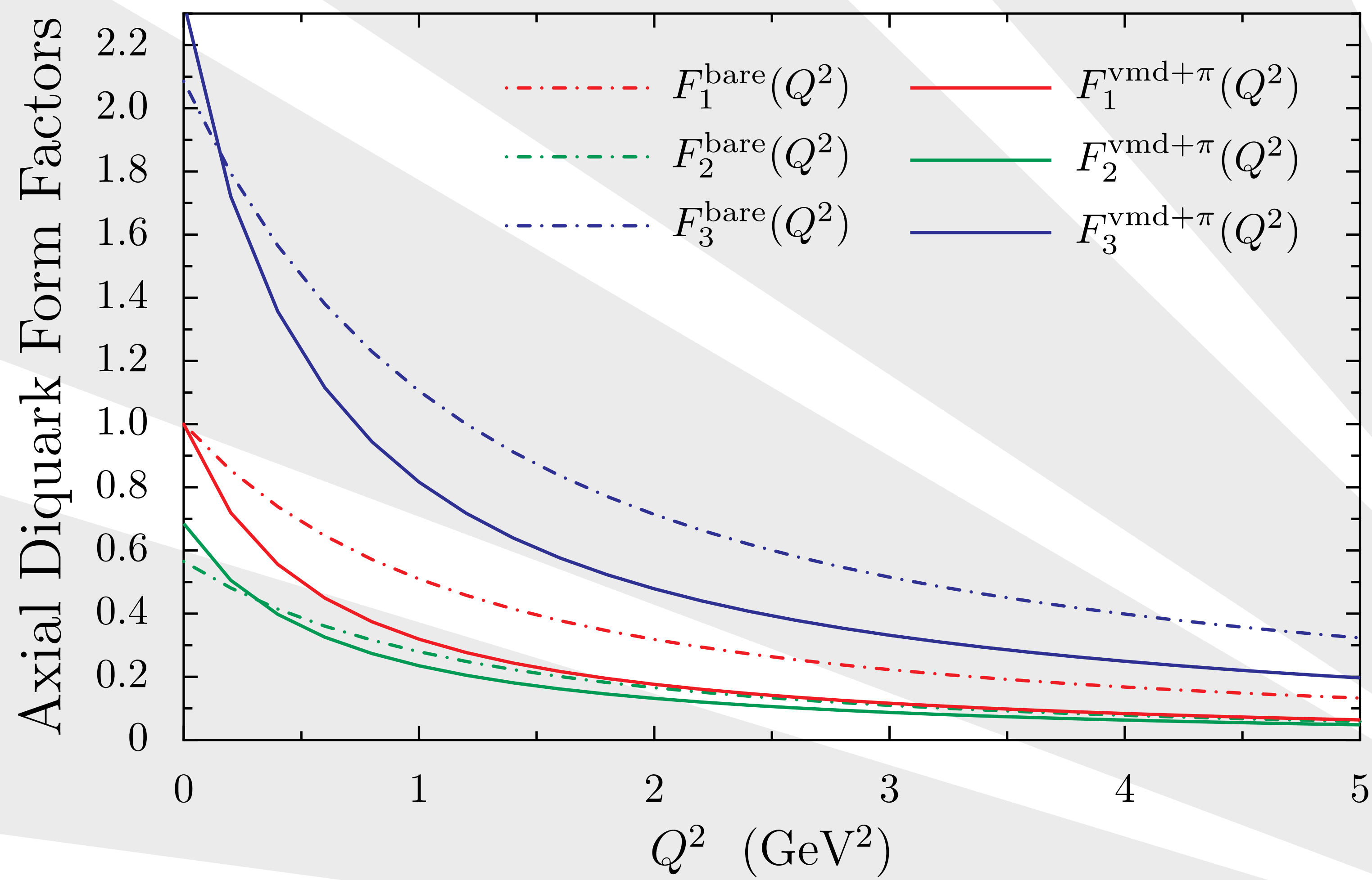
- Sachs Form Factors

$$G_C(Q^2) = F_1(Q^2) + \tfrac{2}{3}\,\frac{Q^2}{4\,M^2}\, G_Q(Q^2), \qquad G_M(Q^2) = F_3(Q^2)$$

$$G_Q(Q^2) = F_1(Q^2) + \left(1 + \frac{Q^2}{4\,M^2}\right) F_2(Q^2) - F_3(Q^2)$$

- NJL Results: $\langle r_E^2 \rangle_\rho = 0.52,\;\; \mu_\rho = 2.08,\;\; \mathcal{Q}_\rho = -0.52$
- DSE Results: $\langle r_E^2 \rangle_\rho = 0.54,\;\; \mu_\rho = 2.01,\;\; \mathcal{Q}_\rho = -0.41$

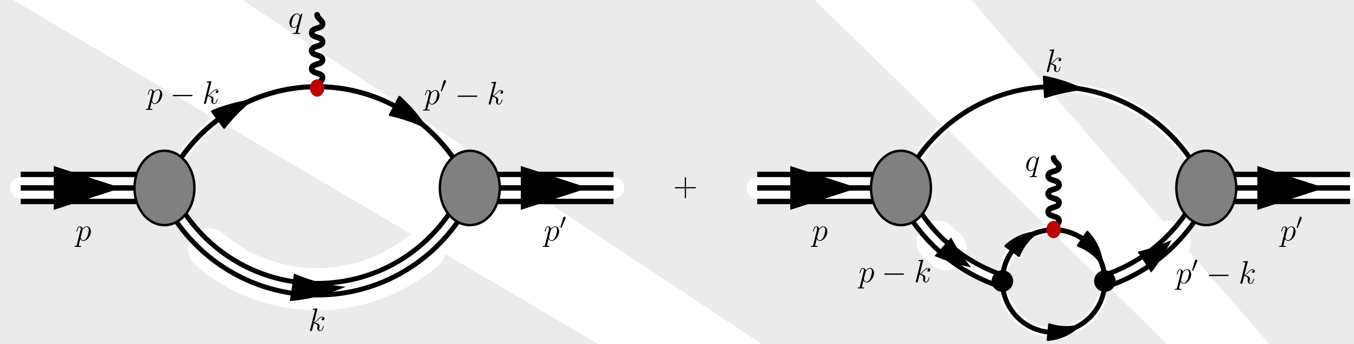
Axial-Vector Diquark Form Factors



- For large Q^2 we find $F_2 \to F_1$.

Nucleon Form Factors – Recall

- Form Factor Feynman diagrams

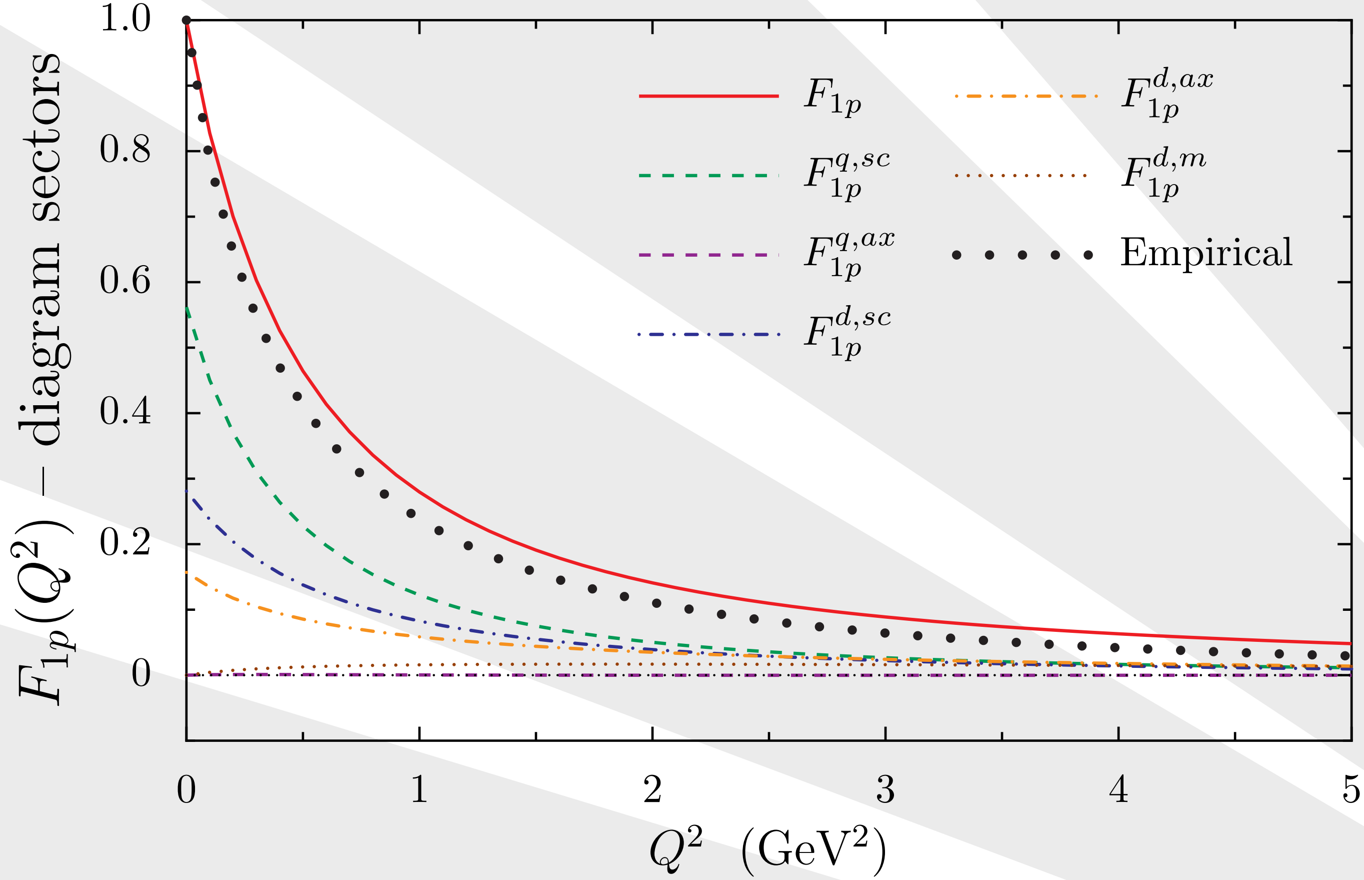


✦ $\bullet = \gamma^{\mu}\, F_{1q}(Q^2) + \frac{i\,\sigma^{\mu\nu} q_{\nu}}{2M}\, F_{2q}(Q^2)$

- Diagrams are expressed in form:

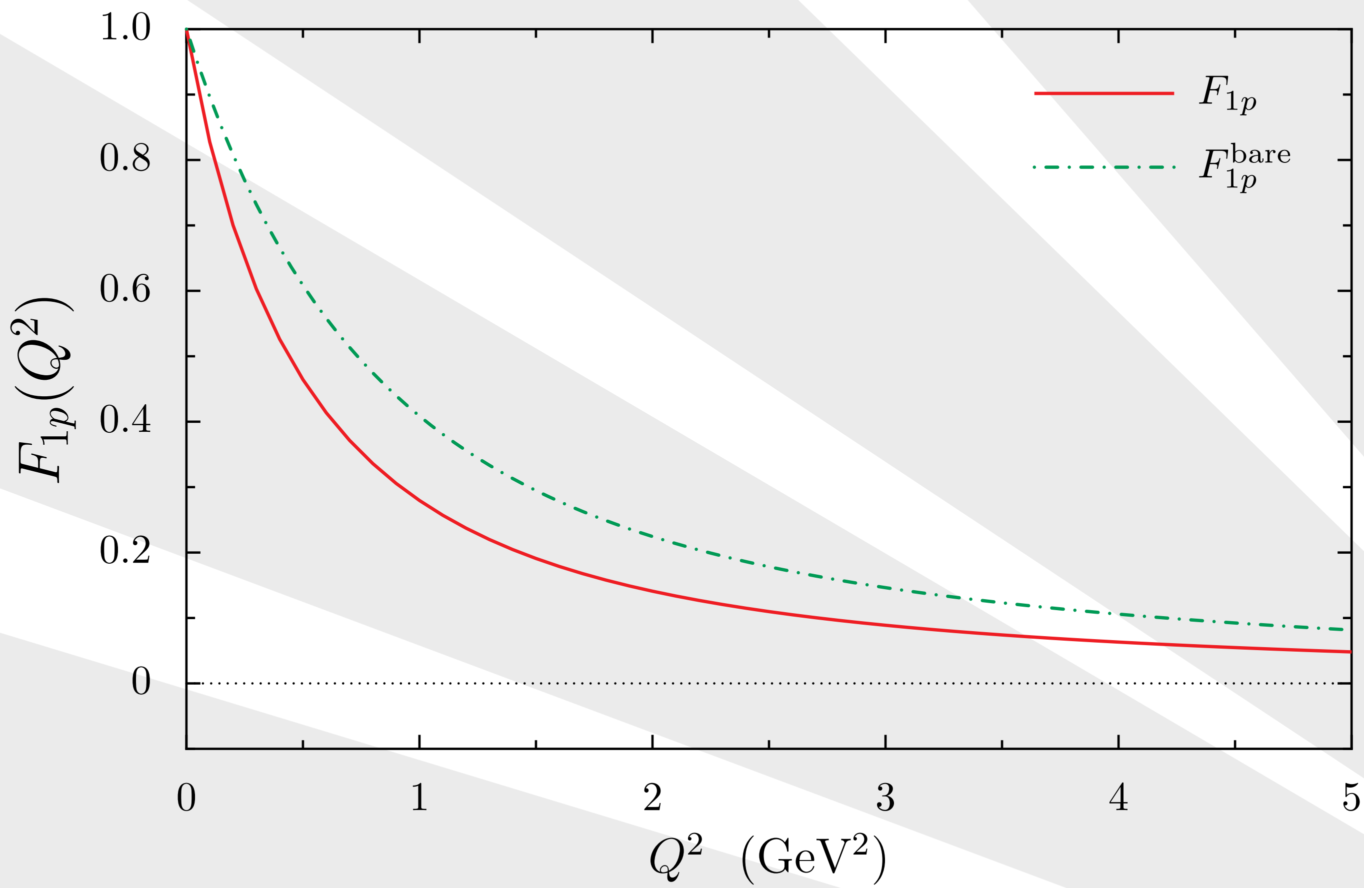
$$\langle J^{\mu}\rangle = \overline{u}_N(p')\left[\gamma^{\mu}\, F_{1N}(Q^2) + \frac{i\,\sigma^{\mu\nu} q_{\nu}}{2M_N}\, F_{2N}(Q^2)\right] u_N(p)$$

Proton Form Factors



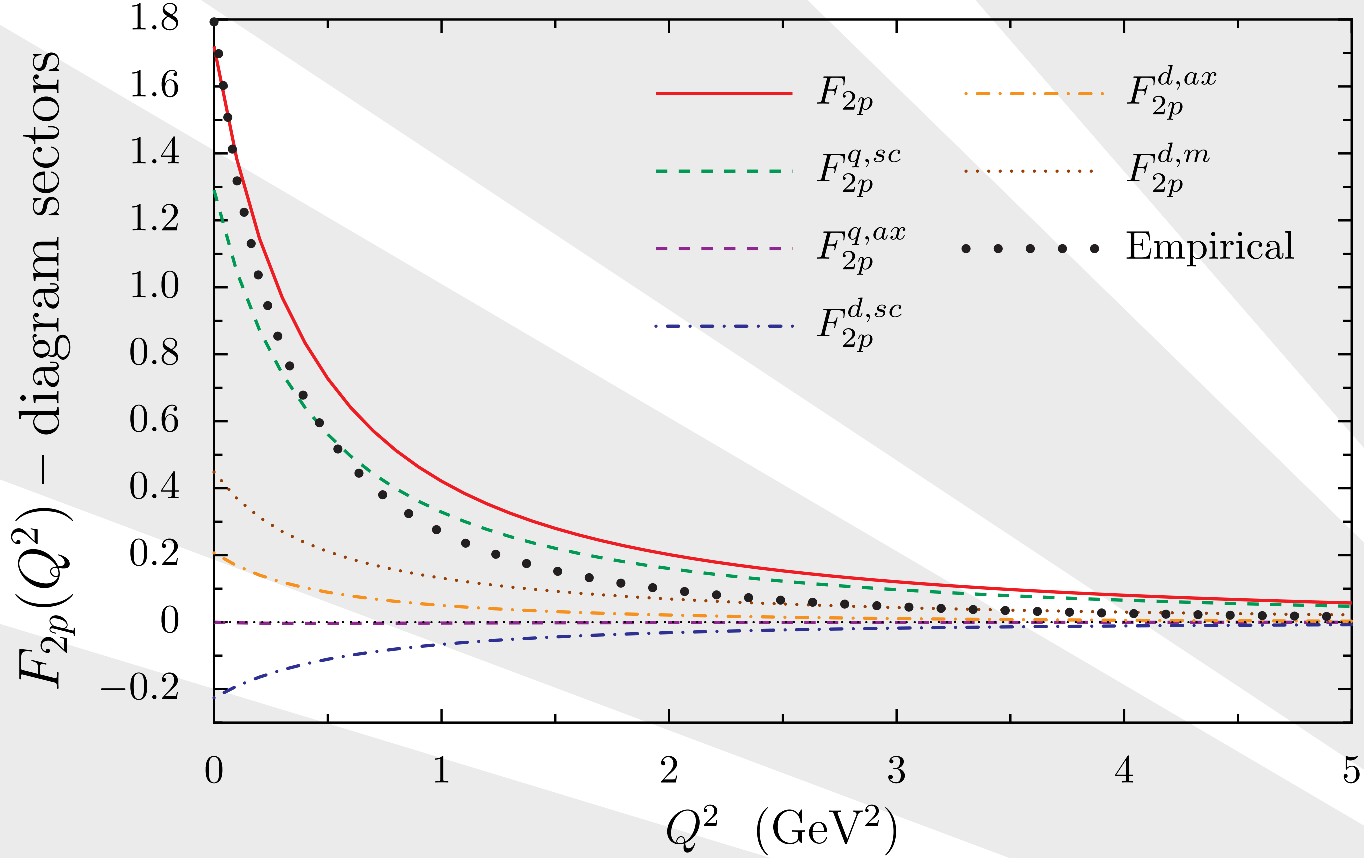
- Arrington, et al, Phys. Rev. C **76**, 035205 (2007)
- NJL: $\langle r_E^2 \rangle_p = 0.58\,\text{fm}^2$, Experiment: $\langle r_E^2 \rangle_p = 0.72\,\text{fm}^2$

Proton Form Factors – no pion



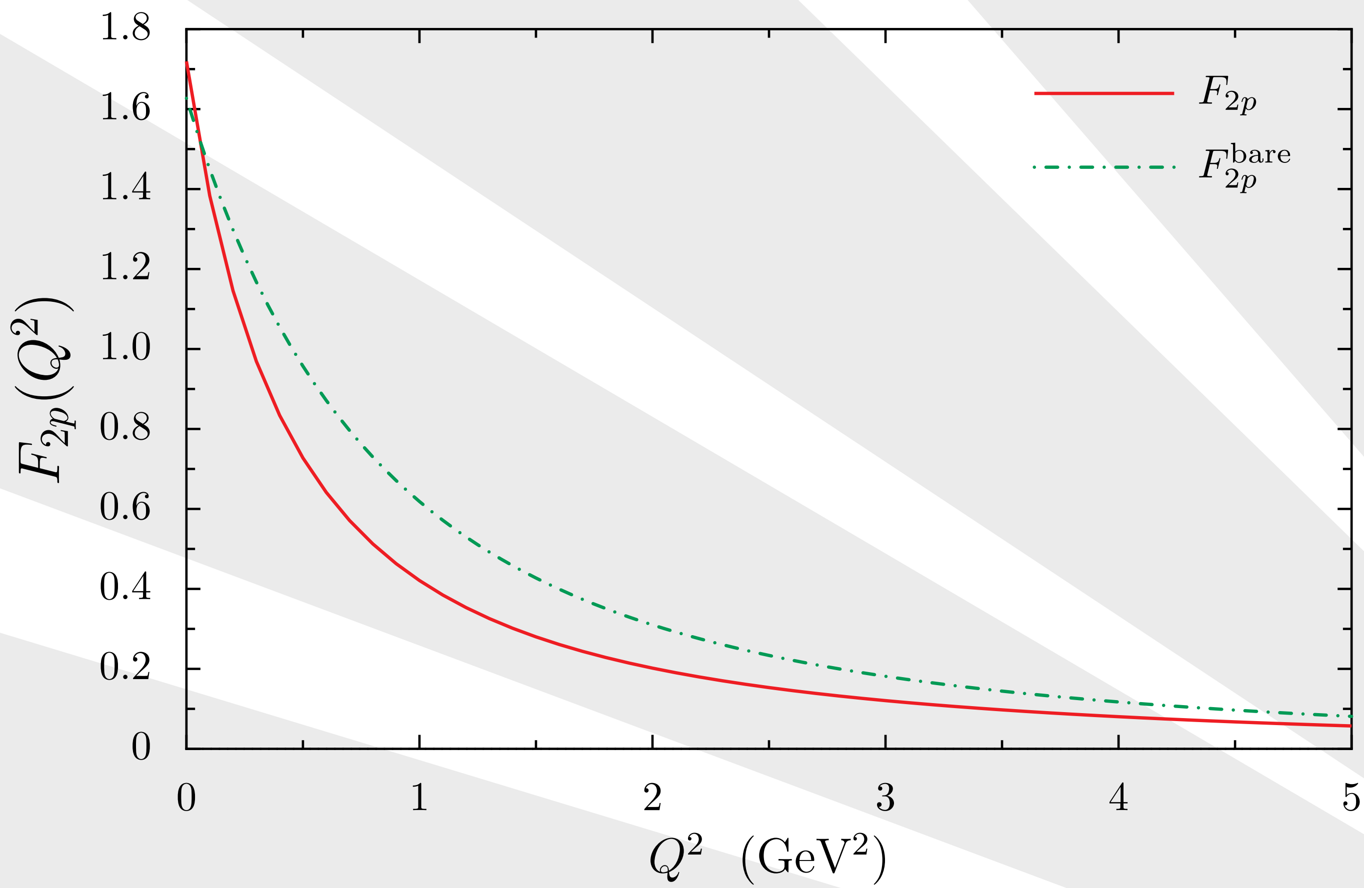
- NJL: $\langle r_E^2 \rangle_p = 0.58\,\text{fm}^2$, NJL$_{\text{bare}}$: $\langle r_E^2 \rangle_p = 0.36\,\text{fm}^2$

Proton Form Factors



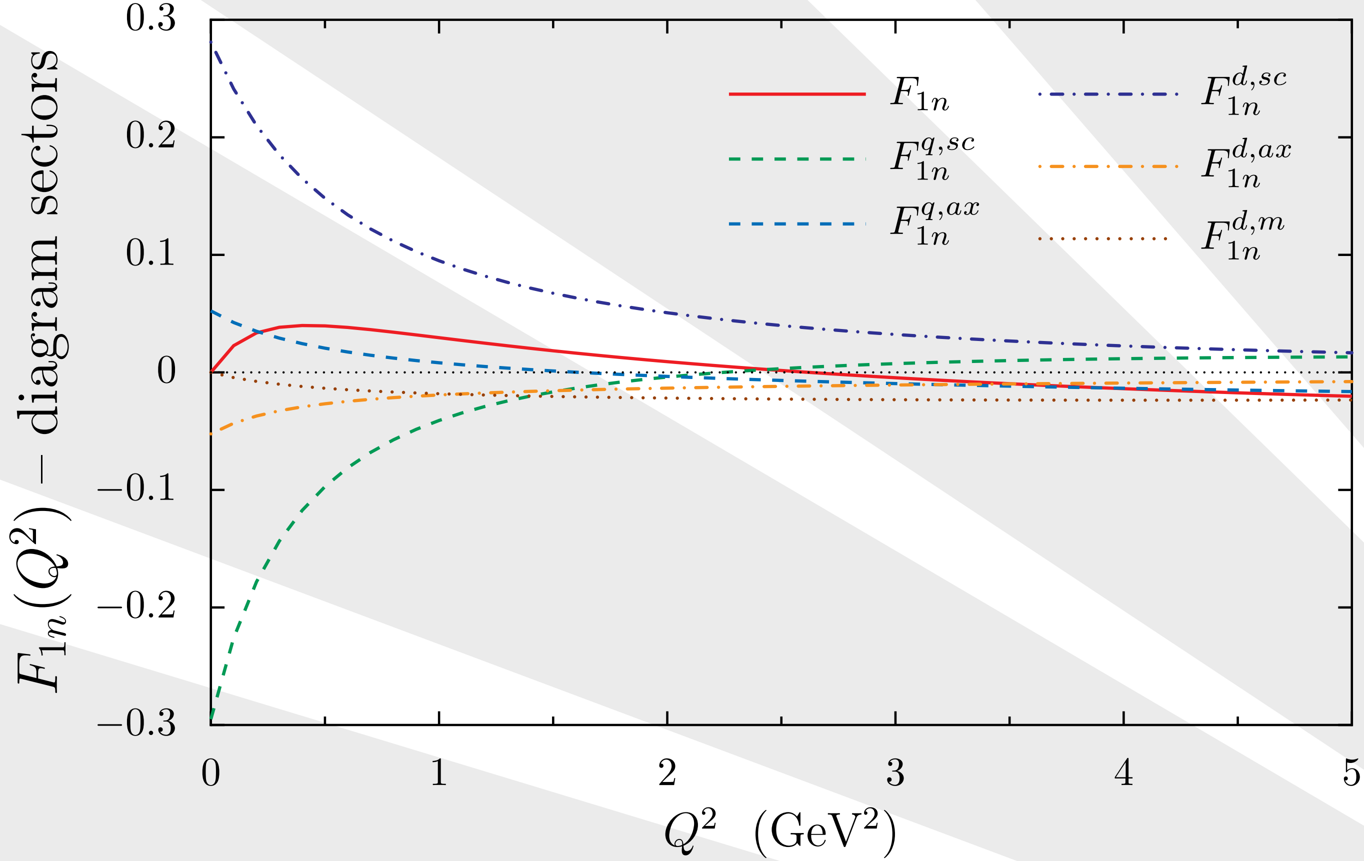
- NJL: $\kappa_p = 1.72$, Experiment: $\kappa_p = 1.79$
- NJL: $\langle r_M^2 \rangle_p = 0.56\,\text{fm}^2$, Experiment: $\langle r_M^2 \rangle_p = 0.71\,\text{fm}^2$

Proton Form Factors – no pion



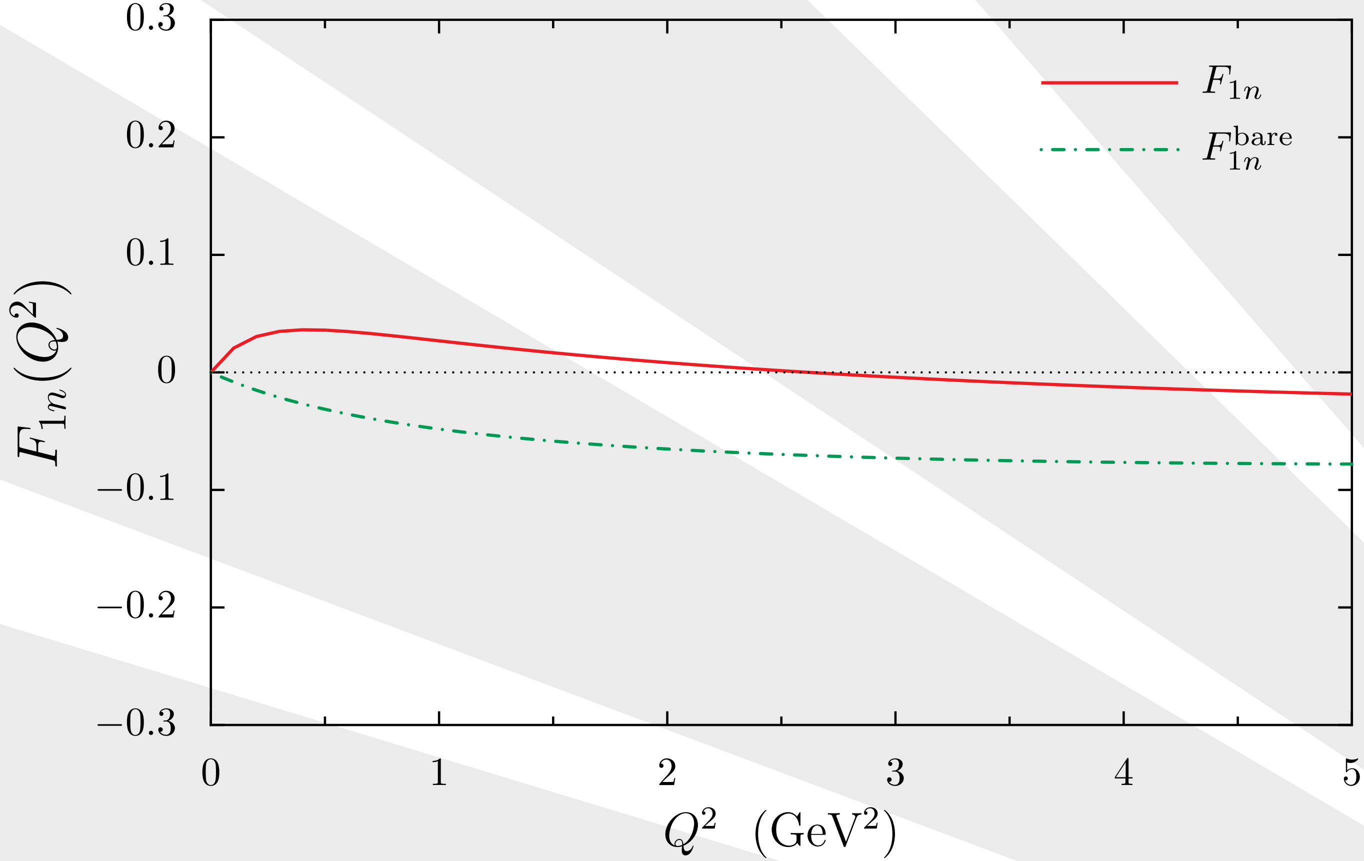
- NJL: $\kappa_p = 1.72$, NJL$_{\mathrm{bare}}$: $\kappa_p = 1.61$
- NJL: $\langle r_M^2 \rangle_p = 0.56\,\mathrm{fm}^2$, NJL$_{\mathrm{bare}}$: $\langle r_M^2 \rangle_p = 0.38\,\mathrm{fm}^2$

Neutron Form Factors



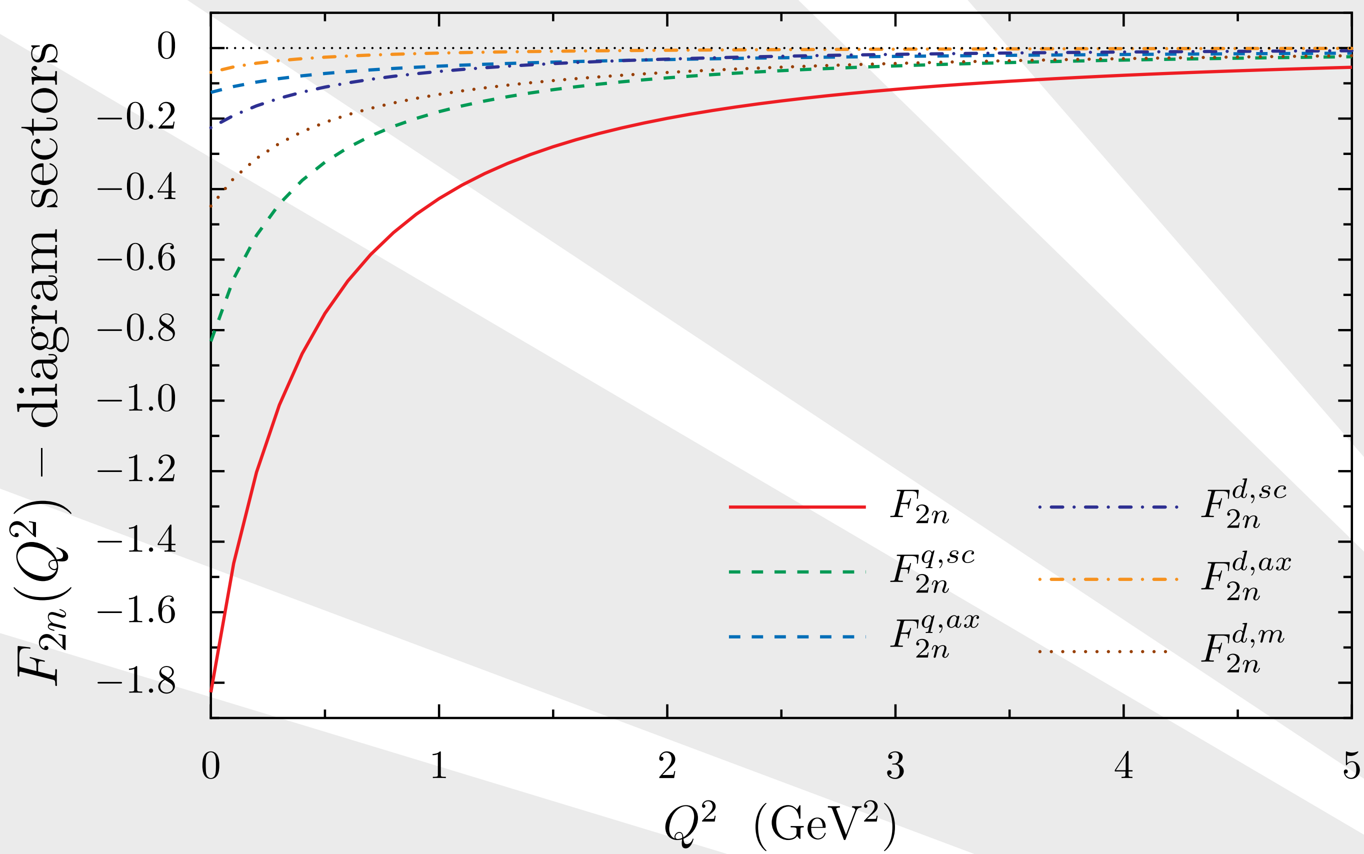
- NJL: $\langle r_E^2 \rangle_n = -0.15\,\text{fm}^2$, Experiment: $\langle r_E^2 \rangle_n = -0.12\,\text{fm}^2$

Neutron Form Factors – no pion



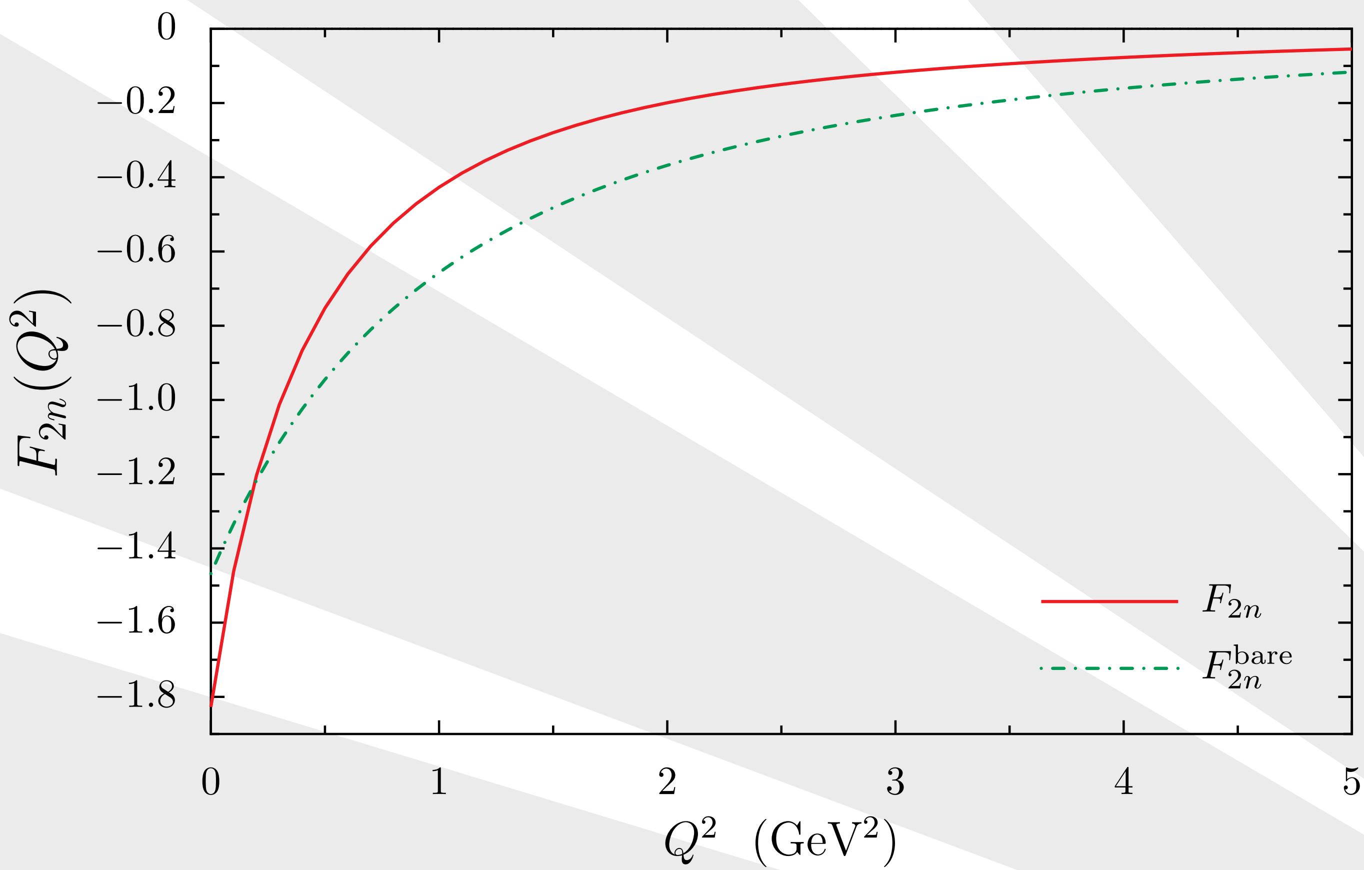
- NJL: $\langle r_E^2 \rangle_n = -0.15$ fm^2, NJL$_{\text{bare}}$: $\langle r_E^2 \rangle_n = -0.07$ fm^2

Neutron Form Factors



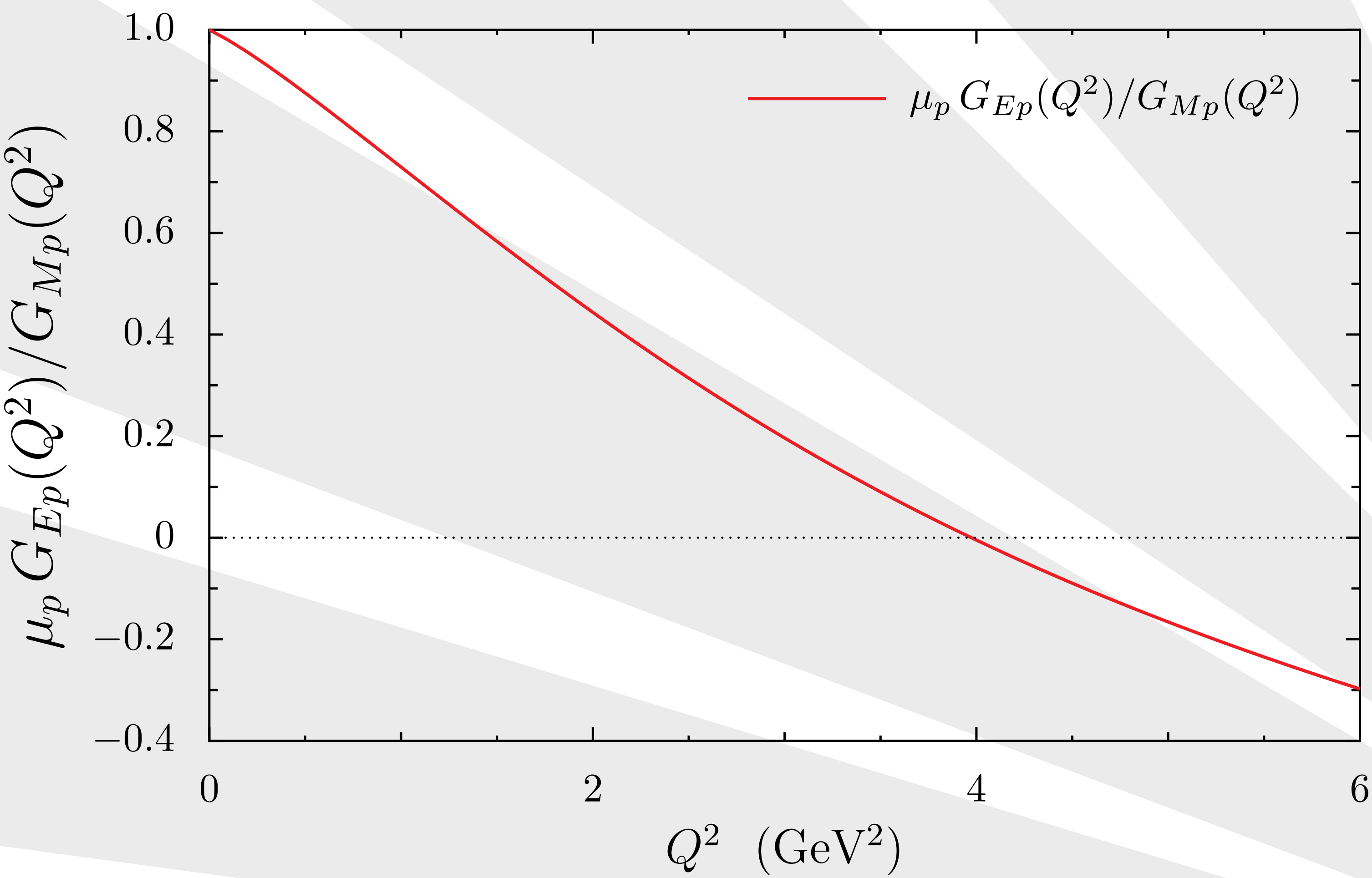
- NJL: $\kappa_n = -1.83$, Experiment: $\kappa_n = -1.91$
- NJL: $\langle r_M^2 \rangle_n = 0.54\,\text{fm}^2$, Experiment: $\langle r_M^2 \rangle_n = 0.79\,\text{fm}^2$

Neutron Form Factors – no pion



- NJL: $\kappa_n = -1.83$, NJL$_{\text{bare}}$: $\kappa_n = 1.47$
- NJL: $\langle r_M^2 \rangle_n = 0.54\,\text{fm}^2$, NJL$_{\text{bare}}$: $\langle r_M^2 \rangle_n = 0.37\,\text{fm}^2$

G_E/G_M *Proton Ratio*



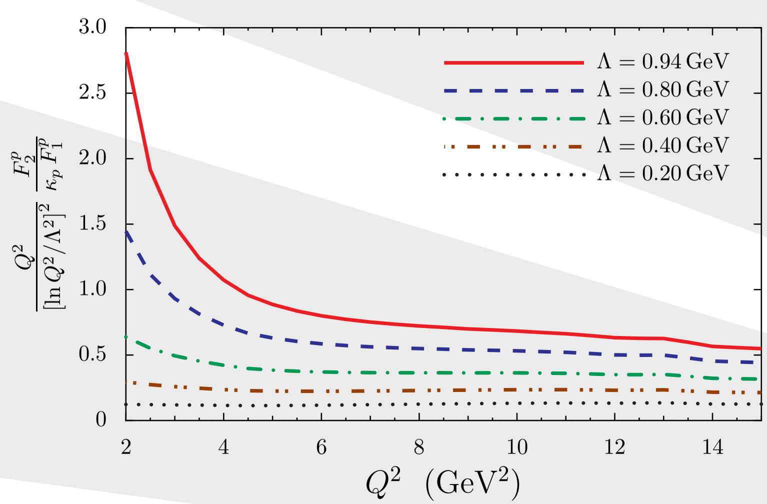
- F_{2p} falls off to slowly

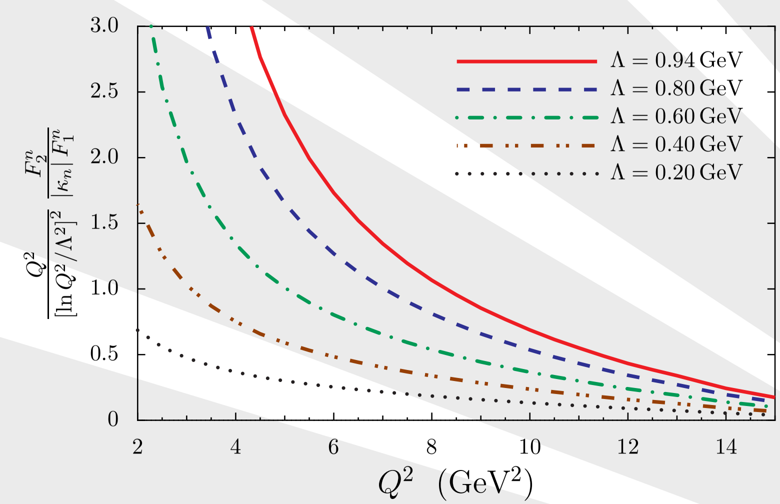
Perturbative QCD Limit – DSE

- pQCD Prediction:

$$\frac{Q^2}{\ln(Q^2/\Lambda^2)^2}\,\frac{F_2(Q^2)}{F_1(Q^2)} = constant, \qquad Q^2 \gg \Lambda^2$$

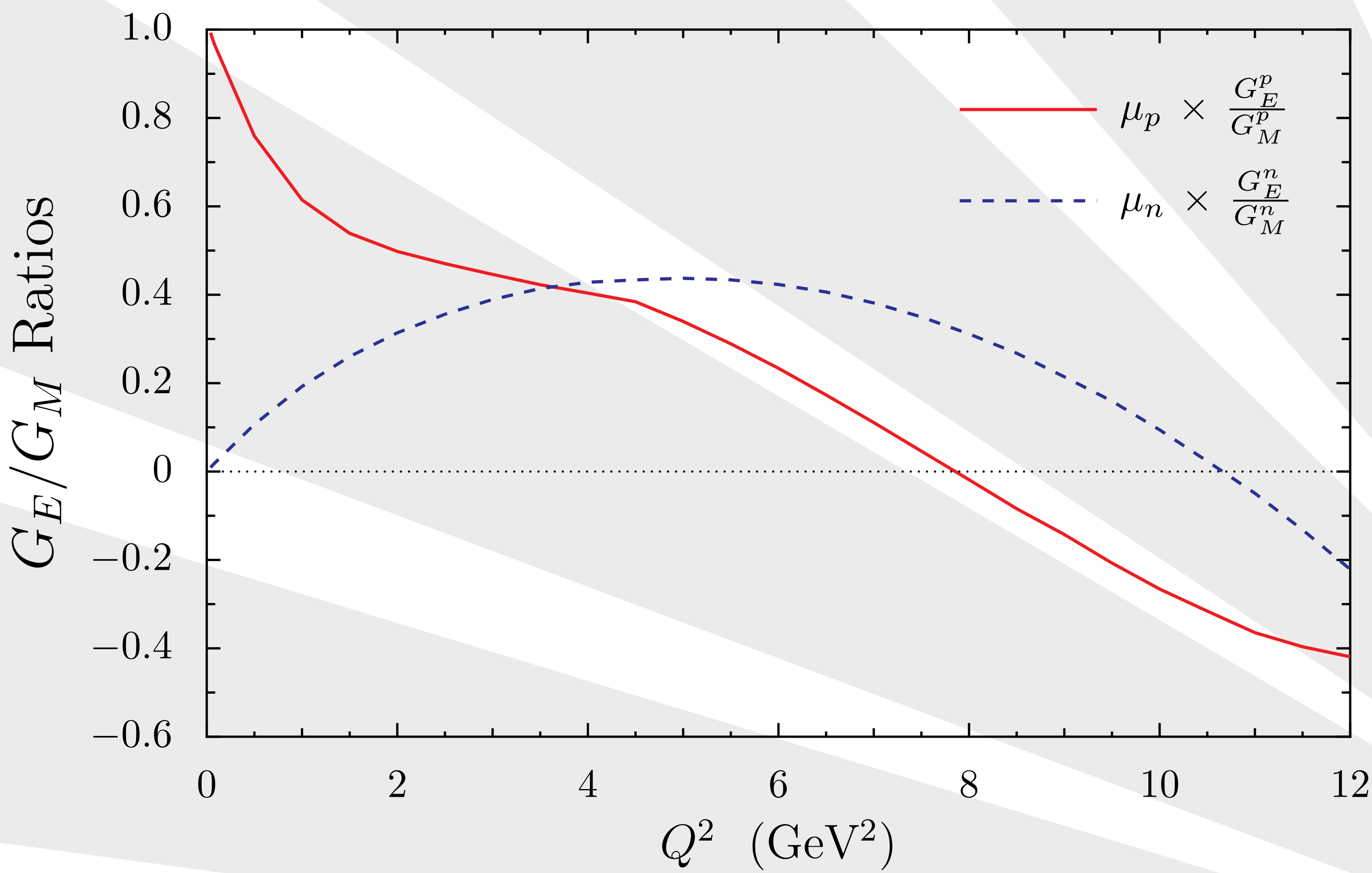
✦ A. V. Belitsky, X. d. Ji and F. Yuan, Phys. Rev. Lett. **91**, 092003 (2003).





- Proton $\rightarrow \mathcal{C}_\Lambda$, Neutron $\rightarrow 0$.

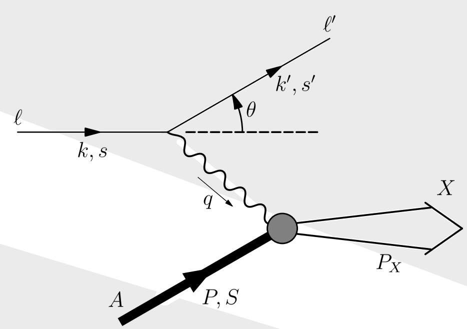
G_E and G_M Ratios

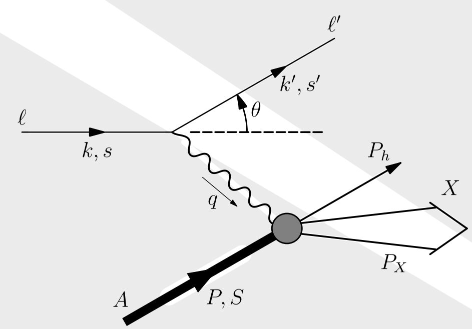


- Proton → 7.8 GeV^2, Neutron → 10.8 GeV^2.

Quark Distributions in the Nucleon

- Three twist-2 parton distributions $(k_\perp = 0)$:
  - Spin-Independent: $q(x)$
  - Helicity: $\Delta q(x)$
  - Transversity: $\Delta_T q(x)$





- All distributions have probability interpretation.
- Essentially contain non-perturbative information.

Definition and Charges

- Light-cone Fourier transforms :

$$q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\xi^-} \langle p, s|\overline{\psi}_q(0)\gamma^+\psi_q(\xi^-)|p, s\rangle_c$$

$$\Delta q(x) = \langle \gamma^+\gamma_5 \rangle, \qquad \Delta_T q(x) = \langle \gamma^+\gamma^1\gamma_5 \rangle$$

- Related to the nucleon axial & tensor charges via

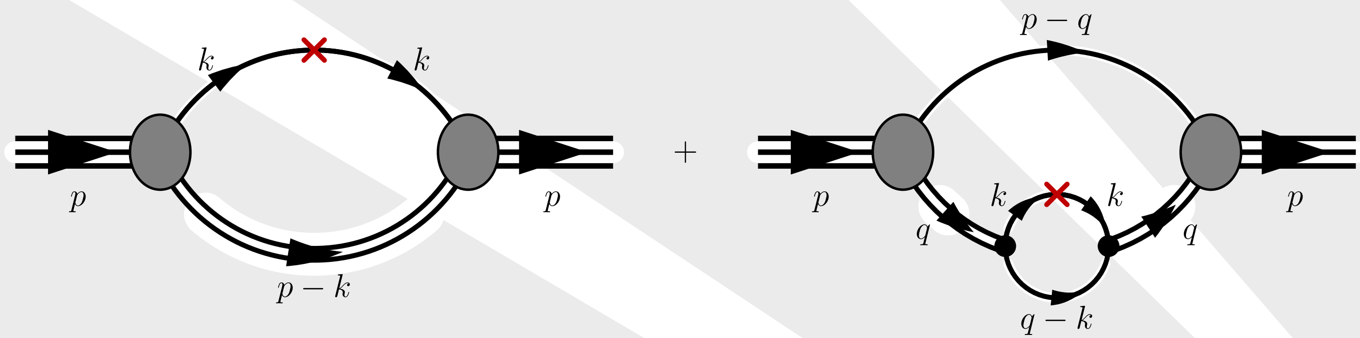
$$g_A = \int dx \Delta u(x) - \Delta d(x), \quad g_T = \int dx \Delta_T u(x) - \Delta_T d(x),$$

- Satisfy positivity constraints and Soffer bound

$$\Delta q(x), \Delta_T q(x) \leqslant q(x), \quad q(x) + \Delta q(x) \geqslant 2\,|\Delta_T q(x)|$$

Nucleon quark distributions

- Associated with a Feynman diagram calculation.

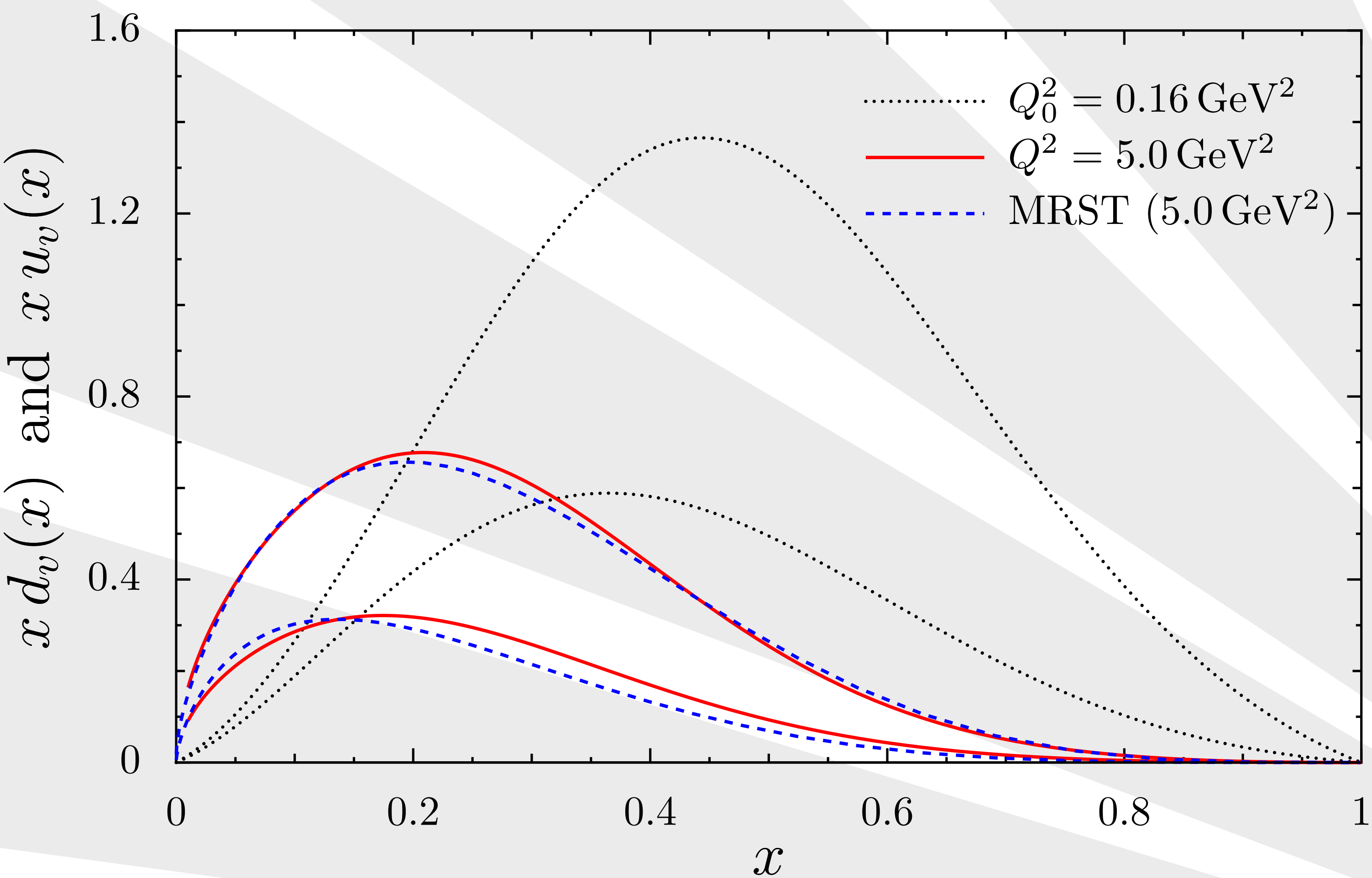


✦ $[q(x), \Delta q(x), \Delta_T q(x)]$

$$\rightarrow \quad \times = \ \delta(x - \tfrac{k^+}{p^+}) \left[\gamma^+, \gamma^+\gamma_5, \gamma^+\gamma^1\gamma_5\right]$$

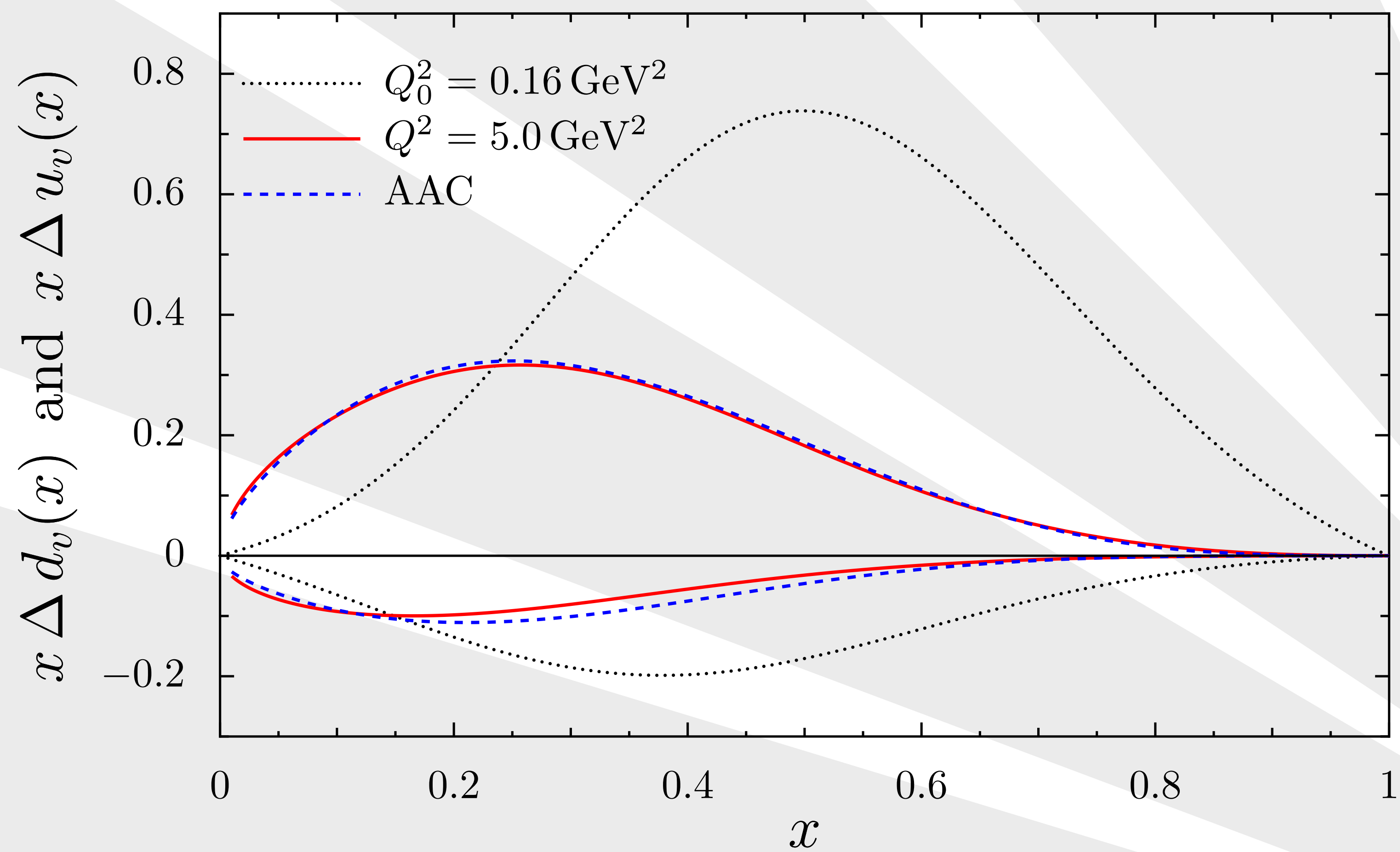
- Satisfies baryon and momentum sum rules.
- Satisfies positivity constraints and Soffer bound.
- Covariant and gives correct support.

$u_v(x)$ ***and*** $d_v(x)$ ***distributions***



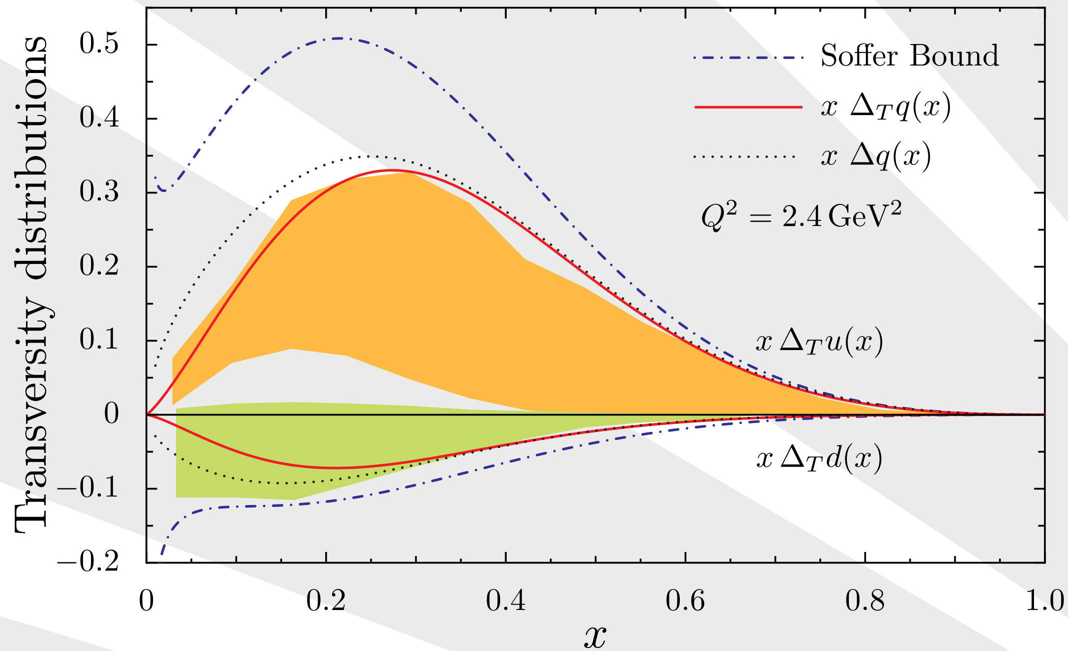
- MRST, Phys. Lett. B **531**, 216 (2002).

$\Delta u_v(x)$ *and* $\Delta d_v(x)$ *distributions*



- M. Hirai, S. Kumano and N. Saito, Phys. Rev. D **69**, 054021 (2004).

$\Delta_T u_v(x)$ *and* $\Delta_T d_v(x)$ *distributions*



- M. Anselmino *et. al.*,Phys. Rev. D **75**, 054032 (2007).
- Non-relativistic limit: $\Delta_T q(x) = \Delta q(x)$
- $M \sim 400$ MeV, large relat. corrections unexpected
- Potential problem for models based concept of "constituent quarks" – maybe running mass

Asymmetric Nuclear Matter: Lagrangian

- Finite Density Lagrangian: $\sigma,\ \omega,\ \rho$ mean fields

$$\mathcal{L} = \overline{\psi}\,(i\,\not{\partial} - M^* - \not{V}\,)\,\psi + \mathcal{L}'_I$$

- σ: isoscalar-scalar – attractive
  ω: isoscalar-vector – repulsive
  ρ: isovector-vector – attractive/repulsive

- Finite density quark propagator

$$S(k) = \frac{1}{\not{k} - M^* - \not{V}}$$

- Quark vector potentials:

$$V_u = \omega_0 + \rho_0 \qquad V_d = \omega_0 - \rho_0$$

Asymmetric Nuclear Matter: Effective Potential

- Hadronization $\rightarrow$ Effective potential

$$\mathcal{E} = \mathcal{E}_V - \frac{\omega_0^2}{4\,G_\omega} - \frac{\rho_0^2}{4\,G_\rho} + \mathcal{E}_p + \mathcal{E}_n$$

$$\mathcal{E}_{p(n)} = 2\int \frac{d^3\vec{p}}{(2\pi^3)} n(k) \left[\sqrt{M_N^2 + \vec{p}^{\,2}_{F_{p(n)}}} + 3\omega_0 \pm \rho_0\right]$$

✦ $\mathcal{E}_V$: Vacuum Energy
$\mathcal{E}_{p(n)}$: Energy of nucleons moving in $\sigma,\ \omega,\ \rho$ fields

- Vector fields

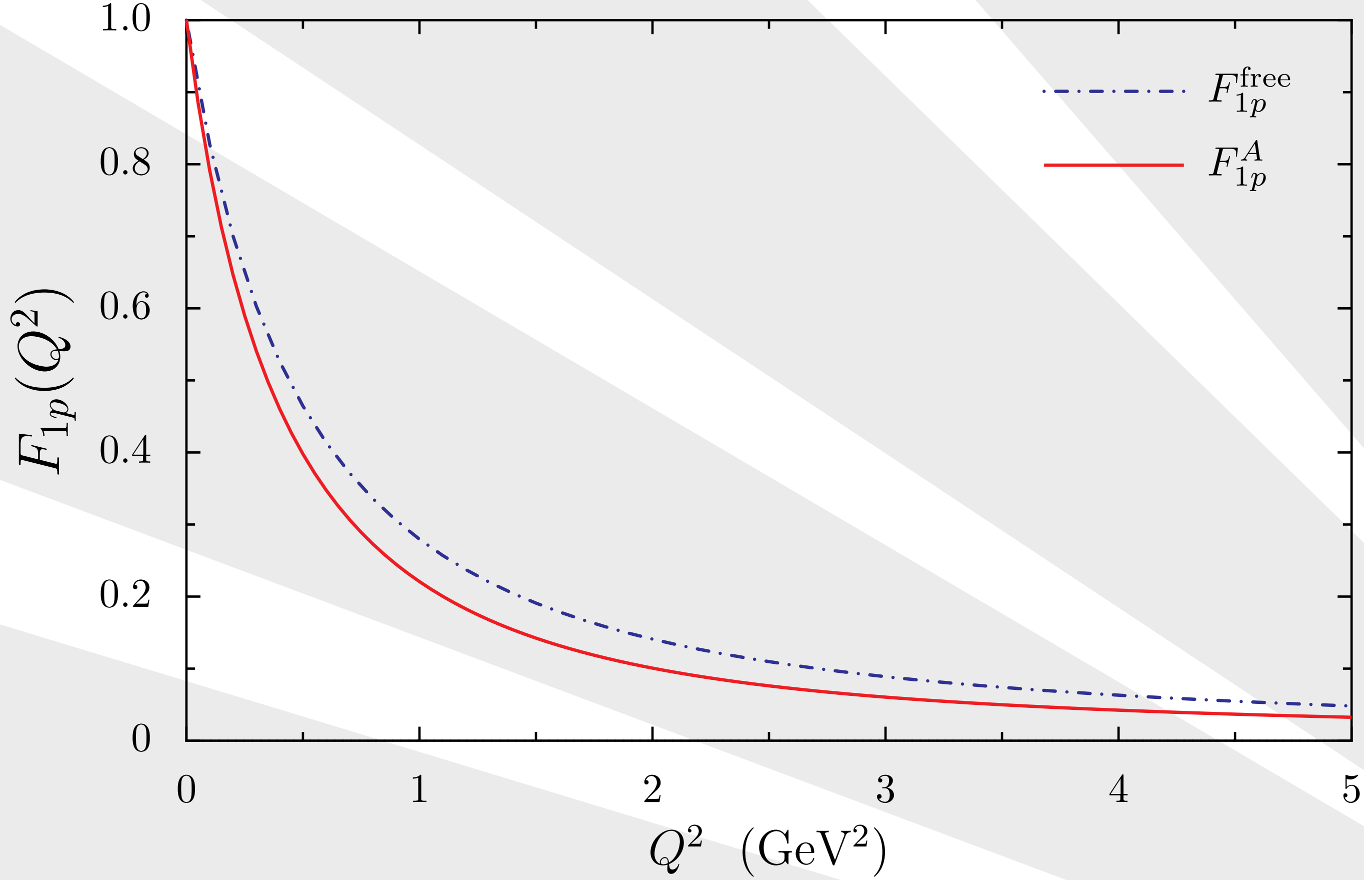
$$\frac{\partial\mathcal{E}}{\partial\omega_0} = 0 \quad \Longrightarrow \quad \omega_0 = 6\,G_\omega\,(\rho_p + \rho_n)$$

$$\frac{\partial\mathcal{E}}{\partial\rho_0} = 0 \quad \Longrightarrow \quad \rho_0 = 2\,G_\rho\,(\rho_p - \rho_n)$$

- Nucleon vector potentials:

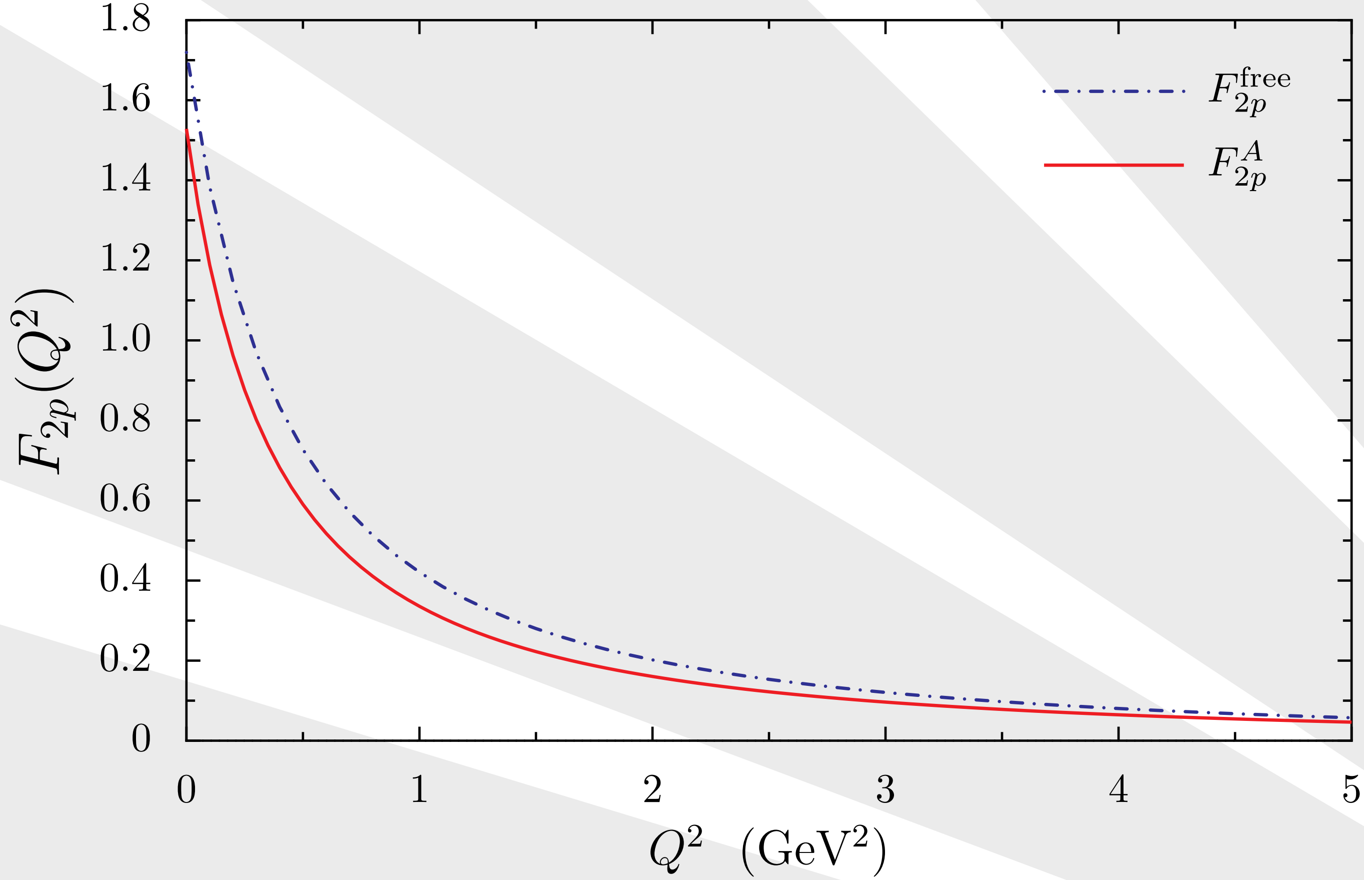
$$V_p = 3\omega_0 + \rho_0 \qquad V_n = 3\omega_0 - \rho_0$$

In-medium Proton Form Factors



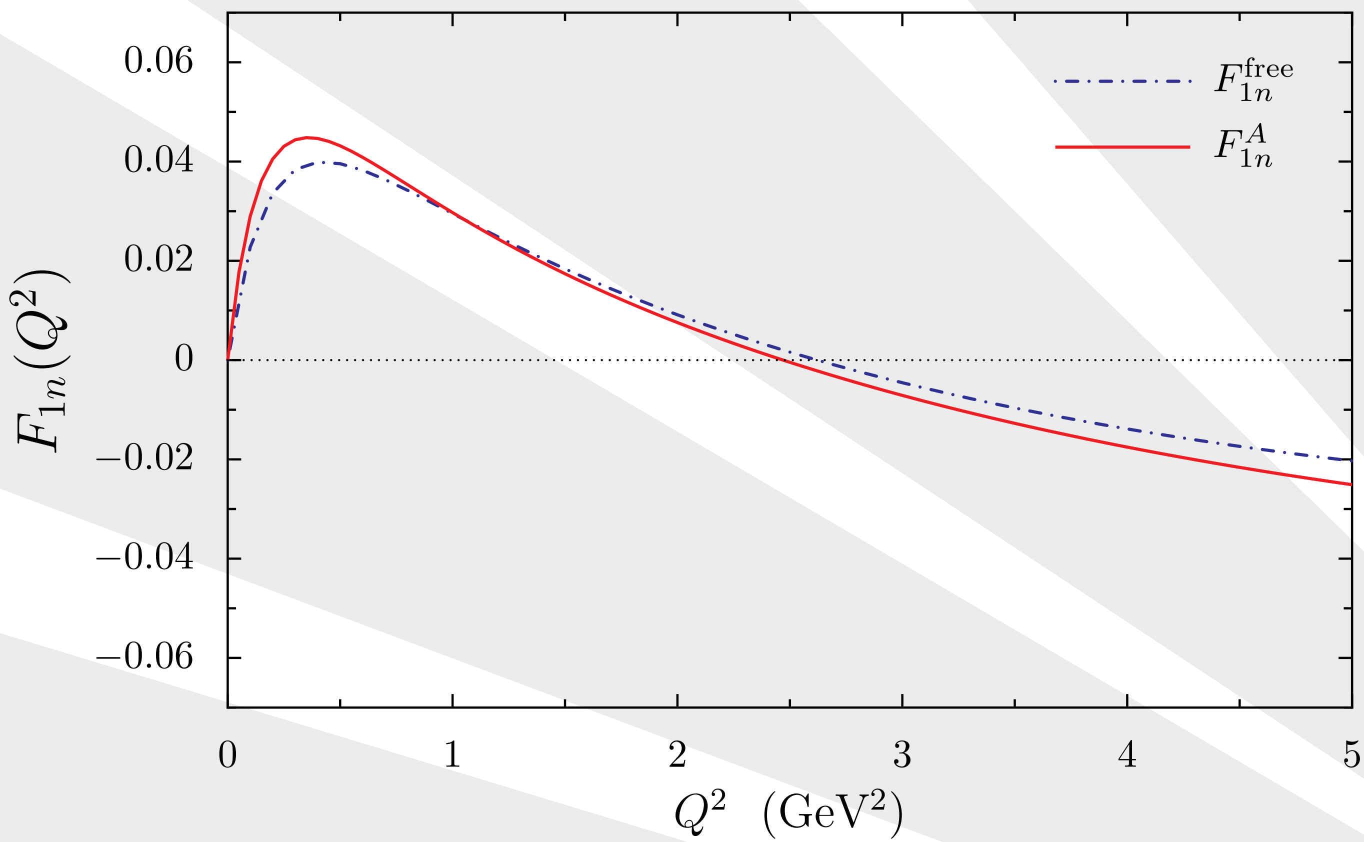
- Free: $\langle r_E^2 \rangle_p = 0.58\,\text{fm}^2$, In-medium: $\langle r_E^2 \rangle_p = 0.66\,\text{fm}^2$

In-medium Proton Form Factors



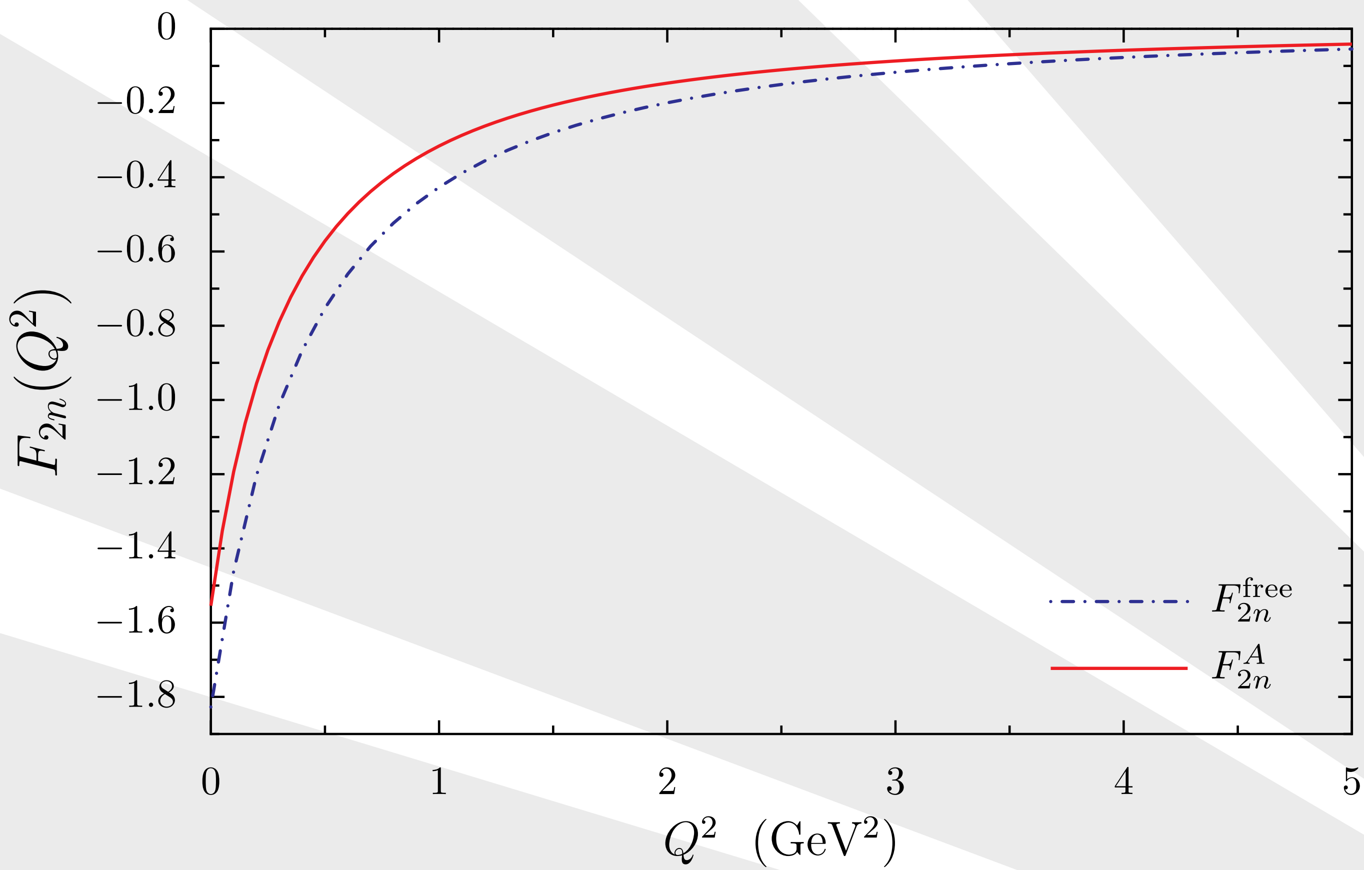
- Free: $\kappa_p = 1.72$, In-medium: $\kappa_p = 1.53$
- Free: $\langle r_M^2 \rangle_p = 0.56\,\mathrm{fm}^2$, In-medium: $\langle r_M^2 \rangle_p = 0.78\,\mathrm{fm}^2$

In-medium Neutron Form Factors



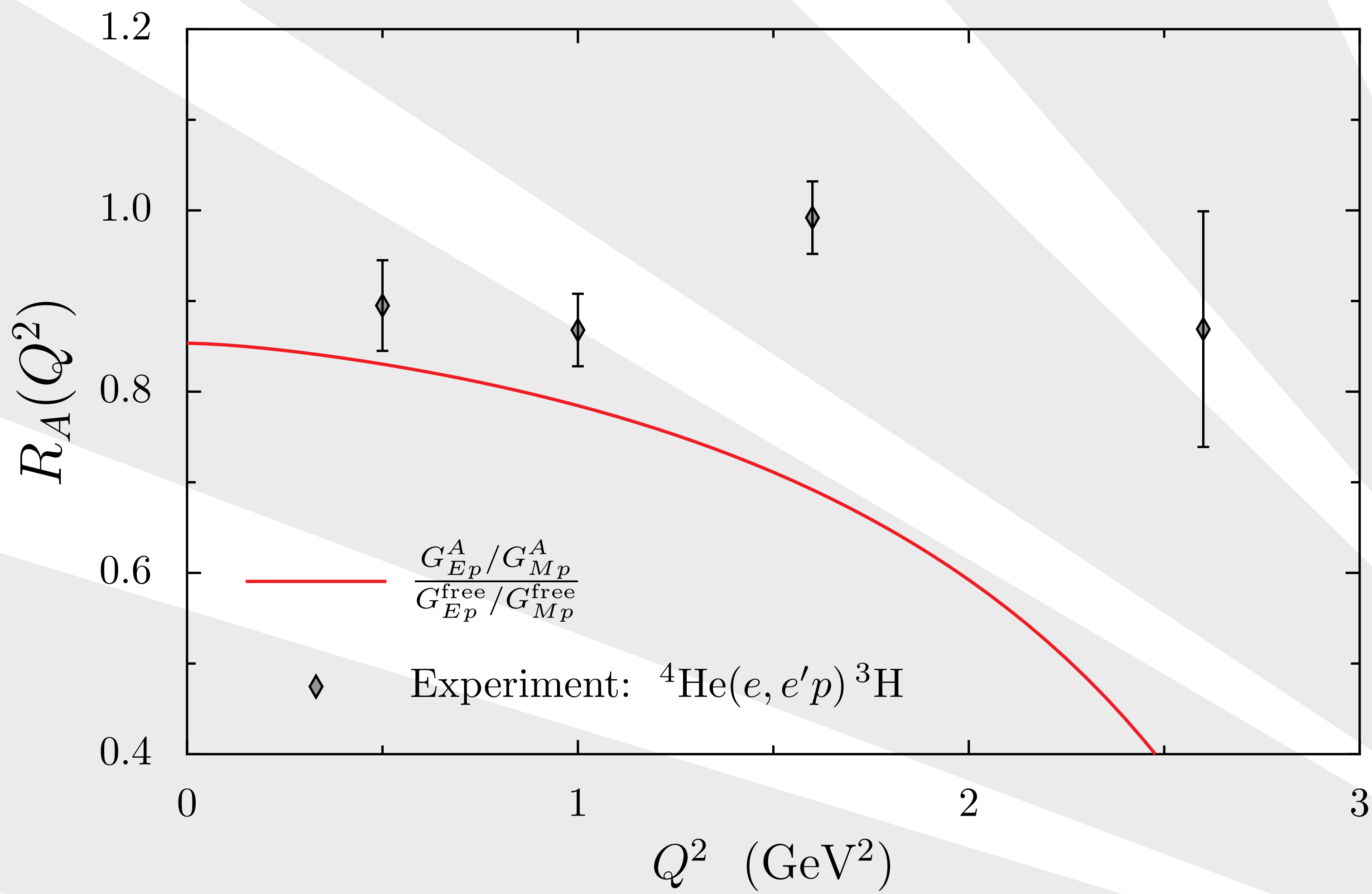
- Free: $\langle r_E^2 \rangle_n = -0.15\,\text{fm}^2$, In-medium: $\langle r_E^2 \rangle_n = -0.18\,\text{fm}^2$

In-medium Neutron Form Factors



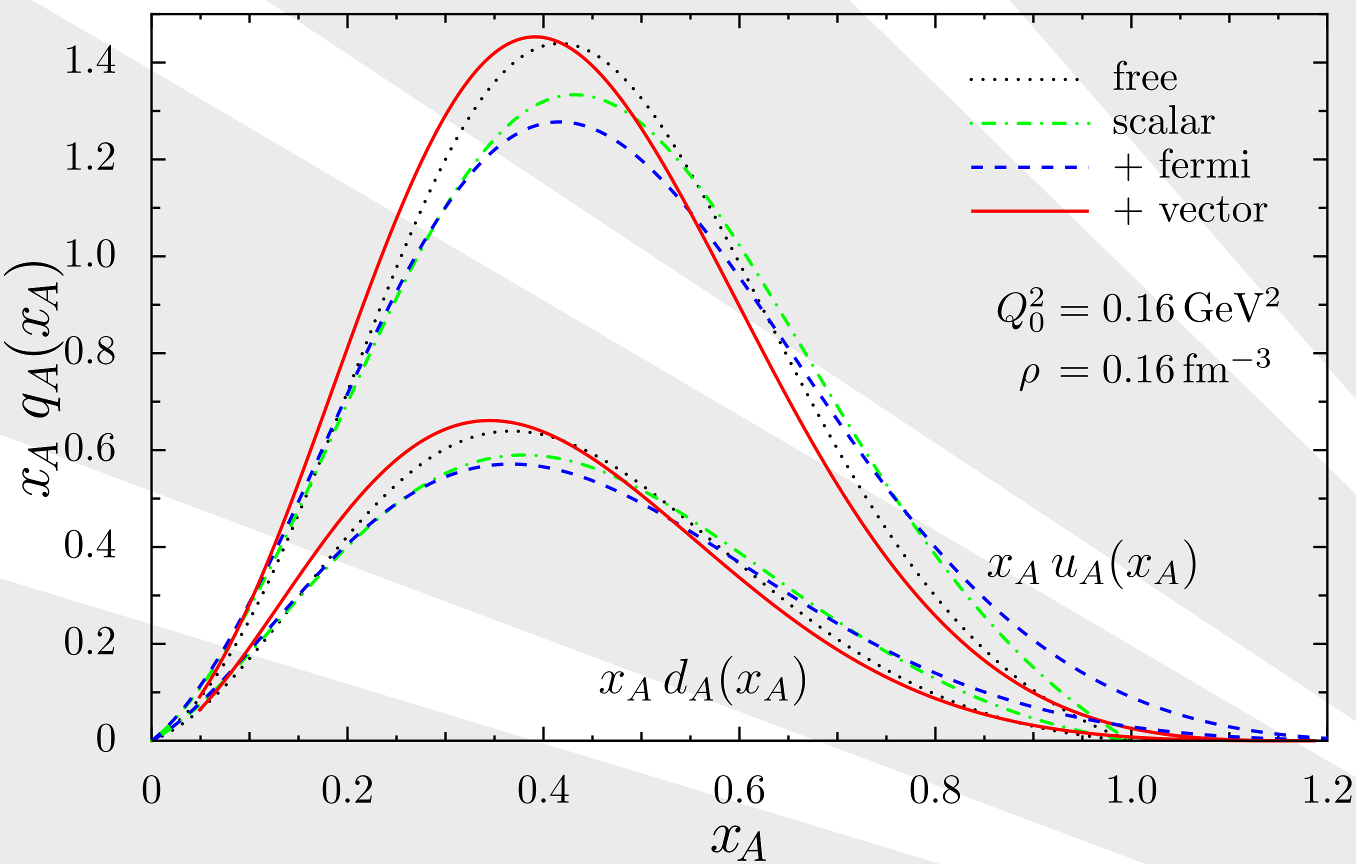
- Free: $\kappa_n = -1.83$, In-medium: $\kappa_n = -1.41$
- Free: $\langle r_M^2 \rangle_n = 0.54\,\text{fm}^2$, In-medium: $\langle r_M^2 \rangle_n = 0.79\,\text{fm}^2$

GE/GM In-medium/free ratio



- S. Strauch *et al.* [Jefferson Lab E93-049 Collaboration],Phys. Rev. Lett. **91**, 052301 (2003)
- Result is for nuclear matter

In-medium Quark Distributions



- Vector field plays critical role: non-local operator
- Fermi motion: $f_p(y_A)=\frac{Z}{A}\,\frac{3}{4}\left(\frac{\overline{M}_N}{p_{Fp}}\right)^3\left[\left(\frac{p_{Fp}}{\overline{M}_N}\right)^2-\left(\frac{\varepsilon_{Fp}}{\overline{M}_N}-y_A\right)^2\right]$

Definition: Polarized EMC effect

- EMC ratio

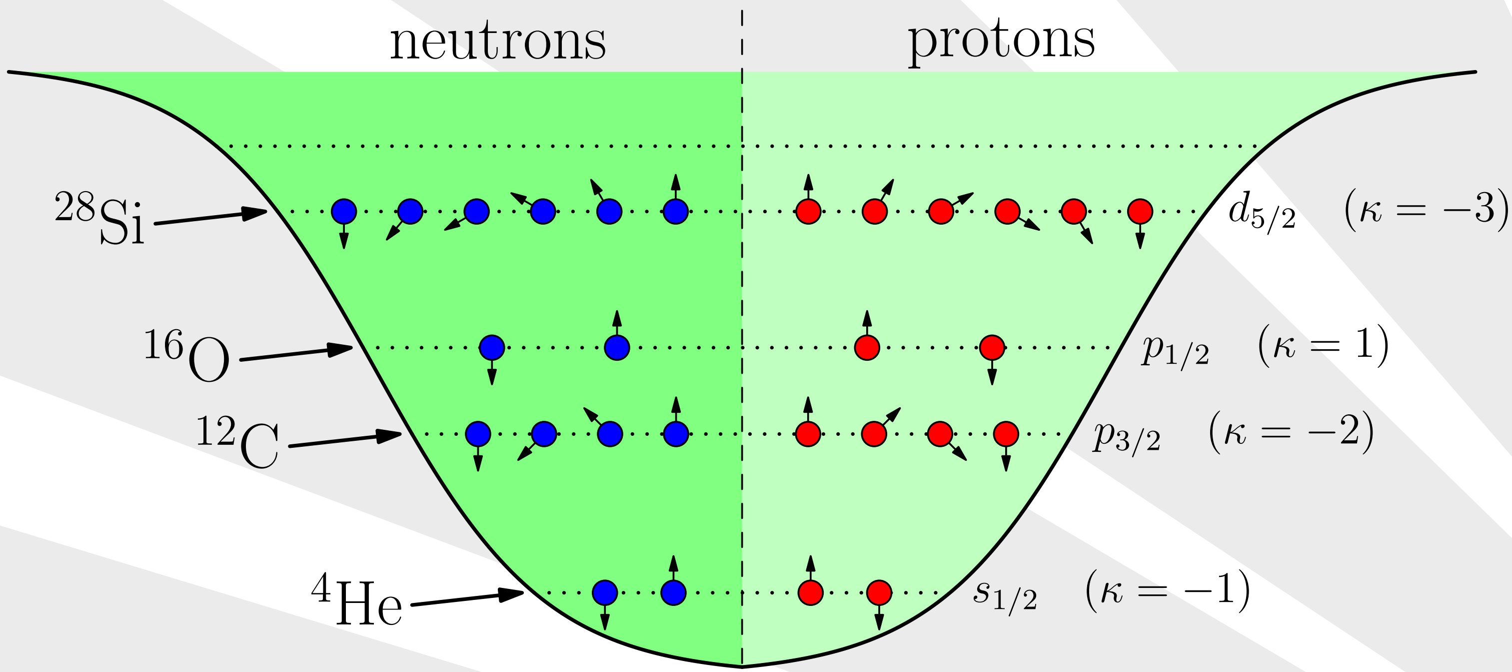
$$R = \frac{F_{2A}}{F_{2A}^{\text{naive}}} = \frac{F_{2A}}{Z\,F_{2p} + (A-Z)\,F_{2n}}$$

- Polarized EMC ratio

$$R_s^{JH} = \frac{g_{1A}^{JH}}{g_{1A,\text{naive}}^{JH}} = \frac{g_{1A}^{JH}}{P_p^{JH}\,g_{1p} + P_n^{JH}\,g_{1n}}$$

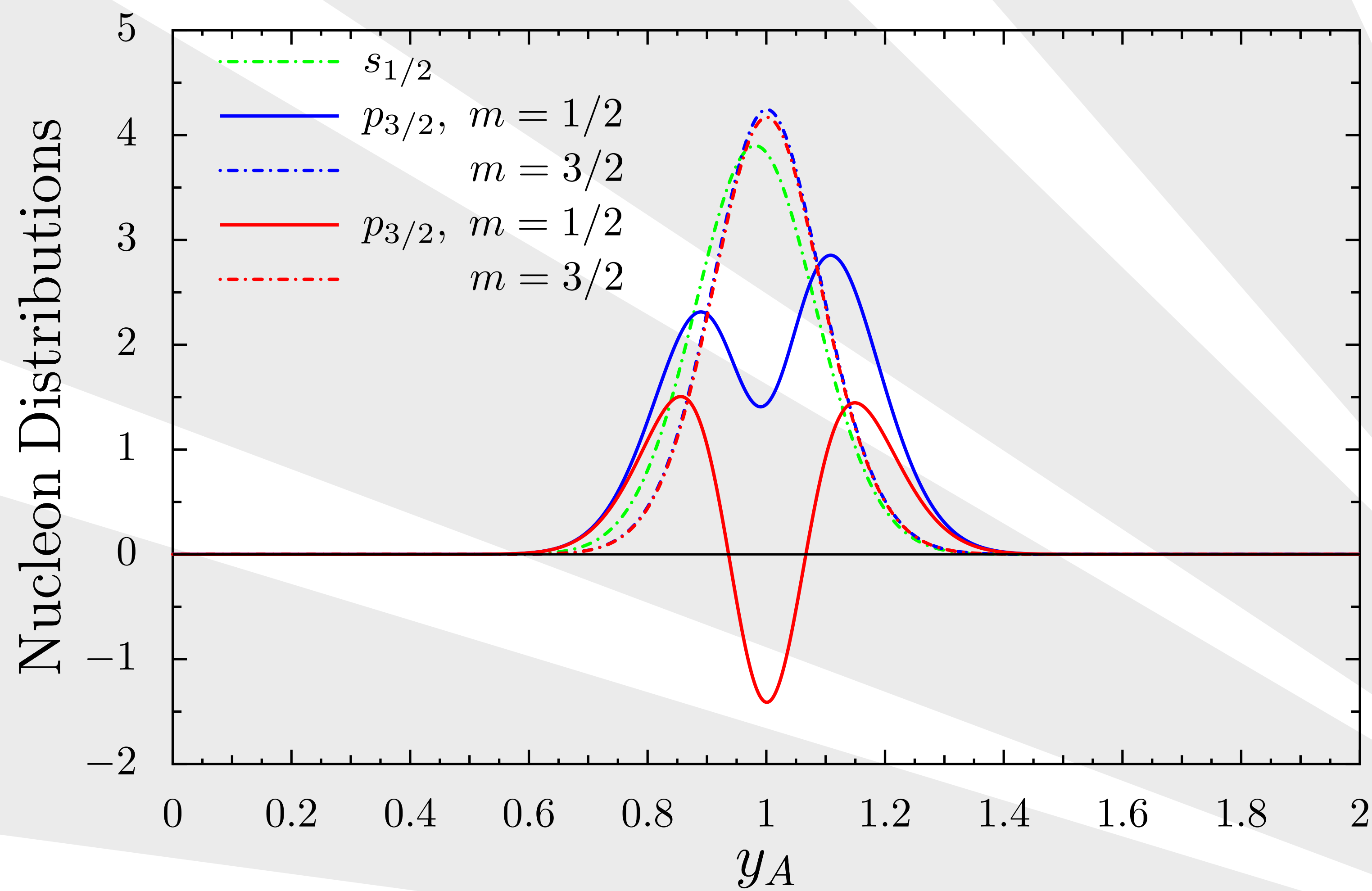
- $1/A$ effect, i.e. $A \lesssim 27$, “proton states”, $\Longrightarrow$ ^{7}Li, ^{11}B, . . .
- QMC for ^{7}Li $\Longrightarrow$ $P_p^{JJ} = 0.86$ & $P_n^{JJ} = 0.04$
- Ratios equal 1 in non-relativistic and no-medium modification limit.

Shell Model

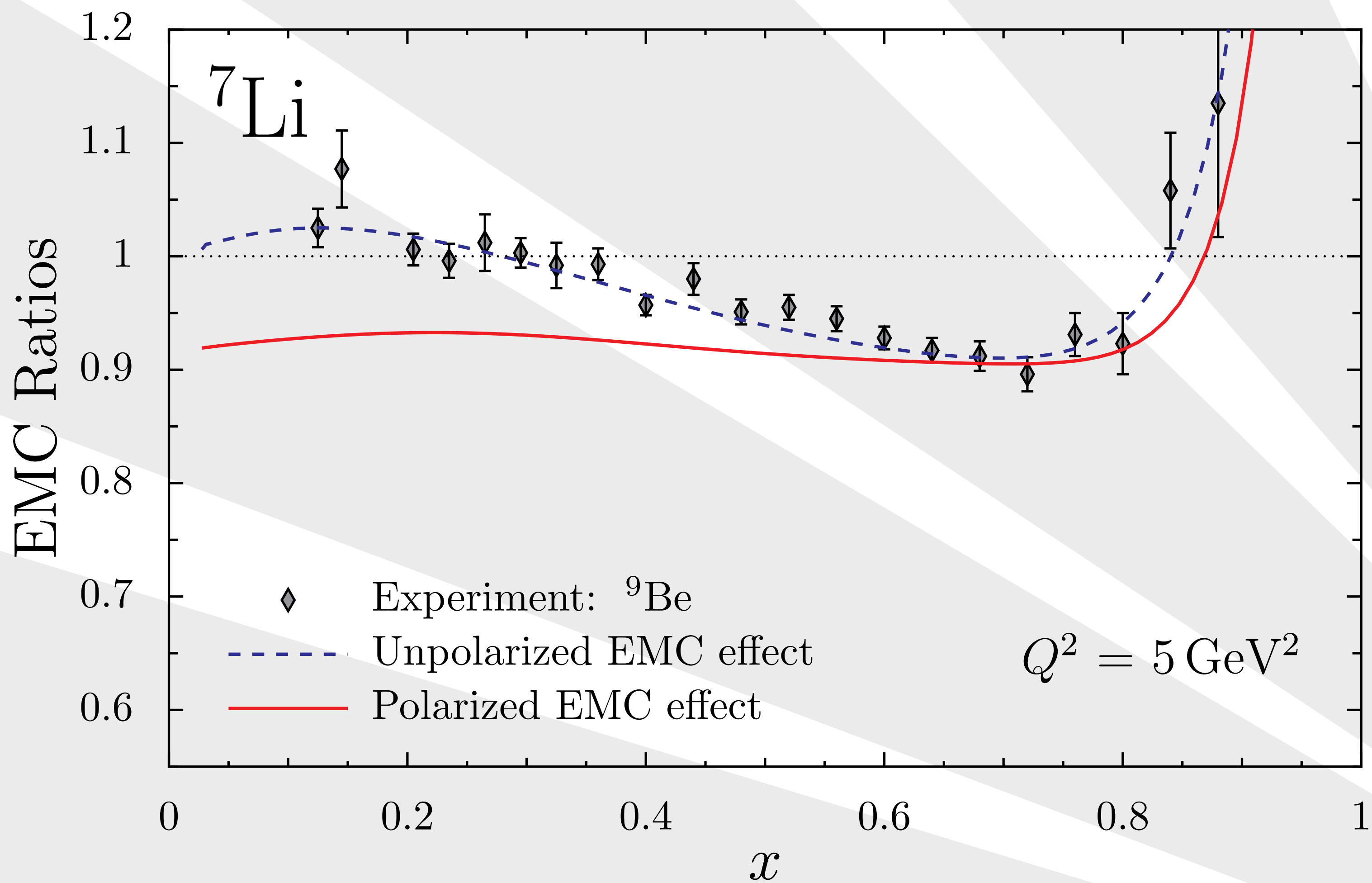


$$q_A^{JH}(x_A) = \sum_{\kappa,m} \int dy_A \int dx \; \delta(x_A - y_A\, x)\, f_{\kappa,m}^{(JH)}(y_A)\; q_\kappa(x)\,.$$

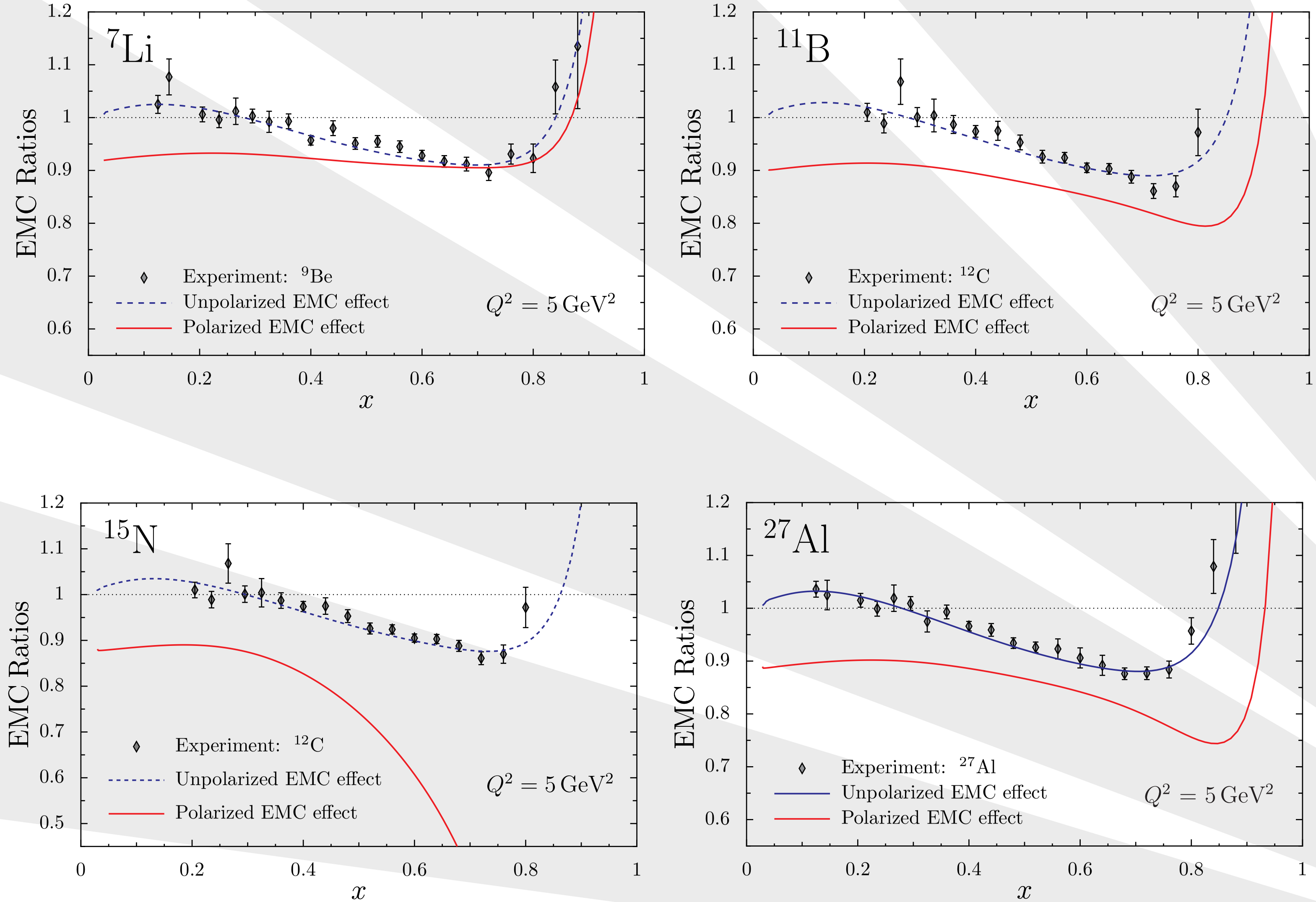
Nucleon distributions: ^{11}B



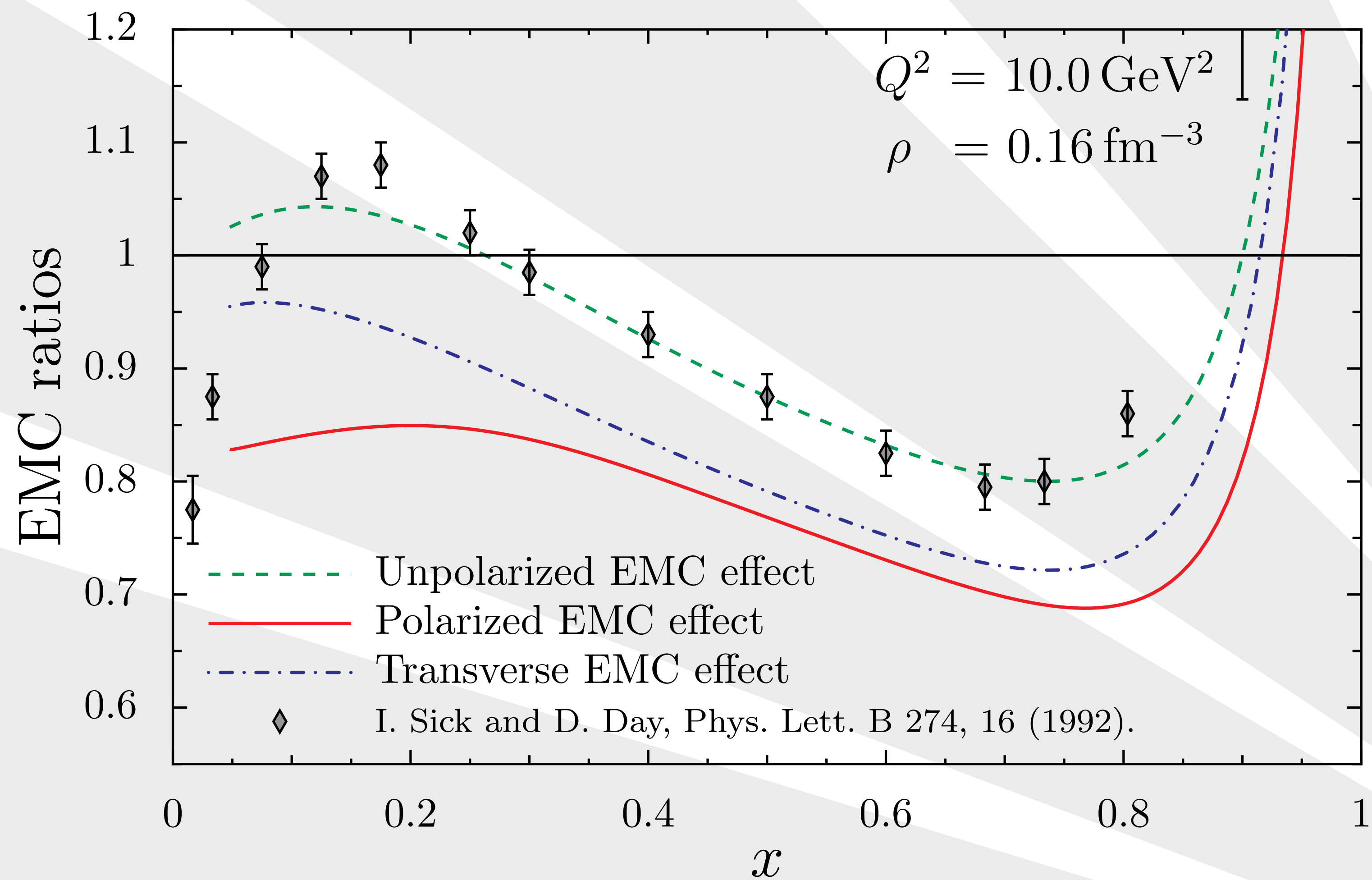
Polarized EMC ratio ^7Li



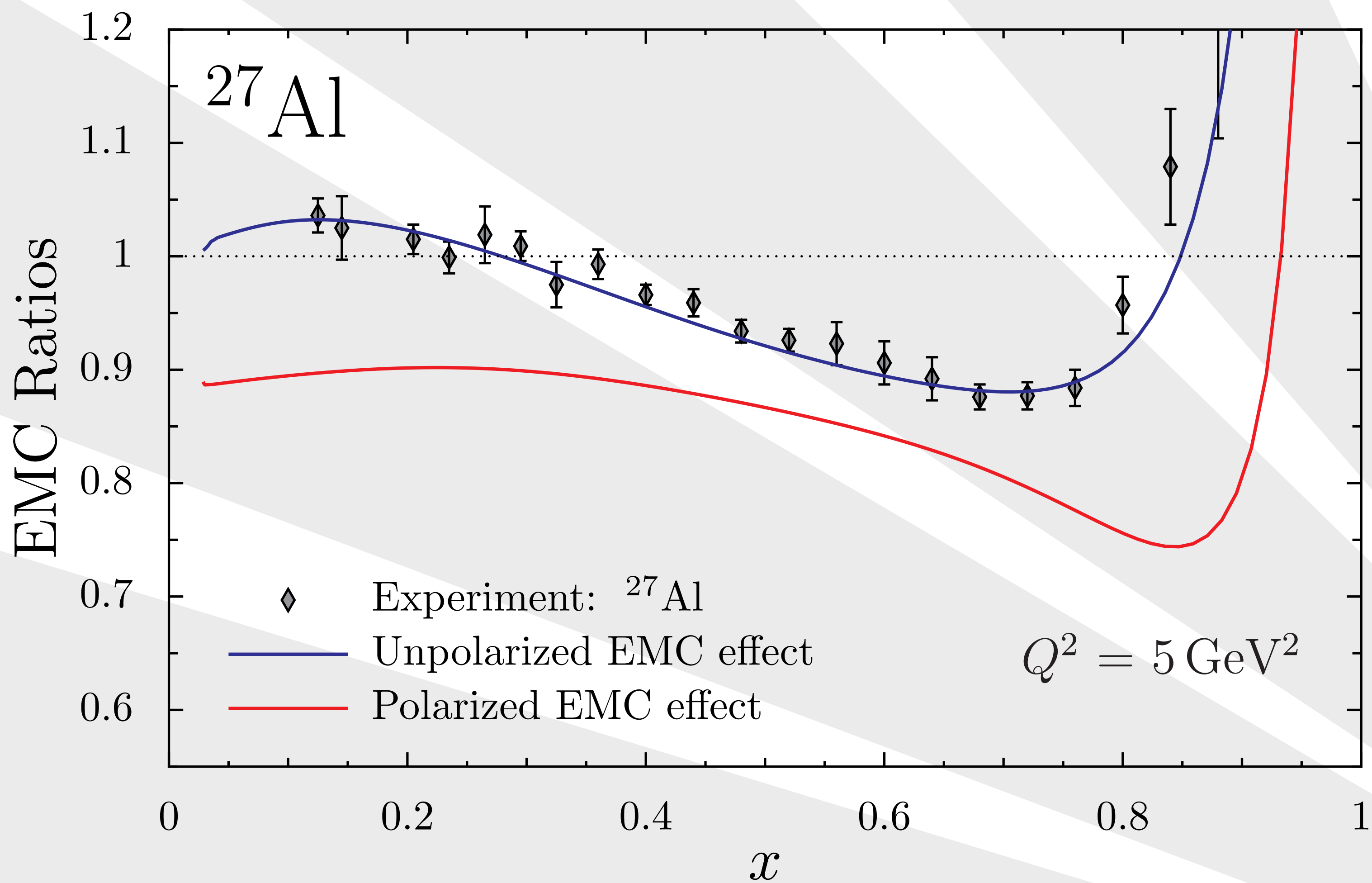
Polarized EMC ratio ^{7}Li, ^{11}B, ^{15}N and ^{27}Al



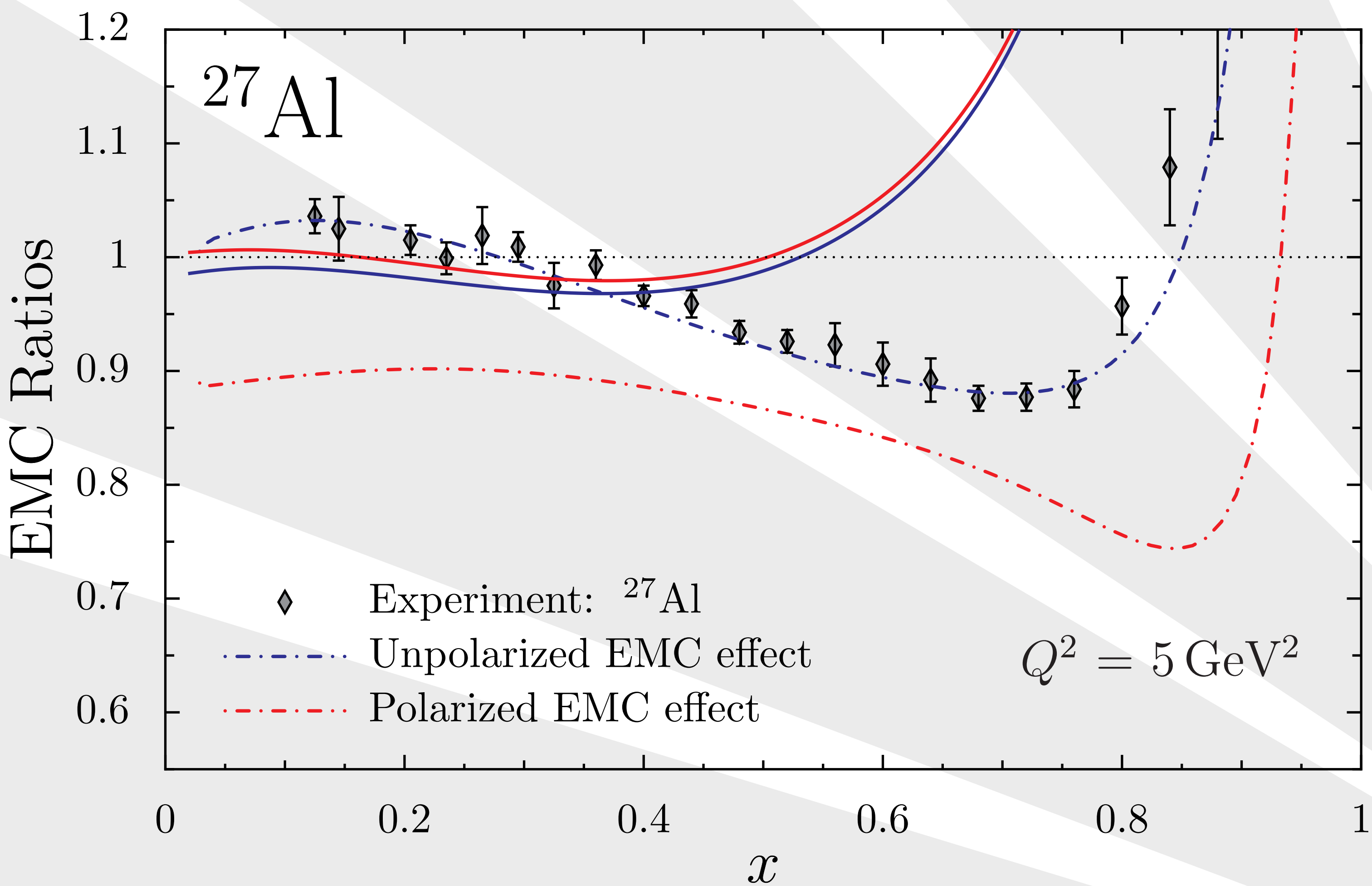
Nuclear Matter



Is there medium modification



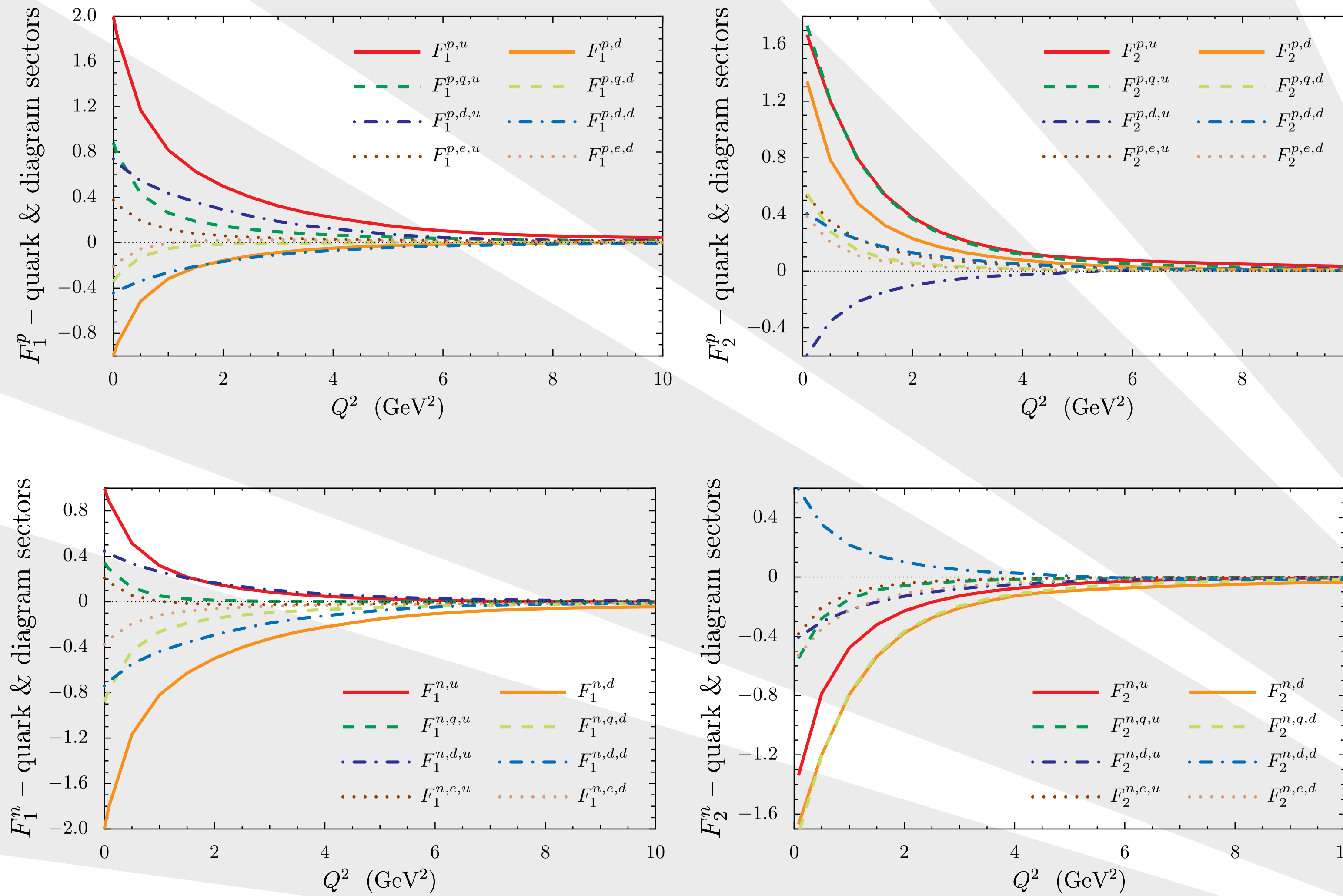
Is there medium modification



Conclusion

- Covariant, Confining, Faddeev formalism:
  - Free/in-medium Form Factors
  - Free/in-medium Quark Distributions
- Form Factors are particularly challenging:
  - Need pions for small Q^2
  - Perturbative limit for large Q^2: $(G_{En}/G_{Mn} \to 0)$
- For Quark Distributions DGLAP compensates
- Can incorporate quarks into many-body physics
- EMC effect explained via binding of quarks to mean scalar and vector fields
- Calculated the Polarized EMC effect in nuclei:
  - may answer “Are nucleons modified in-medium?”

DSE Form Factors: Quark Sector



Calculation

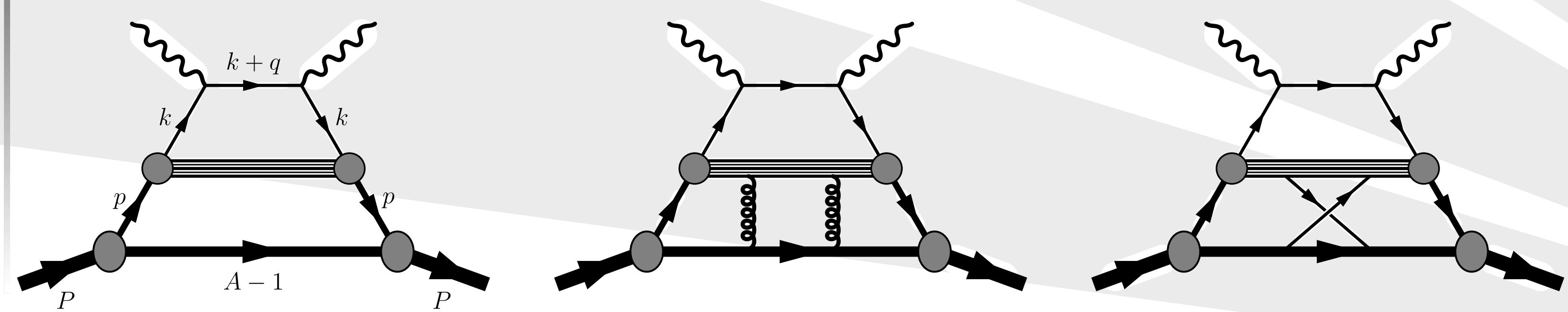
- Finite Nuclei quark distributions

$$q_A^{JH}(x_A) = \frac{P^+}{A}\int \frac{d\xi^-}{2\pi} e^{iP^+ x_A \xi^-/A} \langle A, P, H|\overline{\psi}_q(0)\,\gamma^+\,\psi_q(\xi^-)|A, P, H\rangle.$$

- Using Modified Convolution formalism

$$q_A^{JH}(x_A) = \sum_{\kappa,m}\int dy_A \int dx\ \delta(x_A - y_A\, x)\ f_{\kappa,m}^{(JH)}(y_A)\ q_\kappa(x)\,.$$

- Diagrammatically



(a) (b) (c)

Shell Model: Nucleon distribution functions

- Relativistic single particle shell model
- Nucleon distribution functions

$$f_{\kappa m}(y_A) = \frac{\sqrt{2}\,\overline{M}_N}{A} \int \frac{d^3p}{(2\pi)^3}$$
$$\delta\left(p^3 + \varepsilon_\kappa - \overline{M}_N\, y_A\right) \overline{\Psi}_{\kappa m}(\vec{p})\; \gamma^+\, \Psi_{\kappa\, m}(\vec{p})\,,$$

- Central Potential Dirac eigenfunctions

$$\Psi_{\kappa m}(\vec{p}) = (-i)^{\ell} \begin{bmatrix} F_\kappa(p)\,\Omega_{\kappa m}(\theta,\phi) \\ -G_\kappa(p)\,\Omega_{-\kappa m}(\theta,\phi) \end{bmatrix},$$

  - ✦ Dirac Equation

$$\left[-i\,\vec{\alpha}\cdot\vec{\nabla} + \beta\left[M(r) - V_s(r)\right] + V_v(r)\right]\psi_\kappa(r) = \varepsilon_\kappa\,\psi_\kappa(r)$$

- Assume Wood-Saxon scalar and vector potentials.

Nucleon distributions: Results

- Spin-independent nucleon distribution

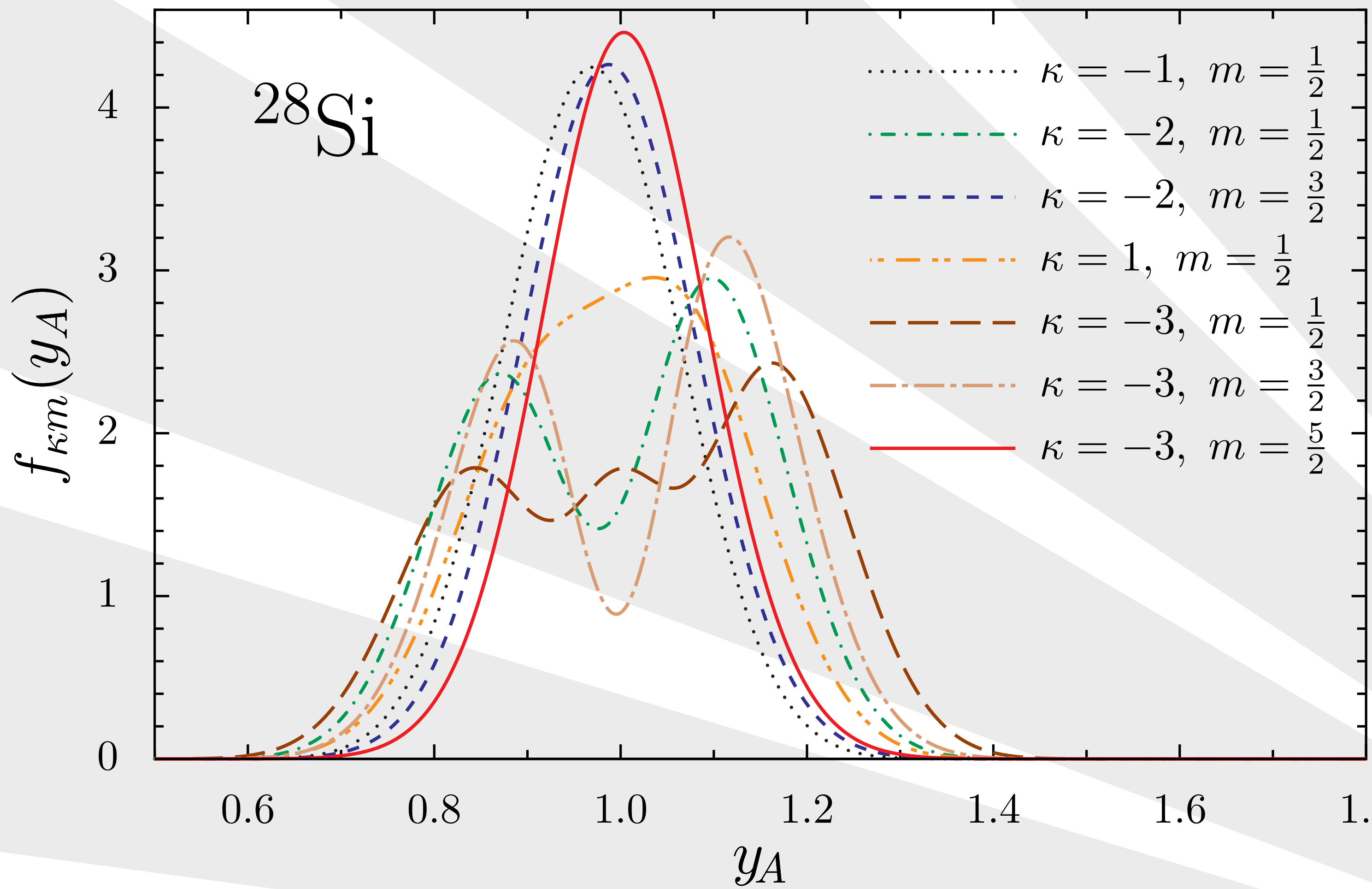
$$f_{\kappa,m}(y_A)=\sum_{k=0,2,\ldots,2j}(-1)^{j-m}\sqrt{2k+1}\begin{pmatrix} j & j & k \\ m & -m & 0\end{pmatrix}$$
$$(-1)^{j+\frac{1}{2}}(2j+1)(2\ell+1)\sqrt{2k+1}\begin{pmatrix} \ell & k & \ell \\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} \ell & k & \ell \\ j & s & j\end{Bmatrix}$$
$$\frac{\overline{M}_N}{16\pi^3}\int_\Lambda^\infty dp\; p\left[F_\kappa(p)^2+G_\kappa(p)^2+\frac{2}{p}\left(\varepsilon_k-\overline{M}_N\, y_A\right)F_\kappa(p)G_\kappa(p)\right]P_k\left(\frac{\overline{M}_N\, y_A-\varepsilon_\lambda}{p}\right)$$

✦ $\Lambda = |\overline{M}_N\, y_A - \varepsilon_\kappa|$

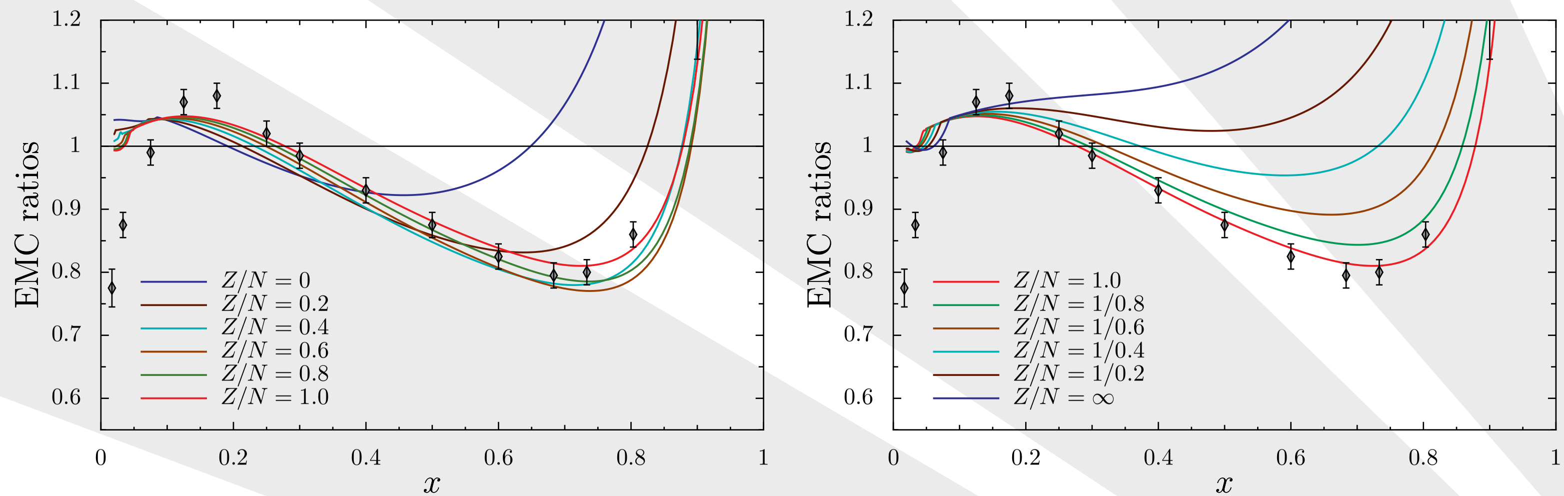
- Infinite nuclear matter

$$f_p(y_A) = \frac{Z}{A}\,\frac{3}{4}\left(\frac{\overline{M}_N}{p_{Fp}}\right)^3\left[\left(\frac{p_{Fp}}{\overline{M}_N}\right)^2 - \left(\frac{\varepsilon_{Fp}}{\overline{M}_N} - y_A\right)^2\right]$$

Nucleon distributions: ^{28}*Si*



EMC Effect: Asymmetric Nuclear Matter



- $R_A = \frac{4\ u_A(x)+d_A(x)}{4\ u_0(x)+d_0(x)}$
- Decreasing EMC effect for $Z/N > 1$
- EMC effect increases for $0.6 < Z/N < 1$
  - ✦ Help explain A dependence of EMC effect.

Nuclear Spin Sum

- Definitions $(H = J)$:

$$\Sigma^{(A)} = \Delta u_A + \Delta d_A \equiv \Sigma \, (P_p + P_n)$$
$$g_A^{(A)} = \Delta u_A - \Delta d_A \equiv g_A \, (P_p - P_n)$$

	Δu	Δd	Σ	g_A	$\Delta_T u$	$\Delta_T d$	g_T
p	0.97	-0.30	0.67	1.267	1.04	-0.24	1.28
^{7}Li	0.91	-0.29	0.62	1.19			
^{11}B	0.88	-0.28	0.60	1.16			
^{15}N	0.87	-0.28	0.59	1.15			
^{27}Al	0.87	-0.28	0.59	1.15			
Nucl. Matt.	0.79	-0.26	0.53	1.05	0.93	-0.23	1.16

- Quark Spin $\Longrightarrow$ orbital angular momentum