Low Q^2 Proton Form Factors

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Hadron Electromagnetic Form Factors
ECT* Trento
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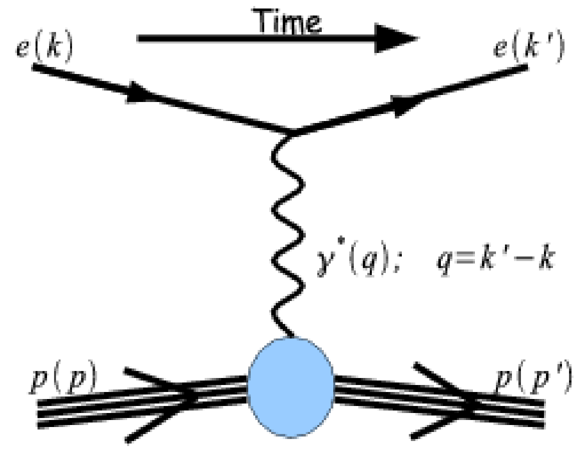
Low Q^2 Proton *Space-like* Form Factors

- Introduction / Review
  - Since this is a form factor meeting, so I will go quickly over the usual introductory material, but quickly
  - I will largely focus on polarizations – no discussion of Coulomb / radiative corrections
- Recent low Q^2 work
- The new Hall A Experiment: E08-007
  - Aims
  - Status
  - Expected Results and Analyses
- Summary

Introduction

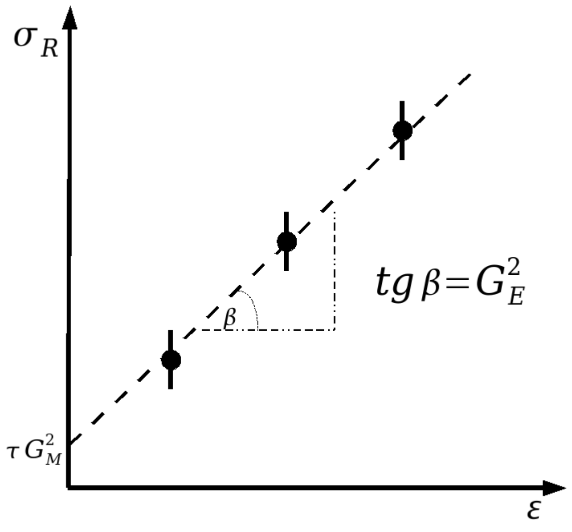
$$J^{\mu} = \bar{u}(p)\left[F_1(Q^2)\gamma^{\mu} + i\frac{\kappa}{2M}F_2(Q^2)\sigma^{\mu\nu}q_{\nu}\right]u(p)$$

- Spin-½ proton 2s+1=2 form factor, usually chosen as either F_1 Dirac and F_2 Pauli or as G_E electric and G_M magnetic
- Form factors describe charge / magnetization distributions
- Rosenbluth technique and fits extract f.f. from cross section:



$$\sigma_R \equiv \epsilon(1+\tau)\frac{d\sigma/d\Omega}{d\sigma_{Mott}/d\Omega} = \epsilon G_{Ep}^2(Q^2) + \tau G_{Mp}^2(Q^2)$$

$$\epsilon^{-1} = 1 + 2(1+\tau)\tan^2\frac{\theta}{2} \qquad \tau = Q^2/4M^2$$



Introduction

$J^\mu = \bar{u}(p)[$

- Or model-dependent extraction: choose convenient f.f. parameterization and fit data at "all" Q^2 simultaneously

- Spin-½ proton 2s+1=2 form factor, usually chosen as either F_1 Dirac and F_2 Pauli or as G_E electric and G_M magnetic
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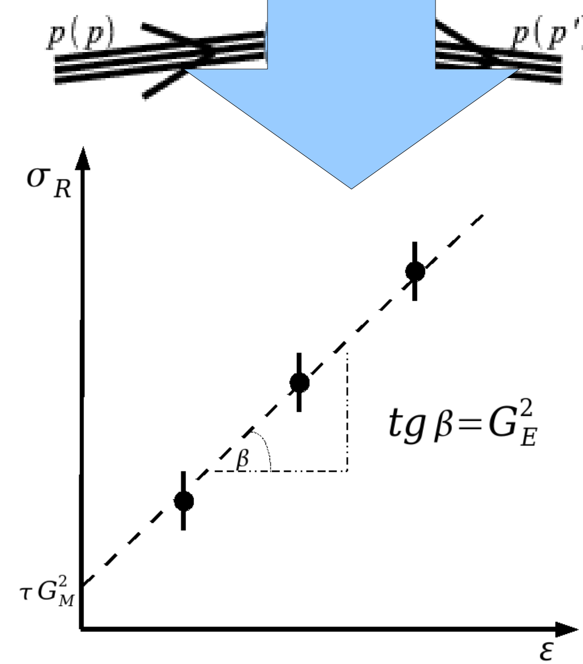
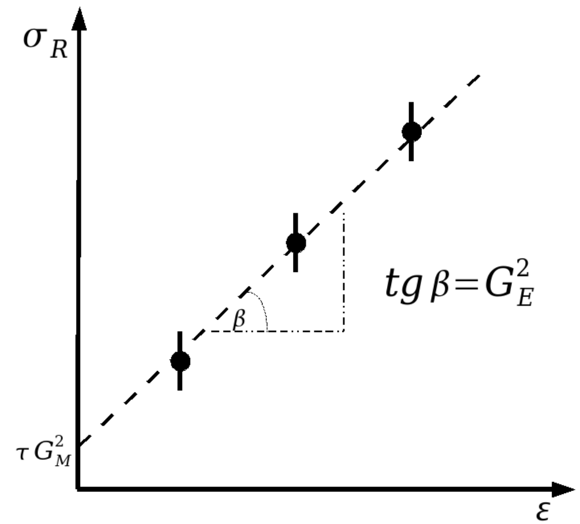


Theory

- Numerous approaches to nucleon form factors, which experimentalists should list and describe with caution
- Theories, phenomenology, fits – theoretically inspired or not
- Some include:
  - vector dominance models
  - (relativistic) constituent quark models
  - quark-meson coupling models
  - lattice QCD
  - perturbative QCD
  - ads/CFT models
  - ... (My apologies if I left yours out / misnamed it.)
- Something of an EMC effect, in the Miller sense

Limits to Rosenbluth

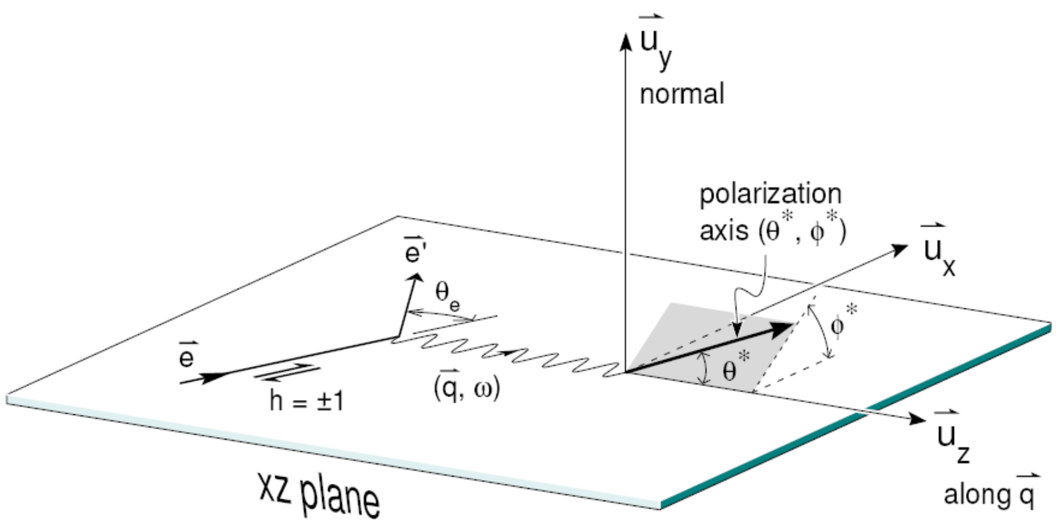
$$\sigma_R \equiv \epsilon(1+\tau)\frac{d\sigma/d\Omega}{d\sigma_{Mott}/d\Omega} = \epsilon G_{Ep}^2(Q^2) + \tau G_{Mp}^2(Q^2)$$



- Rosenbluth does not work well when
  - $\varepsilon G_E^2/\tau G_M^2 \sim 1/\tau^2 \ll 1$ at high Q^2, or
  - $\varepsilon G_E^2/\tau G_M^2 \sim 1/\tau \gg 1$ at low Q^2, except at $\theta = 180^\circ$
- Need extremely high precision (Mainz), or polarization measurements:
  - polarized beam & target asymmetry
  - polarization transfer with FPP

Polarized Beam & Target Asymmetry

$$A_{phys} = \frac{v_z \cos\theta' G_M^2 + v_x \sin\theta' \cos\phi' G_E G_M}{(\epsilon G_{Ep}^2 + \tau G_{Mp}^2)/[\epsilon(1+\tau)]}$$

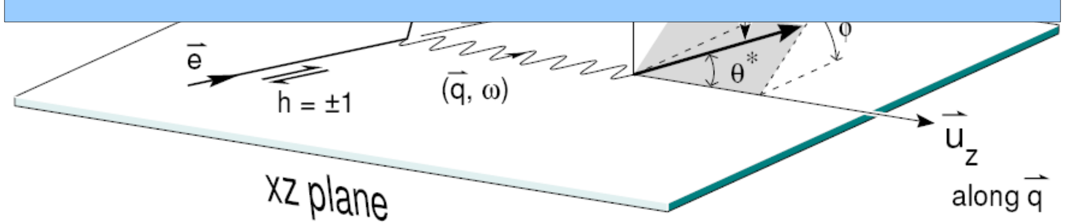


- Following notation of Crawford et al, BLAST article, PRL98 – but note typos in their formula (e.g., G_E, not G_E^2)
- Measuring with two sectors at the same time allowed determination of both $R = \mu_p G_E/G_M$ and of the product $P_{beam}P_{target}$

Polarized Beam & Target Asymmetry

$$A_{phys} = \frac{v_z \cos\theta' G_M^2 + v_x \sin\theta' \cos\phi' G_E G_M}{(\epsilon G_{Ep}^2 + \tau G_{Mp}^2)/[\epsilon(1+\tau)]}$$

$$A_{phys} = \frac{v_z \cos\theta^* G_M^p + v_x \sin\theta^* \cos\phi^* G_M^p G_E^p}{(\tau G_M^p + \epsilon G_E^p)/[\epsilon(1+\tau)]},$$



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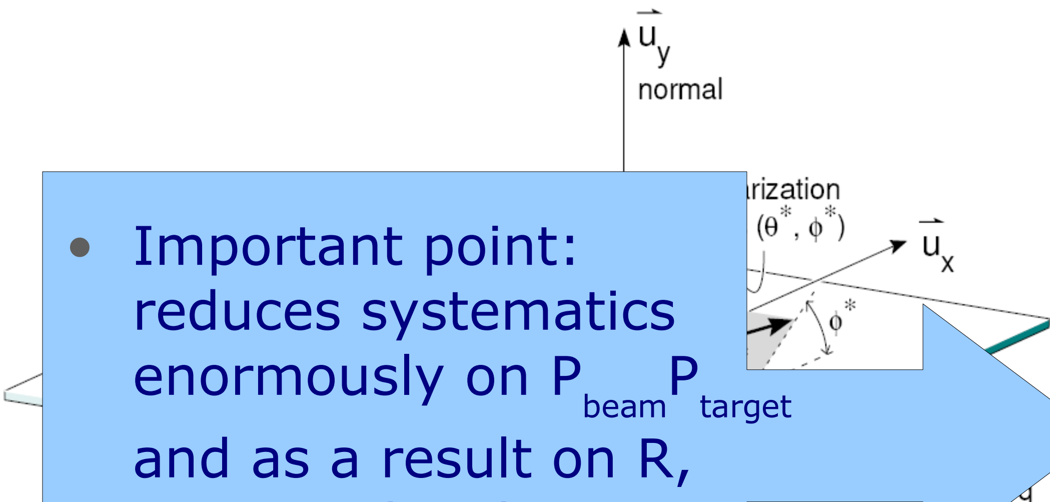


- Important point: reduces systematics enormously on $P_{beam} P_{target}$ and as a result on R, compared with sequential measurements

- Following notation of Crawford et al, BLAST article, PRL98 – but note typos in their formula (e.g., G_E, not G_E^2)
- Measuring with two sectors at the same time allowed determination of both $R = \mu_p G_E/G_M$ and of the product $P_{beam} P_{target}$

Polarization Transfer

$$I_0 P_x = -2\sqrt{\tau(1+\tau)}\tan\left(\frac{\theta_e}{2}\right) G_E^p G_M^p$$

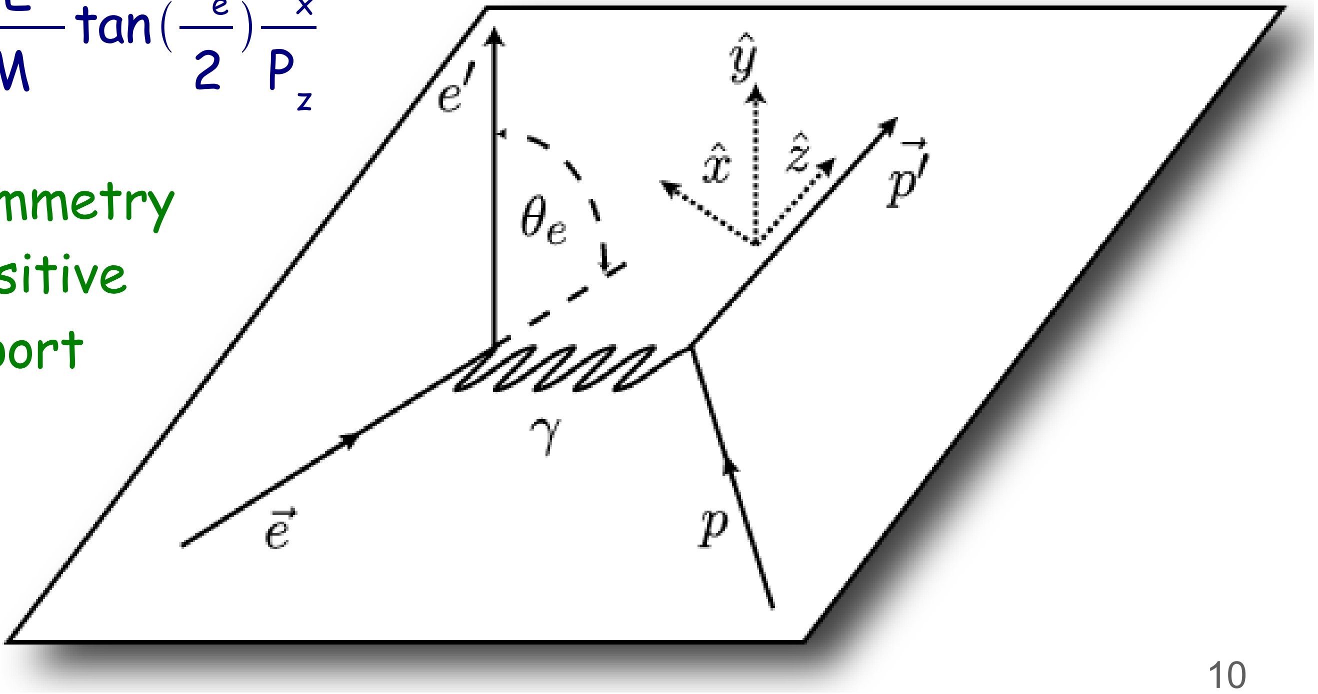
$$I_0 P_z = \frac{E+E'}{M}\sqrt{\tau(1+\tau)}\tan^2\left(\frac{\theta_e}{2}\right) G_{Mp}^2$$

$$I_0 = G_E^2 + \frac{\tau}{\epsilon} G_M^2$$

Py: induced from (imaginary part of) 2γ exchange, small and hard to measure

$$R = \mu_p \frac{G_{Ep}}{G_{Mp}} = -\mu_p \frac{E+E'}{2M}\tan\left(\frac{\theta_e}{2}\right)\frac{P_x}{P_z}$$

FPP azimuthal asymmetry determines R, sensitive only to spin transport



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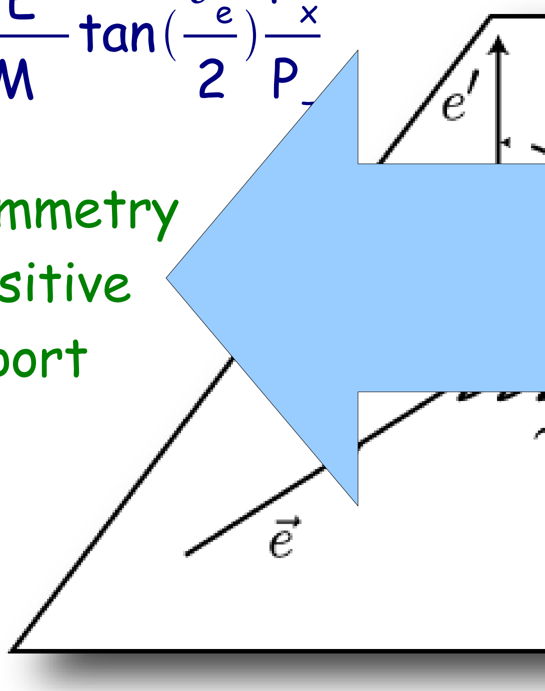

Insensitive to: spectrometer solid angle, target density, trigger and detector efficiencies, beam charge, charge asymmetry, normal radiative corrections, false asymmetries in FPP.

These might affect statistics and size of uncertainty, but not value of data point.

Polarization Transfer

$$I_0 P_x = -2\sqrt{\tau(1+\tau)}\tan\left(\frac{\theta_e}{2}\right) G_E^p G_M^p$$

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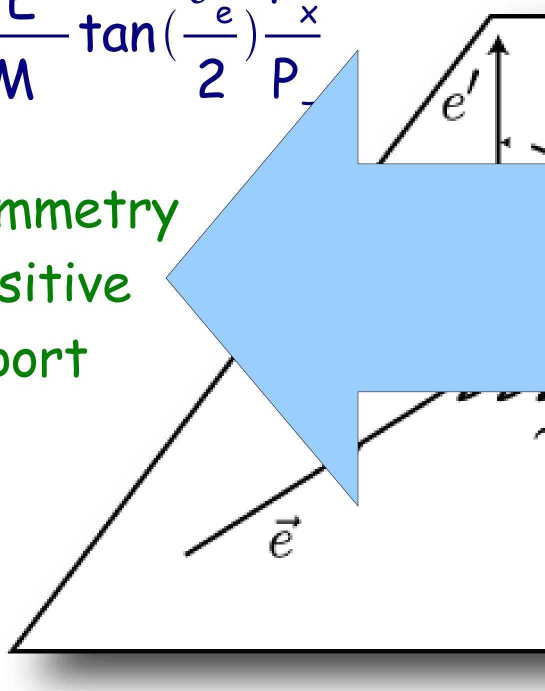
Py: induced from (imaginary part of) 2γ exchange, small and

Minimal sensitivity to helicity-correlated asymmetries (beam energy, position, angle) and box/cross 2γ radiative corrections.

We measure "%" asymmetries, not ppm.

FPP azimuthal asymmetry determines R, sensitive only to spin transport



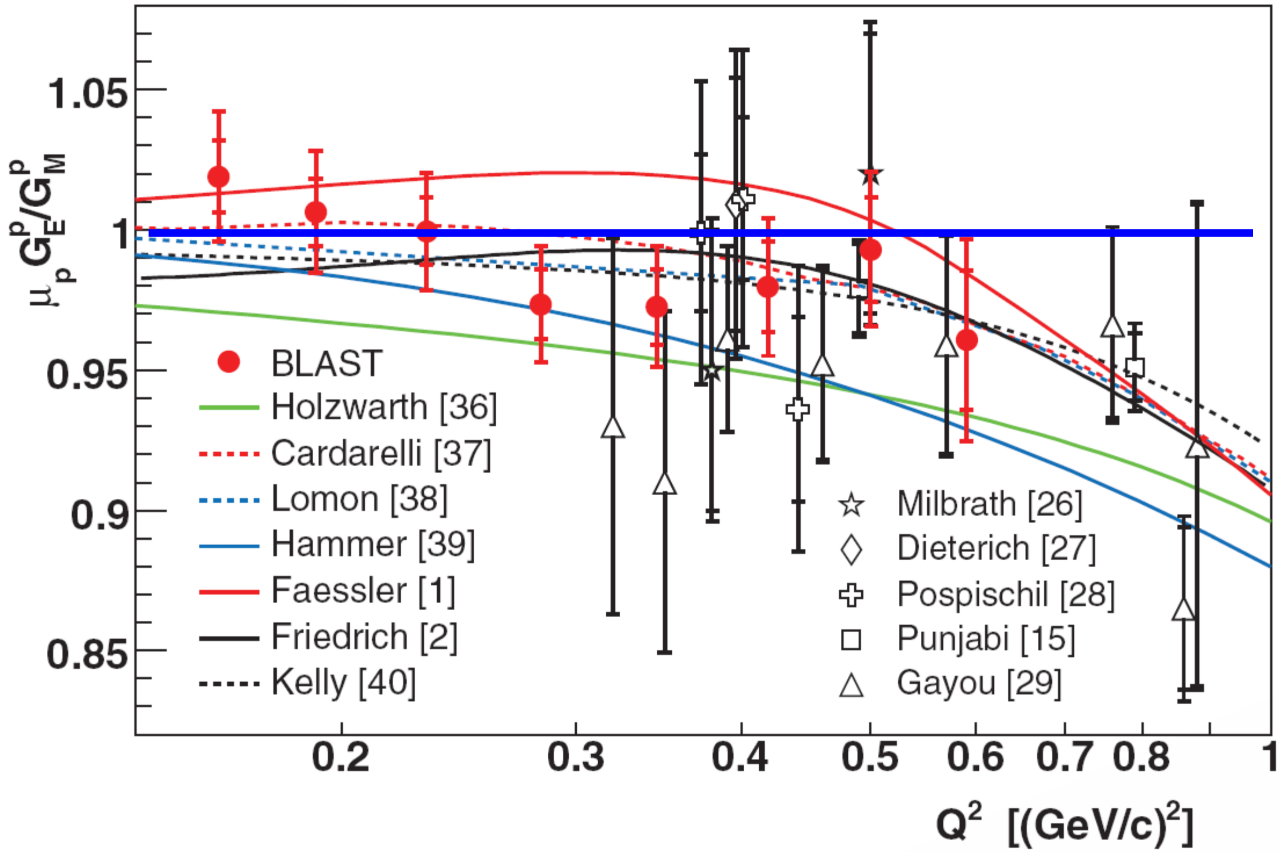
Low Q^2 Proton Ratio Data, early 2007

From Crawford, PRL98, BLAST

R~1 to 0.6 GeV^2

Generally low precision, except for JLab GEp-I (Jones/Punjabi) and BLAST

Why?



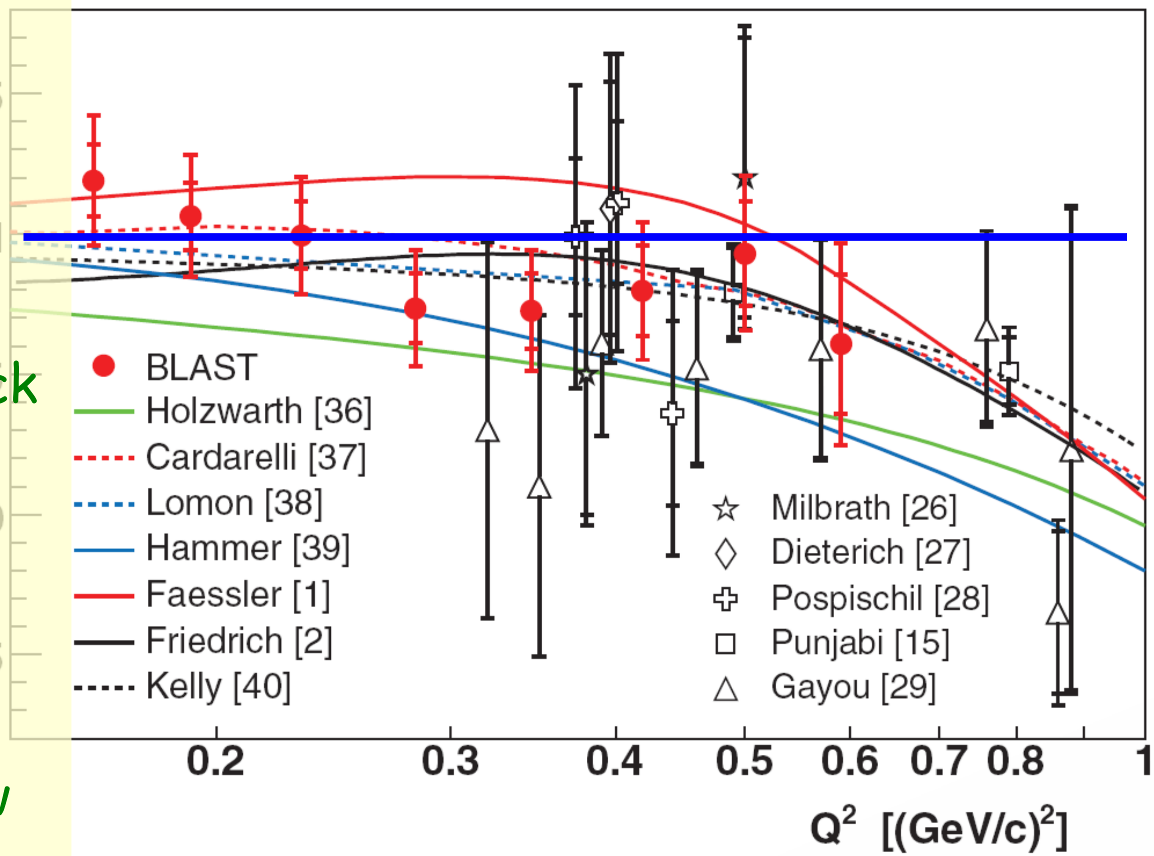
Low Q^2 Proton Ratio Data, early 2007

Rates low at Bates (Milbrath).

Systematics at Mainz (Dieterich, Popischil).

Calibration data or lack of interest at JLab (Gayou, Wijesooriya)

Also, FPP difficult at very low Q^2 due to low proton energy

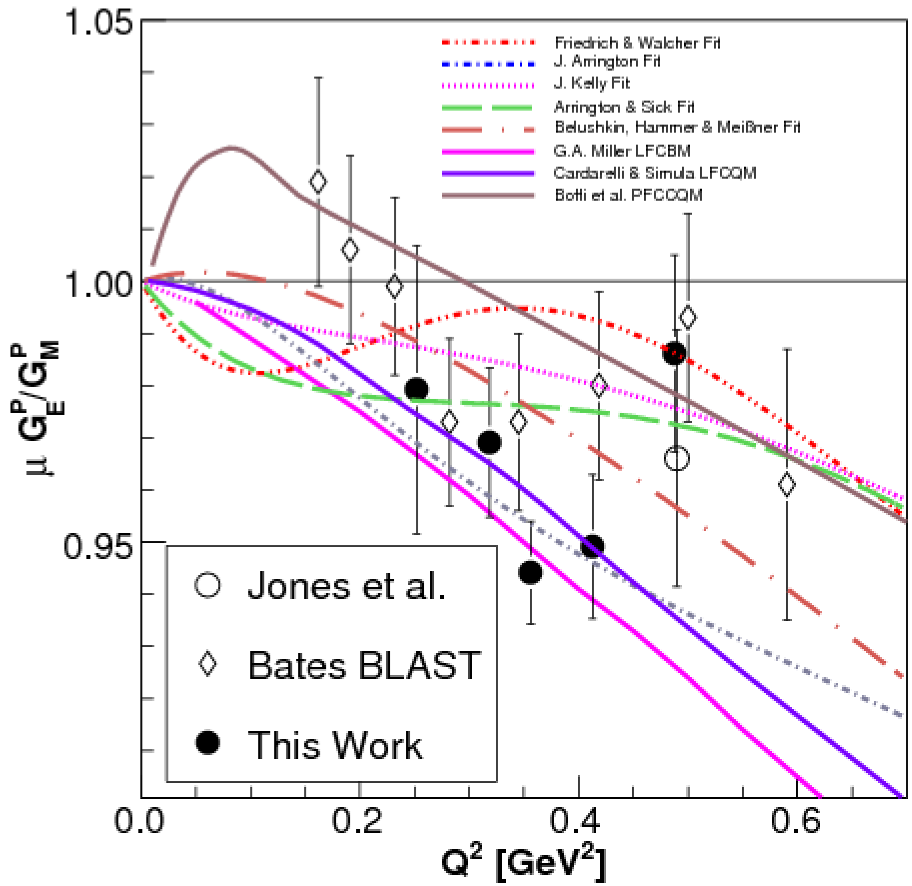


Interest in Low Q^2

- Why had we not done high-precision low-Q^2 FPP determinations of R before?
  - Lack of realization that it is interesting
- What changed our minds?
  - Complement to BLAST A, clearly show R<1
  - Friedrich & Walcher analysis suggesting structures
  - Discussions of hyperfine splitting

Low Q^2 Proton Ratio Data, late 2007

In 2006, we took more FPP calibration data (G. Ron et al PRL 98), but with higher statistics than previous calibrations (Gayou, Wijesooriya, Jiang et al.) to determine false asymmetries well

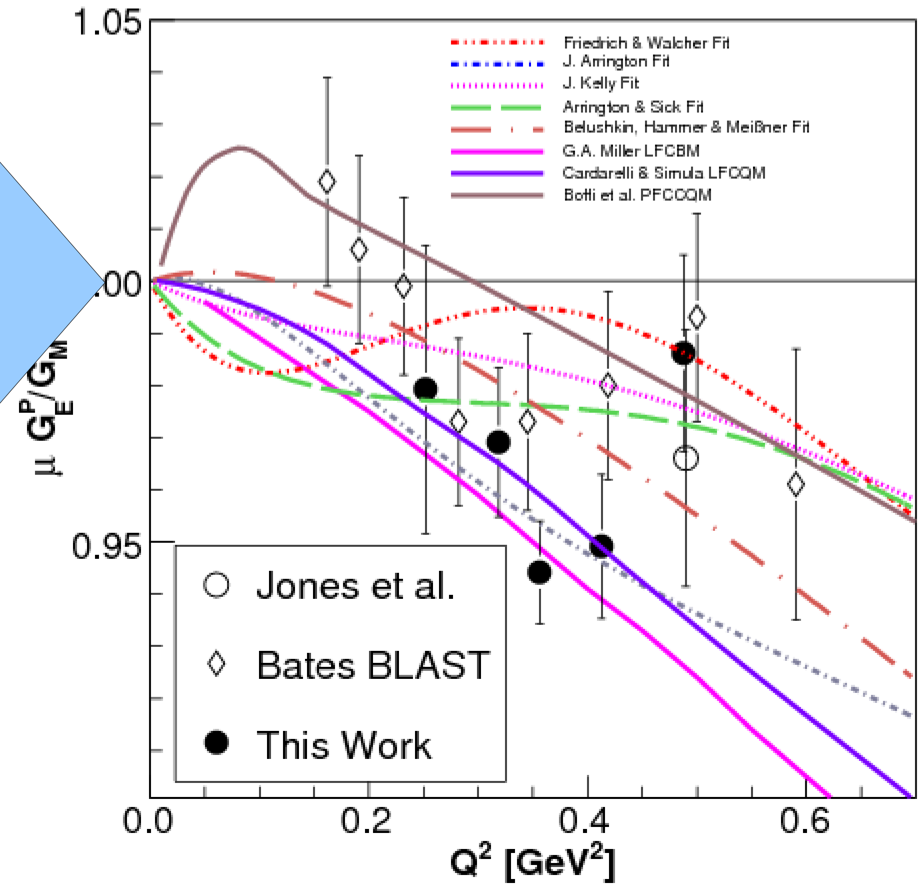


Low Q^2 Proton Ratio Data, late 2007

We find R<1 for Q^2 ~ 0.3 - 0.4 GeV^2

We do not support FW analysis; there is an unfortunate (?) hint of a different structure

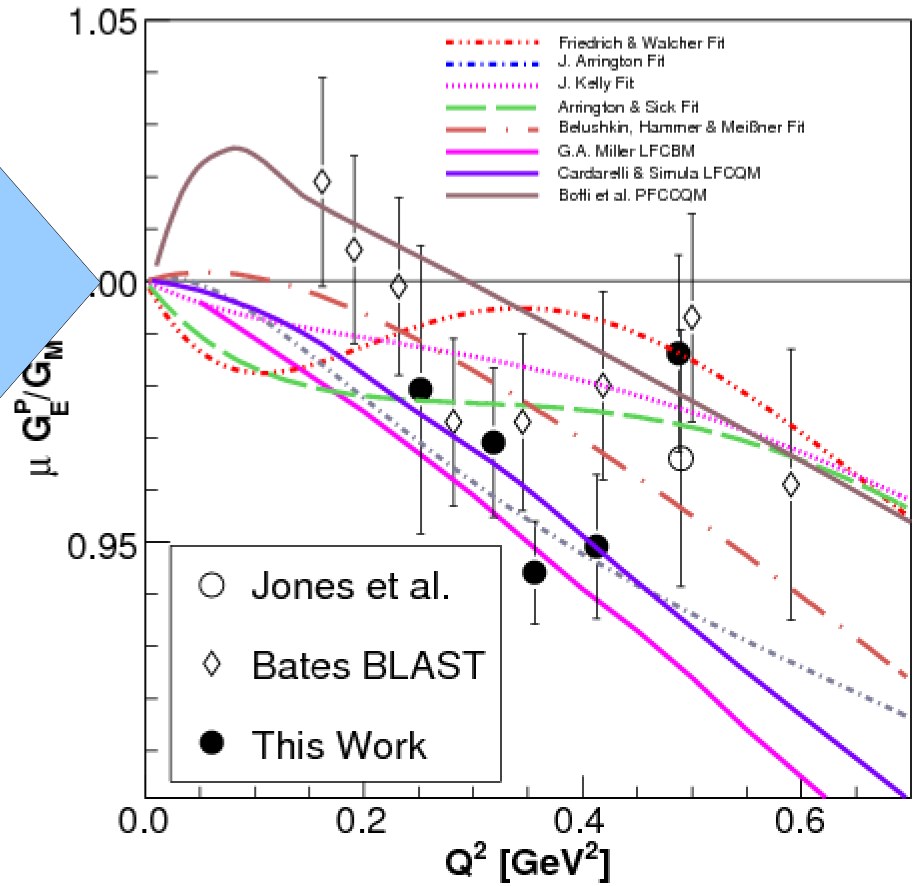
To determine false asymmetries well,



Low Q^2 Proton Ratio Data, late 2007

Belushkin fit and lowest Q^2 points suggest a + slope at $Q^2 = 0$, conventionally implying slightly larger magnetic radius

To determine false asymmetries well



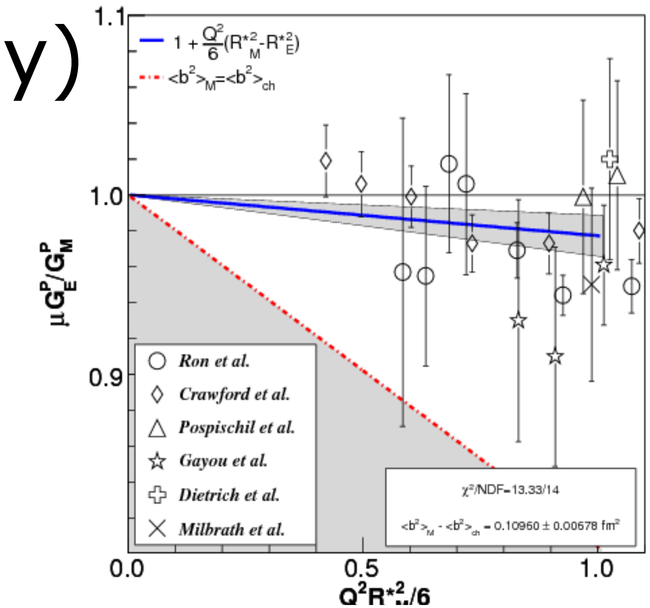
Radii - Miller et al. (Monday)

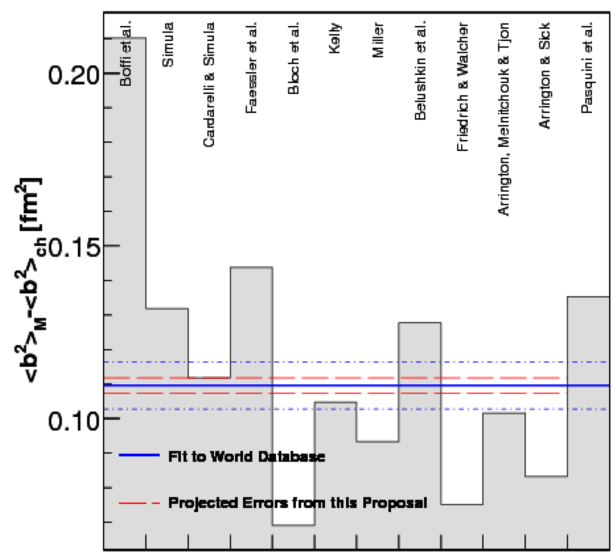
$$\text{As } Q^2 \to 0,\ R \approx 1 - \frac{Q^2}{6}\left(R_M^2 - R_E^2\right)$$

$$b_M^2 - b_E^2 = \frac{2}{3}\frac{\mu}{\kappa}\left(R_M^2 - R_E^2\right) + \frac{\mu}{M^2}$$

While the sign of $R^2_M - R^2_E$ is basically undetermined - is R really linear out to 0.2 or 0.3 GeV^2? - all data and fits indicate $b^2_M - b^2_E > 0$

Fit gives: $R^2_M - R^2_E = -0.014 \pm 0.007$ and $b^2_M - b^2_E = 0.110 \pm 0.007$



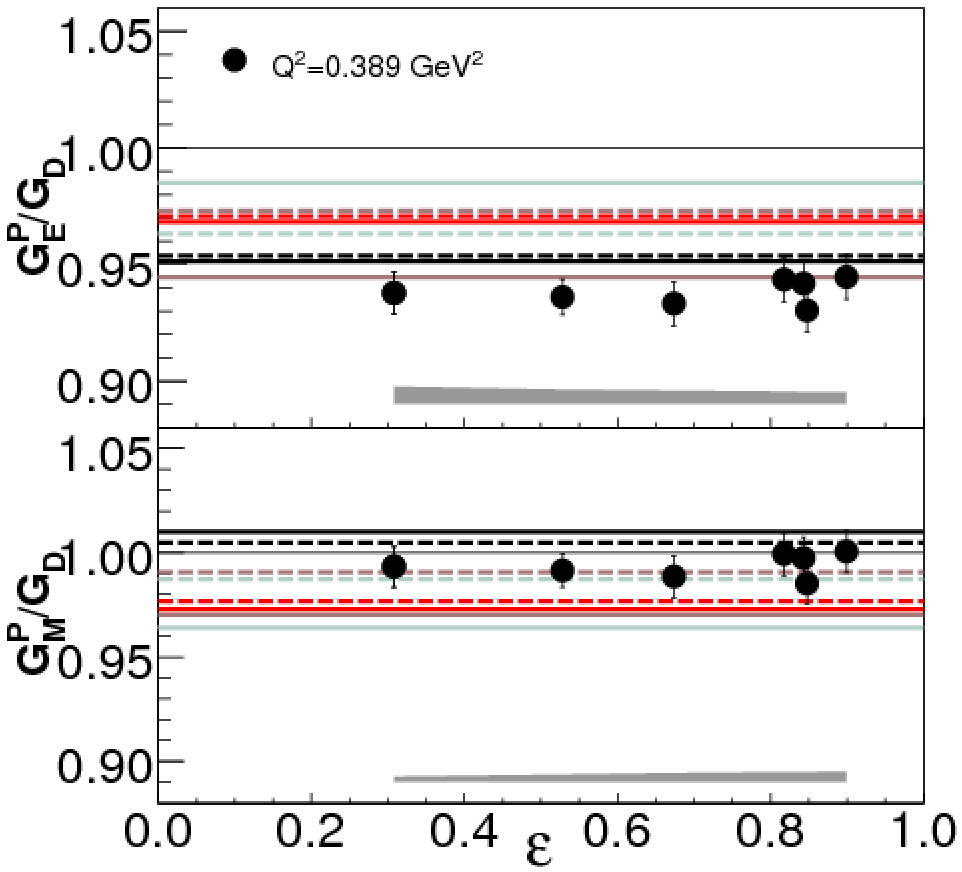


Direct Implications on Separated F.F.

Combining Berger at al. PLB 35, 1971 dσ/dΩ with new FPP data in G. Ron et al PRL 98, we showed fits tend to get G_M about right, but tend to over predict G_E

Table 1.
Differential cross sections: The quoted errors are only random errors. A normalization error of ± 4% has to be added.

$q^2 (f^{-2})$	θ (°)	s_0(GeV)	$\frac{d\sigma}{d\Omega}$ $[10^{-34} \frac{cm^2}{ster}]$	
2	25.25	0.660	32800	± 990
3	25.25	0.815	18570	± 550
3.065	35.15	0.605	8630	± 260
5	25.25	1.064	8410	± 260
	35.15	0.784	4000	± 120
8	25.25	1.364	3610	± 90
10	25.25	1.537	2285	± 46
	31.74	1.249	1328	± 26
	32.27	1.231	1310	± 26
	35.15	1.142	1080	± 22
	50.06	0.848	460.3	± 9.4
	64.72	0.696	252.9	± 4.1
	90.27	0.556	117.8	± 2.3



Interest in Low Q^2

- Why had we not done high-precision low-Q^2 FPP determinations of R before?
  - Lack of realization that low-Q^2 f.f. are interesting
- What changed our minds?
  - Complement to BLAST A, clearly show R<1
  - Friedrich & Walcher analysis
  - Discussions of hyperfine splitting
  - Realization that PV determinations of strange f.f. might be affected at the (0.5-1)σ level
  - Complement new Mainz dσ/dΩ data
  - Unfortunate hints of structure in our data
- After the LEDEX/G. Ron results, we proposed E08-007

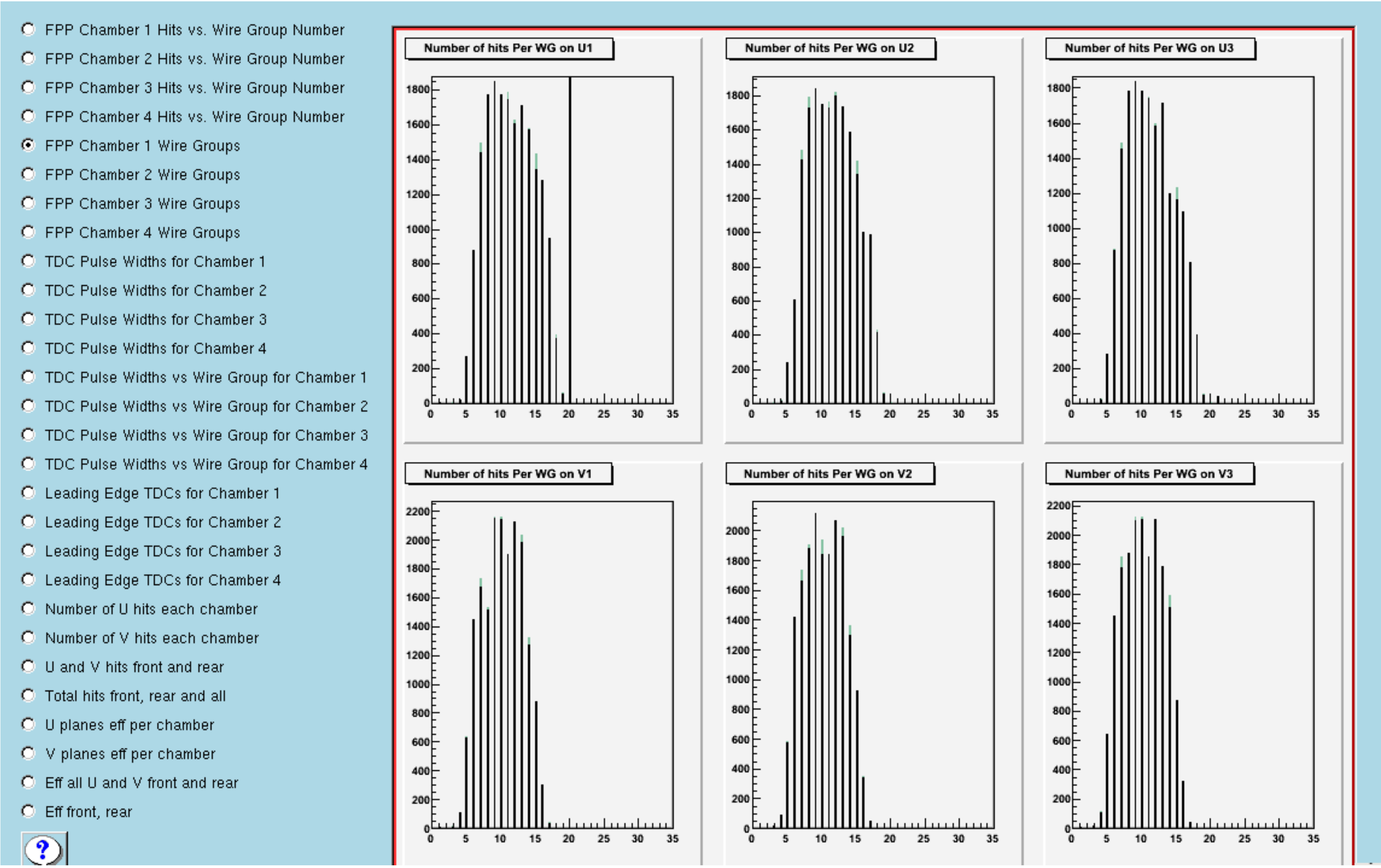
E08-007 Aims

- Direct experimental goal: determine R to
  - Increase range and precision of form factor ratio measurements over recent BLAST data
  - Complement new high-precision Mainz cross section measurements
- Planned uses for data
  - Improved input for fits / testing models of nucleon
  - Improved electric vs magnetic radius
  - Improved input for other physics: hyperfine splitting, strange form factors, etc.

E08-007 Status

- Experiment approved January 2008
- Experiment fit into hole in Hall A schedule
- Started commissioning last Thursday, May 15, 2008
- FPP in HRS in coincidence with BigBite lead glass so that we have clean ep elastic scattering events
  - E_e = 1.194 GeV, Q^2 = 0.25 - 0.7 GeV^2
- Analysis: Guy Ron (Tel Aviv) (still a Ph.D. student, but now a spokesperson as well) and Xiaohui Zhan (MIT)

E08-007 Status - online



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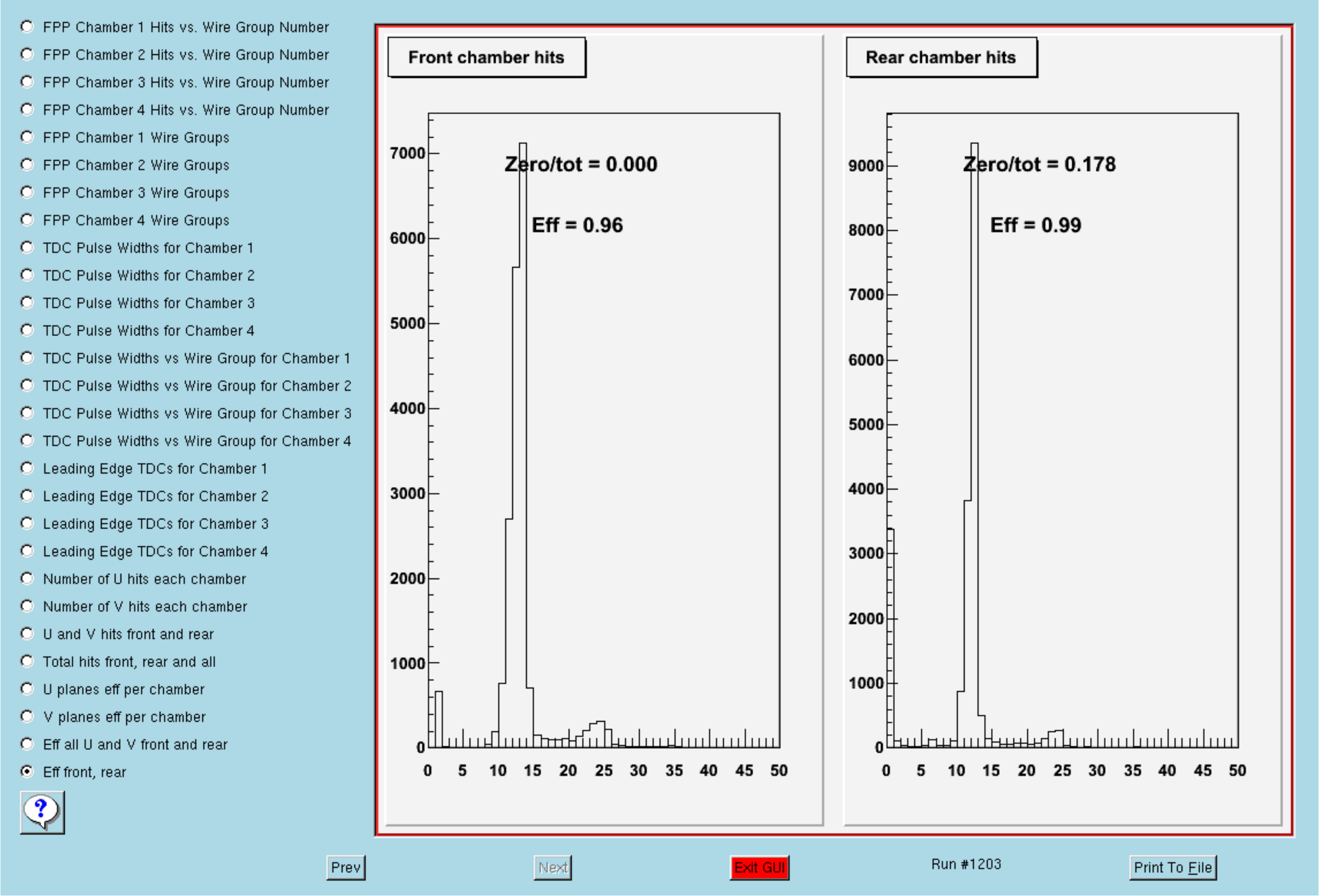

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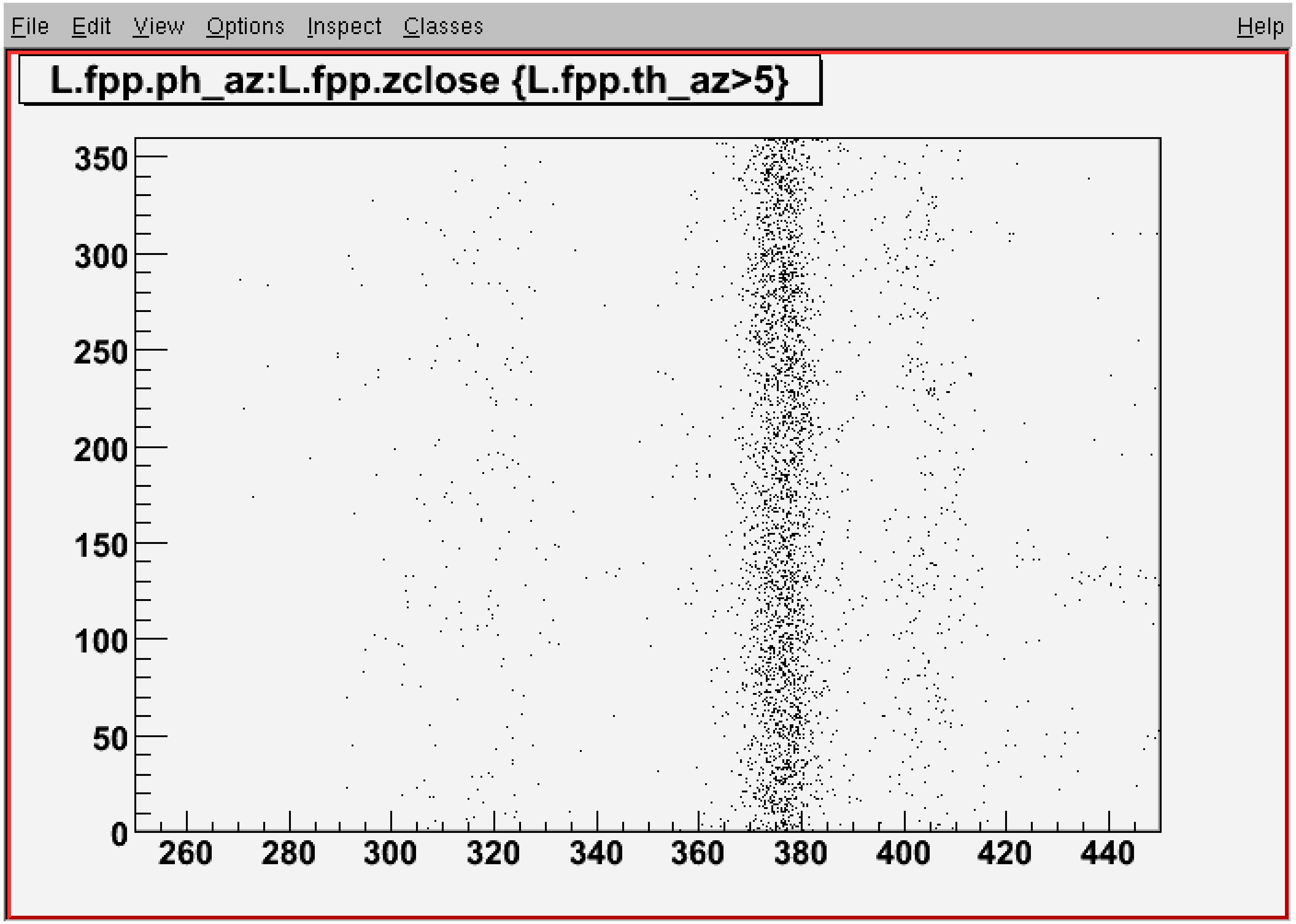

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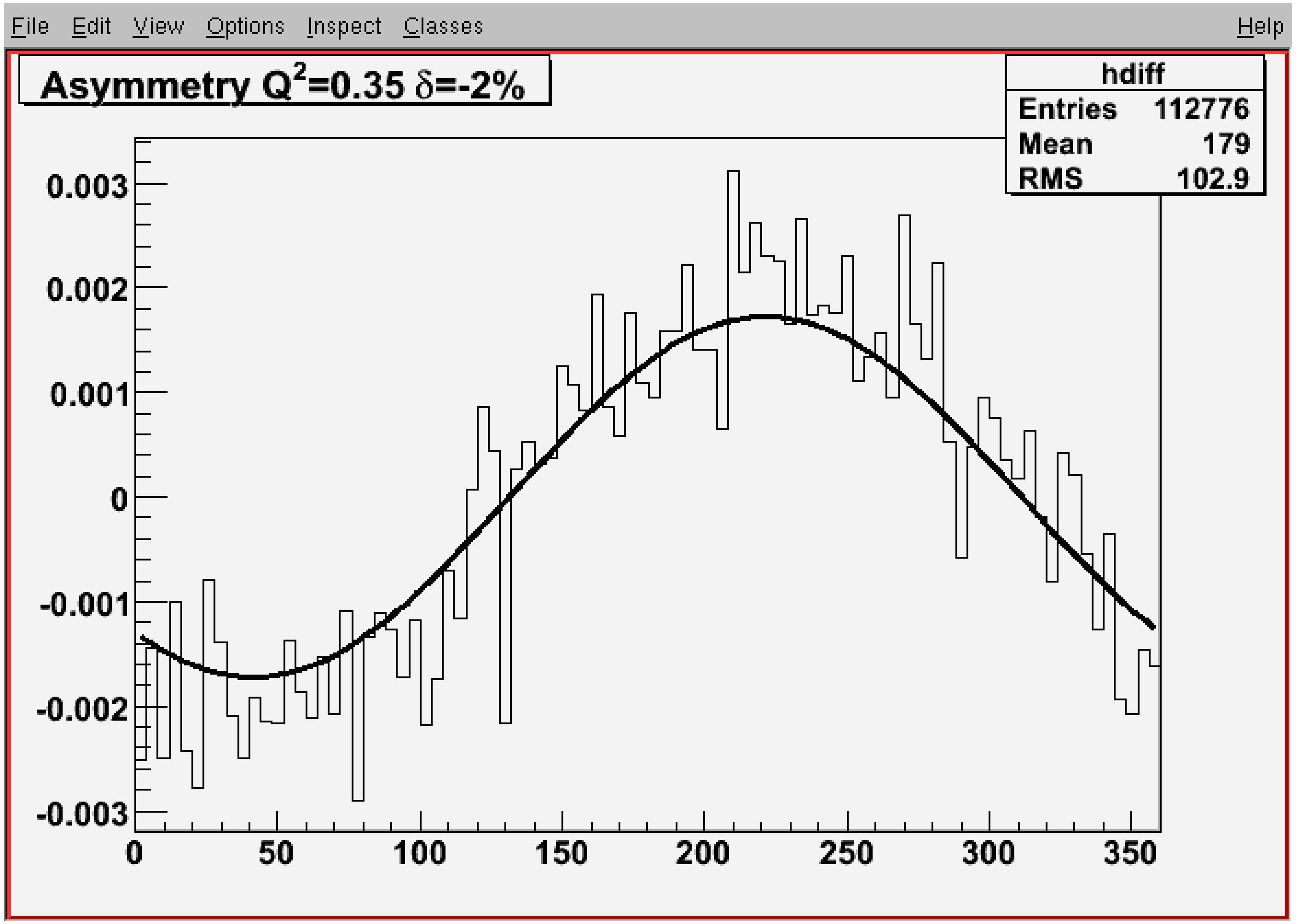
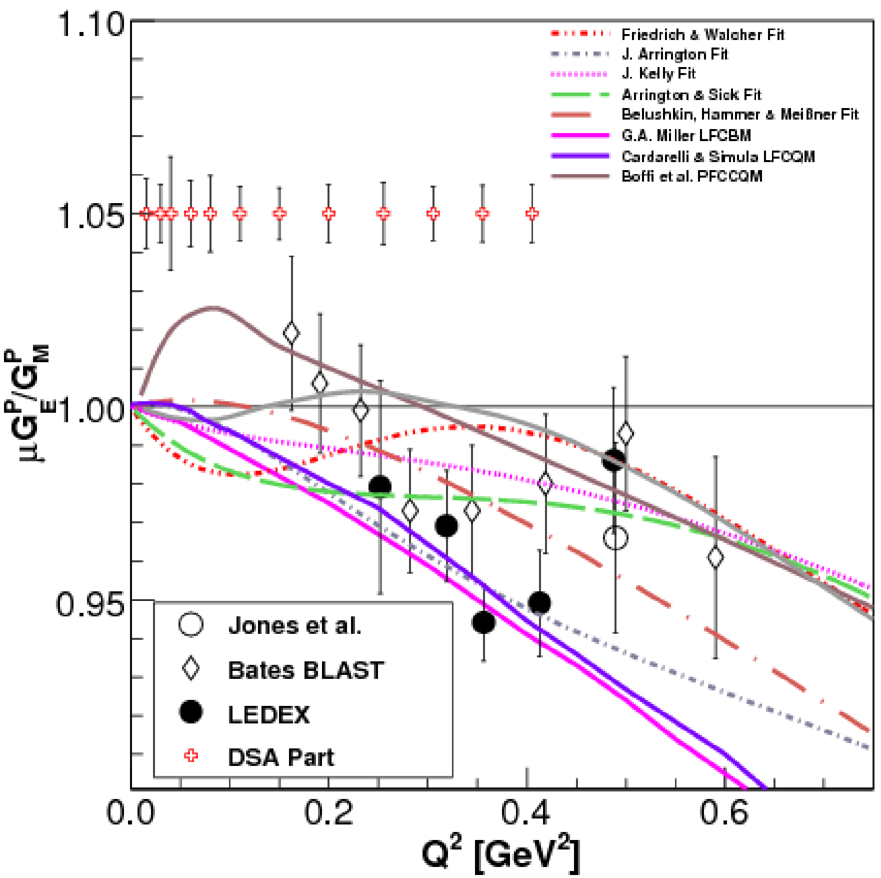
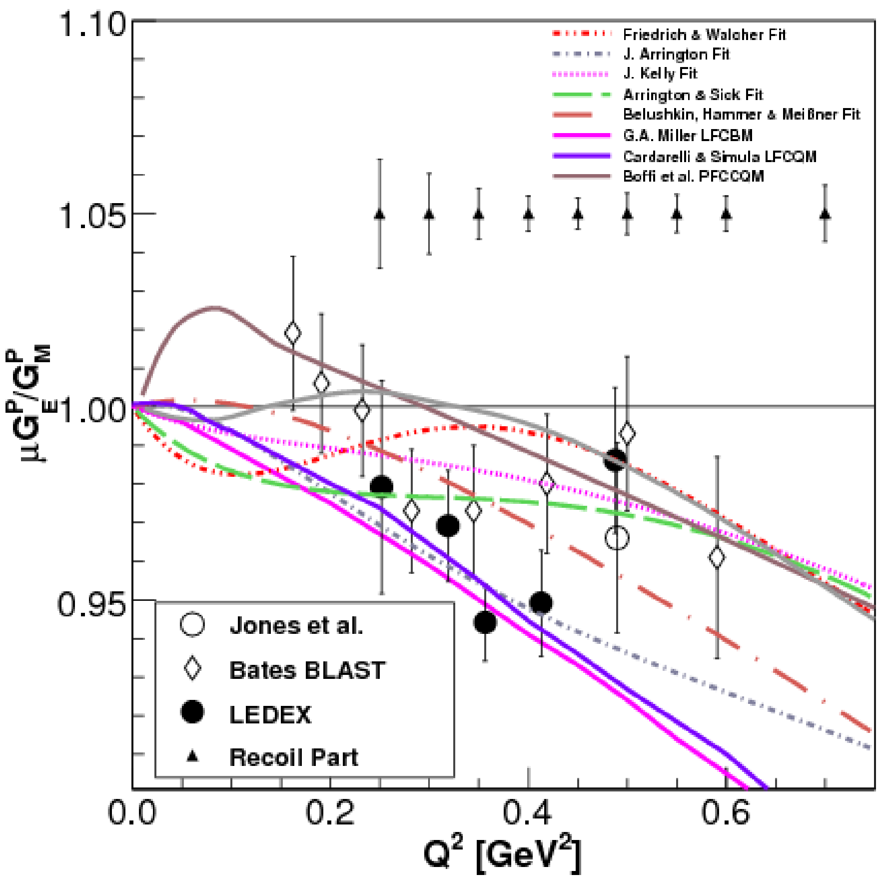


E08-007 Status

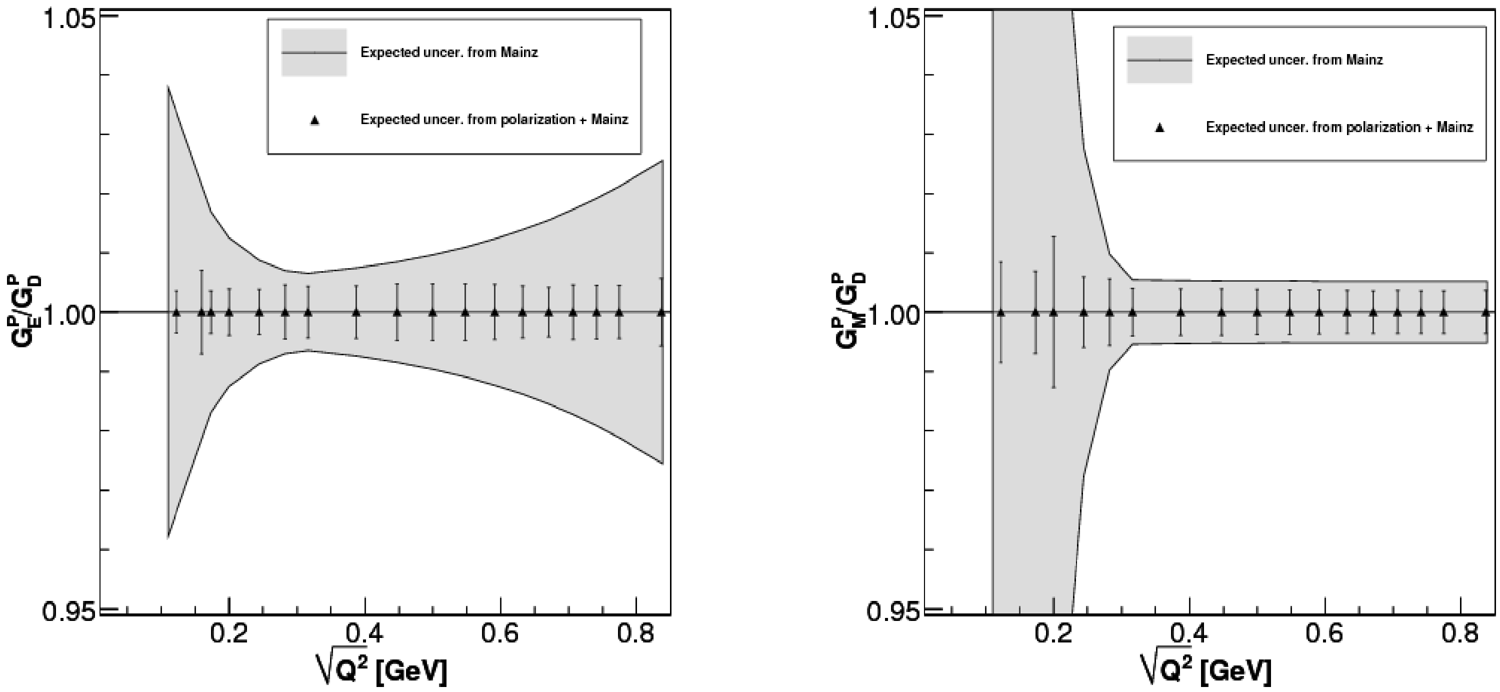
- Finished first point at $Q^2 = 0.35\ GeV^2$
- Now running second point at $Q^2 = 0.30\ GeV^2$
- More FPP measurements to follow, until ~June 7
- Hope to run even lower Q^2, with polarized beam-target asymmetries, ~ 2011
  - Polarized target + 2 HRS spectrometers at identical angles – same Q^2 with two different angles w.r.t. polarization vector reduces sensitivity to $P_{beam}P_{target}$

E08-007 Anticipated DSA/FPP Results





Improvement in Form Factors, in conjunction with Mainz Rosenbluth



- Fits of functional forms improve uncertainties in each case, but introduce model dependence

Hyperfine Splitting

- $E_{HFS} = (1 + \Delta_{QED} + \Delta^P_R + \Delta^p_{hvp} + \Delta^P_{\mu vp} + \Delta^P_{weak} + \Delta_S) E_F^P =$ 1420.405 751 766 7(9) MHz
- Structure term $\Delta_S = \Delta_Z + \Delta_{POL}$, $\Delta_Z = -2\alpha m_e r_z (1+d^{rad}_Z)$
- Zemach radius $r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[G_E(Q^2)\frac{G_M(Q^2)}{(1+\kappa_p)} - 1\right]$
- Some recent articles:
  - Friar and Sick, PLB 579 (2004)
  - Brodsky, Carlson, Hiller, and Hwang, PRL 96 (2005)
  - Friar and Payne, PRC 72 (2005)
  - Nazaryan, Carlson, and Griffioen, PRL 96 (2006)
  - Carlson, Nazaryan, and Griffioen, arXiv:0805.2603v1

Hyperfine Splitting

- Friar and Sick, PLB 579 (2004)
  - Form factors from electron scattering lead to r_Z = 1.086 ± 0.012 fm
    - Continued Fraction Expansion up to 4 fm^{-1}, dipole parameterization for higher Q^2
  - Need Δ_{POL} ~ 3.2 ± 0.5 ppm, somewhat inconsistent with estimate of 1.8 ± 0.8 ppm

Hyperfine Splitting

- Brodsky, Carlson, Hiller, and Hwang, PRL 96 (2006)
  - $\Delta_S = \Delta_Z + \Delta_{POL}$ = -38.62(16) ppm
  - Use Δ_{POL} ~ 1.4 ± 0.3 ppm to obtain Δ_Z = -40.0 ± 0.6 ppm, and r_Z = 1.043 ± 0.016 fm
  - Fits / parameterizations give Δ_Z = -38.8 -> -41.7 ppm, and r_Z = 1.012 -> 1.088 fm
  - Needed Zemach correction between modern fits and the dipole

Hyperfine Splitting

- Nazaryan, Carlson, and Griffioen, PRL 96 (2006)
  - $\Delta_S = \Delta_Z + \Delta_{POL}$ = -38.58(16) ppm, but Δ_Z = -39.32 ppm (dipole) or ~ -41->-42 ppm (Kelly,Sick fits) and Δ_{POL} ~ 1.3 ± 0.3 ppm => perhaps okay to 1 - 2 ppm

Hyperfine Splitting

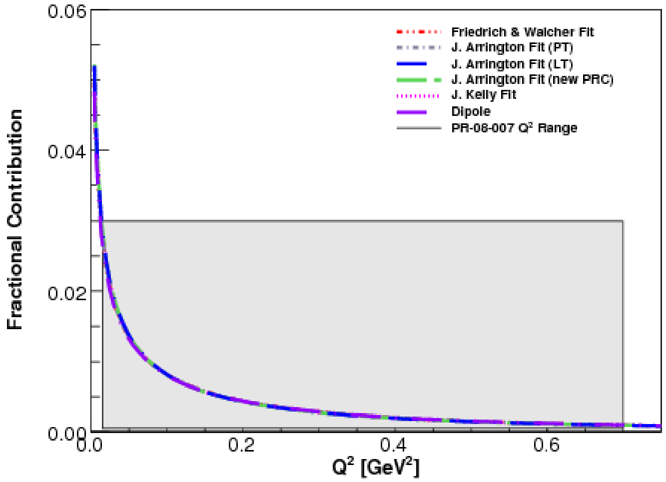
- Carlson, Nazaryan, and Griffioen, arxiv:0805.2603 (2008)
  - Use new CLAS EG1b data to determine g_1 better at low Q^2 and constrain g_2
  - Δ_{POL} ~ 1.88 ± 0.64 ppm
  - Target $\Delta_S = \Delta_Z + \Delta_R + \Delta_{POL}$ = -32.77(1) ppm

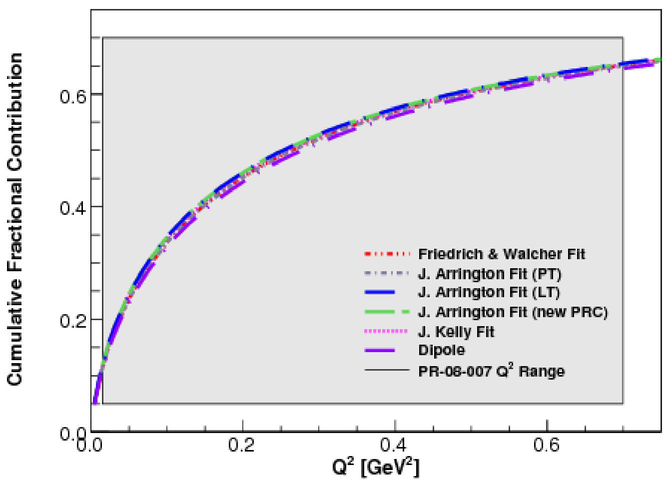
Form factor	r_P (fm)	r_Z (fm)	Δ_Z (ppm)	Δ_R^p (ppm)	Δ_{pol} (ppm)	Δ_S (ppm)
AMT [31]	0.885	1.080	−41.43	5.85	1.88	−33.70
AS [32]	0.879	1.091	−41.85	5.87	1.89	−34.09
Kelly [33]	0.878	1.069	−40.99	5.83	1.89	−33.27
FW [34]	0.808	1.049	−40.22	5.86	2.00	−32.36
dipole	0.851	1.025	−39.29	5.78	1.94	−31.60

Some General Comments

- Δ_{POL} relies on low Q^2 estimates of g_2^p in an unmeasured region -> a better data base is needed (Hall A low Q^2 g_2^p E08-027, expected 2011)
  - But MAID was okay for $g_{1,2}^n$, so probably okay here
- For Δ_Z, uncertainties (and offsets?) in the fits -> a better data base is needed (Mainz+JLab)
- Our limited result at Q^2 = 0.4 GeV^2 suggests G_M is about right, but G_E is 2% smaller than fits – if this were true generally, it would reduce the Zemach correction by about 0.5 ppm, moving it in the "right" direction – but thus is one point, and high Q^2 form factors are largely a guess

Zemach Integrand and Integral





- Zemach integrand is largest for low Q^2, but
- for integral to converge, have to integrate to large Q^2 - even the 12 GeV experiments will be needed
- But existing fits do not vary that much in integral – for any semi-reasonable behavior

Parity Violation

- The usual relations:

$$G^{p}_{E,M}(Q^2)=\frac{2}{3}G^{u}_{E,M}(Q^2)-\frac{1}{3}G^{d}_{E,M}(Q^2)-\frac{1}{3}G^{s}_{E,M}(Q^2)$$

$$A_{\rm th} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \left[\frac{-G_F Q^2}{\pi\alpha\sqrt{2}}\right] \frac{\varepsilon G_E^{p\gamma} G_E^{pZ} + \tau G_M^{p\gamma} G_M^{pZ} - \frac{1}{2}(1 - 4\sin^2\theta_W)\varepsilon' G_M^{p\gamma} G_A^{pZ}}{\varepsilon (G_E^{p\gamma})^2 + \tau (G_M^{p\gamma})^2}$$

$$G_{E,M}^{pZ} = \frac{1}{4}(G_{E,M}^{p\gamma} - G_{E,M}^{n\gamma}) - \sin^2\theta_W G_{E,M}^{p\gamma} - \frac{1}{4} G_{E,M}^{s}$$

- A_{PV} + $G_{E,M}^{p,n\gamma}$ + G_A^{pZ} (calculated) --> $G_{E,M}^{s}$
- G. Ron et al indicated HAPPEX-I shifted by ~0.5σ towards 0 due to smaller G_E
- Similar change in F.F. in HAPPEX-III kinematics would lead to ~1σ shift

Summary

- Interest in low Q^2 form factors for numerous reasons:
  - Structure of proton – comparison with theory / models / parameterizations
  - Possible structures in proton
  - Proton radii, electric and / vs magnetic
  - Input for Zemach radius
  - Input for strange form factors
  - Improved IS/IV form factors, not discussed