A Bethe-Salpeter equation view of pion electromagnetic properties

Andreas Krassnigg

University of Graz, Austria

HEFF, Trento, 22nd May 2008

Work with

A. Höll, C. D. Roberts, S. V. Wright (ANL, PHY)

R. Alkofer, G. Eichmann, M. Joergler, M. Schwinzerl (Univ. Graz)

P. Maris (Iowa State)

Work performed at/supported by/in collaboration with

- Austrian Research Foundation
- Argonne National Laboratory
- University of Graz

Outline

Outline

EM Interaction

Electromagnetic interaction important for our life

EM Interaction

Electromagnetic interaction important for our life

EM Interaction

It even governs our life

EM Interaction

- Need understanding of EM properties of matter

EM Interaction

- Need understanding of EM properties of matter
- Need understanding of EM properties of hadrons

EM Interaction

- Need understanding of EM properties of matter
- Need understanding of EM properties of hadrons
- EM probes of hadrons readily available
- Yield information about hadron structure
- Wide range of momentum transfer
- Information contained in electromagnetic hadron form factors
- Measurable
- Calculable (from QCD?!)

QCD and Hadrons

- Study hadrons as composites of quarks and gluons . . .
- . . . including:
  - Chiral symmetry and $D\chi SB$
  - correct perturbative limit (via $\alpha_p(Q^2)$)
  - quark and gluon confinement
  - Poincaré covariance
- Calculate observables!

QCD and Hadrons

- Study hadrons as composites of quarks and gluons . . .
- . . . including:
  - Chiral symmetry and $D\chi SB$
  - correct perturbative limit (via $\alpha_p(Q^2)$)
  - quark and gluon confinement
  - Poincaré covariance
- Calculate observables!
- Dyson Schwinger Equations: a modern method in relativistic QFT

P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12** (2003) 297
R. Alkofer and L. von Smekal, Phys. Rept. **353** (2001) 281
C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. 45 (2000) S1
A. Holl, C. D. Roberts, S. V. Wright, nucl-th/0601071
C. S. Fischer, J. Phys. G **32** (2006) R253

Dyson-Schwinger Equations

- Euclidean Green functions (also calculated on the lattice) satisfy the [Dyson, Schwinger] equations
- Each function satisfies integral equation involving other functions $\Rightarrow$
- Infinite set of coupled integral equations
- Truncation scheme necessary $\Rightarrow$
- Generating tool for perturbation theory

FWF

Dyson-Schwinger Equations

- Euclidean Green functions (also calculated on the lattice) satisfy the [Dyson, Schwinger] equations
- Each function satisfies integral equation involving other functions ⇒
- Infinite set of coupled integral equations
- Truncation scheme necessary ⇒

- Nonperturbative truncation scheme
- Respect symmetries
- Prove exact (model independent) results
- Devise (sophisticated) models to illustrate them
- Perform reliable calculations of hadron properties
- Propagators and Bethe-Salpeter amplitudes (BSAs) → can be used to calculate observables

FWF

Coming up next . . .

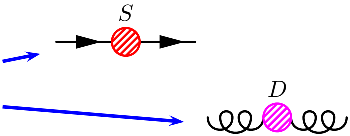
Equations and Solutions

- Quark DSE
- Bethe-Salpeter Equation
- BSE Solution Strategies

Building Blocks

2-point functions

- quark propagator
- gluon propagator



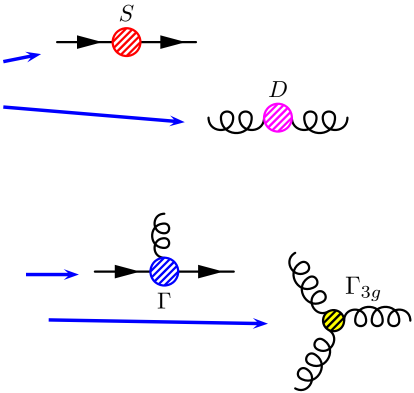
Building Blocks

2-point functions

- quark propagator
- gluon propagator

3-point functions

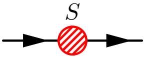
- quark-gluon vertex
- three-gluon vertex . . .

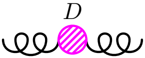


Building Blocks

2-point functions

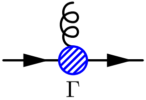
- quark propagator
- gluon propagator

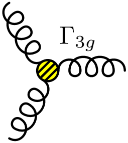




3-point functions

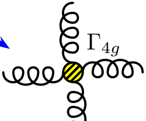
- quark-gluon vertex
- three-gluon vertex ...

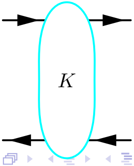




4-point functions

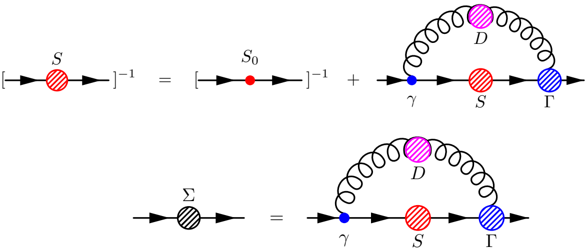
- quark-antiquark scattering kernel
- four-gluon vertex ...





Gap Equation

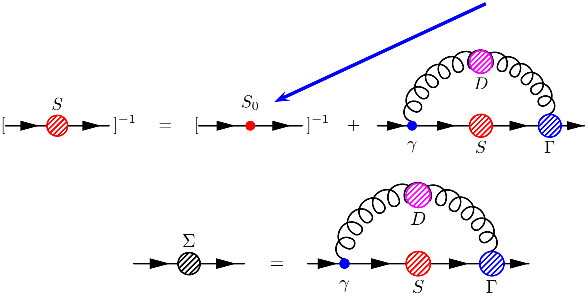
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

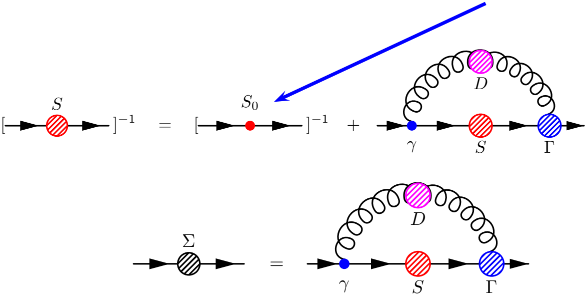
current quark mass m_ζ



Gap Equation

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

current quark mass m_ζ



- Weak coupling expansion reproduces every diagram in perturbation theory, but:
- Perturbation theory: $m_\zeta = 0 \;\Rightarrow\; M(p^2) \equiv 0$

Quark Mass Function

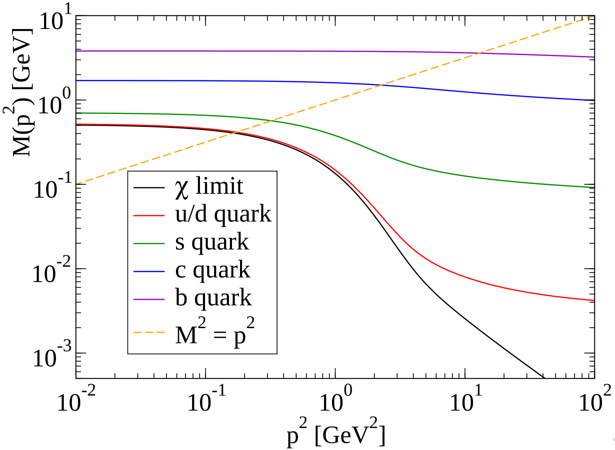
Solution of gap equation: $S(p) = \dfrac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$

P. Maris, C. D. Roberts, Phys. Rev. **C56**, 3369 (1997)

Quark Mass Function

Solution of gap equation: $S(p) = \dfrac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$

P. Maris, C. D. Roberts, Phys. Rev. **C56**, 3369 (1997)

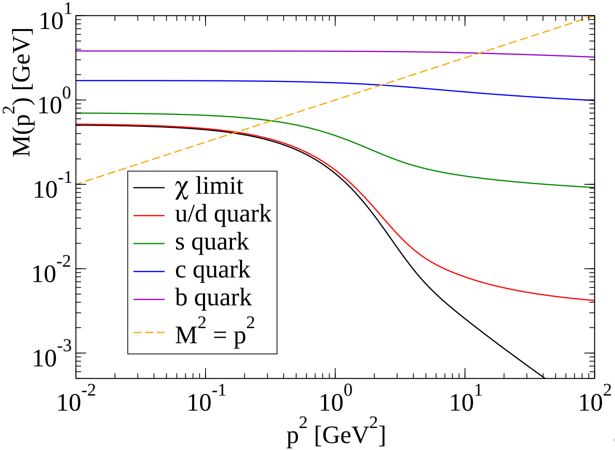


Quark Mass Function

Solution of gap equation: $S(p) = \dfrac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$

P. Maris, C. D. Roberts, Phys. Rev. **C56**, 3369 (1997)

$M^2(p^2) = p^2 \Rightarrow$ Euclidean constituent quark mass M_E

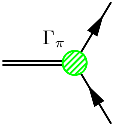


q	M_E/m_ζ
χ	∞
u/d	100
s	7
c	1.7
b	1.2

$\rightarrow D\chi SB$

Example BSA :: Pseudoscalar Meson

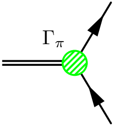
- Pseudoscalar meson Bethe-Salpeter amplitude:



$$\Gamma^j_\pi(k;P) = \tau^j \gamma_5 \left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) \right.$$
$$\left. + \quad \gamma\cdot k\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P) \right]$$

Example BSA :: Pseudoscalar Meson

- Pseudoscalar meson Bethe-Salpeter amplitude:

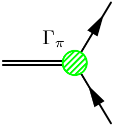


$$\Gamma_\pi^j(k;P) = \tau^j \gamma_5 \left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) \right.$$
$$\left. + \quad \gamma\cdot k\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P) \right]$$

- P: total momentum, k: relative momentum
- Variables: k^2, P^2, $z := \widehat{k}\cdot\widehat{P} \rightarrow$ angle variable

Example BSA :: Pseudoscalar Meson

- Pseudoscalar meson Bethe-Salpeter amplitude:

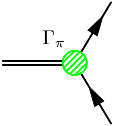


$$\Gamma_\pi^j(k;P) = \tau^j \gamma_5 [iE_\pi(k;P) + \gamma \cdot P F_\pi(k;P) + \gamma \cdot k\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)]$$

- P: total momentum, k: relative momentum
- Variables: k^2, P^2, $z := \widehat{k} \cdot \widehat{P} \rightarrow$ angle variable
- pseudoscalar piece

Example BSA :: Pseudoscalar Meson

- Pseudoscalar meson Bethe-Salpeter amplitude:



$$\Gamma^j_\pi(k;P) = \tau^j \gamma_5 \left[iE_\pi(k;P) + \gamma \cdot P F_\pi(k;P) \right.$$
$$\left. + \quad \gamma \cdot k\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P) \right]$$

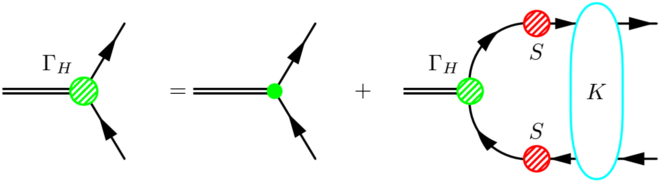
- P: total momentum, k: relative momentum
- Variables: k^2, P^2, $z := \widehat{k} \cdot \widehat{P} \rightarrow$ angle variable
- pseudoscalar piece
- pseudovector pieces:
  - intrinsic orbital angular momentum
  - crucial for Lorentz invariance
  - preserving symmetries (AV WTI)
  - asymptotic behavior of pion em form factor

FWF

Inhomogeneous BSE

- BSE for $q\bar{q}$ or qq bound states ($\chi = S\,\Gamma_H\,S$)

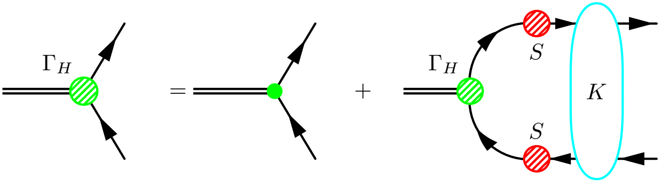
$$\Gamma_H(p;P) = \mathrm{d.\,t.} + \int d^4q\ \chi(q;P)\,K(q,p;P)\,.$$



Inhomogeneous BSE

- BSE for $q\bar{q}$ or qq bound states ($\chi = S\,\Gamma_H\,S$)

$$\Gamma_H(p;P) = \text{d. t.} + \int d^4q\ \chi(q;P)\,K(q,p;P)\,.$$

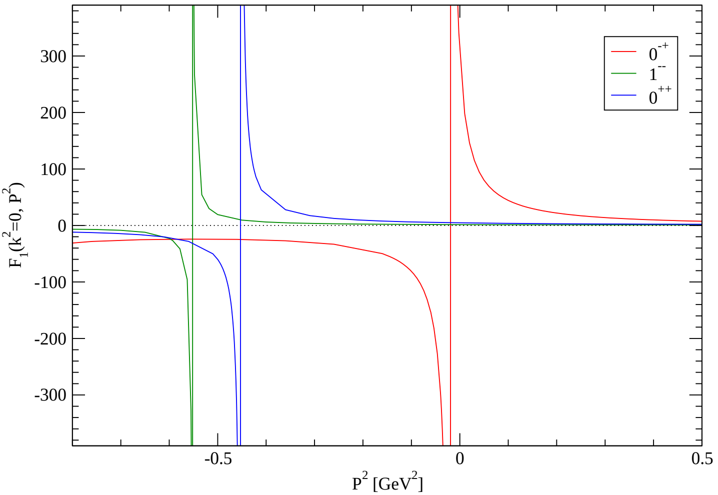


- Gap eq. output $\rightarrow$ BSE input
- Bound state at $P^2 = -m_H^2$:

$$\Gamma_H(q;P) = \frac{r_H\,\Gamma_h(q;P)}{P^2 + m_H^2} + \text{regular terms}$$

Inhomogeneous BSE :: Solution

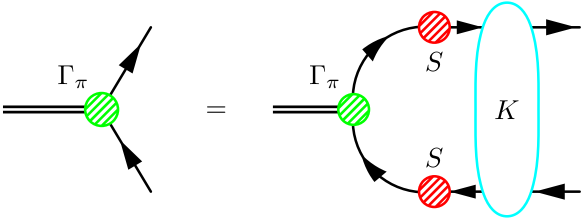
0^{-+}, 0^{++}, and 1^{--} meson amplitudes



Homogeneous BSE

- BSE for $q\bar{q}$ or qq bound states ($\chi = S\,\Gamma_h\,S$)

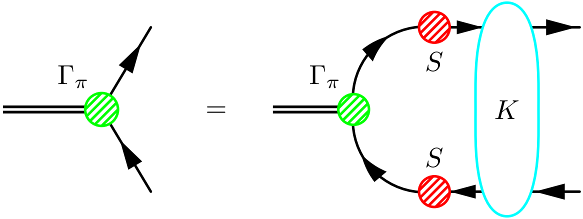
$$\Gamma_{h\;tu}(p;P) \qquad = \int d^4q\;[\chi(q;P)]_{sr}\,K_{rs}^{tu}(q,p;P)\,.$$



Homogeneous BSE

- BSE for $q\bar{q}$ or qq bound states ($\chi = S\,\Gamma_h\,S$)

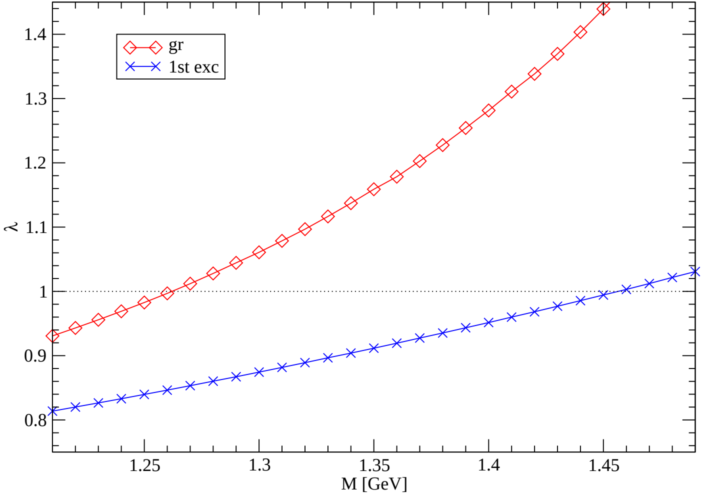
$$\Gamma_{h\;tu}(p;P)\;\lambda(P^2) \;=\int d^4q\;[\chi(q;P)]_{sr}\;K^{tu}_{rs}(q,p;P)\,.$$



- homogeneous $\rightarrow$ eigenvalue equation

Homogeneous BSE :: Solution Strategy

Solution strategy for homogeneous BSE



Coming up next . . .

Symmetries and Exact Results

- AV WTI
- Truncation and Model Building

AV WTI

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^j_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \frac{\tau^j}{2} + i\gamma_5 \frac{\tau^j}{2} S^{-1}(k_-) - 2i\, m(\zeta)\, \Gamma^j_5(k;P),$$

AV WTI

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^j_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \frac{\tau^j}{2} + i\gamma_5 \frac{\tau^j}{2} S^{-1}(k_-) - 2i\, m(\zeta)\, \Gamma^j_5(k;P),$$

- Consequence: Gap and BSE kernels related

AV WTI

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^j_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \frac{\tau^j}{2} + i\gamma_5 \frac{\tau^j}{2} S^{-1}(k_-) - 2i\, m(\zeta)\, \Gamma^j_5(k;P),$$

- Consequence (residues): $f_{\pi_n} m^2_{\pi_n} = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)$; with $n = \mathrm{gr}, \mathrm{exc1}, \ldots$

FWF

AV WTI

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^j_{5\mu}(k;P) = S^{-1}(k_+) i\gamma_5 \frac{\tau^j}{2} + i\gamma_5 \frac{\tau^j}{2} S^{-1}(k_-) - 2i\, m(\zeta)\, \Gamma^j_5(k;P),$$

- Consequence (residues): $f_{\pi_n} m^2_{\pi_n} = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)\,;$
  with $n = \mathrm{gr}, \mathrm{exc1}, \ldots$
- valid for every pseudoscalar meson
- valid for every current quark mass
- $\Rightarrow$ GMOR, PCAC

P. Maris, C. D. Roberts, Phys. Rev. **C56**, 3369 (1997)

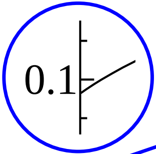
A. Höll, A. K., and C. D. Roberts, Phys. Rev. C **70**, 042203 (2004)

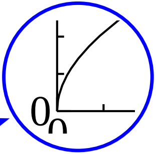
Qualitative Check

- Investigate the chiral limit of $f_{\pi_n} m^2_{\pi_n} = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)$;

Qualitative Check

- Investigate the chiral limit of $f_{\pi_n} m_{\pi_n}^2 = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)$;
- Ground state pion:
  - $m(\zeta) \to 0$
  - $f_{\pi_{gr}}$ finite
  - $m_{\pi_{gr}} \to 0$



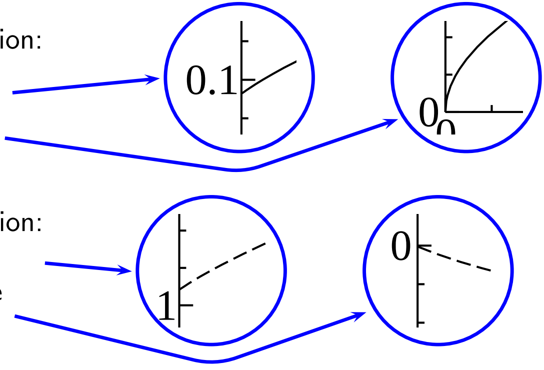


Qualitative Check

- Investigate the chiral limit of $f_{\pi_n} m_{\pi_n}^2 = 2\, m(\zeta)\, \rho_{\pi_n}(\zeta)$;

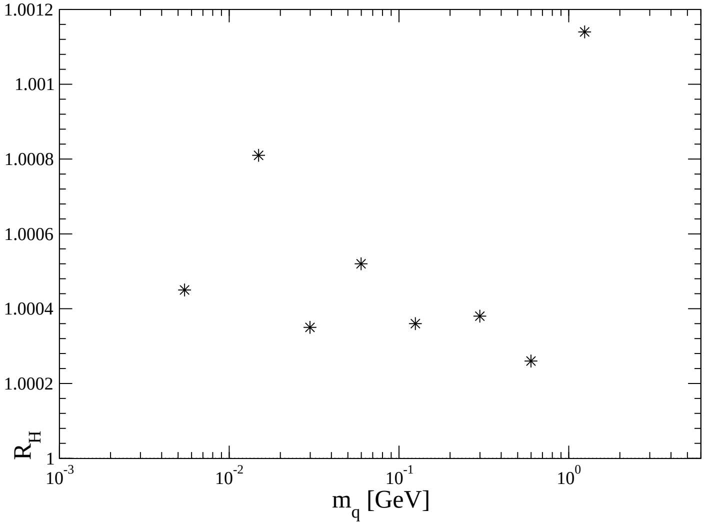
- Ground state pion:
  - $m(\zeta) \longrightarrow 0$
  - $f_{\pi_{gr}}$ finite
  - $m_{\pi_{gr}} \longrightarrow 0$

- Excited state pion:
  - $m(\zeta) \longrightarrow 0$
  - $m_{\pi_{exc1}}$ finite
  - $f_{\pi_{exc1}} \longrightarrow 0$



Quantitative Check

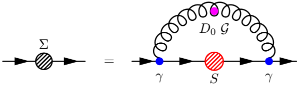
Numerical accuracy of $R_H := \frac{f_{\pi_{\rm gr}}\, m^2_{\pi_{\rm gr}}}{2\, m(\zeta)\, \rho_{\pi_{\rm gr}}(\zeta)}$ for u/d to c quark pseudoscalar mesons



Rainbow-Ladder (RL) Truncation

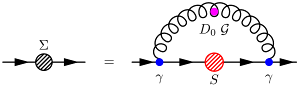
- Satisfy the AVWTI!
- Simplest truncation to do this:

Rainbow-Ladder (RL) Truncation

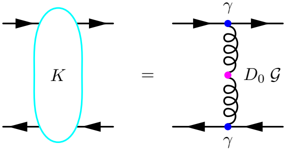
- Satisfy the AVWTI!
- Simplest truncation to do this:
- Rainbow approx. for gap eq. 

Rainbow-Ladder (RL) Truncation

- Satisfy the AVWTI!
- Simplest truncation to do this:
- Rainbow approx. for gap eq.

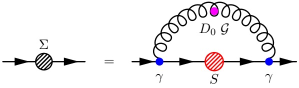


- Ladder approximation for BSE

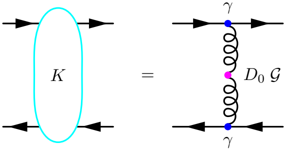


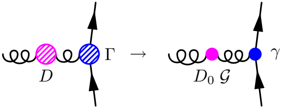
Rainbow-Ladder (RL) Truncation

- Satisfy the AVWTI!
- Simplest truncation to do this:
- Rainbow approx. for gap eq.



- Ladder approximation for BSE





- Bare quark-gluon vertex γ_ν
- Bare gluon prop. $D^{\mathrm{free}}_{\mu\nu}(p-q)$
- Effective coupling $\mathcal{G}$
- Input needed for $\mathcal{G}$: modeling

Effective Coupling

- What do we know?
- Effective running coupling $\mathcal{G}(Q^2)$
- Perturbative QCD determines UV regime
- IR unknown in detail

Effective Coupling

- What do we know?
- Effective running coupling $\mathcal{G}(Q^2)$
- Perturbative QCD determines UV regime
- IR unknown in detail
- IR enhancement necessary for dynamical breaking of chiral symmetry
- Integrated strength is essential
- Precise form at low $Q^2 \;\rightarrow$ model

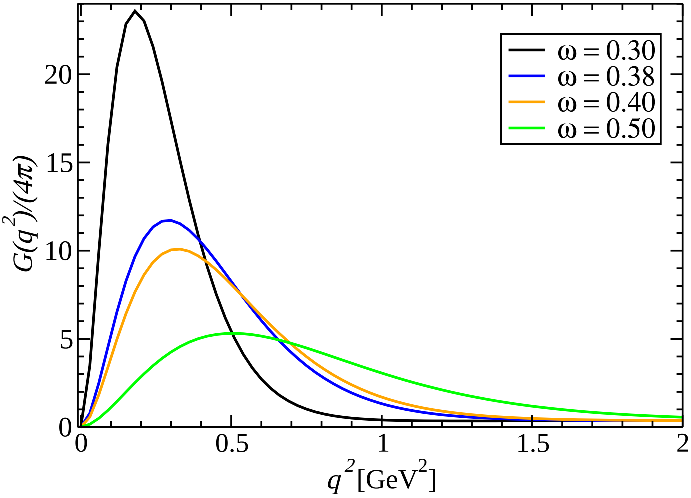
Effective Coupling

- What do we know?
- Effective running coupling $\mathcal{G}(Q^2)$
- Perturbative QCD determines UV regime
- IR unknown in detail
- IR enhancement necessary for dynamical breaking of chiral symmetry
- Integrated strength is essential
- Precise form at low Q^2 $\rightarrow$ model
- IR: two-parameters via Gaussian:
  strength D and width ω
- perturbative α in the UV region
- P. Maris, P. C. Tandy: series of papers following

  P. Maris and P. C. Tandy, Phys. Rev. C **60**, 055214 (1999).
- Successful description of light pseudoscalar and vector mesons
  $\Rightarrow$ see P. C. Tandy's talk

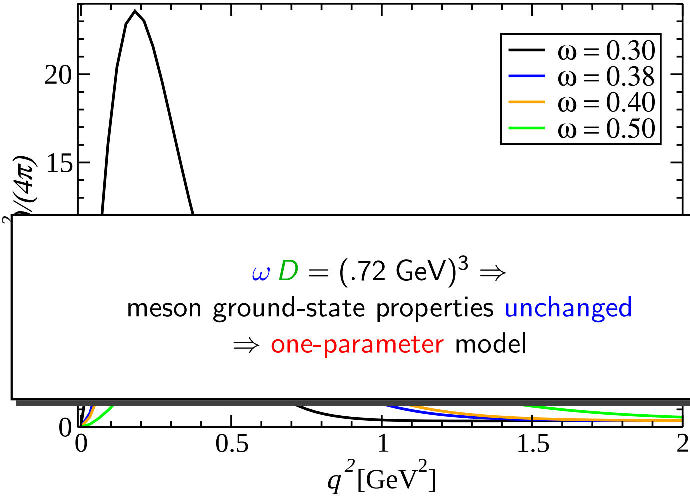
Model Details

- Effective coupling $\mathcal{G}(Q^2)$: ω D =const.



Model Details

- Effective coupling $\mathcal{G}(Q^2)$: $\omega\, D$ =const.



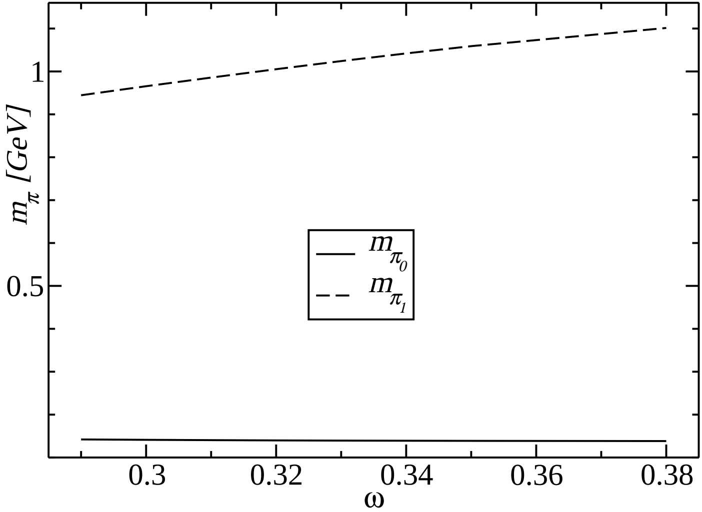
$\omega\, D = (.72\ \text{GeV})^3 \Rightarrow$

meson ground-state properties unchanged

$\Rightarrow$ one-parameter model

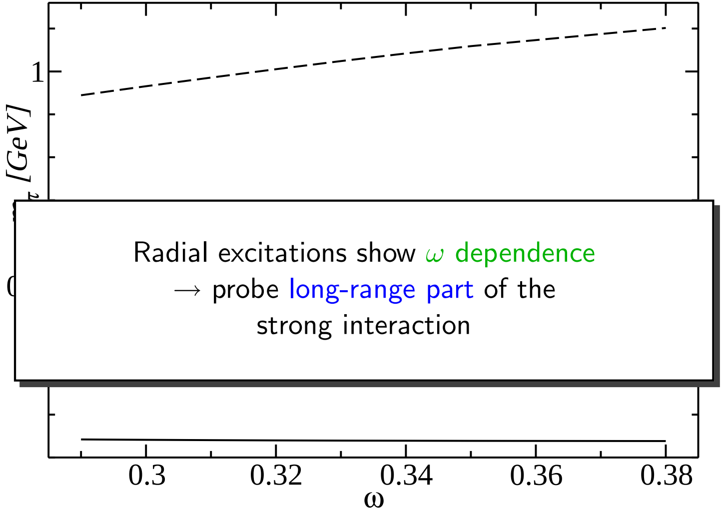
Model Parameter Dependence

$m_{\pi_{gr}}$ and $m_{\pi_{exc1}}$ as functions of ω



Model Parameter Dependence

$m_{\pi_{gr}}$ and $m_{\pi_{exc1}}$ as functions of ω



Radial excitations show ω dependence
$\rightarrow$ probe long-range part of the strong interaction

Model Parameter Dependence

This will be baaaack!!

Coming up next . . .

Pion-Form-Factor Howto

Howto :: Compute the Pion EM FF

What you need:

- Dressed quark propagator $S(p)$
- Pion BSA $\Gamma_\pi(P; q)$
- Quark-photon vertex $\Gamma_\mu(Q; k)$

Howto :: Compute the Pion EM FF

What you need:

- Dressed quark propagator $S(p)$
- Pion BSA $\Gamma_\pi(P;q)$
- Quark-photon vertex $\Gamma_\mu(Q;k)$

Then:

- Arrange the above (in impulse approximation) in triangle diagram
- This construction is consistent with RL truncation
- Put on a PC, stir for a few seconds, and enjoy the result!

Howto :: Compute the Pion EM FF

What you need:

- Dressed quark propagator $S(p)$
- Pion BSA $\Gamma_\pi(P; q)$
- Quark-photon vertex $\Gamma_\mu(Q; k)$

Then:

- Arrange the above (in impulse approximation) in triangle diagram
- This construction is consistent with RL truncation
- Put on a PC, stir for a few seconds, and enjoy the result!
- This is not cooking; it is the exactly consistent thing to use!

FWF

Before We Start :: The Quark-Photon Vertex

- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2

Before We Start :: The Quark-Photon Vertex

- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2
- Bare vertex γ_μ does this (obviously) 

Before We Start :: The Quark-Photon Vertex

- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2

- Bare vertex γ_μ does this (obviously)
- Next: WTI
  $iQ_\mu\Gamma_\mu(Q;k) = \hat{Q}[S^{-1}(k+Q/2) - S^{-1}(k-Q/2)]$

FWF

Before We Start :: The Quark-Photon Vertex

- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2

- Bare vertex γ_μ does this (obviously)
- Next: WTI
  $iQ_\mu\Gamma_\mu(Q;k) = \hat{Q}[S^{-1}(k+Q/2) - S^{-1}(k-Q/2)]$
- γ_μ doesn't satisfy this for dressed quark propagators S
- $\Rightarrow$ violates current conservation

Before We Start :: The Quark-Photon Vertex

- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2
- WTI $\quad iQ_\mu\Gamma_\mu(Q;k) = \hat{Q}[S^{-1}(k+Q/2) - S^{-1}(k-Q/2)]$

Before We Start :: The Quark-Photon Vertex

- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2
- WTI $\quad iQ_\mu\Gamma_\mu(Q;k) = \hat{Q}[S^{-1}(k+Q/2) - S^{-1}(k-Q/2)]$
- Ball-Chiu construction:
  $\Gamma_\mu^{BC}(Q;k) = \Gamma_\mu^{BC}(k_1,k_2) = [A(k_1^2)+A(k_2^2)]/2\,\gamma_\mu +$
  $(k_1+k_2)_\mu([A(k_1^2)-A(k_2^2)][\gamma\cdot k_1+\gamma\cdot k_2]/2 - i[B(k_1^2)-B(k_2^2)])$
- Satisfies both of the above 

Before We Start :: The Quark-Photon Vertex

- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2
- WTI $\quad iQ_\mu \Gamma_\mu(Q;k) = \hat{Q}[S^{-1}(k+Q/2) - S^{-1}(k-Q/2)]$
- Ball-Chiu construction:
  $\Gamma_\mu^{BC}(Q;k) = \Gamma_\mu^{BC}(k_1,k_2) = [A(k_1^2)+A(k_2^2)]/2\,\gamma_\mu +$
  $(k_1+k_2)_\mu([A(k_1^2)-A(k_2^2)][\gamma\cdot k_1+\gamma\cdot k_2]/2 - i[B(k_1^2)-B(k_2^2)])$
- Satisfies both of the above 
- Next: vector-meson pole contributions

Before We Start :: The Quark-Photon Vertex

- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2
- WTI $\quad iQ_\mu\Gamma_\mu(Q;k) = \hat{Q}[S^{-1}(k+Q/2) - S^{-1}(k-Q/2)]$
- Ball-Chiu construction:
  $$\Gamma_\mu^{BC}(Q;k) = \Gamma_\mu^{BC}(k_1,k_2) = [A(k_1^2)+A(k_2^2)]/2\,\gamma_\mu + (k_1+k_2)_\mu([A(k_1^2)-A(k_2^2)][\gamma\cdot k_1+\gamma\cdot k_2]/2 - i[B(k_1^2)-B(k_2^2)])$$
- Satisfies both of the above 
- Next: vector-meson pole contributions
- BC vertex doesn't contain such contributions
- $\Rightarrow$ underestimation of the pion charge radius

FWF

Before We Start :: The Quark-Photon Vertex

- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2
- WTI $iQ_\mu \Gamma_\mu(Q;k) = \hat{Q}[S^{-1}(k+Q/2) - S^{-1}(k-Q/2)]$
- Vector-meson pole contributions

FWF

Before We Start :: The Quark-Photon Vertex

- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2
- WTI $\quad iQ_\mu\Gamma_\mu(Q;k) = \hat{Q}[S^{-1}(k+Q/2) - S^{-1}(k-Q/2)]$
- Vector-meson pole contributions
- The solution of the inhomogeneous BSE for the quark-photon vertex in RL truncation satisfies all the above

Before We Start :: The Quark-Photon Vertex

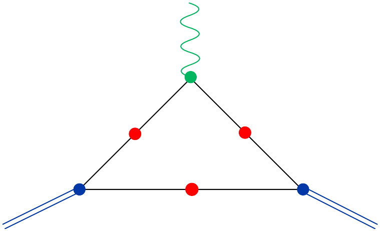
- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2
- WTI $\quad iQ_\mu\Gamma_\mu(Q;k) = \hat{Q}[S^{-1}(k+Q/2) - S^{-1}(k-Q/2)]$
- Vector-meson pole contributions
- The solution of the inhomogeneous BSE for the quark-photon vertex in RL truncation satisfies all the above
- 12 Dirac structures:
  $\Gamma_\mu(Q;k) = \gamma_\mu F_1(Q;k) + k_\mu\, \gamma\cdot k\, F_2(Q;k) + \ldots$
- Amplitudes show pole at $Q^2 = -M_\rho^2$

Before We Start :: The Quark-Photon Vertex

- Restrictions on the quark-photon vertex $\Gamma_\mu(Q;k)$:
- Correct asymptotic behavior for large Q^2
- WTI $\quad iQ_\mu \Gamma_\mu(Q;k) = \hat{Q}[S^{-1}(k + Q/2$
- Vector-meson pole contributions
- The solution of the inhomogeneous BS
  vertex in RL truncation satisfies all the
- 12 Dirac structures:
  $\Gamma_\mu(Q;k) = \gamma_\mu F_1(Q;k) + k_\mu \gamma \cdot k \, F_2(Q$
- Amplitudes show pole at $Q^2 = -M_\rho^2$
- That's great!

More Precisely :: Compute the Pion EM FF

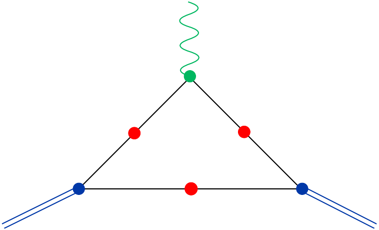
- Dressed quark propagator $S(p)$ from its DSE
- Pion BSA $\Gamma_\pi(P;q)$ from its (homogeneous) BSE
- Quark-photon vertex $\Gamma_\mu(Q;k)$ from its (inhomogeneous) BSE
- Triangle diagram: $F_\pi \sim \int \mathrm{Tr}[\Gamma_\pi S_2 \Gamma_\mu S_3 \Gamma_\pi S_1]$



- Consistent impulse approximation

More Precisely :: Compute the Pion EM FF

- Dressed quark propagator $S(p)$ from its DSE
- Pion BSA $\Gamma_\pi(P;q)$ from its (homogeneous) BSE
- Quark-photon vertex $\Gamma_\mu(Q;k)$ from its (inhomogeneous) BSE
- Triangle diagram: $F_\pi \sim \int \mathrm{Tr}[\Gamma_\pi S_2 \Gamma_\mu S_3 \Gamma_\pi S_1]$



- Consistent impulse approximation
- Improvement is possible, but has to be consistent with truncation of BSEs (see P. C. Tandy's talk)

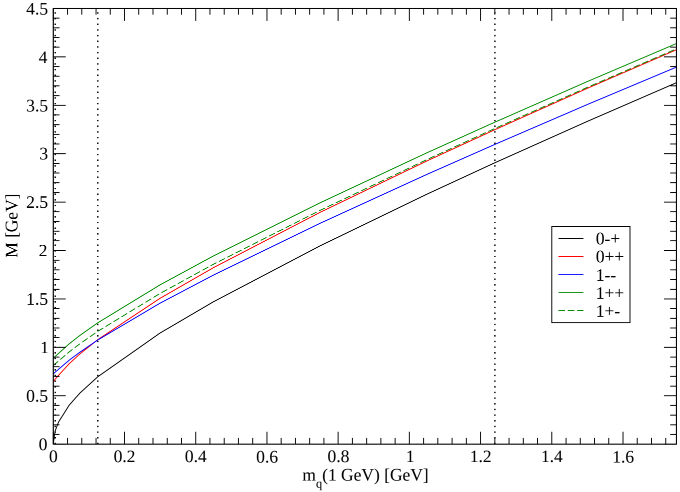
Coming up next . . .

Results

- Spectra et al.
- Electromagnetic Properties

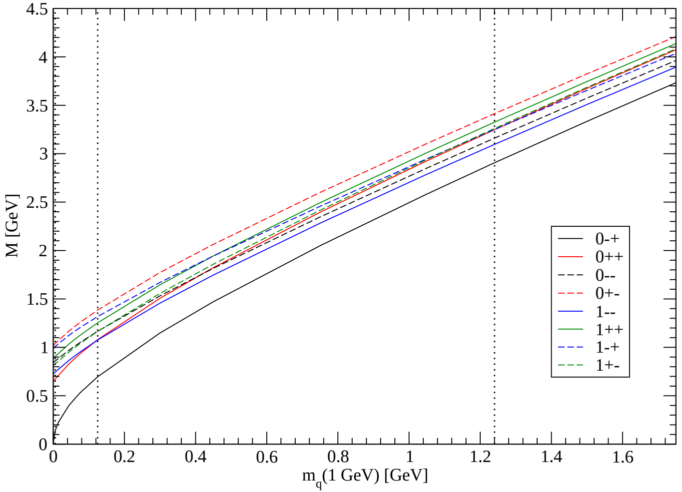
First Things First :: Spectra

Meson masses for $J = 0, 1$ as functions of current quark mass.
Three vertical dotted lines: u/d, s, c quark masses



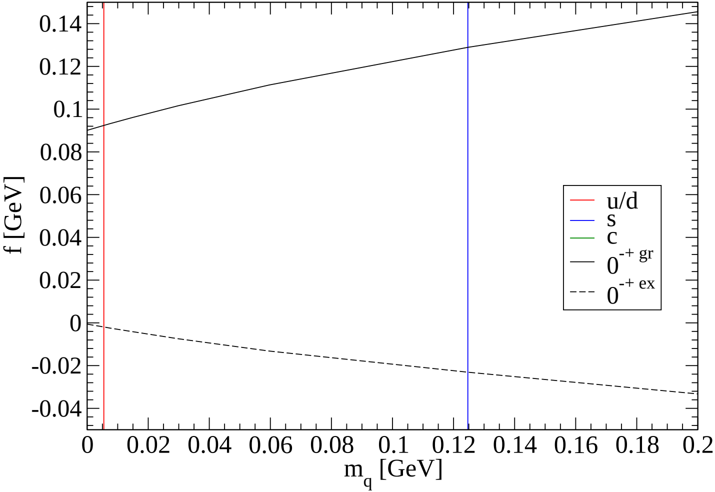
Sideremark :: Exotic Quantum Numbers

Meson masses for $J = 0, 1$ as functions of current quark mass.
Exotic quantum numbers natural in $\bar{q}q$ BSE



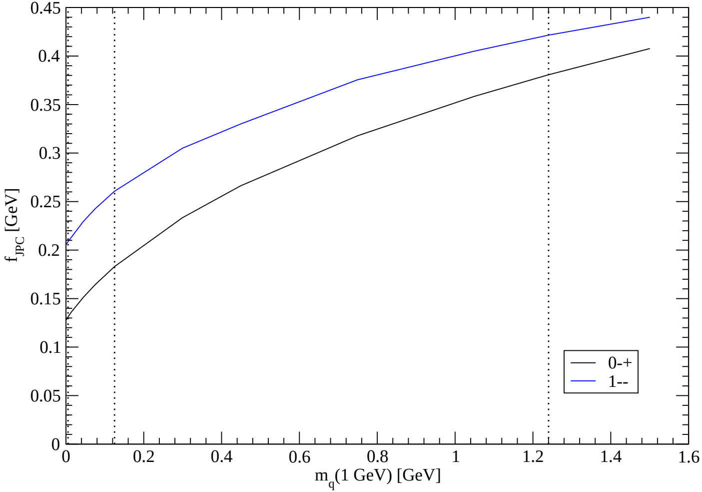
As Promised Earlier :: Leptonic Decay Constants

$f_{0^{-+}_{gr}}$ and $f_{0^{-+}_{exc1}}$ as functions of current quark mass



Leptonic Decay Constants :: Quark-Mass Dependence

$f_{0^{-+}_{gr}}$ and $f_{1^{--}_{gr}}$ as functions of current quark mass



Meson EM Form Factors :: Achievements

- P. Maris, C. D. Roberts, P. C. Tandy, and collaborators used
- Renormalization-group improved rainbow-ladder truncation to calculate:
  - Pion charge radius and electromagnetic form factor
  - Kaon em form factors
  - Vector meson em form factors
  - $\gamma^* \pi \to \gamma$
- With great success
- See also P. C. Tandy's talk

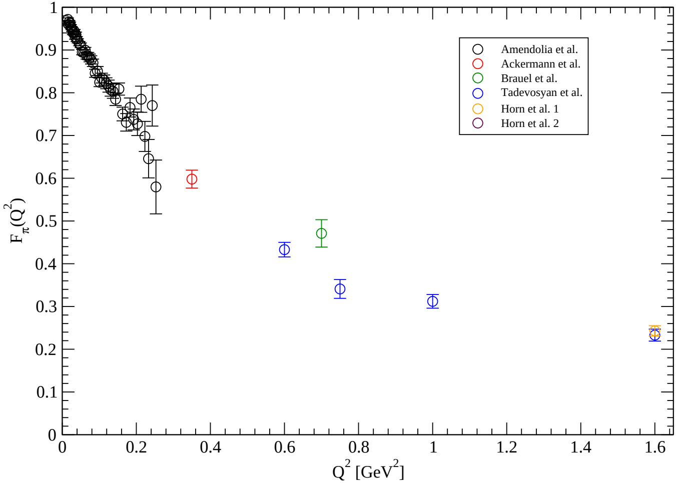
Meson EM Form Factors :: Achievements

- P. Maris, C. D. Roberts, P. C. Tandy, and collaborators used
- Renormalization-group improved rainbow-ladder truncation to calculate:
  - Pion charge radius and electromagnetic form factor
  - Kaon em form factors
  - Vector meson em form factors
  - $\gamma^* \pi \to \gamma$
- With great success
- See also P. C. Tandy's talk
- Present talk:
- Focus on a few interesting details of the approach
- Contrast:
  - Very successful description so far
  - Making room for corrections beyond present truncation

FWF

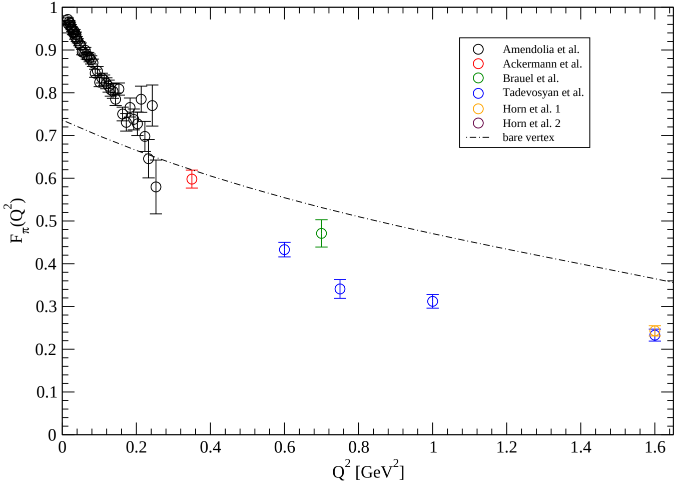
Pion Form Factor :: Dos and Don'ts

$F_\pi(Q^2)$ computed with the bare quark-photon vertex



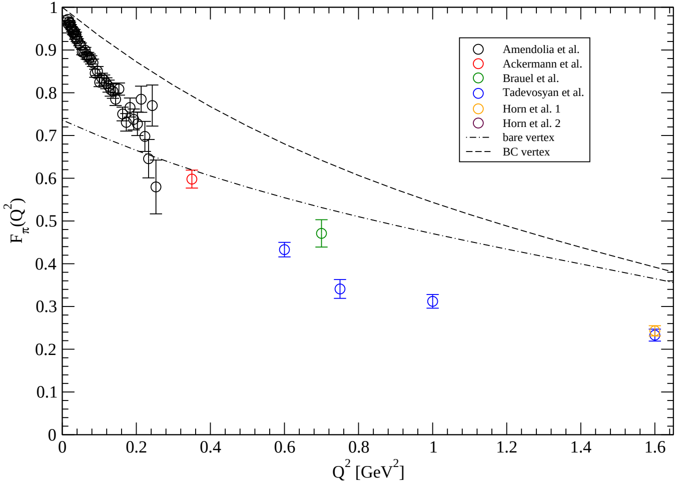
Pion Form Factor :: Dos and Don'ts

$F_\pi(Q^2)$ computed with the bare quark-photon vertex



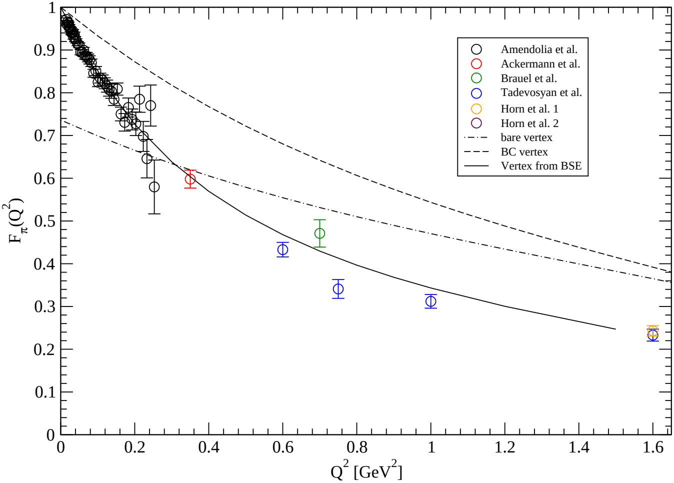
Pion Form Factor :: Dos and Don'ts

$F_\pi(Q^2)$ computed with the Ball-Chiu vertex



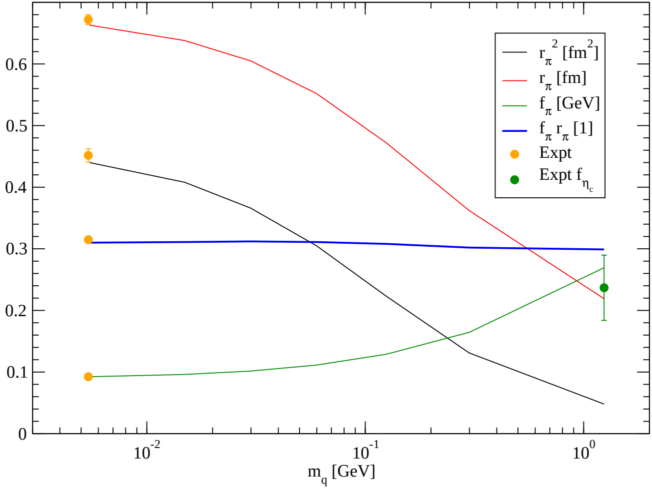
Pion Form Factor :: Dos and Don'ts

$F_\pi(Q^2)$ computed with the full $q - \gamma$ vertex from its (RL)BSE



Pion Charge Radius and Leptonic Decay Constant

r_π and f_π from u to c quark mass



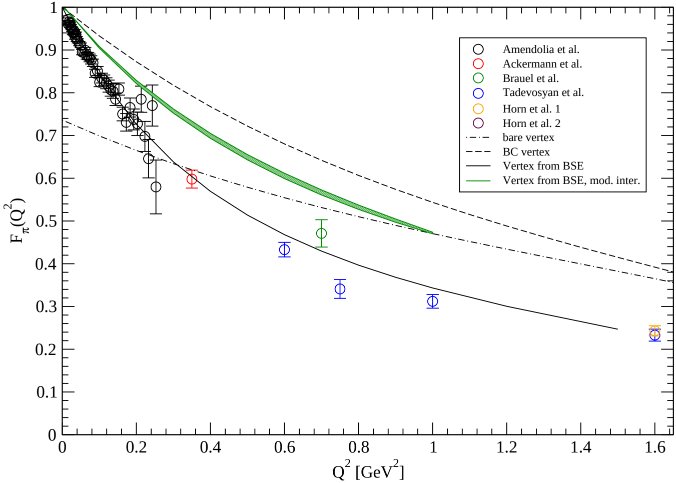
Modified Interaction :: Motivation

G. Eichmann *et al.*, Phys. Rev. C77: 042202(R) (2008)

- RGIRL truncation fitted to pion observables
- Excellent description of pseudoscalar and vector meson ground-state properties
- Had no room for ...
  - systematic nonresonant corrections beyond RL truncation
  - corrections from pion loops etc.
- So, modify interaction parameters to correspond to the actual RL contributions
- This is increasingly accurate (comprehensive) for higher quark masses
- For more details on the modified interaction, see G. Eichmann's talk

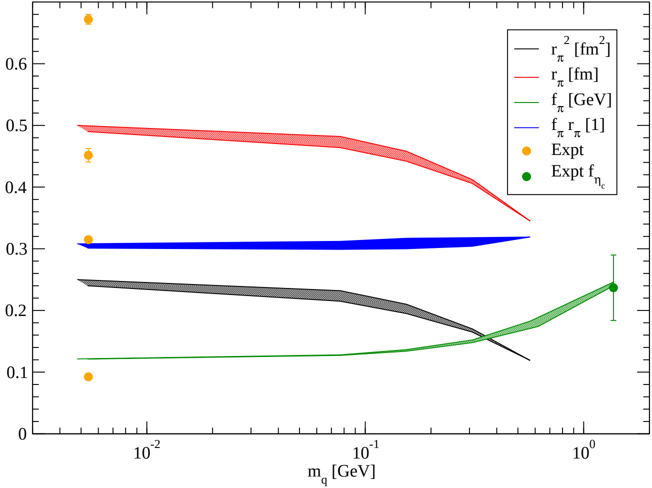
Modified Interaction :: Pion Form Factor

$F_\pi(Q^2)$ computed with the full $q-\gamma$ vertex from its (RL)BSE



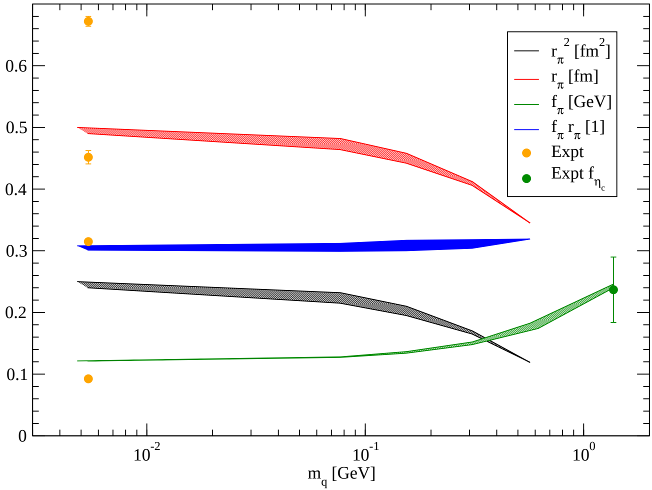
Modified Interaction :: $r_\pi f_\pi$

$r_\pi f_\pi$ from u to c quark mass (PRELIMINARY)



Modified Interaction :: $r_\pi f_\pi$

$r_\pi f_\pi$ from u to c quark mass (PRELIMINARY)



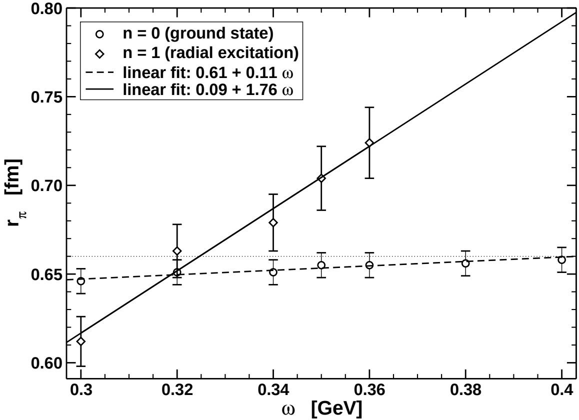
$\Rightarrow$ from $f_{\eta_c} = 0.243$ GeV predict $r_{\eta_c} = 0.25$ fm

Radial Excitations :: Electromagnetic Properties

A. Höll *et al.*, Phys. Rev. C71: 065204, 2005

Estimates

- π_1 charge radius: $r_{\pi_1} \approx 0.93\,\mathrm{fm} \approx 1.4\, r_{\pi_0}$
- radiative decay $\pi_1 \to \gamma\gamma$: $\Gamma_{\pi_1^0 \gamma\gamma} \approx 240$ eV

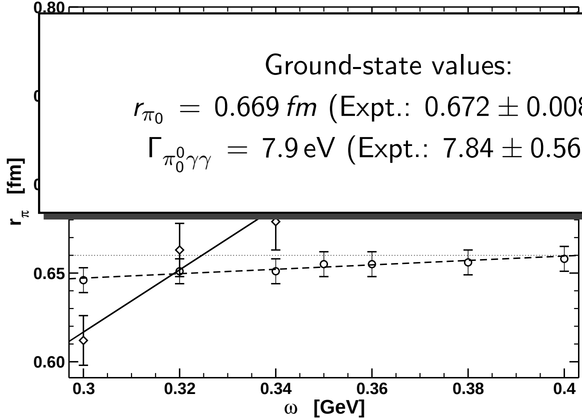


Radial Excitations :: Electromagnetic Properties

A. Höll *et al.*, Phys. Rev. C71: 065204, 2005

Estimates

- π_1 charge radius: $r_{\pi_1} \approx 0.93\,\mathrm{fm} \approx 1.4\, r_{\pi_0}$
- radiative decay $\pi_1 \to \gamma\,\gamma$: $\Gamma_{\pi_1^0\gamma\gamma} \approx 240$ eV



Ground-state values:

$$r_{\pi_0} = 0.669\, fm \text{ (Expt.: } 0.672 \pm 0.008 \text{ fm)}$$

$$\Gamma_{\pi_0^0\gamma\gamma} = 7.9\,\text{eV (Expt.: } 7.84 \pm 0.56\,\text{eV)}$$

FWF

WIP - Wish List

Other work in progress

- Higher J (tensor mesons)
- Higher radial excitations
- Heavy quark sector
- Heavy-light mesons and radial excitations
- Hadronic decays, e. g. $\pi(1300) \to \varrho\,\pi$
- Nucleon properties (e. g. strangeness content)

WIP - Wish List

Other work in progress

- Higher J (tensor mesons)
- Higher radial excitations
- Heavy quark sector
- Heavy-light mesons and radial excitations
- Hadronic decays, e. g. $\pi(1300) \to \varrho\,\pi$
- Nucleon properties (e. g. strangeness content)

Wish list

- Sophisticated meson model beyond RLT
- Good description of axial-vector mesons
- Study states with "exotic" quantum numbers
- Calculate higher-order corrections to RL for π and N

Conclusions

Don't underestimate the power of the electromagnetic interaction.

Conclusions

Summary

- Dyson-Schwinger equations provide a nonperturbative continuum approach to QCD
- BSE describes bound states in a manifestly covariant way
- Present approach: the only symmetry-preserving π calculation applicable to whole range of data in m_q

FWF

Conclusions

Summary

- Dyson-Schwinger equations provide a nonperturbative continuum approach to QCD
- BSE describes bound states in a manifestly covariant way
- Present approach: the only symmetry-preserving π calculation applicable to whole range of data in m_q

Conclusions

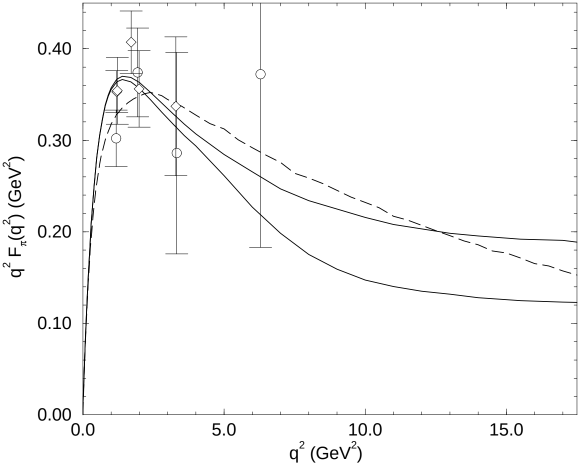
- Excellent description of PS and VE meson ground states in Rainbow-Ladder truncation, including em properties
- Modified interaction leaves room for corrections
- Step beyond RL needed to go for axial vectors, scalars, exotics, radially excited states
- These provide means to study the long-range behavior of the strong interaction between light quarks

The End

Thank you!

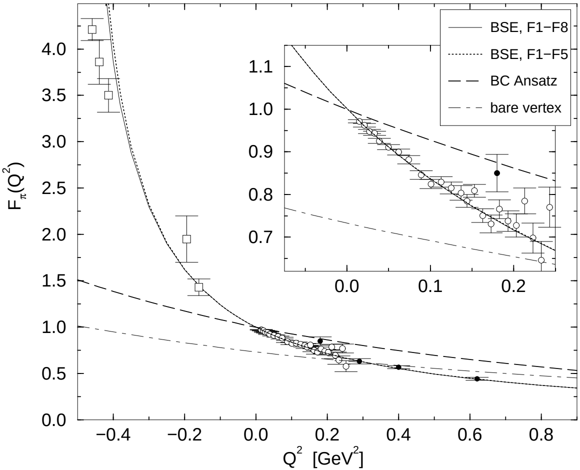
Pion EM FF :: High Q^2

P. Maris and C. D. Roberts, Phys. Rev. C58: 3659, 1998



Pion EM FF :: Timelike Region

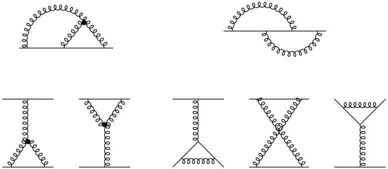
P. Maris and P. C. Tandy, Phys. Rev. C61: 045202, 2000



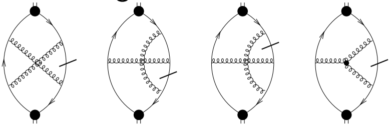
Pion EM FF :: Corrections Beyond RL

P. Maris and P. C. Tandy, Phys. Rev. C62: 055204, 2000

Diagrams in the quark DSE



$\longrightarrow$ diagrams in the norm



$\longrightarrow$ diagrams in the triangle diagram