GPDs, form factors and wide-angle Compton scattering

P. Kroll

Fachbereich Physik, Universität Wuppertal

Trento, May 2008

Outline:

- **Introduction**
- **Analysis of elm. form factors**
- **Moments of GPDs**
- **Physical interpretation**
- **Applications: Compton scattering**
- **The strange form factors**
- **Summary**

based on work done in coll. with M. Diehl, T. Feldmann, R. Jakob, hep-ph/0408173
M. Diehl, T. Feldmann, arXiv:0711.4304

Different views of the structure of the nucleon

The non-relativistic case: refers to proton at rest

Fourier transform $\vec{p} \longrightarrow \vec{r}$

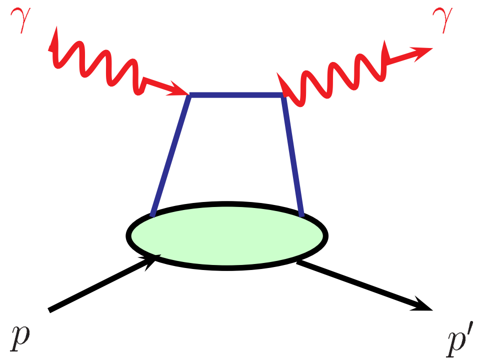
three-dimensional densities

Light-cone physics IMF: refers to rapidly moving proton

Fourier transform $\mathbf{\Delta}_\perp \longrightarrow \vec{b}$

two-dimensional densities in plane transverse to direction of proton momentum

Generalized Parton Distributions



D. Müller et al (94), Ji(97), Radyushkin (97)

occur in

DVES ($\gamma^* p \to \gamma p\,, Mp$) Q^2 large, t small

WAES ($\gamma p \to \gamma p\,, Mp$) Q^2 small, t large

soft physics: GPDs $H^q(x,\xi,t)$, $\widetilde{H}^q$, E^q, $\widetilde{E}^q$ (skewness: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$)

- reduction formulas: $H^q(x,0;0) = q(x)$; $\widetilde{H}^q(x,0;0) = \Delta q(x)$
- sum rules: $h^q_{10}(t) = \int_{-1}^{1} dx H^q(x,\xi,t)$; $F_1 = \sum_q e_q h^q_{10}$;
  $E^q \to F_2^q$; $\widetilde{H}^q \to F_A^q$; $\widetilde{E}^q \to F_P^q$
- polynomiality: e.g. $\int_{-1}^{1} dx\, x^{n-1} H^q(x,\xi,t) = \sum_{i=0}^{[n/2]} h^q_{n,i}(t)\xi^i$
- universality, evolution, positivity constraints, Ji's sum rule

Interpretation

Fourier transform:

$$q(x, \xi, \mathbf{b}) = \int \frac{d^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\mathbf{b}\cdot\boldsymbol{\Delta}} H^q(x, \xi, t = -\Delta^2)$$

and analogue for the other GPDs

Burkhardt: density interpretation of $\xi = 0$ GPDs in mixed representation of long. momentum and transverse position space (IMF)

$q(x, \xi = 0, \mathbf{b})$ gives probability to find a quark q with long. momentum fraction x at transverse position $\mathbf{b}$

GPD analysis - what can be done?

DFJK hep-ph/0408173 (similar Guidal et al hep-ph/0410251)
analogue to PDF analyses

use all available data on $G_M^p\,, G_M^n\,, G_E^p\,, G_E^n\,(\Rightarrow F_1^p\,, F_1^n\,, F_2^p\,, F_2^n)\,, F_A$

exploit sum rules at $\xi = 0$

$$F_1^{p(n)}(t) = \int_0^1 dx \Big[e_{u(d)}\, H_v^u(x,t) + e_{d(u)}\, H_v^d(x,t)\Big] \qquad F_2 \Rightarrow E_v$$

$$F_A(t) = \int_0^1 dx \Big[\widetilde{H}_v^u(x,t) - \widetilde{H}_v^d(x,t)\Big] + 2\int_0^1 dx \Big[\widetilde{H}^{\bar{u}}(x,t) - \widetilde{H}^{\bar{d}}(x,t)\Big]$$

$H_v = H^q - H^{\bar{q}}$

$s - \bar{s}$ (see later), $c - \bar{c}$ and sea quark contribution to F_A neglected, probably very small

in order to determine $H_v^{u,d}\,, \widetilde{H}_v^{u,d}\,, E_v^{u,d}$

in a strict mathematical sense an ill-posed problem BUT

Parameterisation of the GPDs

BlueANSATZ: $H_v^q(x,t) \;=\; q_v(x)\,\exp\left[f_q(x)t\right]$

$$f_q = \left[\alpha' \log(1/x) + B_q\right](1-x)^{n+1} + A_q\, x(1-x)^n$$

$\alpha' = 0.9\,\mathrm{GeV}^{-2}$ (fixed); $n = 1, 2;$ $q_v(x)$ from CTEQ6 (INPUT)

(Guidal et al: $B_q = A_q = 0$, α' free, $n = 0$)

Motivation: for large $-t$ and x: overlaps of Gaussian LC wavefunctions

$$H_v^q(x,t) \;\rightarrow\; \exp\left[a^2 t \frac{1-x}{2x}\right] q_v(x)$$

low $-t$, very small x: Regge behaviour expected

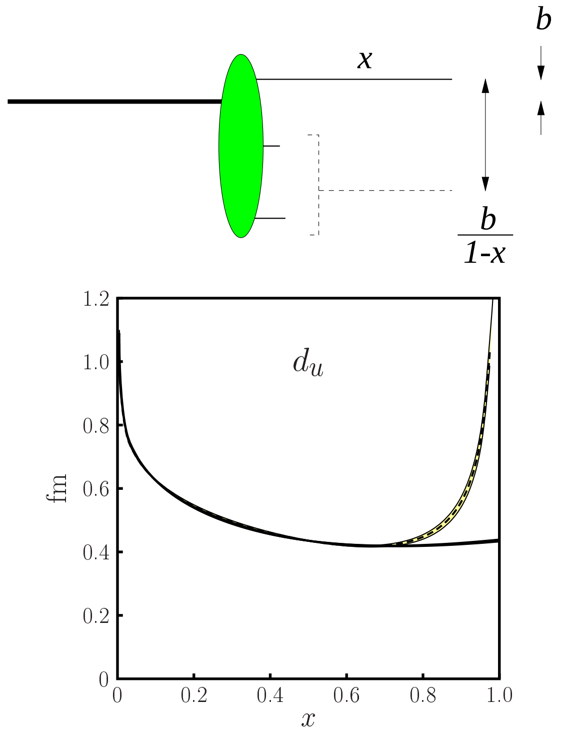
$$H_v^q(x,t) \;\rightarrow\; x^{-\alpha(0)} \exp\left[\alpha' t\, \log(1/x)\right]$$

criteria for good parameterization (met by ansatz):

- simplicity and consistency with theor. and phenom. constraints
- plausible interpretation of parameters (if possible)
- stability with respect to variation of PDFs
- stability under evolution (scale dep. of GPDs can be absorbed into parameters)

$n = 1$ or 2?

Fourier transform to impact parameter plane: $q_v(x, \mathbf{b})$



$\mathbf{b}$ transverse distance between struck quark and hadron's center of momentum:

$\sum_i x_i \mathbf{b_i} = 0$ (chosen, $\quad \sum_i x_i = 1$)

$\mathbf{b}/(1-x)$ relative distance between struck quark and cluster of spectators

average distance: $d_q = \frac{\sqrt{\langle b^2 \rangle_x^q}}{1-x}$

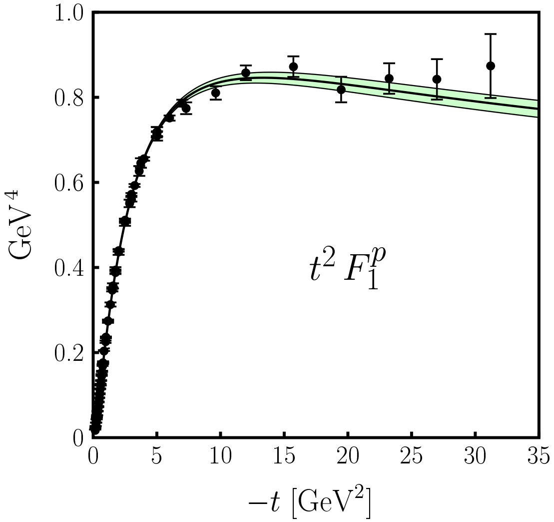
d_q provides estimate of size of hadron

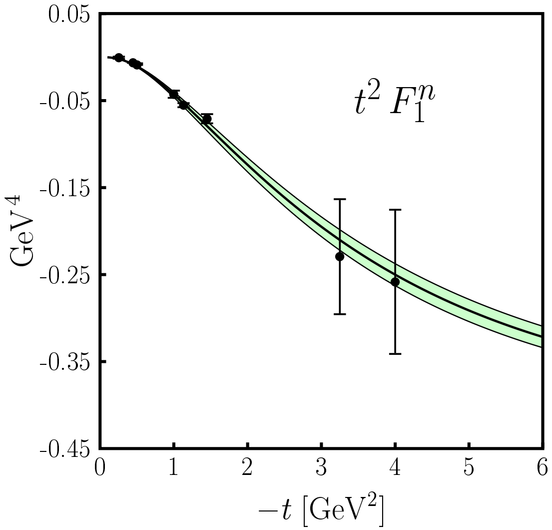
For $x \to 1 \quad d_q \to \infty$ for $n = 1$ while d_q remains finite for $n = 2$

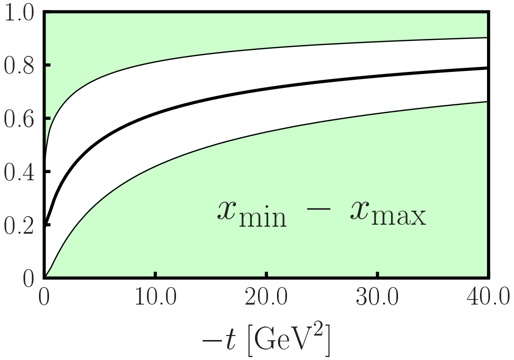
expected for system subject to confinement

Dirac form factors

$$F_1^{p(n)} = \int_0^1 dx\,[e_{u(d)} H_v^u + e_{d(u)} H_v^d]$$





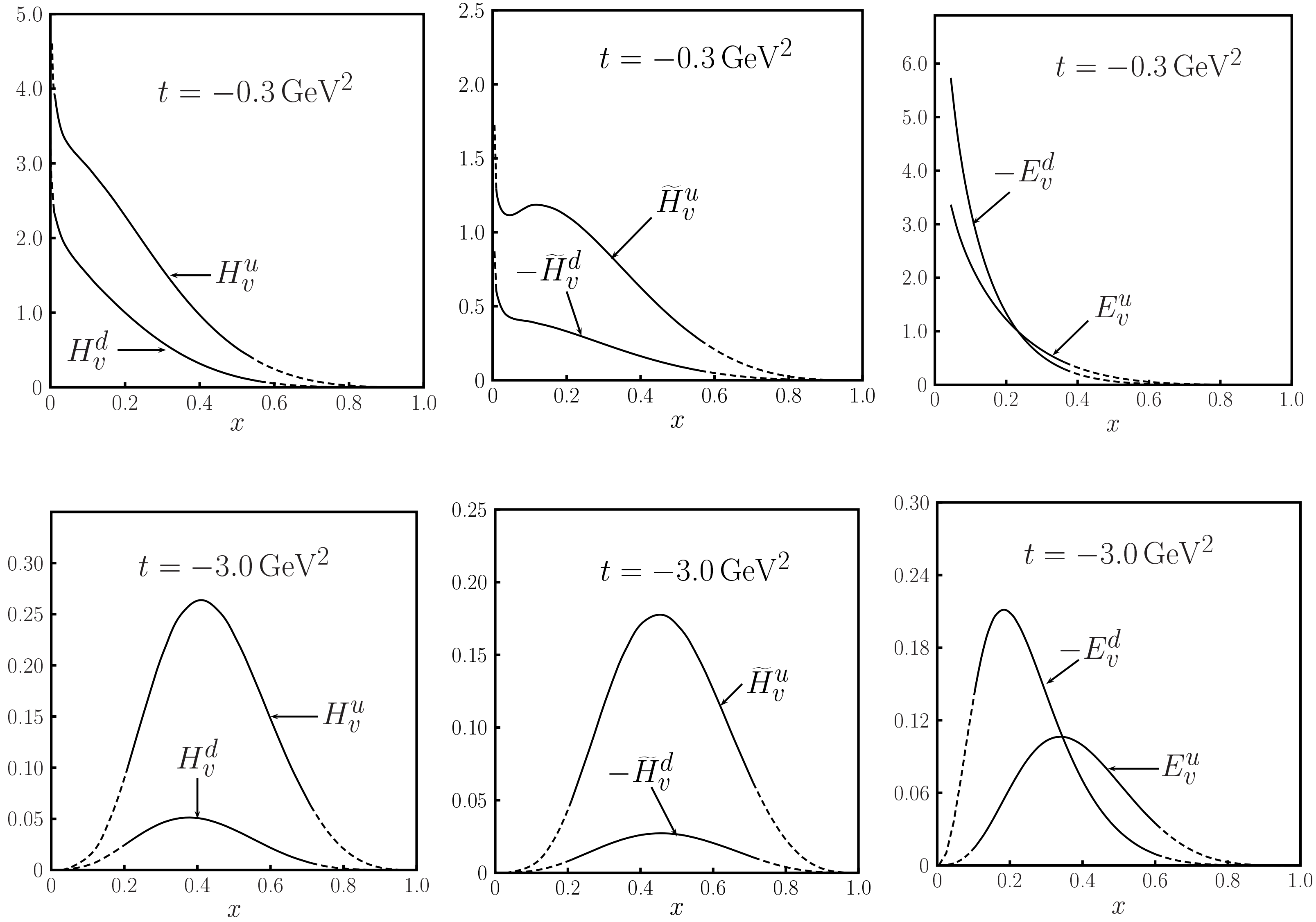


$$\int_{x_{max}(0)}^{1(x_{min})} dx \sum e_q H_v^q(x,t) = 5\% F_1^p(t)$$

parameters for $n=2$ fit ($\mu = 2\,\mathrm{GeV}$):

$$B_u = B_d = (0.59 \pm 0.03)\,\mathrm{GeV}^{-2}\,,\ A_u = (1.22 \pm 0.02)\,\mathrm{GeV}^{-2}\,,\ A_d = (2.59 \pm 0.29)\,\mathrm{GeV}^{-2}$$

Results on the GPDs (at $\mu^2 = 4\,\mathrm{GeV}^2$)

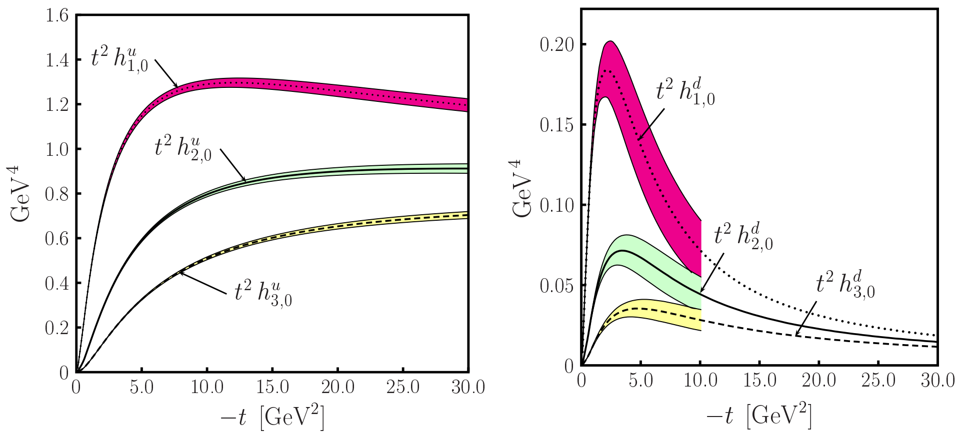


large $-t$: GPDs large in narrow region of large x

Physics potential of GPDs

- moments
- parton angular momenta from Ji's sum rule
- transverse localization of partons
- soft physics input for hard exclusive scattering
  (e.g. RCS, construction of $E(x, \xi, t)$ with double distr. ansatz)

Moments

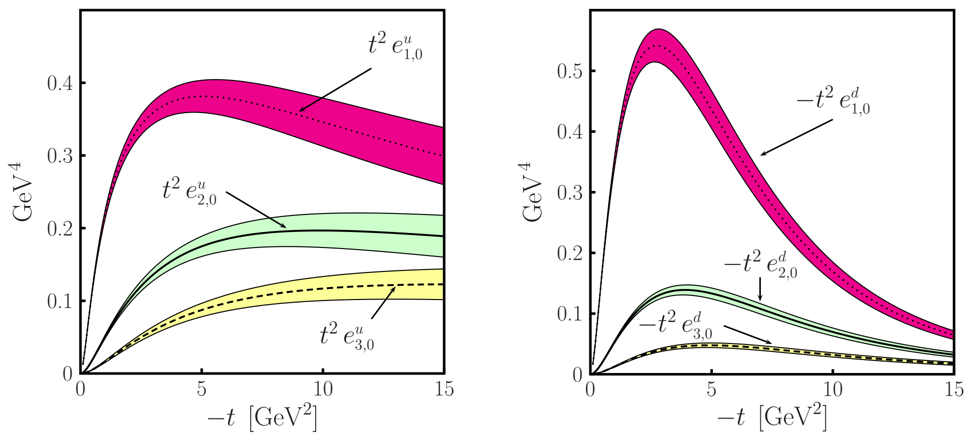


u-moments behave similar to F_1^p

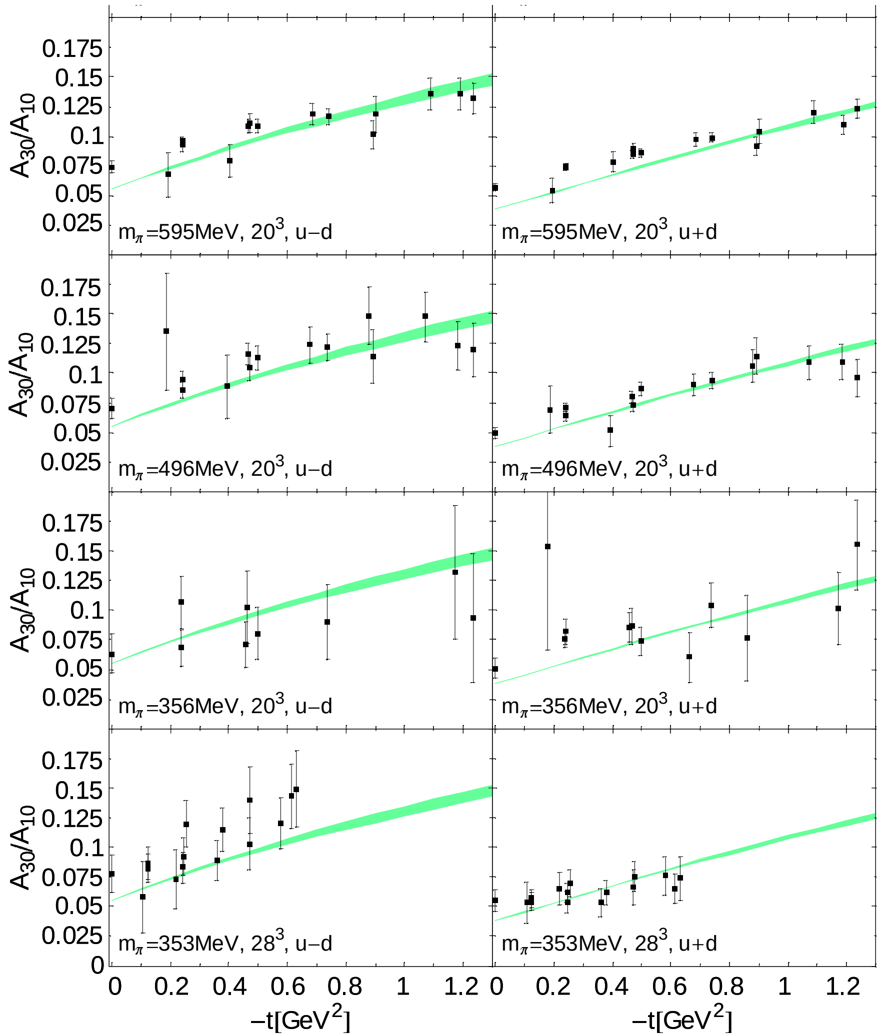
d-moments small, die out rapidly

almost negligible for $-t > 3\,\mathrm{GeV}^2$

explains why F_1^n negative



d moments of E also seem to drop faster with t then u-moments



Comparison with lattice

Haegler *et al* (07)

$$A_{30}(t) = \int_{-1}^{1} dx x^2 H^a(x, \xi = 0, t)$$

for $u - d$ and $u + d$

for different values of m_π

scaled by A_{10} (fails)

t dependence correctly predicted

$\widetilde{H}$ moments of similar quality

Ji's sum rule

$$\langle J_q \rangle \;=\; \int_{-1}^{1} dx\, x \left[H^q(x,\xi,t=0) + E^q(x,\xi,t=0)\right]$$

valence quark contribution to sum rule respective to orbital angular momentum for our parameterization

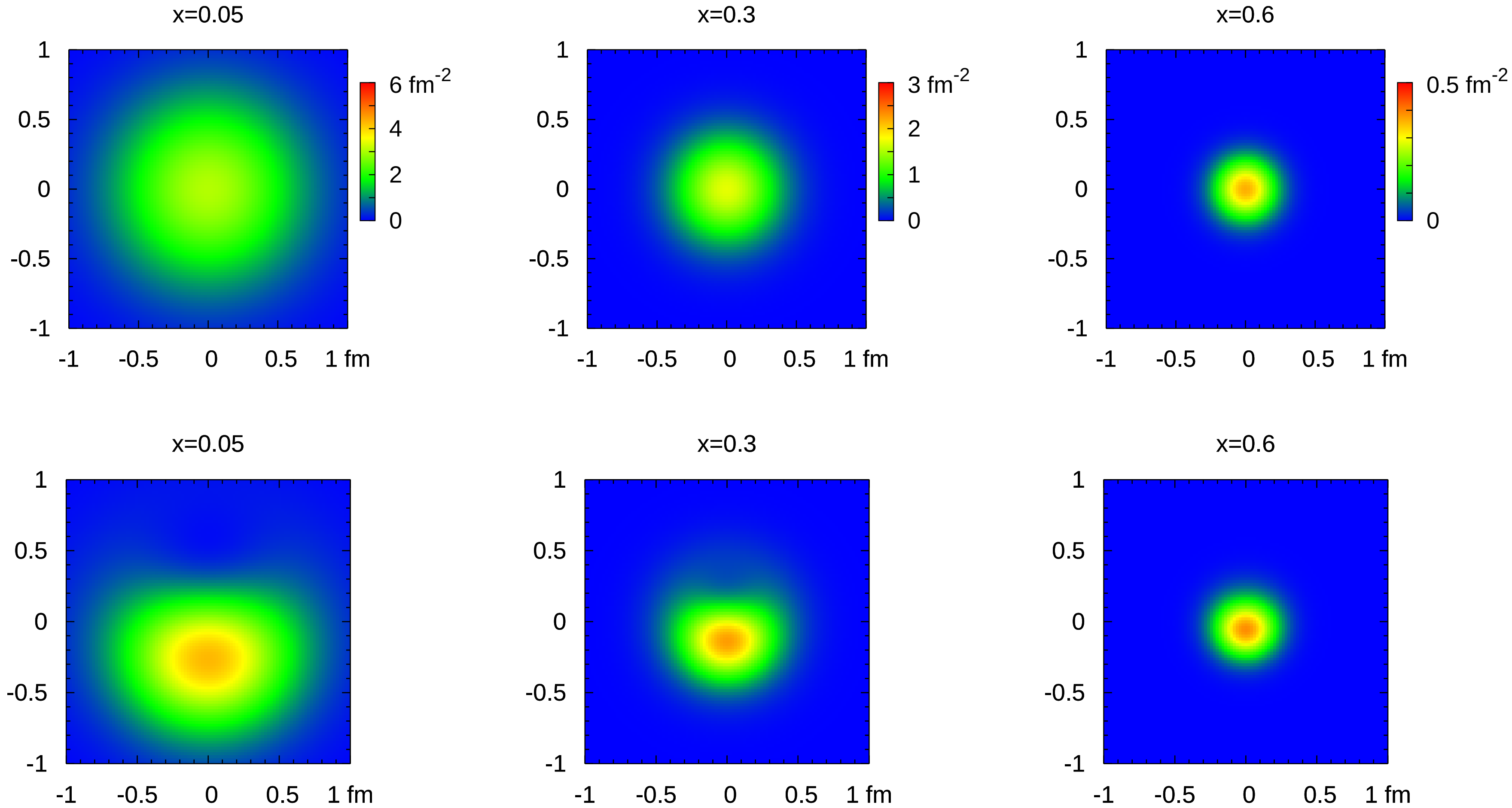
$$\langle L_v^q \rangle \;=\; \frac{1}{2}\int_0^1 dx \left[x e_v^q(x) + x q_v(x) - \Delta q_v(x)\right]$$

e_v^q from our analysis, q_v known from CTEQ6 PDFs
Δq_v known from axial-vector couplings of nucleon and hyperons

$$\langle L_v^{u+d} \rangle = -(0.04 - 0.12) \qquad \langle L_v^{u-d} \rangle = 0.39 - 0.46 \qquad (\text{at } \mu = 2\,\text{GeV})$$

$e_{2,0}^u(t=0)$ and $e_{2,0}^d(t=0)$ almost equal in magnitude but opposite in sign
contributions from E cancel to a large extent in sum
Lattice (Hägler et al (07)): $\langle L^{u+d} \rangle = 0.006 \pm 0.038\,,\ \langle L^{u-d} \rangle = -0.395 \pm 0.038$
at m_π(phys) (sea quark contributions seem to be small)

Tomography of d_v quarks



$$q_v^X(x,\mathbf{b}) = q_v(x,\mathbf{b}) - \frac{b^y}{m}\frac{\partial}{\partial \mathbf{b}^2} e_v^q(x,\mathbf{b})$$

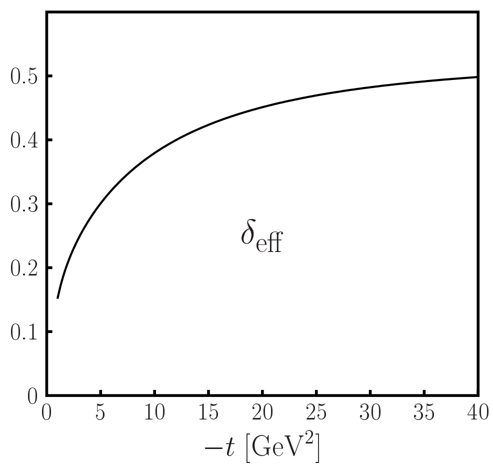
flavor separation in pol. proton

e_v contains non-zero orbital angular momentum

Feynman mechanism

k, k' momenta of active parton (before and after it is struck)
l momentum of spectator system; Λ typical hadronic scale
soft region: $1-x \sim \Lambda/\sqrt{|t|}$, $|k^2|, |k'^2| \sim \Lambda\sqrt{|t|}$ Feynman mechanism applies
ultrasoft: $1-x \sim \Lambda/|t|$, $|k^2|, |k'^2| \sim \Lambda^2$

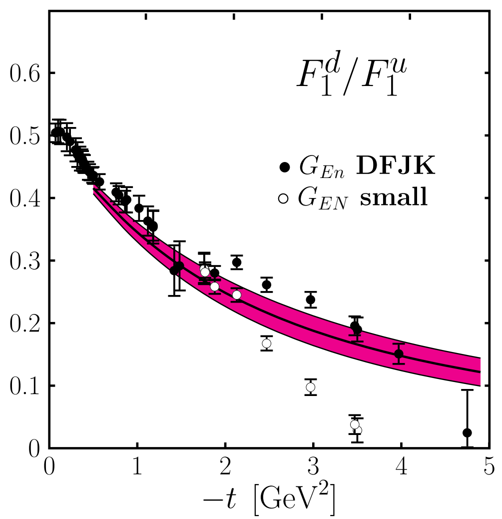
large t: dom. of narrow region of large x, approx.: $q_v \sim (1-x)^{\beta_q}$, $f_q \sim A_q(1-x)^n$
Saddle point method: $1 - x_s = \left(\frac{n}{\beta_q} A_q |t|\right)^{-1/n}$ $h^q_{1,0} \sim |t|^{-(1+\beta_q)/n}$ (similar to DY)
$n = 2$: x_s in soft and sensitive x region $\Rightarrow$ early onset of power beh. ($t \simeq 5\,\mathrm{GeV}^2$)
$n = 1$: x_s in ultrasoft region (suspect - confin.!)
to have x_s in sensitive region requires large t
power behaviour of $h^q_{1,0}$ does not set in before
$-t \simeq 30\,\mathrm{GeV}^2$



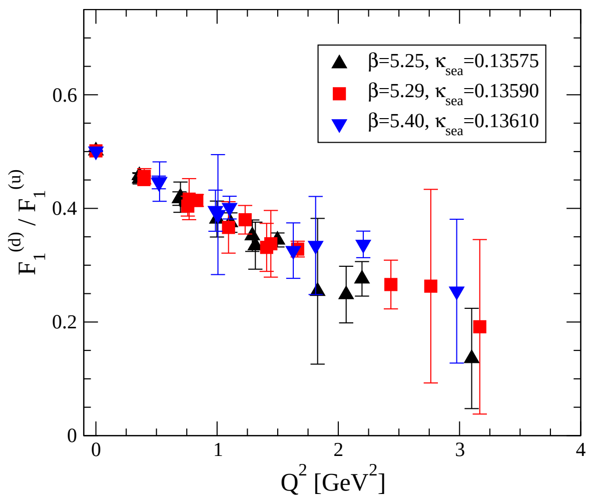
Testing the power laws:
eff. power $\delta_{\rm eff} = t\frac{d}{dt}\log[1 - \langle x\rangle_t]$

soft region: $\delta_{\rm eff} = 1/2$ ultrasoft: $\delta_{\rm eff} = 1$ $n = 1, 2$ practically same $\delta_{\rm eff}$ in fit
fit implies dominance of Feynman mechanism

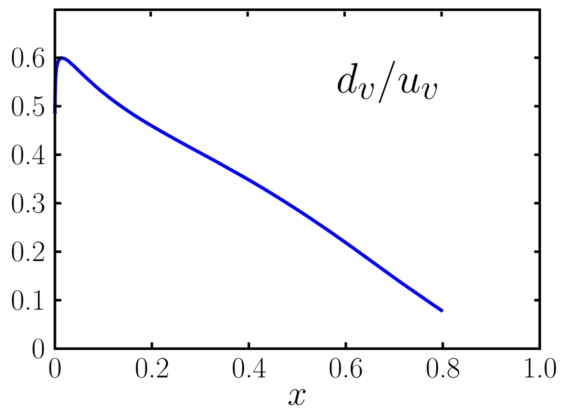
Ratio of d and u quark contributions to Dirac FF



form factor data



lattice (Göckeler et al (06))
$N_F = 2$, $m_\pi = 340\,\mathrm{MeV}$



CTEQ6M pdfs
at $Q^2 = 4\mathrm{GeV}^2$

behaviour of FF: $h_{1,0}^q \sim |t|^{-(1+\beta_q)/2}$

CTEQ6 PDFs: $\beta_u \simeq 3.4$, $\beta_d \simeq 5$ $x \Longleftrightarrow t$ correlation

dominance of u over d in FF at large t corresponds to that in PDFs at large x

except G_E^n is much larger than expected (wait for E02-013 data)

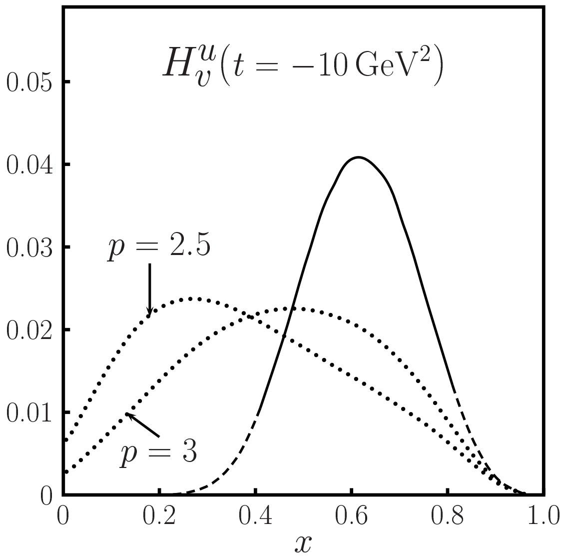
Power law behaviour

to cover a large range of possibilities: $H_v^q = q_v(x)\left[1 - \frac{t f_q(x)}{p}\right]^{-p}$

finite p: • not stable under evolution $\quad p \to \infty$: exponential behaviour as before

• connection to Regge limit lost

• reasonable fits to data obtained for $p \gtrsim 2.5$



broader shape of H

$H(x=0,t)$ remains finite

i.e. small x contribute to FF

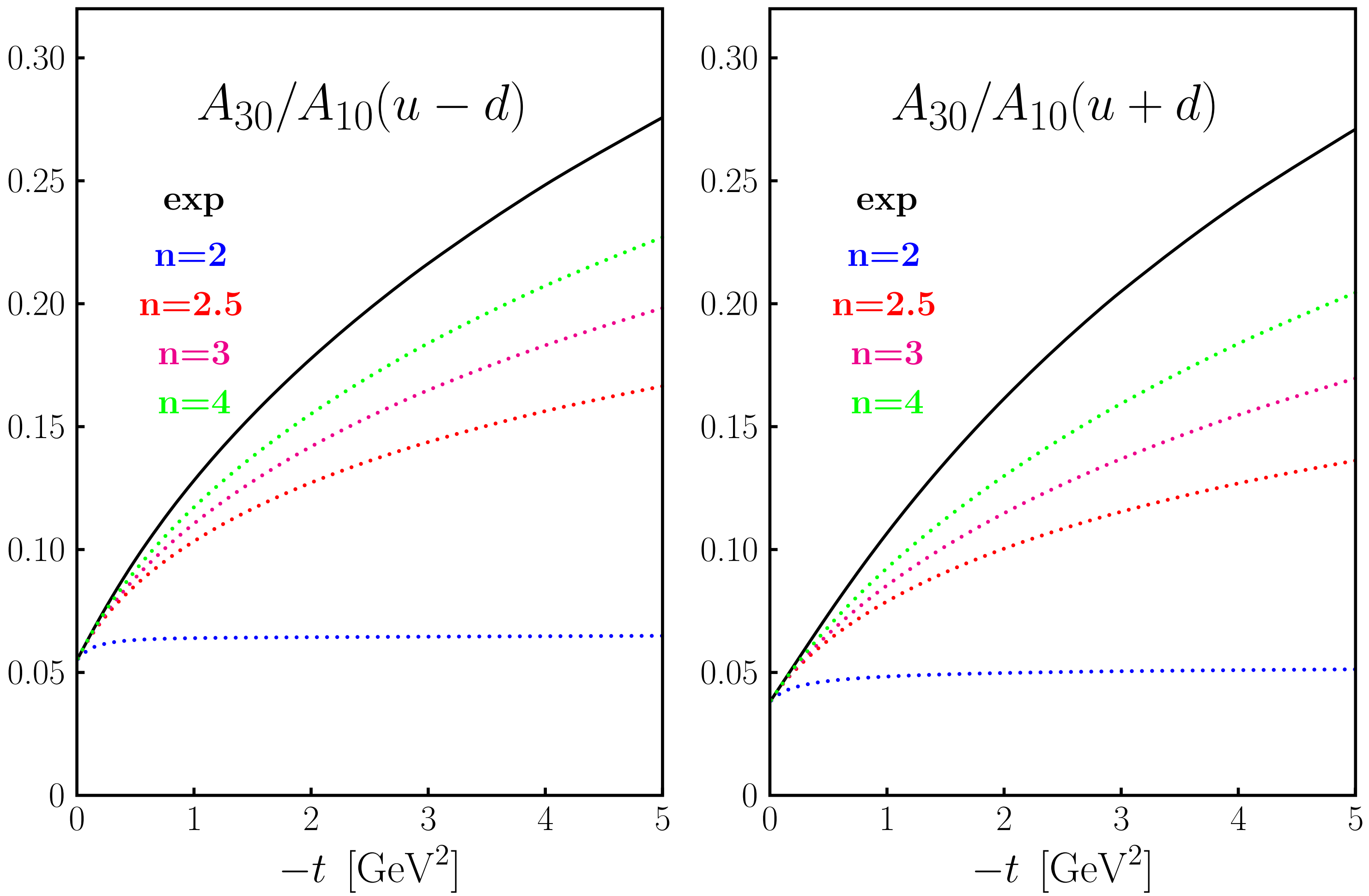
at large t

significant ambiguity: correlated $x - t$ dependence of GPDs not fixed by present data

theoretical input (bias) needed: combination of Regge behaviour at small x and t with dynamics of soft Feynman mechanism at large t is a physical attractive feature

lattice may discriminate between moments from exp. and low p powers

Comparison of moments from exp. and power parameterizations



lattice results shown for $-t < 1.2\,\mathrm{GeV}^2$

The handbag contribution to WACS

work in frame where $\xi = 0$:

if $s, -t, -u \gg \Lambda^2$ (i.e. for c.m.s. scattering angles near 90°)

($\Lambda \sim \mathcal{O}(1\mathrm{GeV})$ is a typical hadronic scale)

Compton amplitudes factorize into - subprocess amplitudes $\gamma q \to \gamma q$

and - $1/x$ moments of GPDs (Compton FFs)

$$R_V(t) \simeq e_u^2 \int_0^1 \frac{dx}{x} H_v^u(x, \xi = 0, t) + e_d^2 \int_0^1 \frac{dx}{x} H_v^d(x, \xi = 0, t)$$

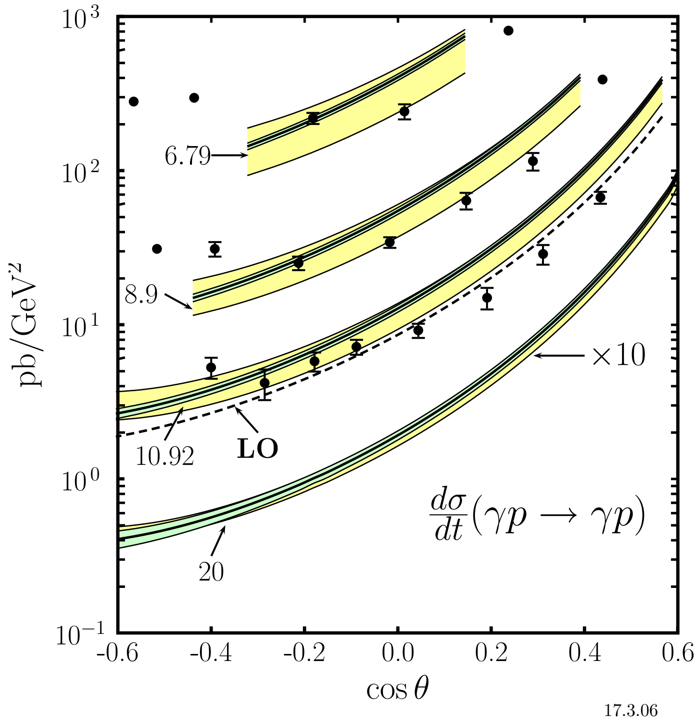
and analogously $\widetilde{H} \to R_A$, $E \to R_T$

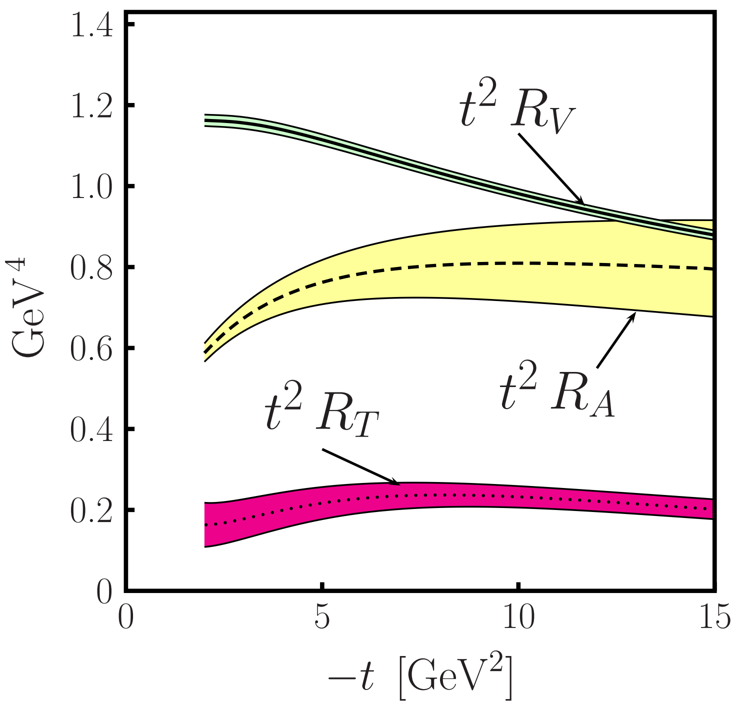
sea quarks neglected

Radyushkin (98), Diehl *et al* (99), Huang *el al* (01)

work out Compton FFs from GPDs and ....

The Compton cross section

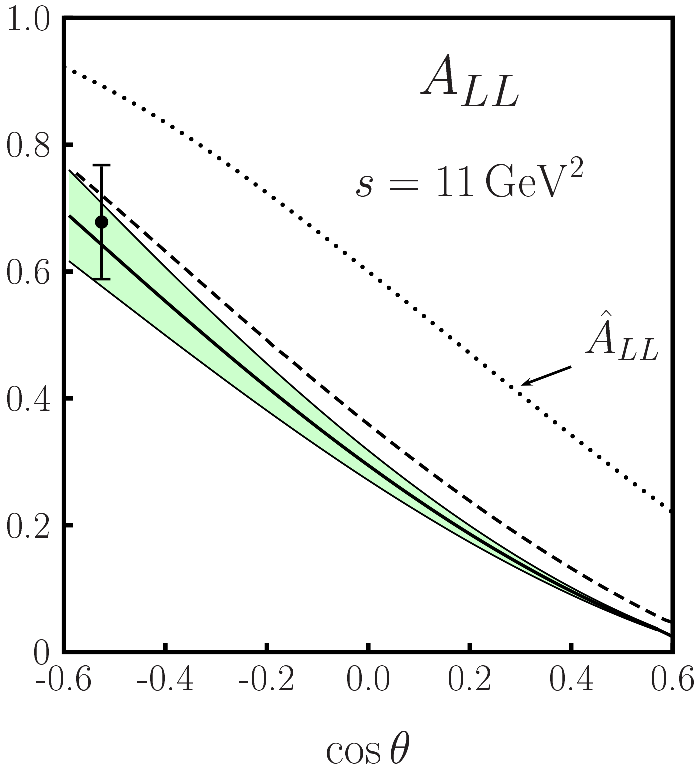


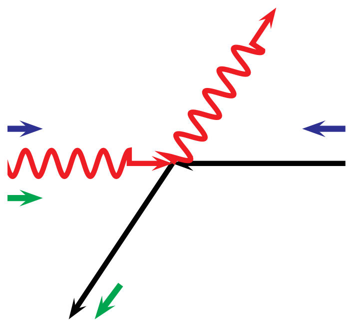


$$\frac{d\sigma}{dt} = \frac{d\hat{\sigma}}{dt}\Big\{\frac{1}{2}\,\frac{(s-u)^2}{s^2+u^2}\,[R_V^2(t) + \frac{-t}{4m^2}\,R_T^2(t)] + \frac{1}{2}\,\frac{(s+u)^2}{s^2+u^2}\,R_A^2(t)\Big\} + \mathcal{O}(\alpha_s)$$

$\frac{d\hat{\sigma}}{dt}(s,t)$ Klein-Nishina cross section data: JLab E99-114 (07)

Helicity correlation A_{LL}, K_{LL}





$$\hat{A}_{LL} = \frac{s^2-u^2}{s^2+u^2}$$

$$A_{LL} = K_{LL} \simeq \hat{A}_{LL}\,\frac{R_A}{R_V}$$

JLab E99-114 (05): at $E_\gamma = 3.23\mathrm{GeV}$: $\cos\theta = -0.5$, $t(u) = -4\,(-1.14)\,\mathrm{GeV}^2$

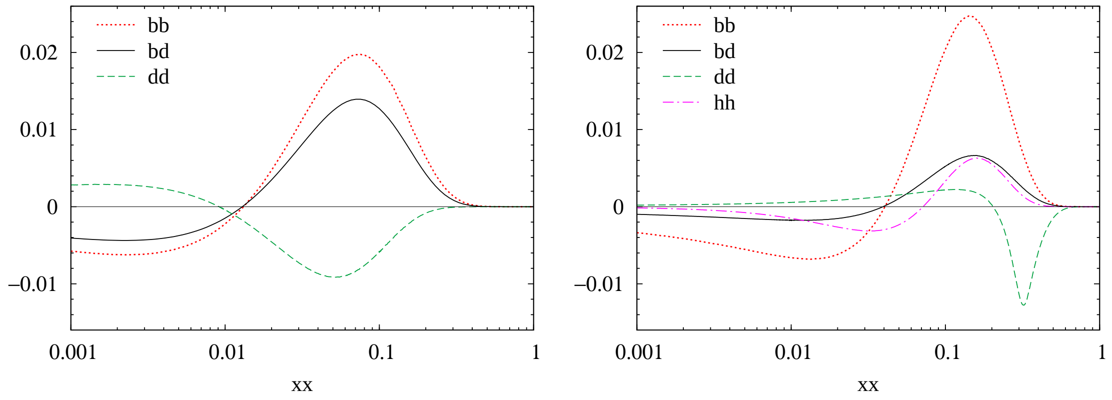
Miller (04): const. quark model with massive quarks $A_{LL} \neq K_{LL}$ for $\theta \gtrsim 90^\circ$
needed helicity corr. at higher energies and set of scattering angles

$s(x) \neq \overline{s}(x)$?

There are s and $\bar{s}$ in the nucleon (DIS) but $\Longrightarrow \int_0^1 dx[s(x) - \overline{s}(x)] = 0$

CTEQ(07) PDFs: weak indication for $s(x) \neq \overline{s}(x)$ MSTW (07) similar (prel.)

new exp information - NuTeV(01) dimuon production in $\nu, \bar{\nu}$ DIS



Diff. version of CTEQ6.5S and CTEQ6 assumed to have precisely one zero crossing

$$\text{Moments at } \mu = 2\,\text{GeV: } \langle x(s - \bar{s})\rangle = \begin{cases} 2.0 \times 10^{-3} & (\text{set bd}), \\ -0.94 \times 10^{-3} & (\text{set dd}), \\ 2.9 \times 10^{-3} & (\text{set bb}), \end{cases}$$

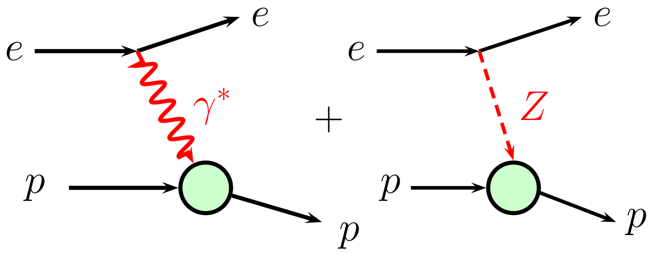
pert. evolution generates a small $s(x) - \bar{s}(x)$ asymmetry Catani et al (04)

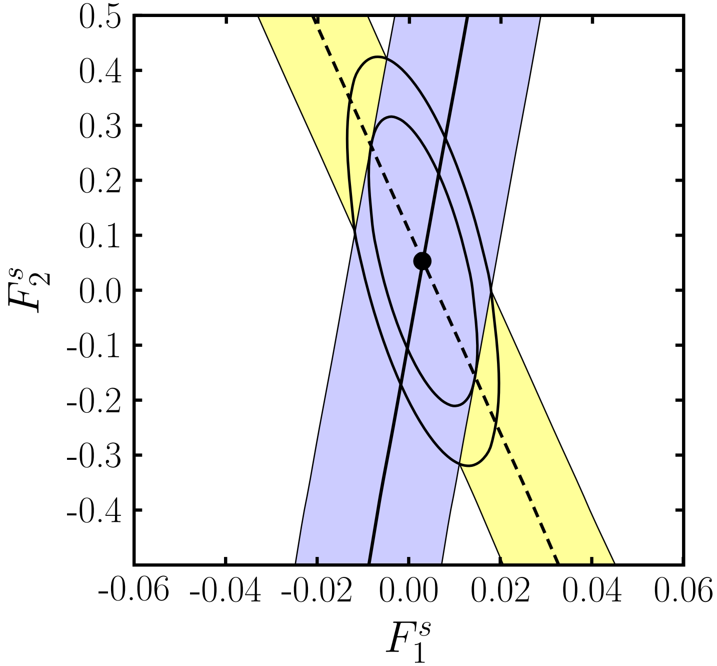
Meson cloud pictures, chiral quark models also asymmetries generated

but wide spread of results, differ in signs

Strangeness form factor

extracted from parity violation in elastic ep scatt. (g0, HAPPEX, A4, SAMPLE)
combination of $G^s_{E,M}$, conversion to lin. comb. F^s_1, F^s_2





HAPPEX(07) (ignore diff. in t values):
$F^s(t \simeq -0.1\,\mathrm{GeV}^2) = 0.003(12) \quad F^s_2 \simeq 0.05(26)$
Young et al (06) (fit to all data below $-0.3\,\mathrm{GeV}^2$)
$F^s_1(t) = -t \times 0.02(11)\,\mathrm{GeV}^{-2} \qquad F^s_2(t) = -0.01(25)$

Table 1: Data for the strange form factors at low $-t$. Statistical and systematic errors have been added in quadrature. We quote results for G_M^s or $G_E^s + \eta\, G_M^s$ and the equivalent ones for $F_1^s + \eta' F_2^s$.

experiment	$-t$	G_E^s, G_M^s	F_1^s, F_2^s
SAMPLE	0.100	$G_M^s = 0.37(34)$	$F_1^s + F_2^s = 0.37(34)$
A4	0.23	$G_E^s + 0.225\, G_M^s = 0.039(34)$	$F_1^s + 0.130\, F_2^s = 0.032(28)$
HAPPEX	0.477	$G_E^s + 0.392\, G_M^s = 0.014(22)$	$F_1^s + 0.184\, F_2^s = 0.010(16)$
A4	0.108	$G_E^s + 0.106\, G_M^s = 0.071(36)$	$F_1^s + 0.068\, F_2^s = 0.064(33)$
HAPPEX	0.091	$G_E^s = -0.038(43)$	$F_1^s - 0.026\, F_2^s = -0.038(43)$
HAPPEX	0.099	$G_E^s + 0.080\, G_M^s = 0.030(28)$	$F_1^s + 0.048\, F_2^s = 0.028(26)$
HAPPEX	0.077	$G_E^s = 0.002(16)$	$F_1^s - 0.022\, F_2^s = 0.002(16)$
HAPPEX	0.109	$G_E^s + 0.090\, G_M^s = 0.007(13)$	$F_1^s + 0.054\, F_2^s = 0.006(12)$

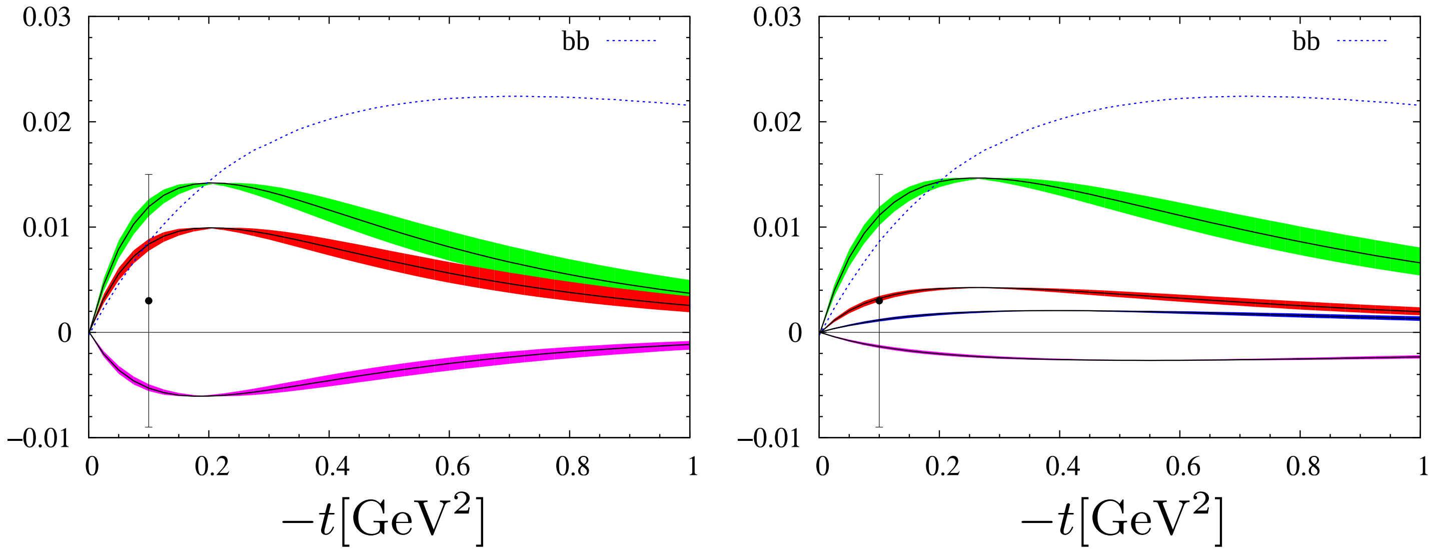
Relating $s - \bar{s}$ to strange Dirac form factor

ansatz (as for non-strange sector) $H^s(x,t) - H^{\bar{s}}(s,t) = \Big[s(x) - \bar{s}(x)\Big] e^{t f_s(x)}$

$f_s(x) = \alpha'(1-x)\ln 1/x$ (small x appr. of general ansatz)

$\langle b^2 \rangle_x = \frac{\int d^2b\, b^2 [s(x,b) - \bar{s}(x,b)]}{\int d^2b [s(x,b) - \bar{s}(x,b)]} = 4 f_s(x)$ vanishes at least like $(1-x)^2$ for $x \to 1$

as required for finite average transverse size of parton configuration (Burkardt (04))



$F_1^s(t)$ blue curve $-0.5 F_1^n(t)$

Neutron form factor

strange contr. to proton form factor negligible but neutron ff too?

$$F_1^n(t) = e_u F_1^d(t) + e_d F_1^u(t) + e_s F_1^s(t)$$

$e_s F_s^s$ at most 1/6 of neutron form factor, may have some effects at small $-t$

f_s being a decreasing fct. $\Longrightarrow e^{tf_s}$ increasingly suppresses small x

at large $-t$: $F_1^s \sim (-t)^{(1+\beta_s)/2}$ for $-t \to \infty$
where $s - \bar{s} \sim (1-x)^{\beta_s}$ for $x \to 1$
$s, \bar{s}$ falls off faster than u_v, d_v with x, likely $s - \bar{s}$ too
$\longrightarrow |F_1^s(t)|$ decrease faster than $|F_1^n(t)| (\sim 1/t^2)$ whith $-t$
neglect of strange contr. in form factor analysis reasonable

Summary

- A first attempt to extract the GPDs from form factor data in analogy to the analyses of the PDFs
- On the basis of physically motivated parameterizations information on $H, \tilde{H}, E$ for valence quarks and at $\xi = 0$ has been obtained
- Strangeness form factor analyzed analogously
- Parameterization is not unique but results are theoretically consistent and imply the physics of the Feynman mechanism at large t
- Some surprising results have been found:

  e.g. elm. FF are dominated by u-quarks at larger t

  e.g. orbital angular momentum carried by valence quarks small but non-zero
- Polarized and unpolarized WACS can be predicted now and found to be in fair agreement with experiment