Nucleon Form Factors in Relativistic Constituent Quark Models

Thomas Melde

Project P19035, Structure of Baryon Resonances

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Collaborators

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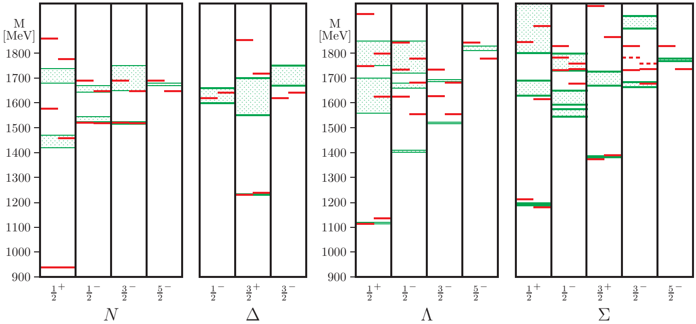
- *INFN, Sezione di Padova*
  L. Canton

Additional Funding for Collaboration

FWF *DK-W1203 Doktoratskolleg*
"Hadronen im Vakuum, in Kernen und in Sternen"

Outline

CQM Spectra Comparison



Left Red: One-Gluon Exchange, Right Red: Goldstone-Boson Exchange,

Green: Experiment (PDG)

Why Relativistic Quantum Mechanics?

- Requirements of special relativity are satisfied
- Finite number of degrees of freedom
- Description of composite particles
- Large class of admissible interactions
- Few-body calculations are tractable

Challenges

- Connection to local quantum field theory?
- No microscopic locality
  (can be replaced by macroscopic locality)
- Definition of suitable spectator-models

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Procedure to formulate RQM

Poincaré Invariance

- Symmetries of special relativity $\longrightarrow$ Poincaré group
  - Translations with four-momentum P^μ
  - Lorentz transformations with rotations **J** and boosts **K**
- Generators adhere to a set of commutation relations
- Properties must be guarantued by systems in question

Free system

- Representation for one-body system straightforward
  - Define mass operator
  - Define spin operator
- Multi-particle representation given by combining single-particle representations
- Suitable coupling gives total momentum, mass and spin

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Poincaré Invariance

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. . . (constraints on interactions) are not easily fulfilled and provide the real difficulty in the problem of constructing a theory of a relativistic dynamical system. . .

Dirac, Forms of relativistic dynamics, Rev. Mod. Phys. **21** *(1949), 392*

Interacting Systems

What is the problem?

- Generally, interaction in *all ten generators*
- (Interacting) generators $\longrightarrow$ commutation relations
- Commutators $\longrightarrow$ non-linear constraints for interaction
- Example: $\left[P^j, K^k\right] = H\delta^{jk}$

Bakamjian-Thomas construction

- Generators for free system
- Define set of auxiliary operators (includes M)
- Add suitable interaction
- Reconstruct generators for interacting system

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Relativistic Transition Amplitude (Point Form)

$$\left\langle V', M', J', \Sigma' \right| \hat{O} \left| V, M, J, \Sigma \right\rangle = \frac{2}{MM'} \sum_{\sigma_i \sigma_i'} \sum_{\mu_i \mu_i'} \int d^3\vec{k}_2 d^3\vec{k}_3 d^3\vec{k}_2' d^3\vec{k}_3'$$

$$\sqrt{\frac{(\omega_1+\omega_2+\omega_3)^3}{2\omega_1 2\omega_2 2\omega_3}} \sqrt{\frac{(\omega_1'+\omega_2'+\omega_3')^3}{2\omega_1' 2\omega_2' 2\omega_3'}}$$

$$\Psi^\star_{M'J'\Sigma'}\left(\vec{k}_i'; \mu_i'\right) \prod_{\sigma_i'} D^{\star\frac{1}{2}}_{\sigma_i'\mu_i'} \left\{R_W\left[k_i'; B\left(V'\right)\right]\right\}$$

$$\left\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' \right| \hat{O}_{\mathrm{rd}} \left| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle$$

$$\prod_{\sigma_i} D^{\frac{1}{2}}_{\sigma_i\mu_i} \left\{R_W\left[k_i; B\left(V\right)\right]\right\} \Psi_{MJ\Sigma}\left(\vec{k}_i; \mu_i\right)$$

$$2MV_0\delta^3\left(M\vec{V} - M'\vec{V}' - \vec{Q}\right)$$

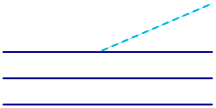
Point-Form Spectator Model (PFSM)

in Relativistic Quantum Mechanics (RQM)

RQM: Hamiltonian $\rightarrow$ invariant mass operator (in rest-frame)

Point-Form: kinematic subgroup is Lorentz-group

Spectator Model:



- Meson couples to quark 1, quarks 2 and 3 are spectators

momentum transfer: $p_1^\mu - p_1'^\mu = \tilde{q}^\mu \neq q^\mu = P^\mu - P'^\mu$

$\rightarrow$ PFSM is effective many-body operator!

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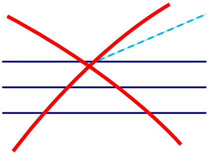

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momentum transfer: $p_1^\mu - p_1'^\mu = \tilde{q}^\mu \neq q^\mu = P^\mu - P'^\mu$

$\longrightarrow$ PFSM is effective many-body operator!

Strategy to define PFSM

- Formal definition of vertex and spectator-constraints
- Adjust definition to satisfy global constraints and sensible limit-behaviours for transition amplitudes

T. Melde et al., Phys. Rev. D 76, 074020 (2007)

$\longrightarrow$ Poincaré-invariance of Form Factors

Spectator Electromagnetic Current

$$\left\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' \right| \hat{J}^{\mu}_{\mathrm{rd}} \left| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle$$

$$= 3\mathcal{N} \left\langle p_1', \sigma_1' \right| \hat{J}^{\mu}_{\mathrm{spec}} \left| p_1, \sigma_1 \right\rangle$$

$$2p_{20}\delta\left(\vec{p}_2 - \vec{p}_2'\right) 2p_{30}\delta\left(\vec{p}_3 - \vec{p}_3'\right) \delta_{\sigma_2\sigma_2'}\delta_{\sigma_3\sigma_3'}$$

Formal Single Particle Current

$$\left\langle p_1', \sigma_1' \right| \hat{J}^{\mu}_{\mathrm{spec}} \left| p_1, \sigma_1 \right\rangle$$

$$= e_1 \bar{u}\left(p_1', \sigma_1'\right) \left[f_1(\tilde{Q}^2)\gamma^{\mu} + \frac{i}{2m_1} f_2(\tilde{Q}^2)\sigma^{\mu\nu}\tilde{q}_{\nu} \right] u\left(p_1, \sigma_1\right)$$

structureless quarks: $f_1(\tilde{Q}^2) = 1$ and $f_2(\tilde{Q}^2) = 0$

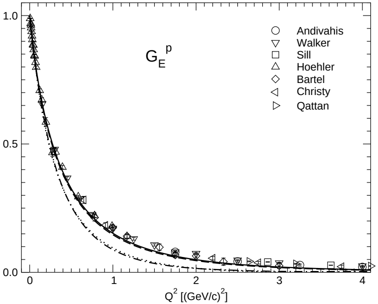
Nucleon Sachs Form factors

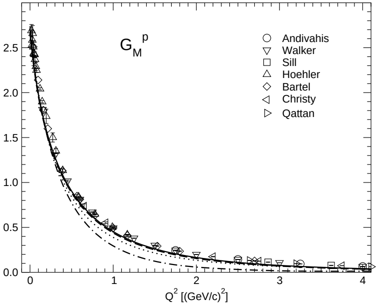
Definition in Breit Frame

$$F^{\nu}_{\Sigma',\Sigma}\left(Q^2\right) = \left\langle V', m_{\mathrm{N}}, \frac{1}{2}, \Sigma' \middle| \hat{J}^{\nu}_{\mathrm{rd}} \middle| V, m_{\mathrm{N}}, \frac{1}{2}, \Sigma \right\rangle$$

$$\begin{aligned} F^{0}_{\Sigma',\Sigma}\left(Q^2\right) &= 2MG_{\mathrm{E}}\left(Q^2\right)\delta_{\Sigma',\Sigma} \\ \vec{F}_{\Sigma',\Sigma}\left(Q^2\right) &= iQG_{\mathrm{M}}\left(Q^2\right)\chi^{\dagger}_{\Sigma'}\left(\vec{\sigma}\times\vec{e}_z\right)\chi_{\Sigma} \end{aligned}$$

Form Factors of the Proton (CQM-Comparison)

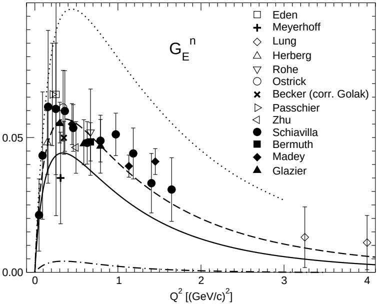




Full: GBE Dashed: BCN Dashed-Dotted: Conf CQM

Dotted: II CQM (Bonn Group)

Form Factors of the Neutron (CQM-Comparison)



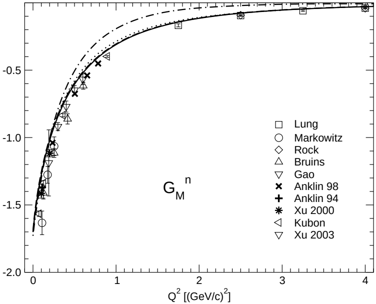


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Table: Magnetic moments of proton and neutron (in n.m.).

	PFSM			BS	Experiment
Nucleon	GBE	BCN	CONF	II	
p	2.70	2.74	2.65	2.74	2.79
n	-1.70	-1.70	-1.73	-1.70	-1.91

Table: Charge radii of proton and neutron (in fm^2) .

	PFSM			BS	Experiment
Nucleon	GBE	BCN	CONF	II	
p	0.824	1.029	0.766	0.67	0.766
n	-0.135	-0.263	-0.009	-0.11	-0.116

What about the $\mathcal{N}$?

$$\left\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' \right| \hat{J}^{\mu}_{\mathrm{rd}} \left| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle$$
$$= 3\mathcal{N} \left\langle p_1', \sigma_1' \right| \hat{J}^{\mu}_{\mathrm{spec}} \left| p_1, \sigma_1 \right\rangle$$
$$2p_{20}\delta\left(\vec{p}_2 - \vec{p}_2'\right) 2p_{30}\delta\left(\vec{p}_3 - \vec{p}_3'\right) \delta_{\sigma_2\sigma_2'}\delta_{\sigma_3\sigma_3'}$$

Motivation for General Structure

T. Melde, et al. Eur. Phys. J. A 25, 97 (2005)
T. Melde, et al. Phys. Rev. D 76, 074020 (2007)

$Q^2 = 0 \longrightarrow$ genuine one-body operator

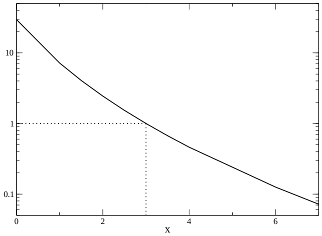
No Interaction $\longrightarrow$ genuine one-body operator

Sensible nonrelativistic limit

Parameterization of $\mathcal{N}$

$$\mathcal{N}(x,y) = \left(\frac{M}{\sum_i \omega_i}\right)^{xy} \left(\frac{M'}{\sum_i \omega'_i}\right)^{x(1-y)}$$

Charge normalization, $G_E^p(0)$



$\longrightarrow x = 3$

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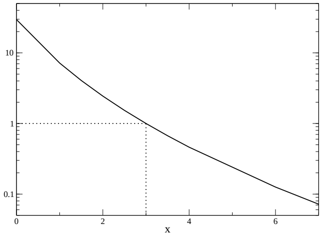
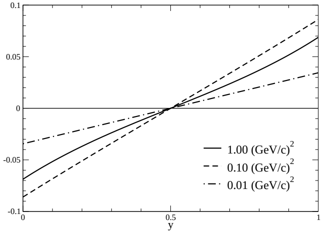


$$\longrightarrow x = 3$$

Parameterization of $\mathcal{N}$

$$\mathcal{N}(3, y) = \left(\frac{M}{\sum_i \omega_i}\right)^{3y} \left(\frac{M'}{\sum_i \omega_i'}\right)^{3(1-y)}$$

Time reversal invariance $\left\langle \hat{J}^{\mu=3} \right\rangle$ (in Breit frame)



$$\mathcal{N}(z) = \frac{1}{2}\left[\left(\frac{M}{\sum_i \omega_i}\right)^{3z} \left(\frac{M'}{\sum_i \omega_i'}\right)^{3(1-z)} + \left(\frac{M'}{\sum_i \omega_i'}\right)^{3z} \left(\frac{M}{\sum_i \omega_i}\right)^{3(1-z)}\right]$$

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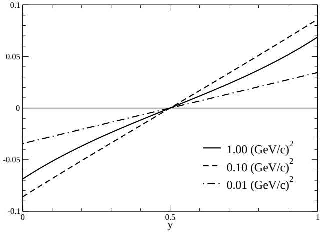

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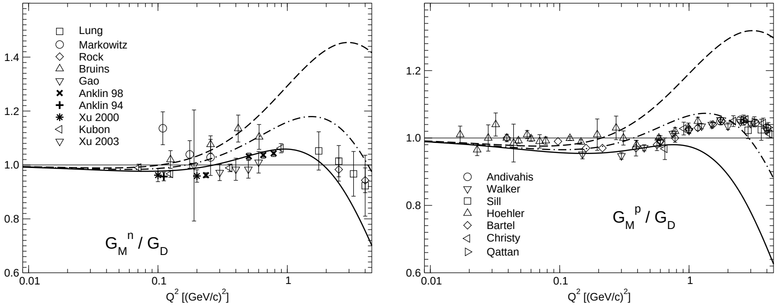
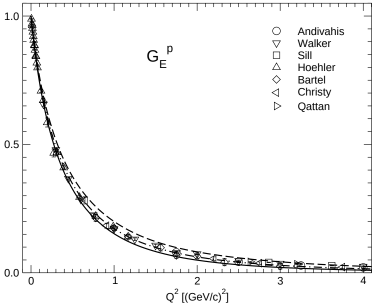
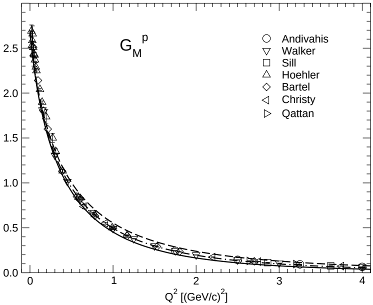
Magnetic Form Factor to Dipole Ratios



Top: $\mathcal{N}_{ari}$ $(z = 0)$ Middle: $\mathcal{N}_{fix}$ $(z = \frac{1}{6})$ Bottom: $\mathcal{N}_{ari}$ $(z = 0.5)$

Form Factors of the Proton





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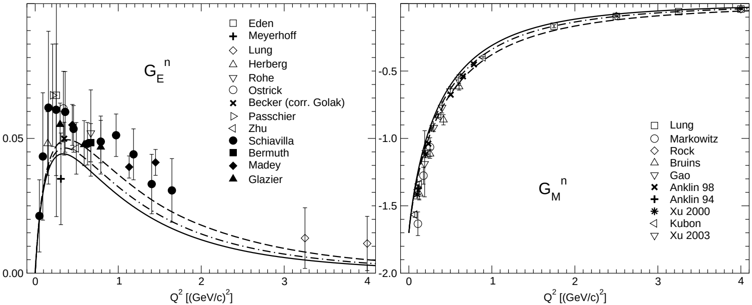
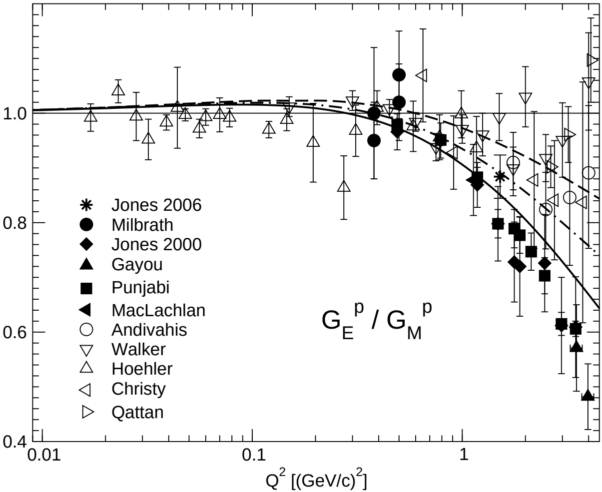
Form Factors of the Neutron



Electric/Magnetic Form Factor Ratio of the Proton



Relativistic Transition Amplitude (Instant Form)

$$
\begin{aligned}
F^{\mu}_{\Sigma',\Sigma}\left(Q^2\right) &= 2\sqrt{EE'}\sum_{\sigma_i\sigma_i'}\sum_{\mu_i\mu_i'}\int d^3\vec{k}_2 d^3\vec{k}_3 d^3\vec{k}_2' d^3\vec{k}_3' \frac{1}{\sqrt{E_{\text{free}}E'_{\text{free}}}} \\
&\quad \sqrt{\frac{\sum\omega_i}{2\omega_1 2\omega_2 2\omega_3}}\sqrt{\frac{\sum\omega_i'}{2\omega_1' 2\omega_2' 2\omega_3'}} \\
&\quad \Psi^{\star}_{M\frac{1}{2}\Sigma'}\left(\vec{k}_i';\mu_i'\right)\prod_{\sigma_i'} D^{\star\frac{1}{2}}_{\sigma_i'\mu_i'}\left\{R_W\left[k_i';B\left(v'\right)\right]\right\} \\
&\quad \left\langle p_1',p_2',p_3';\sigma_1',\sigma_2',\sigma_3'\right|\hat{J}^{\mu}_{\text{rd}}\left|p_1,p_2,p_3;\sigma_1,\sigma_2,\sigma_3\right\rangle \\
&\quad \prod_{\sigma_i} D^{\frac{1}{2}}_{\sigma_i\mu_i}\left\{R_W\left[k_i;B\left(v\right)\right]\right\}\Psi_{M\frac{1}{2}\Sigma}\left(\vec{k}_i;\mu_i\right)
\end{aligned}
$$

Spectator Electromagnetic Current

$$\left\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' \right| \hat{J}^{\mu}_{\mathrm{rd,IFSM}} \left| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle = 3e_1 \bar{u}\left(p_1', \sigma_1'\right) \gamma^{\mu} u\left(p_1, \sigma_1\right) 2p_{20}\delta^3\left(\vec{p}_2 - \vec{p}_2'\right) 2p_{30}\delta^3\left(\vec{p}_3 - \vec{p}_3'\right) \delta_{\sigma_2\sigma_2'}\delta_{\sigma_3\sigma_3'}$$

Formal Single Particle Current

$$\left\langle p_1', \sigma_1' \right| \hat{J}^{\mu}_{\mathrm{spec}} \left| p_1, \sigma_1 \right\rangle = e_1 \bar{u}\left(p_1', \sigma_1'\right) \left[f_1(\tilde{Q}^2)\gamma^{\mu} + \frac{i}{2m_1} f_2(\tilde{Q}^2)\sigma^{\mu\nu}\tilde{q}_{\nu} \right] u\left(p_1, \sigma_1\right)$$

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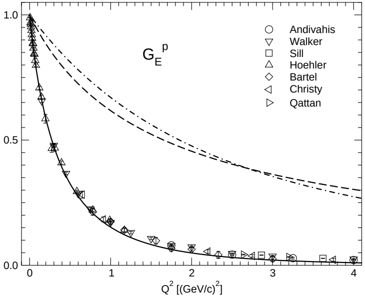
Form Factors of the Proton



Full: PFSM Dashed: IFSM Dashed-Dotted: EEM

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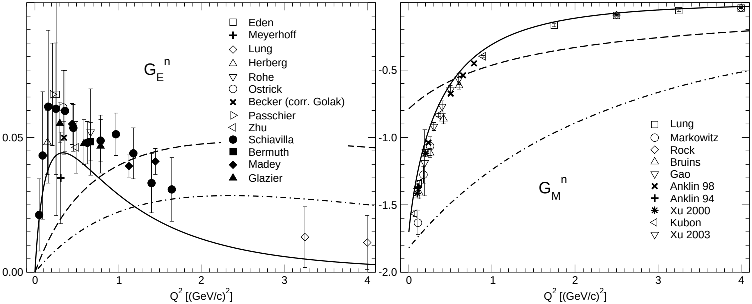


Full: PFSM   Dashed: IFSM   Dashed-Dotted: EEM

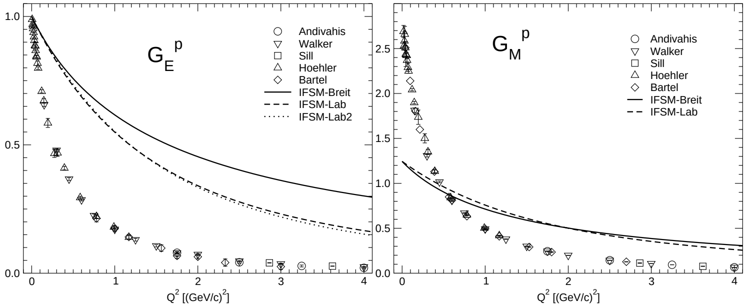
Table: Magnetic moments of proton and neutron (in n.m.).

	GBE CQM			Experiment
Nucleon	IFSM	PFSM	NRIA	
p	1.24	2.70	2.74	2.79
n	-0.79	-1.70	-1.82	-1.91

Table: Charge radii of proton and neutron (in fm^2) .

	GBE CQM			Experiment
Nucleon	IFSM	PFSM	NRIA	
p	0.156	0.824	0.102	0.766
n	-0.020	-0.135	-0.009	-0.116

Form Factors of the Proton (IFSM-Comparison)



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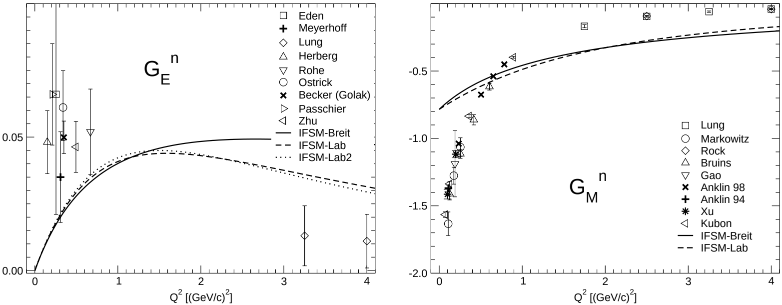


Summary

- PFSM *generally* reproduces data w/o extra parameters
- Model uncertainty is reasonably small, at least for small Q^2
- IFSM and EEM are far off the data
- CQM-dependance small (with hyperfine interaction)

Additional Results

- PFSM results for magnetic moments and charge radii for Hyperon ground states of equal quality.
- Axial and induced pseudoscalar form factor reasonable
- Partial Decay Widths are *consistently* too small

Caveat$\longrightarrow$

Caveat

- Standard CQMs yield **bare** results!
- Resonances are described as *eigenstates*

Implications

- Decay results can not reproduce experimental data

$\longrightarrow$ Hadronic Dressing needs to be accounted for

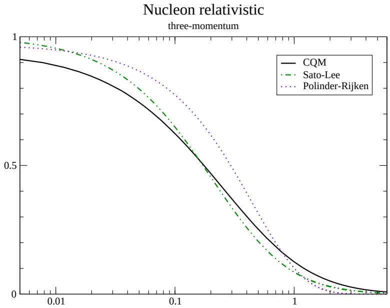
$\longrightarrow$ Even the Baryon Masses are expected to shift

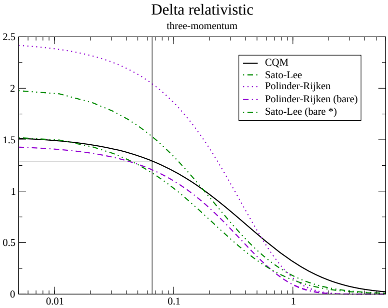
Possible Way Out?

- Compare CQM results with **bare inputs** in phenomenological models (e.g. Sato-Lee, Polinder-Rijken)

$\longrightarrow$ Hadronic Dressing with CQM inputs along similar lines

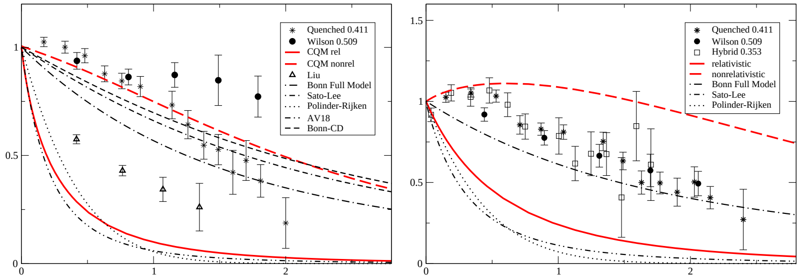
Strong Form Factor of the Nucleon and Delta





More Details in Talk by L. Canton

Strong Form Factor of the Nucleon and Delta



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