Faddeev calculation of $K^{-}d$ scattering length

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Outline

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- $\bar{K}$-nucleus interaction
- Importance of the K^-d scattering length

Formalism

- Used approximations
- Three-body Faddeev equations

Input

- NN potentials
- $\bar{K}N$ potentials

Results and discussion

- Dependence of a_{K^-d} on REZ $\bar{K}N$ potential
- Dependence of a_{K^-d} on two-body inputs
- Comparison with other calculations
- Predicted values of the kaonic deuterium ϵ and Γ

Summary

$\bar{K}N$ interaction

- strongly attractive as well as strongly absorptive near threshold
- subthreshold $I = 0$ resonance $\Lambda(1405)$ assumed a $\bar{K}N$ quasibound state and a resonance in the $\pi\Sigma$ channel

Experimental data

- cross sections of low-energy K^-p scattering and reactions
- position and width of the $\Lambda(1405)$ resonance
- threshold branching ratios
- K^-p scattering length from the kaonic hydrogen $1s$ level shift ϵ and width Γ by applying the Deser-Trueman formula

$$\epsilon + \mathrm{i}\frac{\Gamma}{2} = 2\alpha^3\mu^2 a_{K^-p}$$

$\bar{K}$-nucleus interaction

- also strongly attractive, as deduced from analysis of kaonic atoms
- depth of 150 - 200 MeV (density dependent optical potentials) vs. 50 - 60 MeV (chiral models of $\bar{K}N$ interaction + coupled channels)
- depth of the $\bar{K}$-nucleus potential related to the existence and possible width of $\bar{K}$-nuclear states

At present, the issue of $\bar{K}$-nuclear quasibound states is far from being resolved and more experimental as well as theoretical explorations, including the study of the $\bar{K}N$ interaction, are necessary

Importance of the K^-d scattering length

- precise values of a_{K^-d} and a_{K^-p} are essential for extracting the K^-n scattering length, for our better understanding of low-energy $\bar{K}N$ interaction and for extrapolating to $\bar{K}$-nuclear systems
- SIDDHARTA (Frascati) is going to measure ϵ and Γ of the kaonic hydrogen (precision of several eV)
- SIDDHARTA will also perform measurements of the kaonic deuterium ($\rightarrow$ K^-d scattering length)

Used approximations

Approximations in three-body calculations

- single-channel calculations
- s-wave approximation, no relativistic corrections (low-energy scattering)
- no Coulomb interaction
- calculations performed in momentum and isospin basis ($m_p = m_n$, $m_{K^-} = m_{\bar{K}_0}$)

Three-body Faddeev equations (1)

Three-body Faddeev equations in the AGS form

$$U_{\beta\alpha} = (1 - \delta_{\beta\alpha}) G_0^{-1} + \sum_{\gamma \neq \beta} T_\gamma G_0 U_{\gamma\alpha}$$

Three-body transition operators $U_{\beta\alpha}$ describe elastic and rearrangement processes

$$U_{11} : 1 + (23) \to 1 + (23)$$
$$U_{21} : 1 + (23) \to 2 + (31)$$
$$U_{31} : 1 + (23) \to 3 + (12)$$

From time-independent Schrödinger equation for separable potential $V = |g\rangle\lambda\langle g|$

$$|\psi_\alpha^B\rangle = \lambda_\alpha N_\alpha^B G_0^{(2)}(z^{(2)})|g_\alpha\rangle$$

The notation used in calculations

$$\begin{aligned} X_{\beta\alpha}(z) &\equiv \langle g_\beta | G_0(z) U_{\beta\alpha}(z) G_0(z) | g_\alpha \rangle \\ Z_{\beta\alpha}(z) &\equiv \langle g_\beta | G_0(z) | g_\alpha \rangle \end{aligned}$$

Three-body Faddeev equations (2)

After partial wave decomp. one-dimensional integral equations

$$X^{I_\beta,I_\alpha}_{\beta\alpha}(k_\beta,k'_\alpha;z) = (1-\delta_{\beta\alpha})Z^{I_\beta,I_\alpha}_{\beta\alpha}(k_\beta,k'_\alpha;z) +$$

$$4\pi\sum_{\gamma\neq\beta}\sum_{I_\gamma}\int_0^\infty \mathrm{d}\tilde{k}_\gamma\tilde{k}^2_\gamma Z^{I_\beta,I_\gamma}_{\beta\gamma}(k_\beta,\tilde{k}_\gamma;z)\tau^{I_\gamma}_\gamma\left(z-\frac{\tilde{k}^2_\gamma}{2\mu^{red}_\gamma}\right)X^{I_\gamma,I_\alpha}_{\gamma\alpha}(\tilde{k}_\gamma,k'_\alpha;z)$$

I_γ isospin in two-particle subsystem

$$\alpha,\beta,\gamma \to K,N_1,N_2$$

Operators and wave functions totally antisymmetric under exchange of two undistinguishable nucleons (isospin basis).

The K^-d scattering length

$$a_{K^-d} = -(2\pi)^2\mu^{red}_K(N^B_K)^2\lambda^2_K X^{0,0}_{KK}(k_K=0,k'_K=0;z\to 0)$$

note that $I_{\mathrm{deuteron}} = 0$

PEST *NN* potential

- separable form of Paris potential

$$\lambda_{I=0} = \lambda_{I=1} = -1, \quad g^I(p) = \frac{1}{2\sqrt{\pi}} \sum_{i=1}^{6} \frac{c_i^I}{p^2 + \left(\beta_i^I\right)^2}$$

- repulsive for $r < 0.8\,\mathrm{fm}$
- reproduces deuteron binding energy $E_B^D = -2.2249$ MeV and *NN* scattering lengths

$$a(^3S_1) = -5.422\,\mathrm{fm}$$
$$a(^1S_0) = 17.534\,\mathrm{fm}$$

Energy dependent *NN* potential (Garcilazo)

$$V^I(\mathbf{p}, \mathbf{p}'; E) = \lambda_I(E) g^I(p) g^I(p')$$

$$g^I(p) = \frac{\gamma_I}{p^2 + \alpha_I^2}, \quad \lambda_I(E) = -\tanh\left(1 - \frac{E}{E_c^I}\right)$$

- $\lambda_I(E)$ are negative (positive) for $E < E_c^I$ ($E > E_c^I$) and finite as E goes to $\pm\infty$
- attractive at low energies and repulsive at high energies
- $E_c^{I=0} = 0.816\,\mathrm{fm}^{-1}$ and $E_c^{I=1} = 0.767\,\mathrm{fm}^{-1}$
- γ_I and α_I ($I = 0, 1$) determined from *NN* scattering lengths and effective ranges given by PEST
- $E_B^D = -2.228$ MeV

Chiral based $\bar{K}N$ potential (Cieplý) (1)

- effective chiral model with the s-wave meson-baryon lagrangian up to the second order
- flavor-matrices Φ and B of the lightest meson and baryon octets:

$$\phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

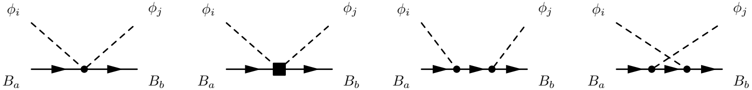
$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

Chiral based $\bar{K}N$ potential (Cieplý) (2)

- 10 coupled channels:

$\pi^0\Lambda$	$\sim$ 1250 MeV
$\pi^0\Sigma^0$, $\pi^-\Sigma^+$, $\pi^+\Sigma^-$	$\sim$ 1330 MeV
K^-p, $\bar{K}^0n$	$\sim$ 1430 MeV
$\eta\Lambda$, $\eta\Sigma^0$	$\sim$ 1700 MeV
$K^0\Xi^0$, $K^+\Xi^-$	$\sim$ 1800 MeV

- Schematic picture (taken from Borasoy, Nissler, Weise - 2005):



$\mathcal{O}(q^1)$ contact — $\mathcal{O}(q^2)$ contact — direct s-term — crossed u-term

- effective separable potentials constructed to match chiral ϕ,B amplitudes up to $\mathcal{O}(q^2)$
- reduced to an equivalent single-channel complex potential

REZ $\bar{K}N$ potential (Shevchenko) (1)

- separable $\bar{K}N$ potential (our own) with formfactors

$$g^I(p) = \frac{1}{p^2 + \beta_I^2}, \quad I = 0, 1$$

- range parameters $\beta_I = 3.5\,\mathrm{fm}^{-1}$ (Shevchenko et al.)
- strength parameters $\lambda_I,\ I = 0, 1$ complex (due to $\bar{K}N \to \pi\Sigma$)
- $\lambda_{I=0}$ determined from mass and width of the $\Lambda(1405)$ resonance
- $\lambda_{I=1}$ determined from a_{K^-p} using

$$a_{K^-p} = \frac{(a_{I=0} + a_{I=1})}{2}$$

REZ $\bar{K}N$ potential (Shevchenko) (2)

- PDG values $m_\Lambda = 1405$ MeV and $\Gamma_\Lambda = 50$ MeV give

$$E_B^{\bar{K}N} = (-29.5 - \mathrm{i}\,25)\,\mathrm{MeV} \gg E_B^{D} = -2.22\,\mathrm{MeV}$$

- two available experimental values of a_{K^-p}:

$$a_{K^-p}^{KEK} = (-0.78 \pm 0.15 \pm 0.03) + \mathrm{i}\,(0.49 \pm 0.25 \pm 0.12)\,\mathrm{fm}$$

$$a_{K^-p}^{DEAR} = (-0.468 \pm 0.090 \pm 0.015) + \mathrm{i}\,(0.302 \pm 0.135 \pm 0.036)\,\mathrm{fm}$$

- it is hard to reproduce simultaneously K^-p cross sections and $a_{K^-p}^{DEAR}$ value $\Rightarrow$ we preferred $a_{K^-p}^{KEK}$ value

Dependence of a_{K^-d} on $\Lambda(1405)$ (1)

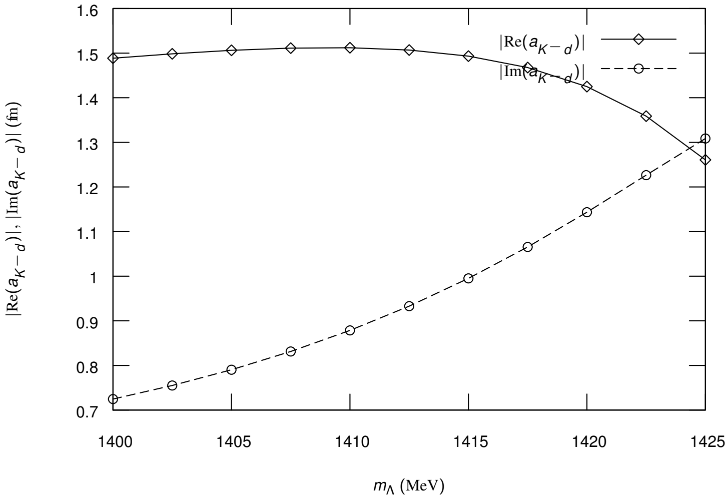


Fig.: The scattering length a_{K^-d} as a function of the mass m_Λ of the $\Lambda(1405)$ resonance. Here the *NN* potential PEST was used, $\Gamma_\Lambda = 50$ MeV and the K^-p scattering length is fixed at the KEK value.

Dependence of a_{K^-d} on $\Lambda(1405)$ (2)

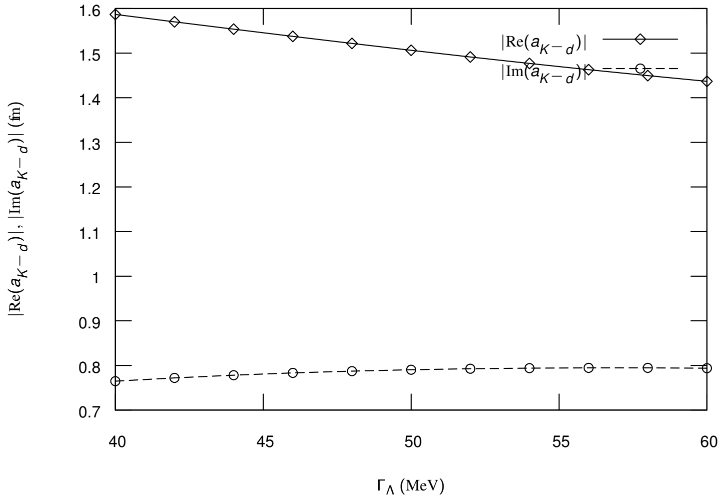


Fig.: The scattering length a_{K-d} as a function of the width Γ_Λ of the $\Lambda(1405)$ resonance. Here the *NN* potential PEST was used, $m_\Lambda = 1405$ MeV and the $K^- p$ scattering length is fixed at the KEK value.

Dependence of a_{K-d} on a_{K-p} (1)

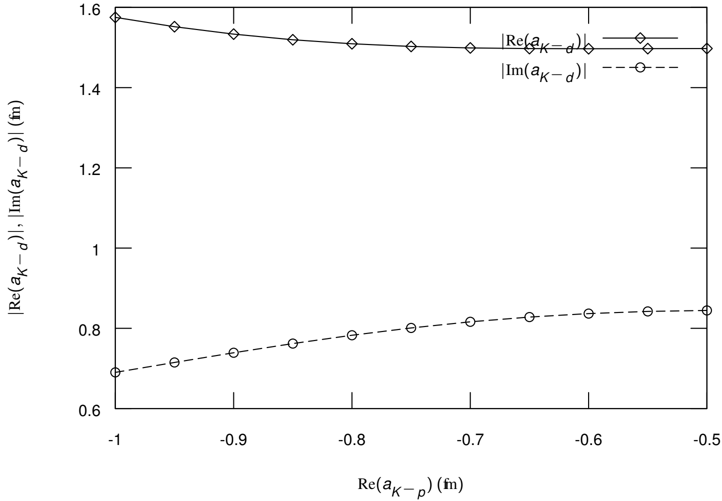


Fig.: The scattering length a_{K-d} as a function of the real part of the two-body $K^- p$ scattering length a_{K-p}. The imaginary part of a_{K-p} is fixed at the KEK value, the NN potential PEST was used, $m_\Lambda = 1405$ MeV and $\Gamma_\Lambda = 50$ MeV were considered.

Dependence of a_{K^-d} on a_{K^-p} (2)

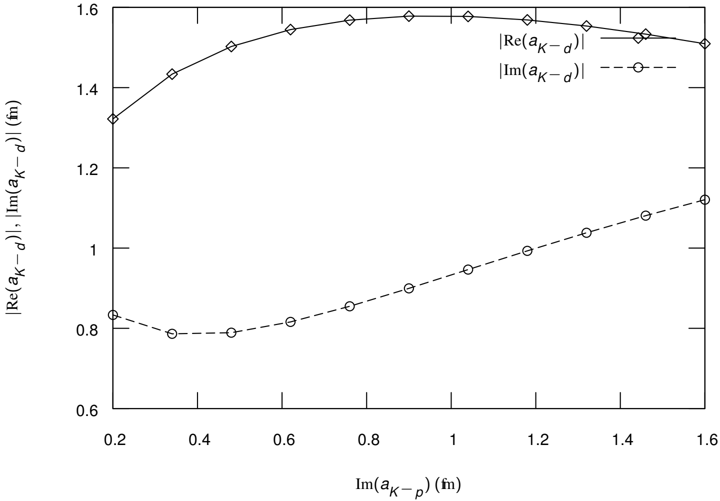


Fig.: The scattering length a_{K^-d} as a function of the imaginary part of the two-body $K^- p$ scattering length a_{K^-p}. The real part of a_{K^-p} is fixed at the KEK value, the *NN* potential PEST was used, $m_\Lambda = 1405$ MeV and $\Gamma_\Lambda = 50$ MeV were considered.

Dependence of a_{K-d} on $\beta_I^{\bar{K}N}$

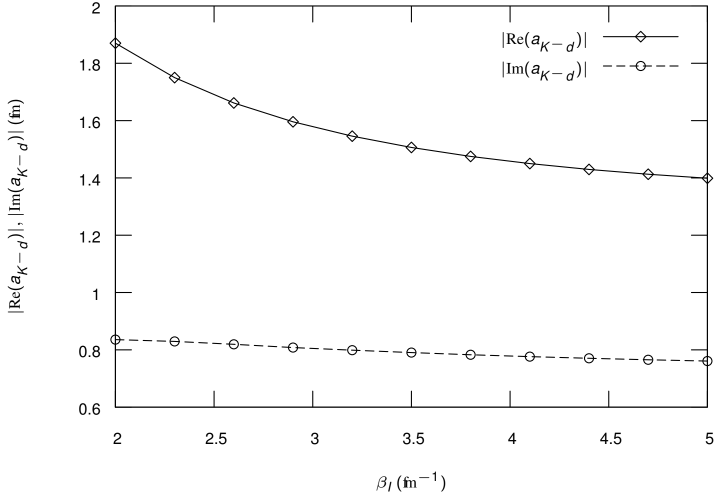


Fig.: The scattering length a_{K-d} as a function of the β_I parameter ($\beta_0 = \beta_1$). Here the *NN* potential PEST was used, $m_\Lambda = 1405$ MeV, $\Gamma_\Lambda = 50$ MeV and the $K^- p$ scattering length is fixed at the KEK value.

Dependence of a_{K^-d} on two-body inputs

Results:

Tab.: The K^-d scattering length in fm calculated using various two-body inputs: NN potentials horizontally, $\bar{K}N$ potentials vertically. REZ $\bar{K}N$ potential holds $m_\Lambda = 1405$ MeV, $\Gamma_\Lambda = 50$ MeV and $a^{KEK}_{K^-p}$.

	PEST	E-dep NN
REZ $\bar{K}N$	$-1.51 + \mathrm{i}\,0.79$	$-1.46 + \mathrm{i}\,0.78$
chiral $\bar{K}N$ (KEK fit)	$-1.78 + \mathrm{i}\,1.84$	$-1.62 + \mathrm{i}\,1.57$
chiral $\bar{K}N$ (DEAR fit)	$-1.66 + \mathrm{i}\,1.88$	$-1.53 + \mathrm{i}\,1.55$

- very sensitive to $\bar{K}N$ potential!

Comparison with other calculations

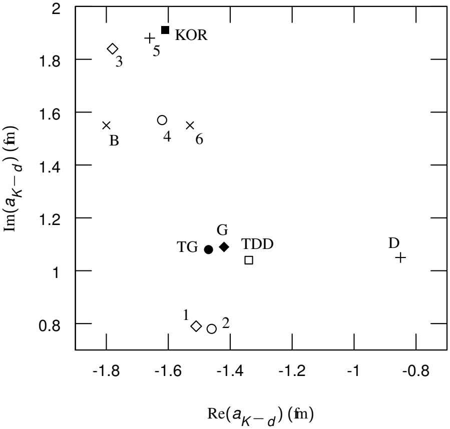


Fig.: The $K^- d$ scattering lengths calculated using various two-body inputs: REZ $\bar{K}N$ and PEST (1), REZ $\bar{K}N$ and E-dep (2), chiral $\bar{K}N$ (KEK fit) and PEST (3), chiral $\bar{K}N$ (KEK fit) and E-dep (4), chiral $\bar{K}N$ (DEAR fit) and PEST (5), chiral $\bar{K}N$ (DEAR fit) and E-dep (6). Results of previous calculations: Bahaoui et al. (B), Torres & Dalitz & Deloff (TDD), Deloff (D), Gal (G), Kamalov & Oset & Ramos (KOR), Toker & Gal (TG).

Predicted values of the kaonic deuterium ϵ and Γ

- possible to calculate the kaonic hydrogen 1*s* level shift ϵ and width Γ by using the Deser-type formula at next-to-leading order in isospin breaking

$$\epsilon^d - \mathrm{i}\frac{\Gamma^d}{2} = -2\alpha^3\mu_r^2 a_{K^-d}[1 - 2\alpha\mu_r a_{K^-d}(\ln\alpha - 1) + ...]$$

- REZ $\bar{K}N$ potential gives

$$\epsilon^d_{\mathrm{REZ}} \doteq 730\,\mathrm{eV}, \Gamma^d_{\mathrm{REZ}} \doteq 470\,\mathrm{eV}$$

- chiral $\bar{K}N$ potential gives

$$\epsilon^d_{\mathrm{chiral}} \doteq 1020\,\mathrm{eV}, \Gamma^d_{\mathrm{chiral}} \doteq 890\,\mathrm{eV}$$

Summary

- we calculated the $K^- d$ scattering length using the Faddeev equations in the AGS form
- NN potentials: PEST + E-dep (Garcilazo)
- $\bar{K}N$ potentials: REZ potential (Shevchenko) (reproduces a_{K^-p} and $\Lambda(1405)$) + chiral based potential (Cieplý)
- $\bar{K}N$ interaction is much stronger than NN interaction in $\bar{K}$-nuclear systems
- a_{K^-d} depends strongly on $I = 0$ $\bar{K}N$ interaction (Λ resonance), weakly on $I = 1$ $\bar{K}N$ interaction $\Rightarrow$ $I = 0$ $\bar{K}N$ interaction is more important for the $K^- d$ system
- plausible value of $\beta_I^{\bar{K}N}$ necessary
- chiral based potentials give high imaginary part of a_{K^-d}
- using the Deser-type formula the values ϵ and Γ of the kaonic deuterium calculated