

**ECT\*, Trento**

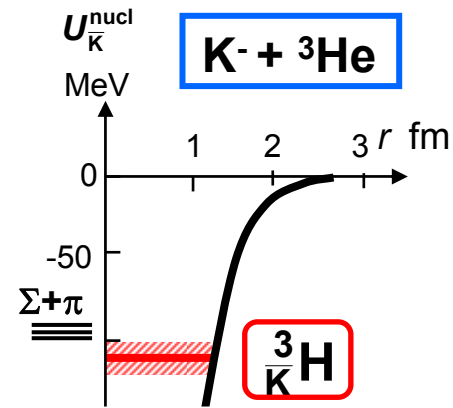
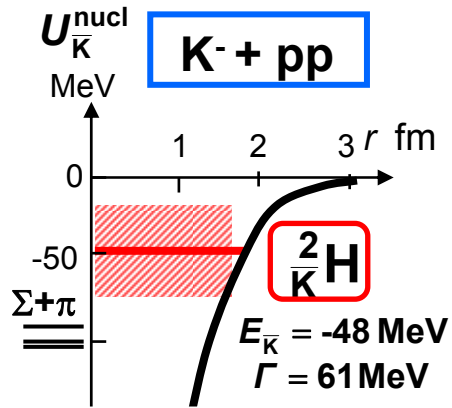
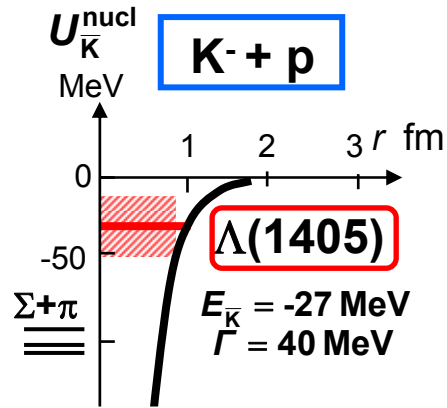
Oct. 12-16, 2009

# **Single-Pole Nature of $\Lambda(1405)$ and Structure of $K^-pp$**

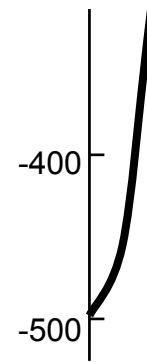
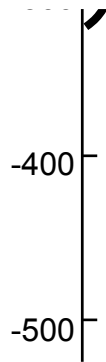
**Yoshinori AKAISHI,**

**Jafar ESMAILI, Toshimitsu YAMAZAKI**

# " $\Lambda(1405)$ Ansatz"

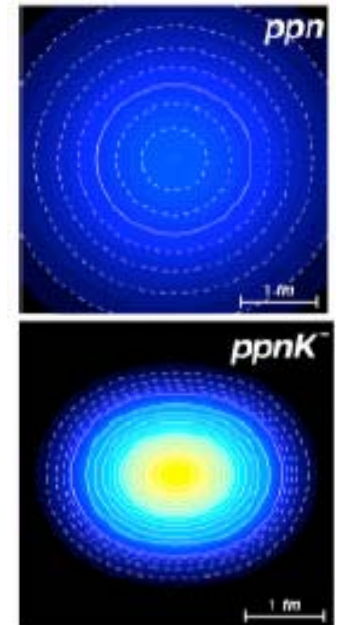


N.V. Shevchenko, A. Gal & J. Mares, Phys. Rev. Lett. 98 (2007) 082301  
 $E = -55 \sim -70 \text{ MeV}$ ,  $\Gamma = 90 \sim 110 \text{ MeV}$   
 Y. Ikeda & T. Sato, Phys. Rev. C 76 (2007) 035203  
 $E = -80 \text{ MeV}$ ,  $\Gamma = 73 \text{ MeV}$



T. Yamazaki & Y. Akaishi, Phys. Lett. B 535 (2002) 70  
 Y. Akaishi & T. Yamazaki, Phys. Rev. C 65 (2002) 044005

A. Dote et al.



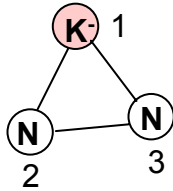
Shrinkage!

# Variational wave function of K<sup>-</sup>pp

## ATMS

Amalgamation of **T**wo-body correlations into **M**ultiple **S**cattering process

$$\Psi = \left[ \left\{ f^{l=0}(r_{12}) \hat{P}_{12}^{l=0} + f^{l=1}(r_{12}) \hat{P}_{12}^{l=1} \right\} f_{NN}(r_{23}) f(r_{31}) + f(r_{12}) f_{NN}(r_{23}) \left\{ f^{l=0}(r_{31}) \hat{P}_{31}^{l=0} + f^{l=1}(r_{31}) \hat{P}_{31}^{l=1} \right\} \right] |T = 1/2\rangle$$



$$\hat{P}_{12}^{l=0} = \frac{1 - \vec{r}_K \vec{r}_N}{4}, \quad \hat{P}_{12}^{l=1} = \frac{3 + \vec{r}_K \vec{r}_N}{4}$$

$$|T = 1/2\rangle = \sqrt{\frac{3}{4}} \left[ \left[ (\bar{K}_1 N_2)^{0,0} p_3 \right] \right] + \sqrt{\frac{1}{4}} \left[ \left[ -\sqrt{\frac{1}{3}} (\bar{K}_1 N_2)^{1,0} p_3 + \sqrt{\frac{2}{3}} (\bar{K}_1 N_2)^{1,1} n_3 \right] \right]$$

$\Lambda^* p$

## Euler-Lagrange equation

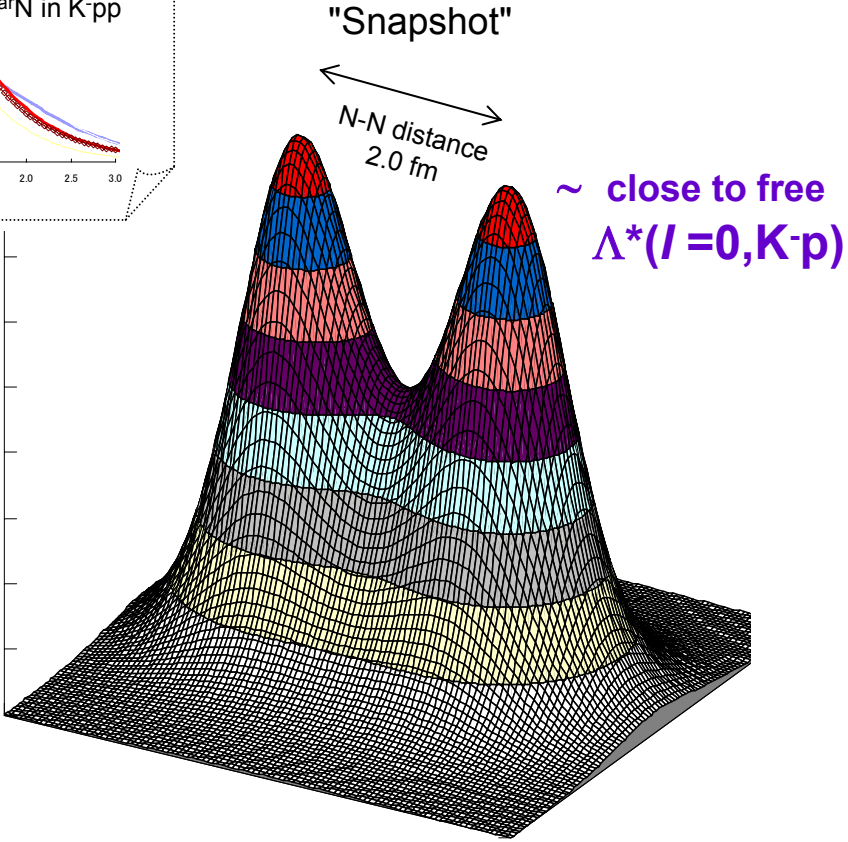
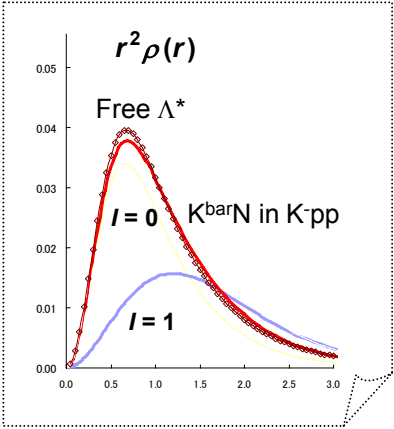
$$\delta_f \{ \langle \Psi | H | \Psi \rangle - \lambda \langle \Psi | \Psi \rangle \} = 0$$

$$v_{KN}^{T=0}(r) = \{ -595 - i83 \}_{\text{MeV}} \exp\left\{ - (r/0.66_{\text{fm}})^2 \right\}$$

$$v_{KN}^{T=1}(r) = \{ -175 - i105 \}_{\text{MeV}} \exp\left\{ - (r/0.66_{\text{fm}})^2 \right\}$$

$$v_{NN}(r) = 2000_{\text{MeV}} \exp\left\{ - (r/0.447_{\text{fm}})^2 \right\} - 270_{\text{MeV}} \exp\left\{ - (r/0.942_{\text{fm}})^2 \right\} - 5_{\text{MeV}} \exp\left\{ - (r/2.5_{\text{fm}})^2 \right\}$$

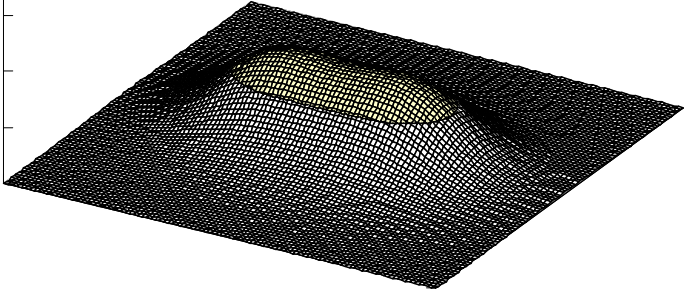
# K- distribution in K $\bar{p}p$



“Kaonic hydrogen molecule”

EXA05 : P. Kienle

Covalent part



$$\Psi_{\pm} = \phi_a \pm \phi_b$$

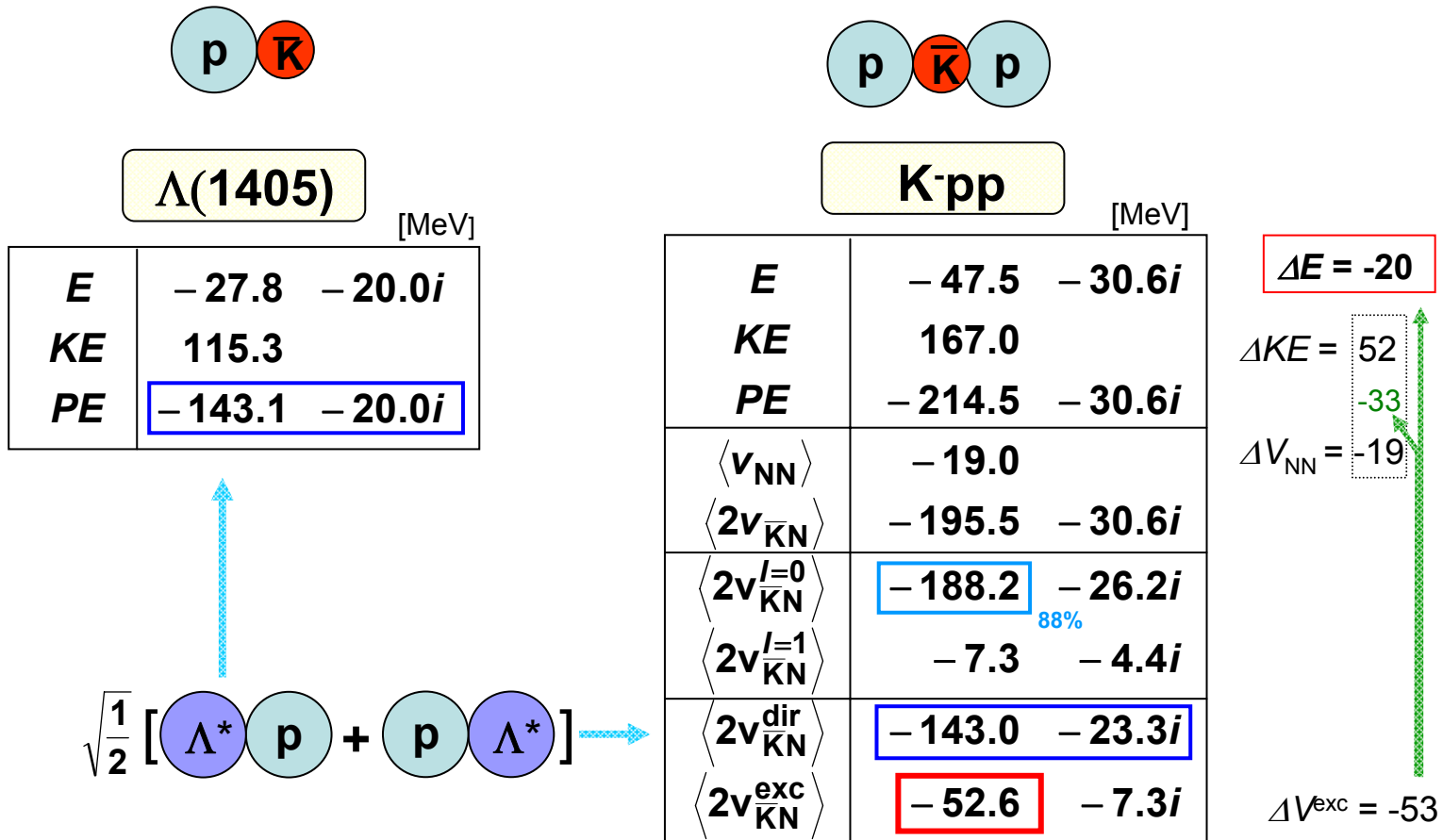
$$\int d\vec{r} |\Psi_{\pm}|^2 = \int d\vec{r}_a |\phi_a(\vec{r}_a)|^2 + \int d\vec{r}_b |\phi_b(\vec{r}_b)|^2 \pm \int d\vec{r} \left[ \phi_a^*(\vec{r}_a) \phi_b(\vec{r}_b) + \phi_b^*(\vec{r}_b) \phi_a(\vec{r}_a) \right]$$

Direct integral

Covalent bonding

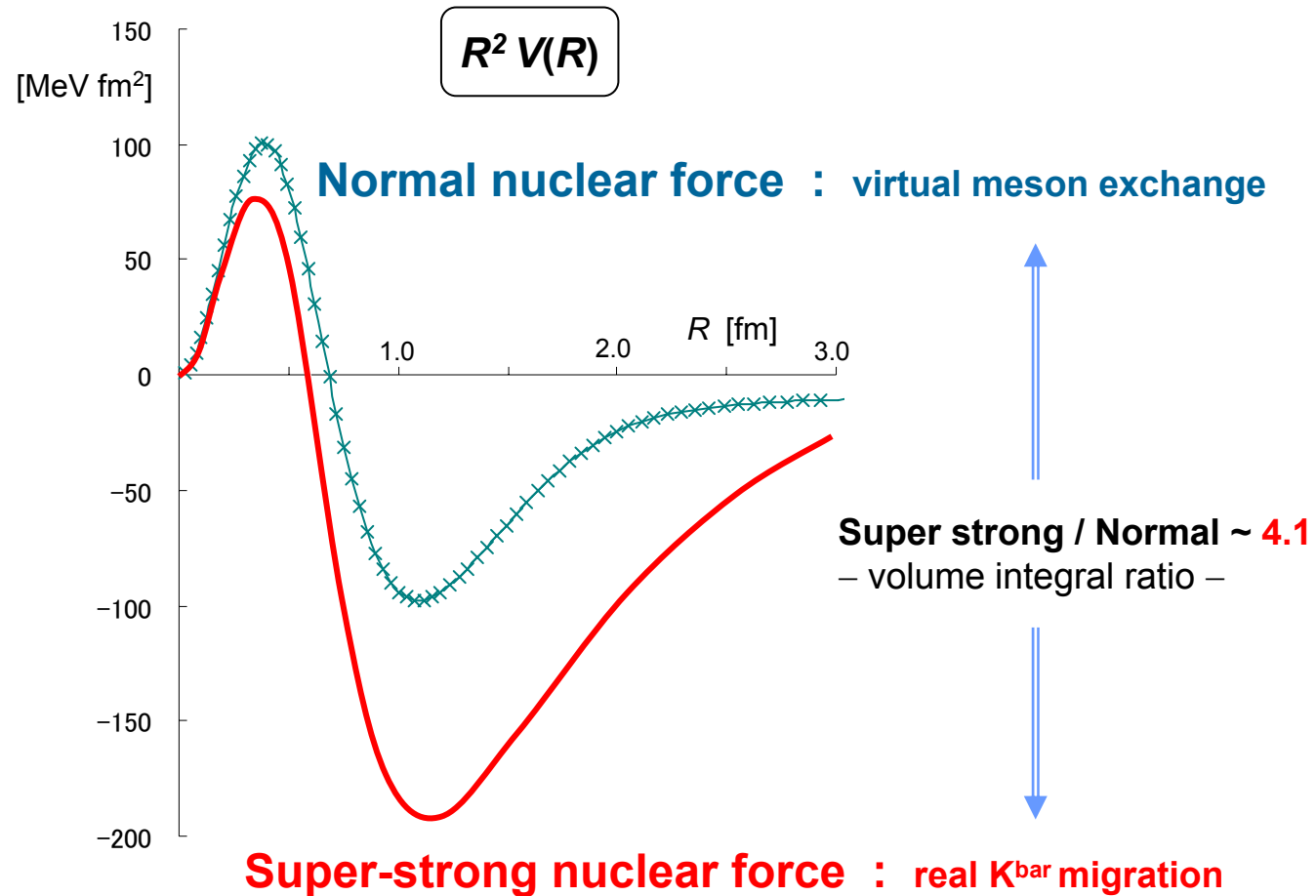
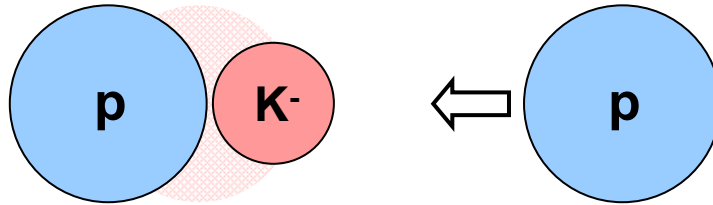
Exchange integral

# Heitler-London picture of $K^-pp$

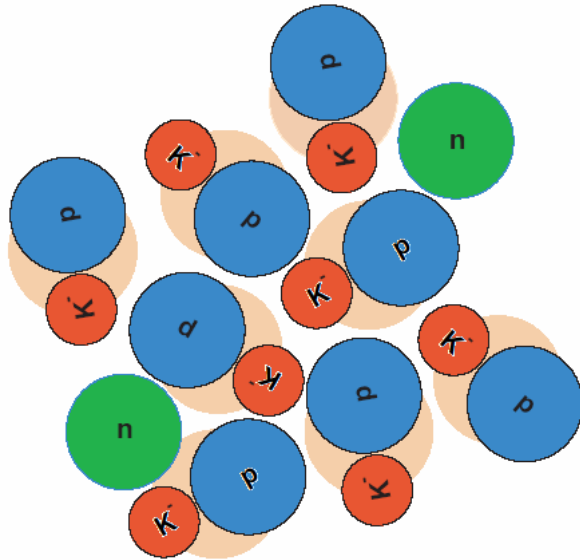


Real  $K^{\text{bar}}$  migrating attraction

# Adiabatic p-p potential in $K^-pp$



$\Lambda^* = (\mathbf{K-p})_{I=0}$  condensed matter



**Migrating real kaons!**  
 — a new paradigm —

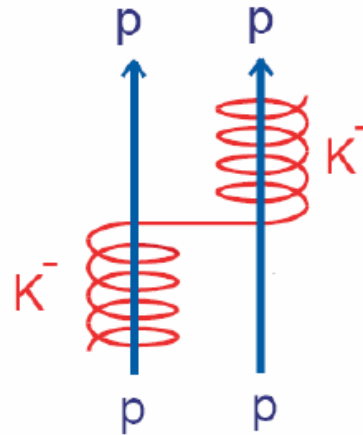
New!

DISTO data on K-pp

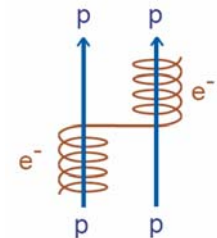
T. Yamazaki et al.  
 on 15 Oct.

FINUDA data on K-pp

S. Piano et al.  
 This morning!

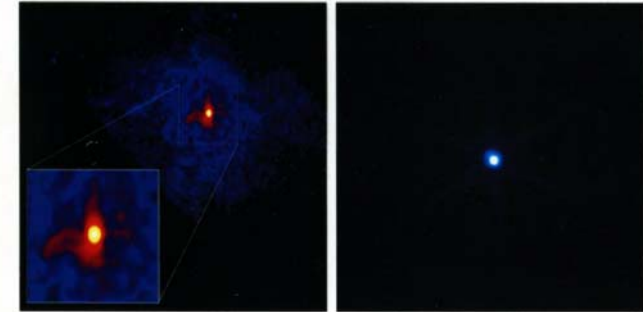


Revival of Heitler-London-Heisenberg picture



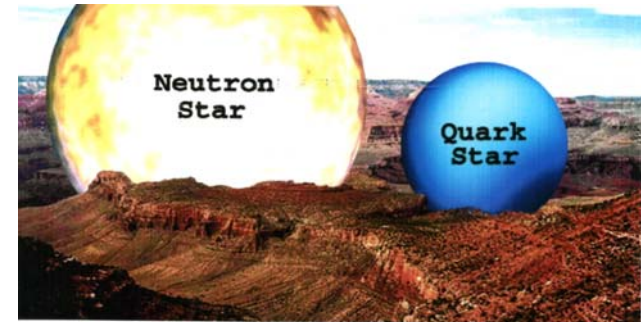
T. Yamazaki & Y. Akaishi,  
 Proc. Japan Academy, B 83 (2007) 144

NASA's Chandra X-ray



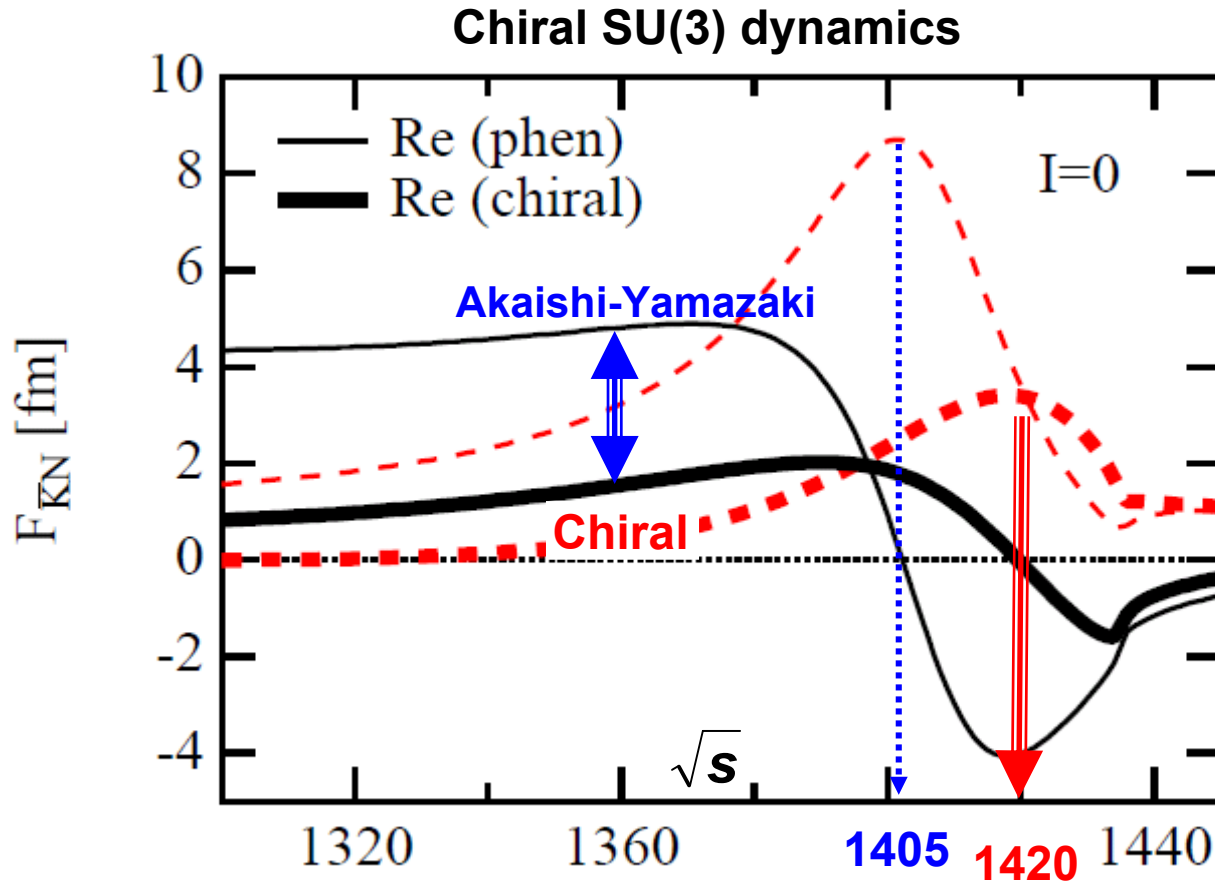
3C59

RX J1856



# $K^{\text{bar}}N$ scattering amplitude

T. Hyodo and W. Weise, Phys. Rev. C 77 (2008) 035204



"ORB": E. Oset, A. Ramos & C. Bennhold, Phys. Lett. B 527 (2002) 99

"HNJH": T. Hyodo, S.I. Nam, D. Jido & A. Hosaka, Phys. Rev. C 68 (2003) 018201

"BNW": B. Borasoy, R. Nissler & W. Weise, Eur. Phys. J. A 25 (2005) 79

"BMN": B. Borasoy, U.G. Meissner & R. Nissler, Phys. Rev. C 74 (2006) 055201

"JOORM": D. Jido, J.A. Oller, E. Oset, A. Ramos & U.G. Meissner, Nucl. Phys. A 725 (2003) 181

# Double pole structure of $\Lambda(1405)$

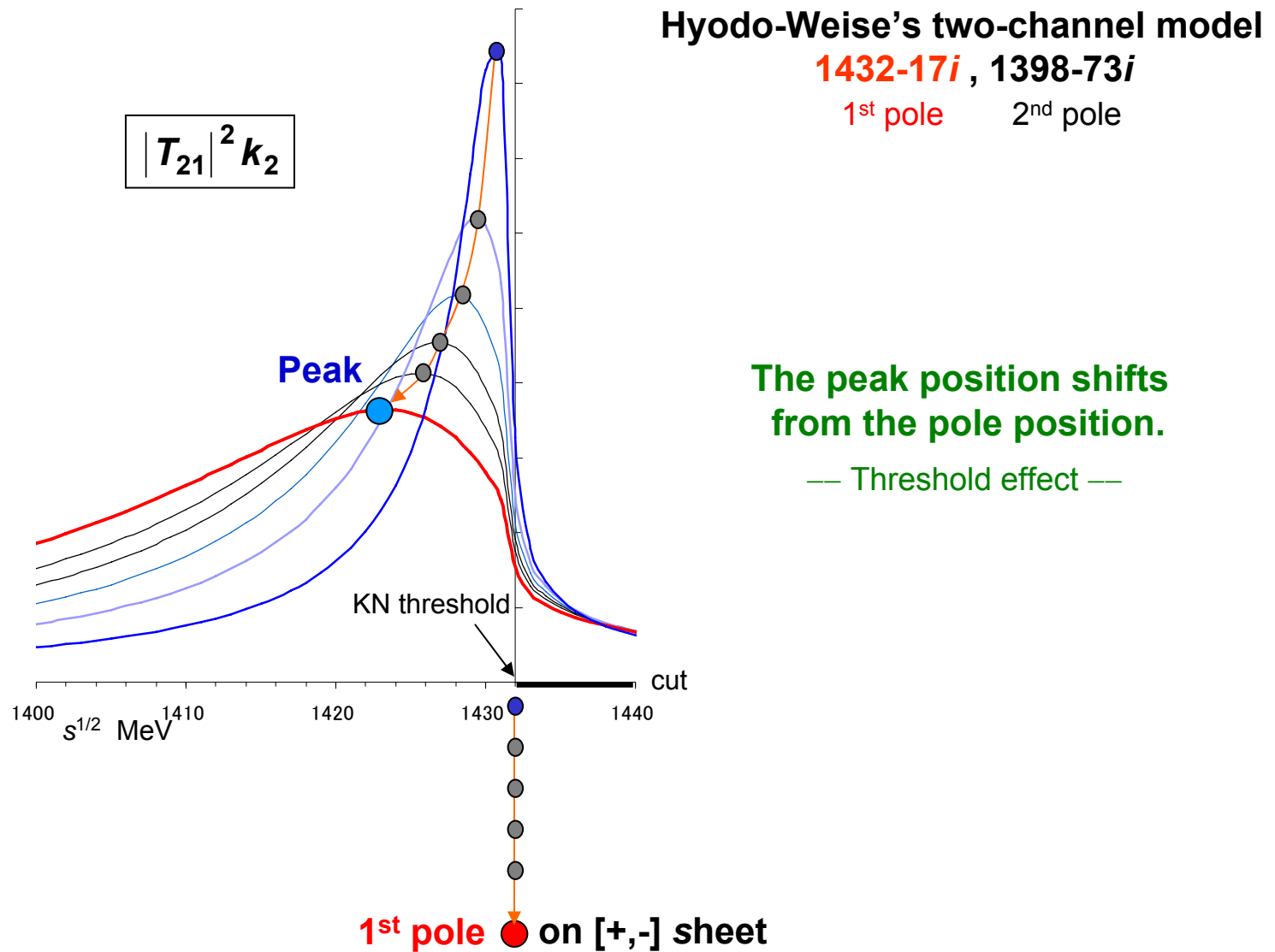
D. Jido, J.A. Oller, E. Oset, A. Ramos & U.G. Meissner, Nucl. Phys. A 725 (2003) 181



$\Lambda(1405)$  consists of two poles, one of which is not  $K\bar{p}$  but  $\Sigma\pi$  pole.

What effects on experimental observables?

# Peak position of the 1<sup>st</sup> pole in $M_{\Sigma\pi}$ spectrum



# Chiral SU(3) dynamics and the 2<sup>nd</sup> pole

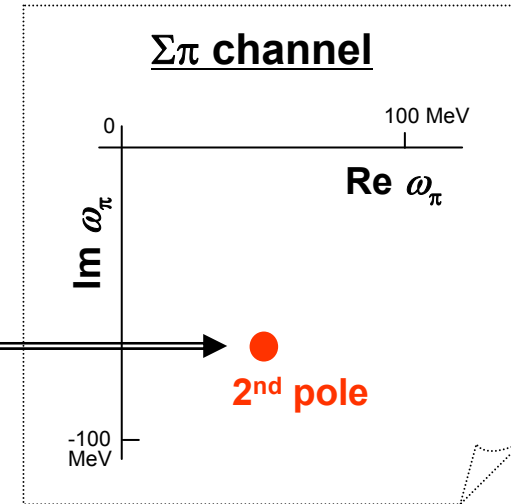
## Weinberg-Tomozawa term

$$-c_{ij} \frac{\omega_i + \omega_j}{4f^2}$$

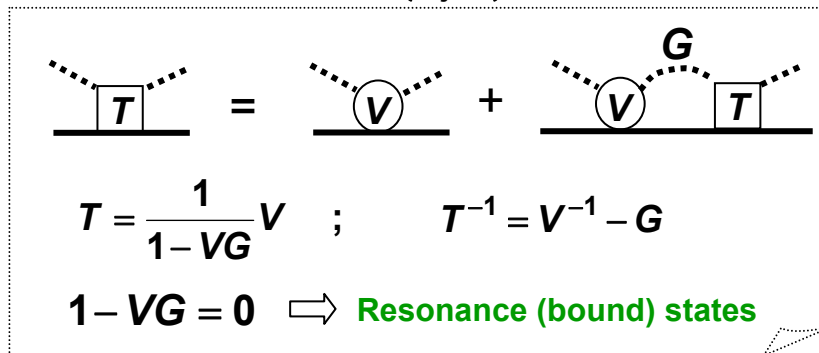
$$c_{ij}^{l=0} = \begin{bmatrix} 3 & -\sqrt{\frac{3}{2}} \\ -\sqrt{\frac{3}{2}} & 4 \end{bmatrix} \begin{array}{l} \bar{K}N \\ \pi\Sigma \end{array}$$

$$\omega_i \leftarrow \sqrt{s} - M_i$$

Energy dependent interaction  
with a large positive imaginary part  
(source term)



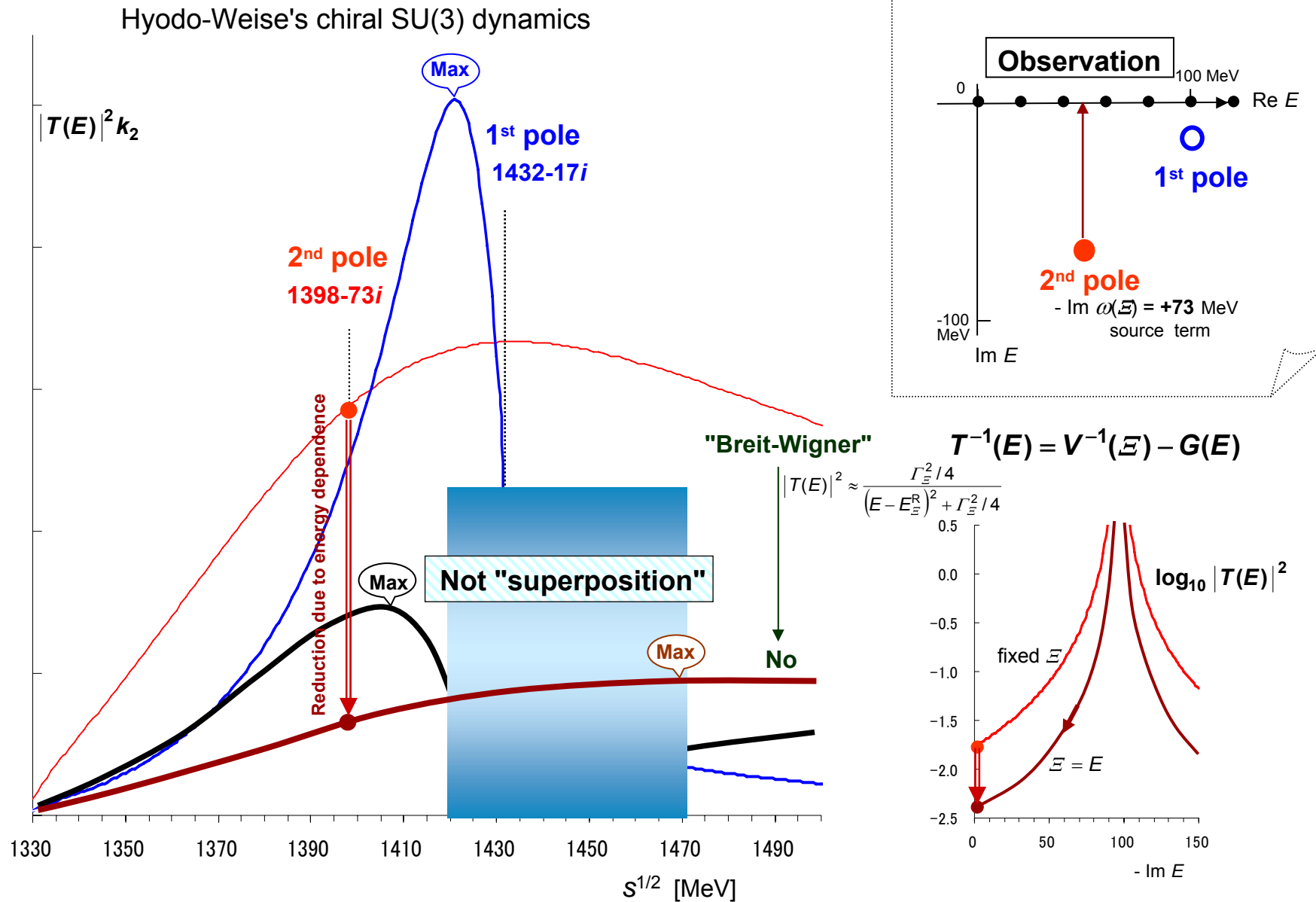
$$V = \{V_{ij}^{WT}\} \quad \text{Total 10 channels}$$



	2 <sup>nd</sup> pole	1 <sup>st</sup> pole
Full ch.	1400- <i>i</i> 76	1428- <i>i</i> 17
2 ch.	1398- <i>i</i> 73	1432- <i>i</i> 17
1 ch.	1388- <i>i</i> 96	

Hyodo-Weise

# $\Sigma\pi$ invariant mass spectrum of $\bar{K}N$ - $\Sigma\pi$ coupled system



# Generalized optical potential

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} + \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix} \begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}$$

$$V_{11}^{\text{opt}}(E_1) = V_{11} + V_{12} \frac{G_2(E_2)}{1 - V_{22}G_2(E_2)} V_{21} \quad , \quad V_{12}^{\text{opt}}(E_2) = \frac{1}{1 - V_{22}G_2(E_2)} V_{12}$$

$$V_{21}^{\text{opt}}(E_1) = \frac{1}{1 - V_{11}G_1(E_1)} V_{21} \quad , \quad V_{22}^{\text{opt}}(E_2) = V_{22} + V_{21} \frac{G_1(E_1)}{1 - V_{11}G_1(E_1)} V_{12}$$

$$E_1 = E_2 - Q$$

$$T_{11}(E_1) = \frac{1}{1 - V_{11}^{\text{opt}}(E_1)G_1(E_1)} V_{11}^{\text{opt}}(E_1) \quad , \quad T_{12}(E_1) = \frac{1}{1 - V_{11}^{\text{opt}}(E_1)G_1(E_1)} V_{12}^{\text{opt}}(E_2)$$

$$T_{21}(E_2) = \frac{1}{1 - V_{22}^{\text{opt}}(E_2)G_2(E_2)} V_{21}^{\text{opt}}(E_1) \quad , \quad T_{22}(E_2) = \frac{1}{1 - V_{22}^{\text{opt}}(E_2)G_2(E_2)} V_{22}^{\text{opt}}(E_2)$$

$$\frac{1 - V_{11}^{\text{opt}}(E_1)G_1(E_1)}{1 - V_{22}^{\text{opt}}(E_2)G_2(E_2)} = \frac{1 - V_{11}G_1(E_1)}{1 - V_{22}G_2(E_2)}$$

# Feshbach theory

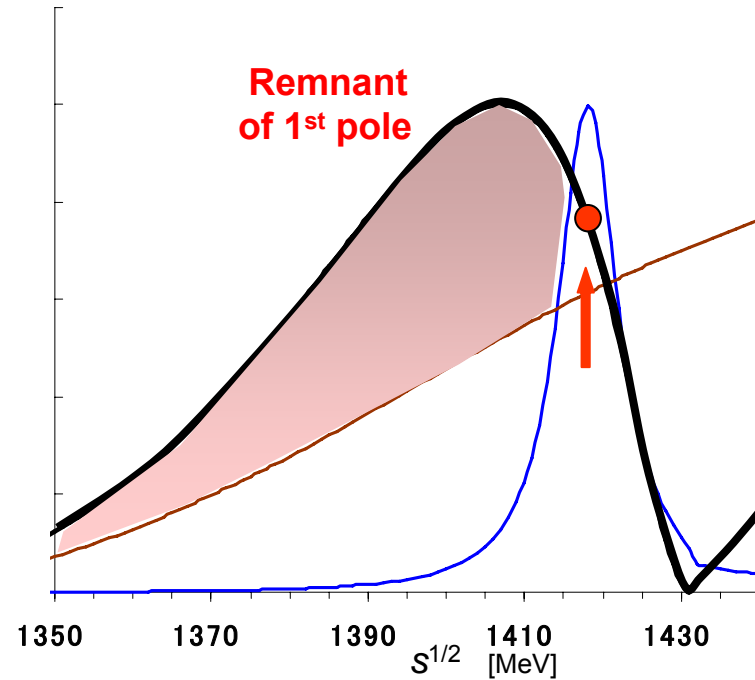
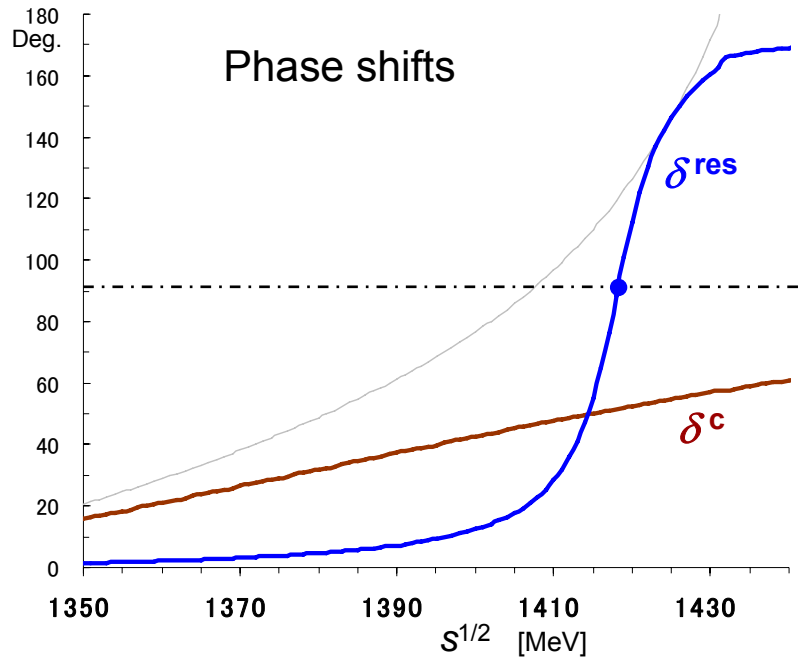
1;  $\bar{K}N$   
2;  $\Sigma\pi$

$$V_{22}^{\text{opt}} = V_{22} + V_{21} \frac{G_1}{1 - V_{11}G_1} V_{12}$$

2<sup>nd</sup> pole ↓  $\delta^c$       ↓  $\delta^{\text{res}}$

$$|T_{22}(E)|^2 \approx \left| e^{i\delta^c} \sin \delta^c + \frac{\Gamma/2}{E - E_R + i\Gamma/2} \right|^2$$

Interference

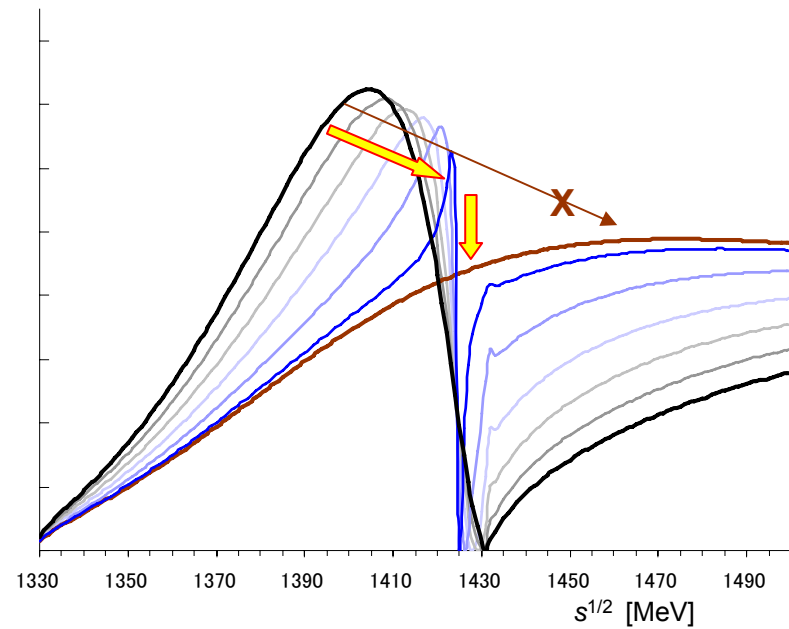
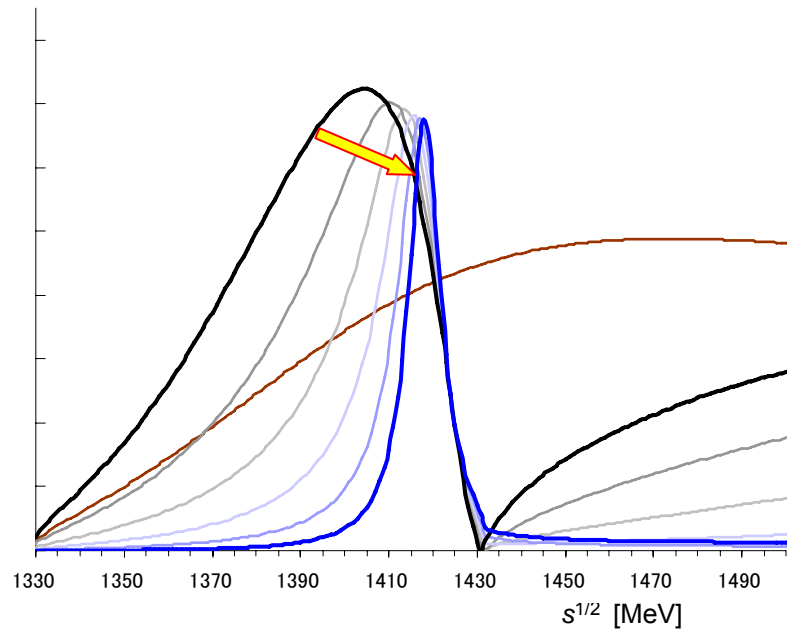


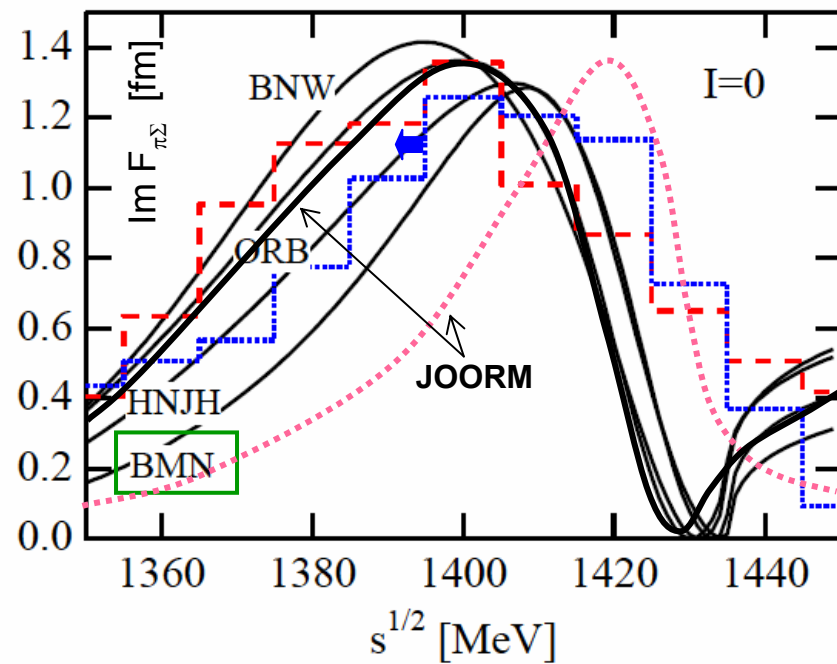
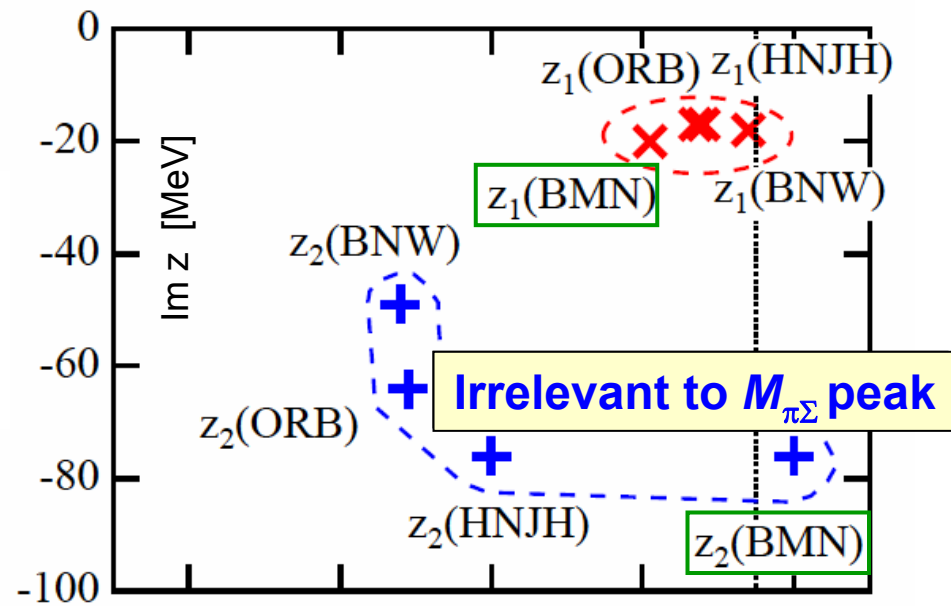
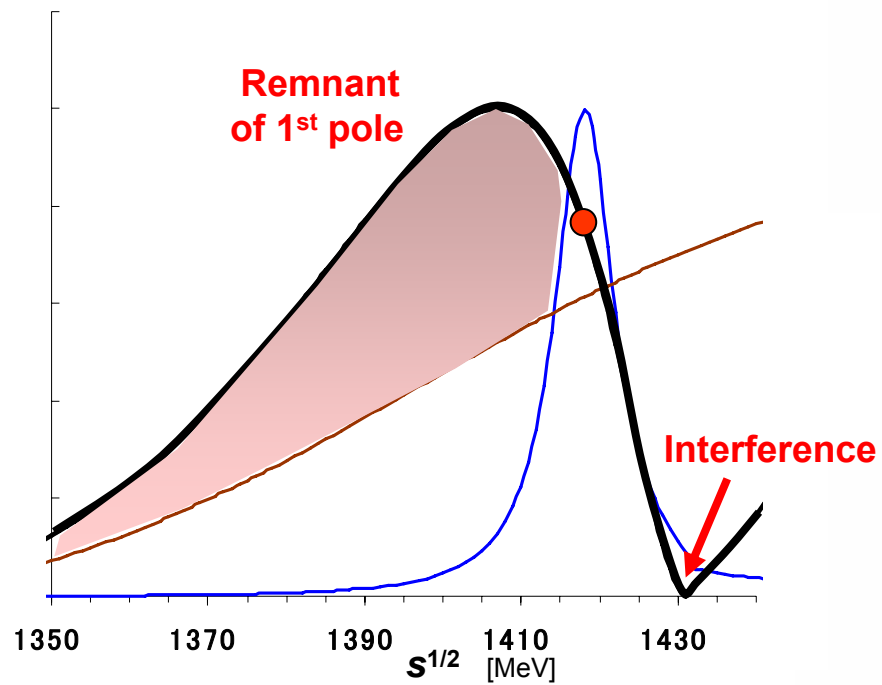
# $\Sigma\pi$ - $\Sigma\pi$ invariant-mass spectrum

1;  $\bar{K}N$   
2;  $\Sigma\pi$

$$V_{22}^{\text{opt}} = V_{22} + V_{21} \frac{G_1}{1 - V_{11}G_1} V_{12}$$

2<sup>nd</sup> pole  
↓ 0 ↓ 0



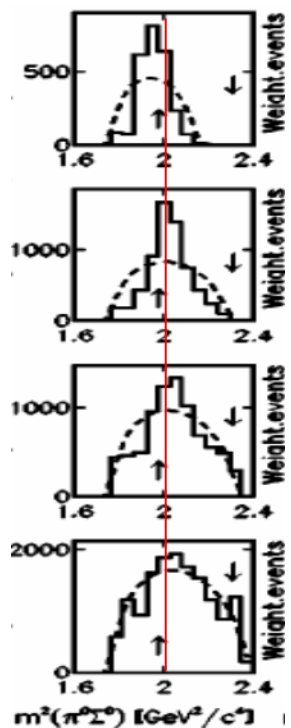


Logically impossible to stand !

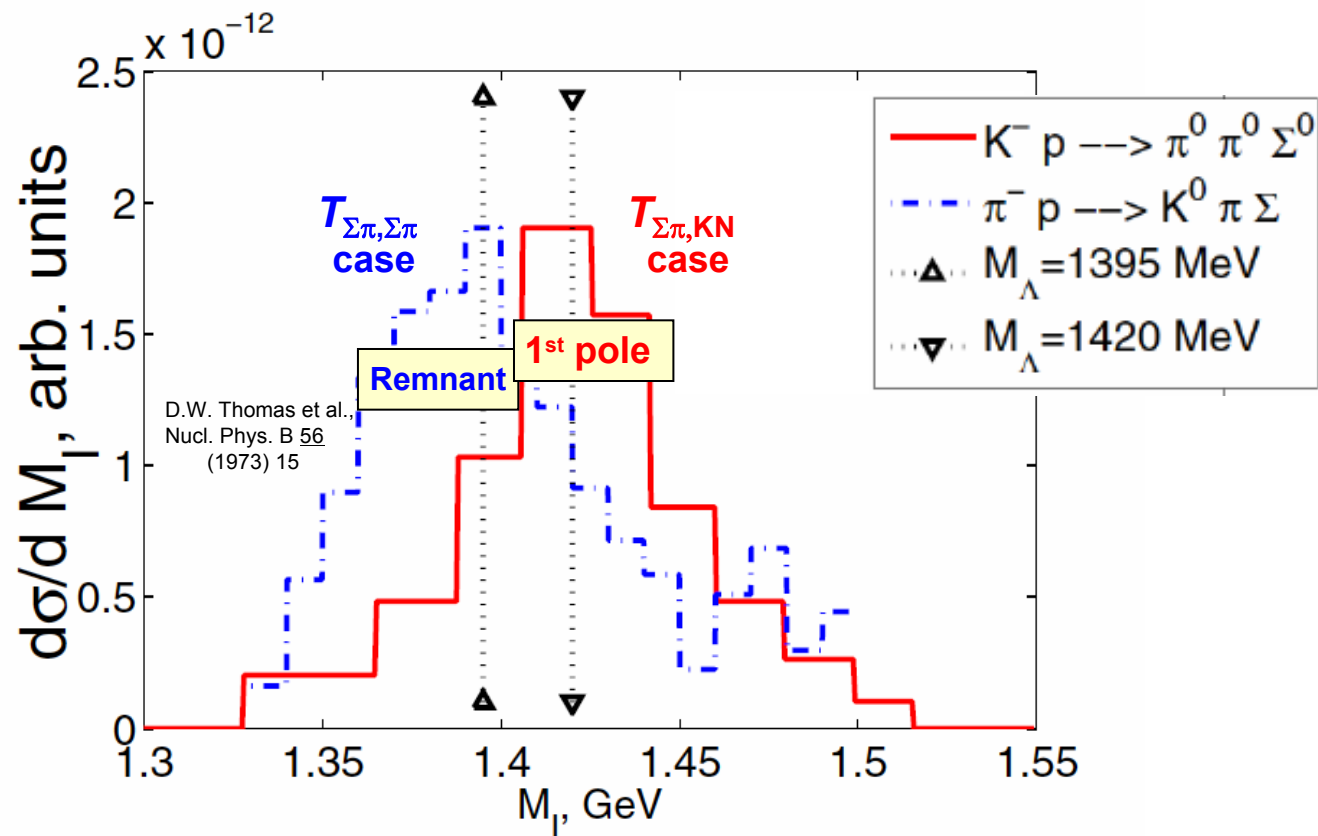


## Evidence for two-pole structure of $\Lambda(1405)$

V.K. Magas, E. Oset and A. Ramos, Phys. Rev. Lett. **95** (2005) 052301

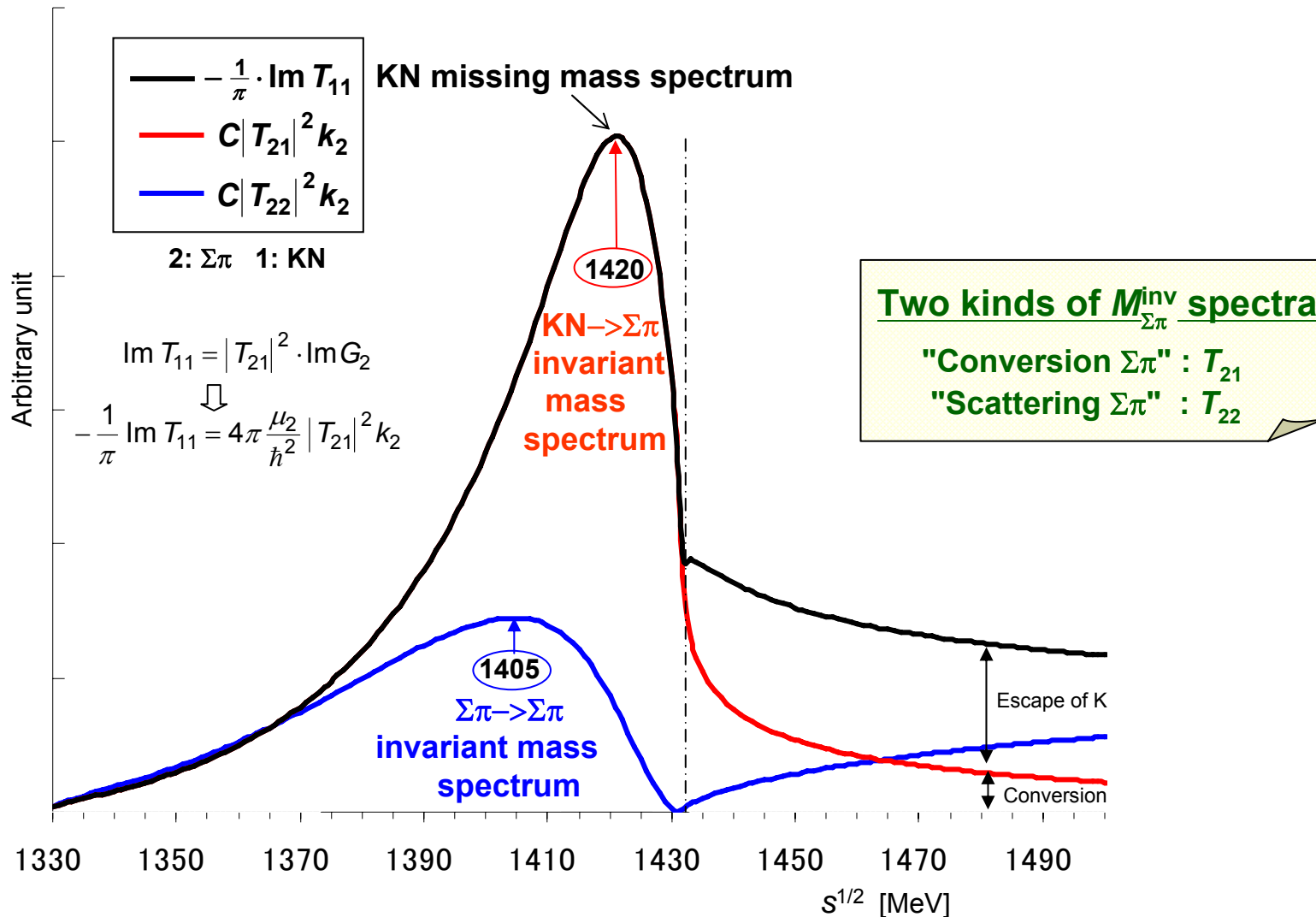


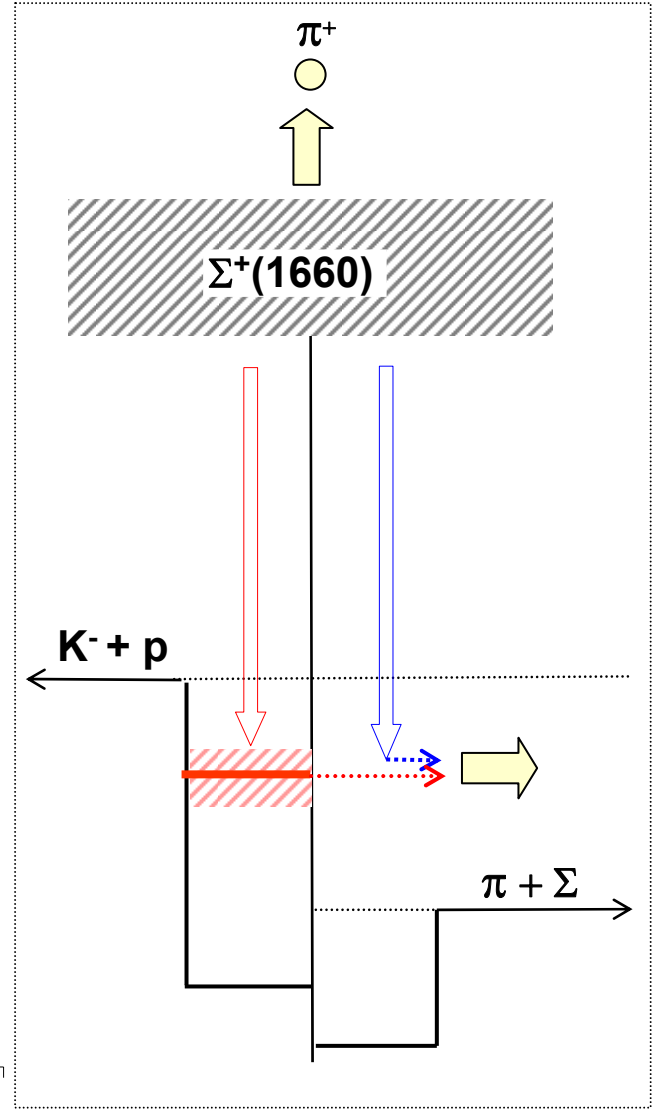
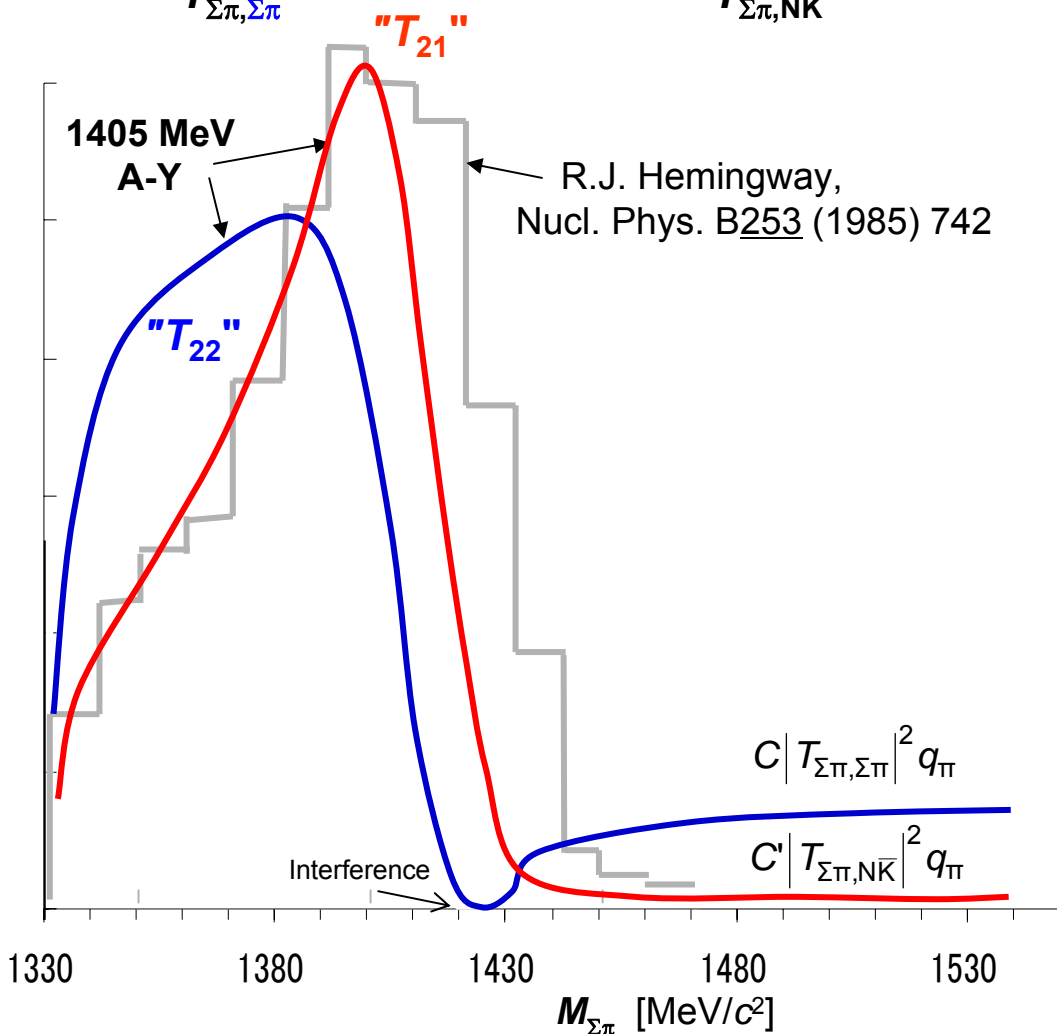
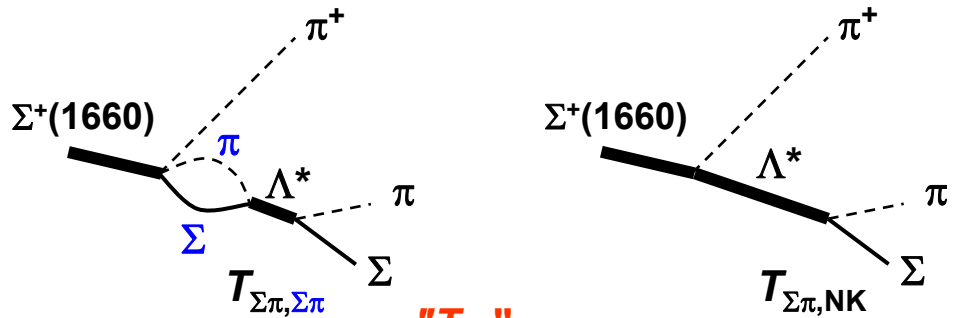
S. Prakhov et al.,  
Phys. Rev. C **70** (2004)  
034605



# Observables of $\bar{K}N-\Sigma\pi$ coupled system

Hyodo-Weise's chiral SU(3) dynamics





# $\Sigma\pi$ invariant-mass spectrum

1;  $\bar{K}N$   
2;  $\Sigma\pi$

" $T_{21}/T_{22}$ " problem

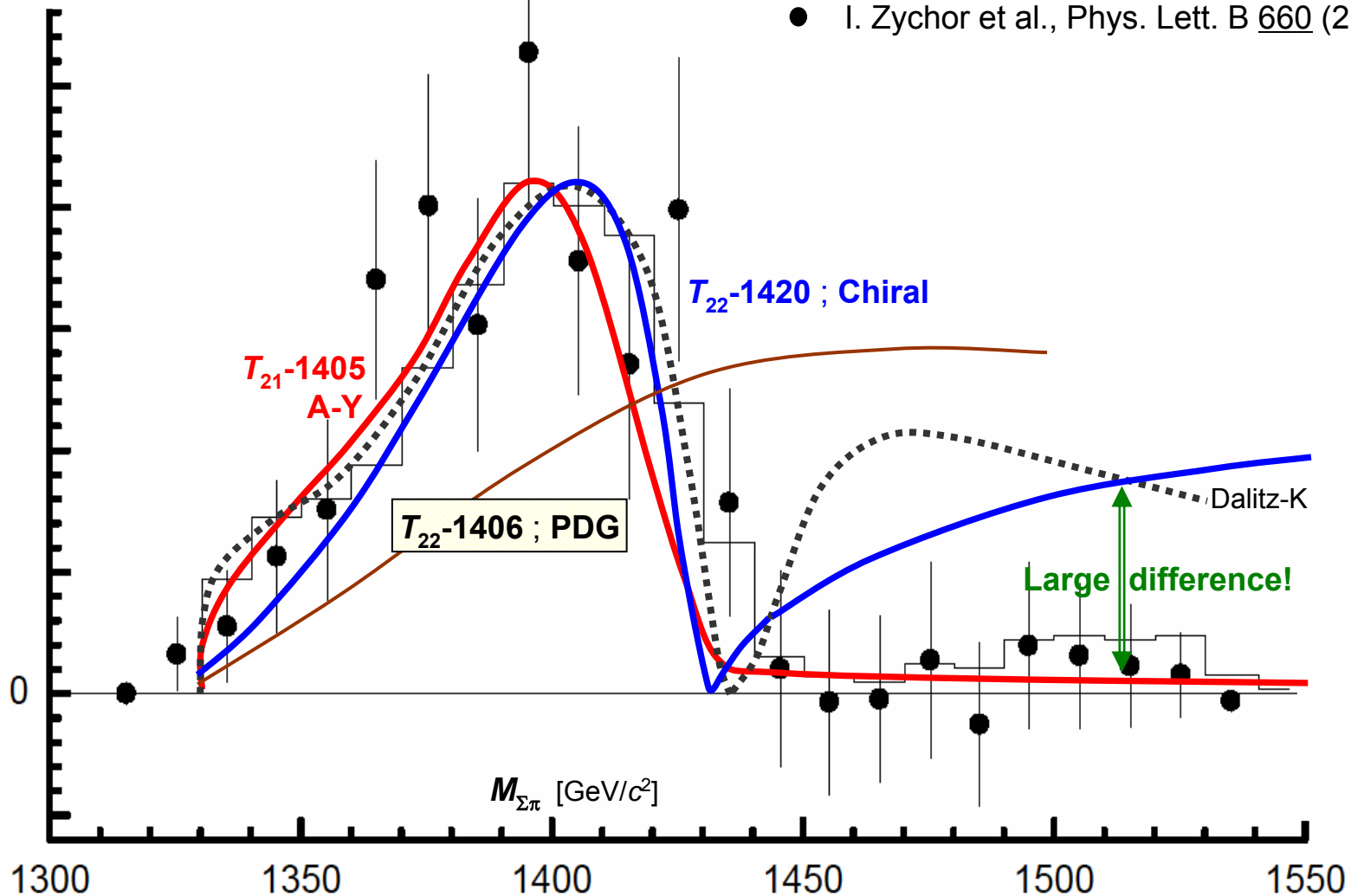
$$|\Sigma^+\pi^+\rangle = \sqrt{\frac{1}{3}}|I=0\rangle + \sqrt{\frac{1}{2}}|I=1\rangle + \sqrt{\frac{1}{6}}|I=2\rangle$$

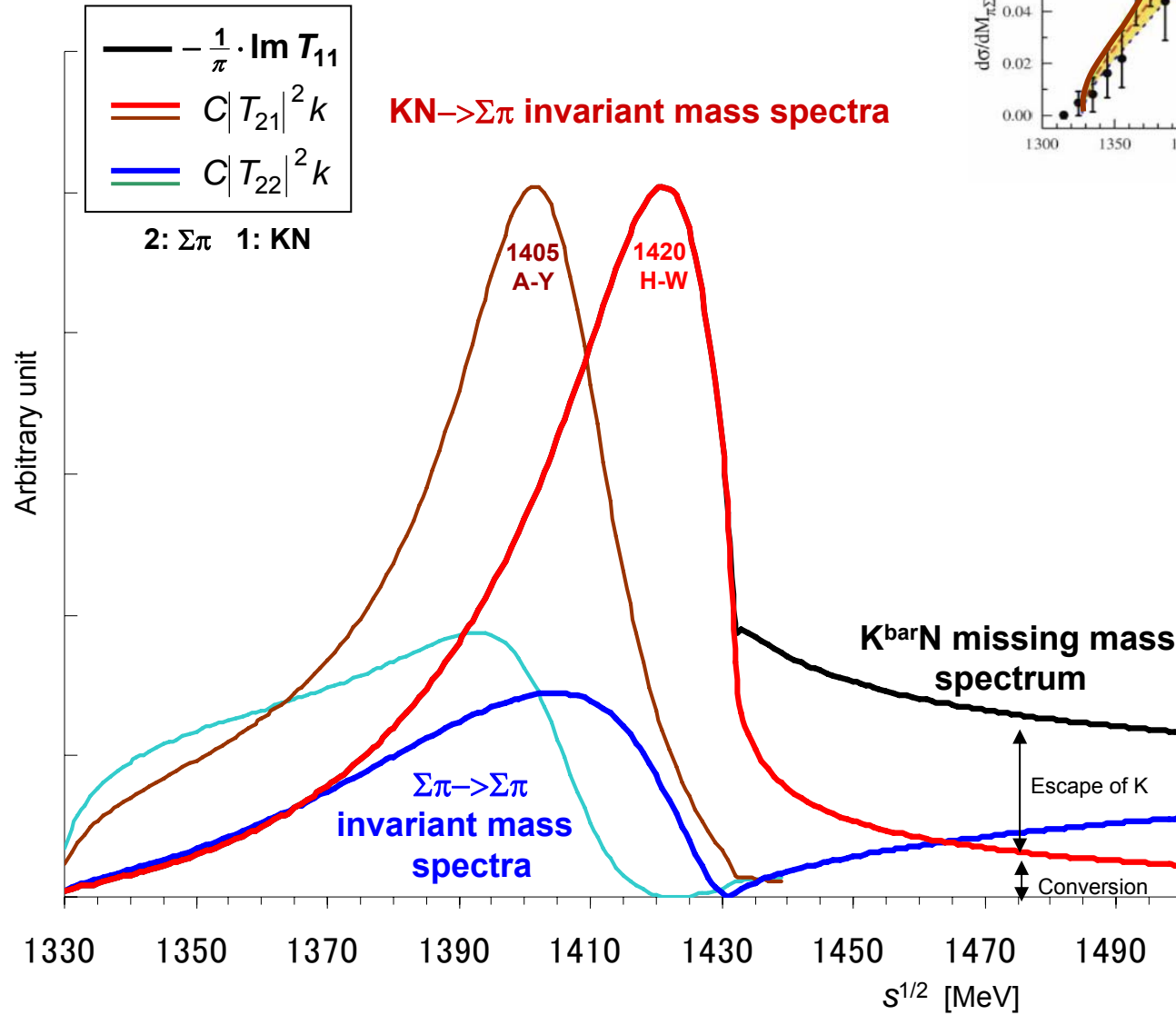
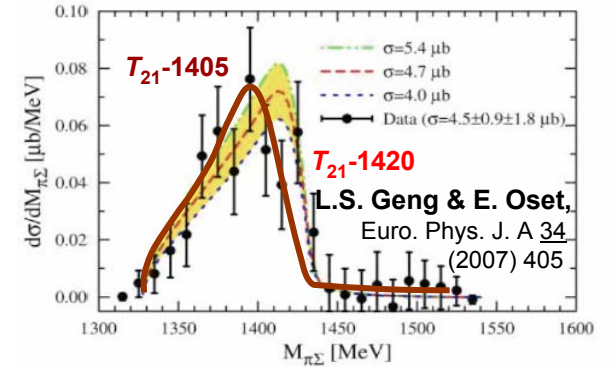
$$|\Sigma^0\pi^0\rangle = -\sqrt{\frac{1}{3}}|I=0\rangle + \sqrt{\frac{2}{3}}|I=2\rangle$$

$$|\Sigma^-\pi^+\rangle = \sqrt{\frac{1}{3}}|I=0\rangle - \sqrt{\frac{1}{2}}|I=1\rangle + \sqrt{\frac{1}{6}}|I=2\rangle$$

— R.J. Hemingway, Nucl. Phys. B253 (1985) 742

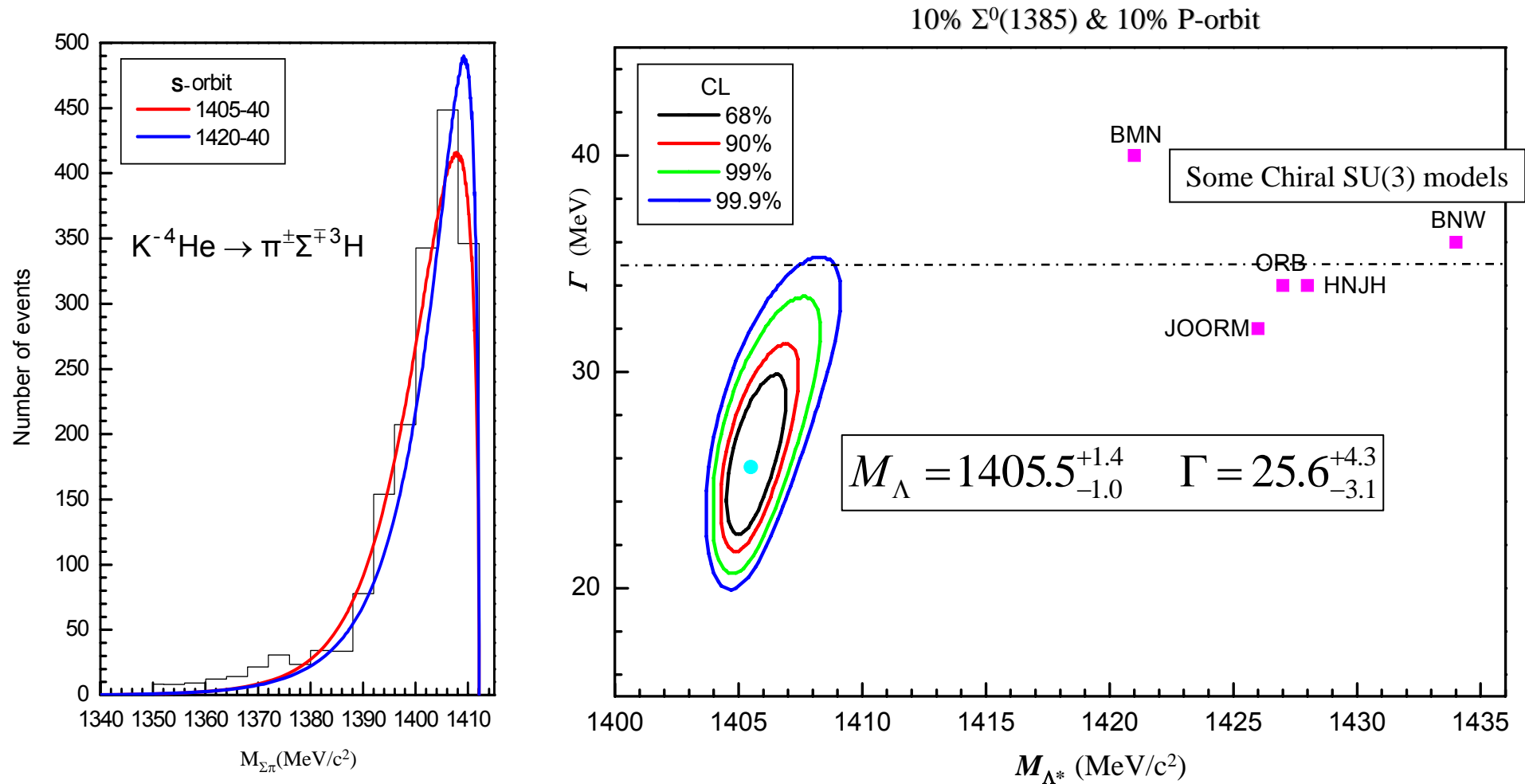
● I. Zychor et al., Phys. Lett. B 660 (2008) 167



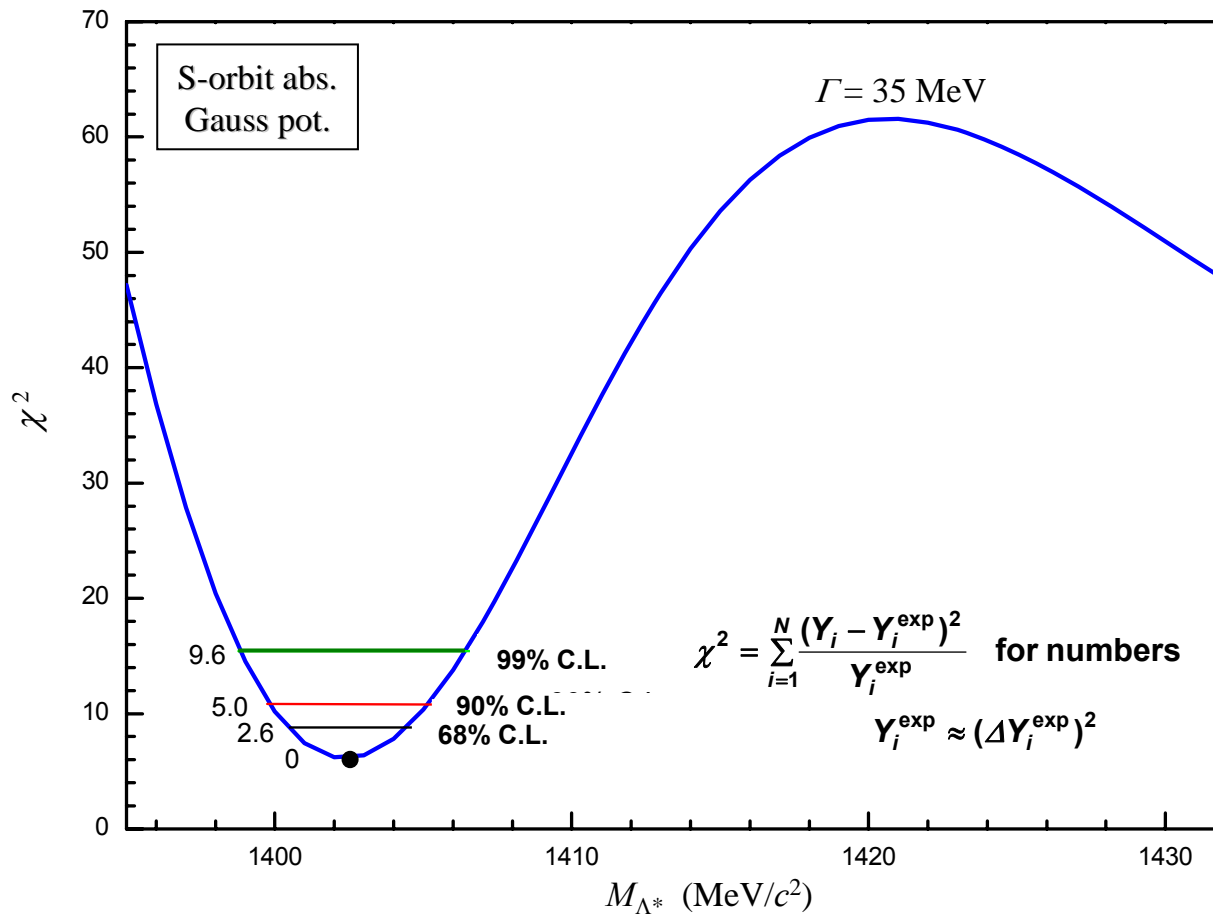


# $\Sigma\pi$ invariant mass from stopped $K^-$ on ${}^4\text{He}$

Data : B. Riley et al., Phys. Rev. D 11 (1975) 3065



# $\chi^2$ values for $\Sigma\pi$ invariant-mass spectra



# K-D atom : n-spectator resonant process

$$\frac{d^2\Gamma}{dk_\Sigma dk_n} = \text{const} \cdot \left| \tilde{g}(k') T_{\Sigma\pi, pK^-}(k_2, \gamma_1) \tilde{g}(k_n/2) \right|^2 k_\Sigma k_n E_\pi \left| F_0(k_n) \right|^2$$

$$X_0 = \cos\theta_{\Sigma n} = \frac{(M_d c^2 + m_K c^2 - E_\Sigma - E_n)^2 - \{m_\pi^2 c^4 + \hbar^2 c^2 (k_\Sigma^2 + k_n^2)\}}{2\hbar^2 c^2 k_\Sigma k_n}$$

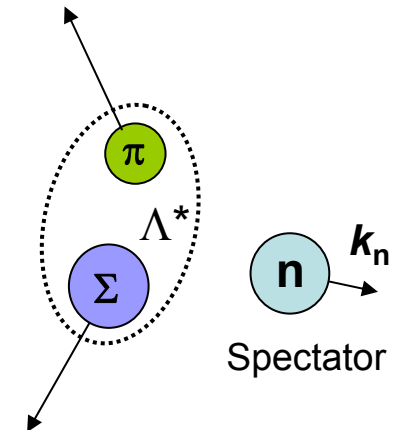
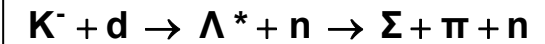
$|X_0| \leq 1$  is kinematically allowed.

$$\tilde{g}(k') = \frac{\Lambda^2}{\Lambda^2 + k'^2}, \quad k' = \sqrt{k_\Sigma^2 + \frac{1}{4}k_n^2 + k_\Sigma k_n X_0}$$

$$E_\pi = \sqrt{m_\pi^2 c^4 + \hbar^2 c^2 k_\pi^2}, \quad k_\pi = \sqrt{k_\Sigma^2 + k_n^2 + 2k_\Sigma k_n X_0}$$

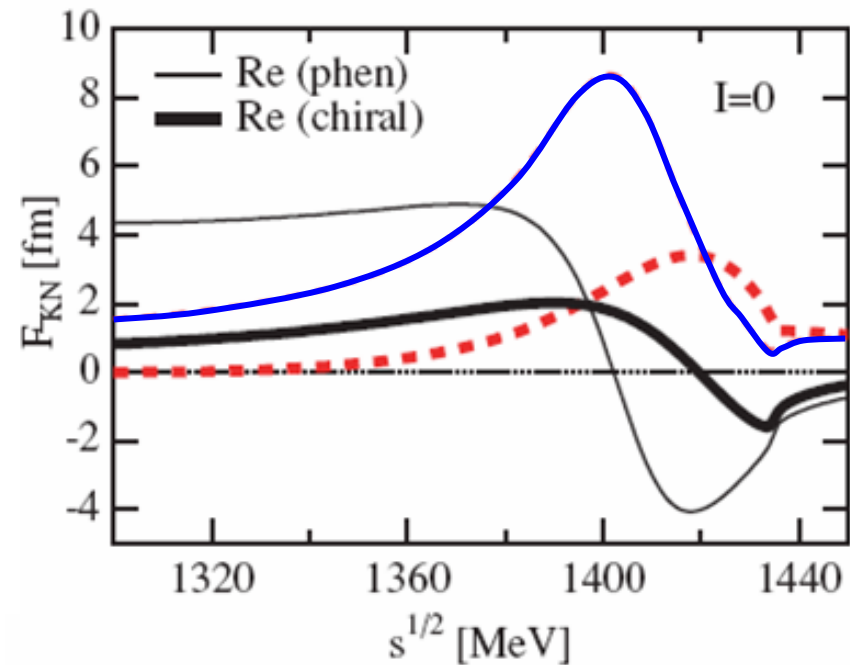
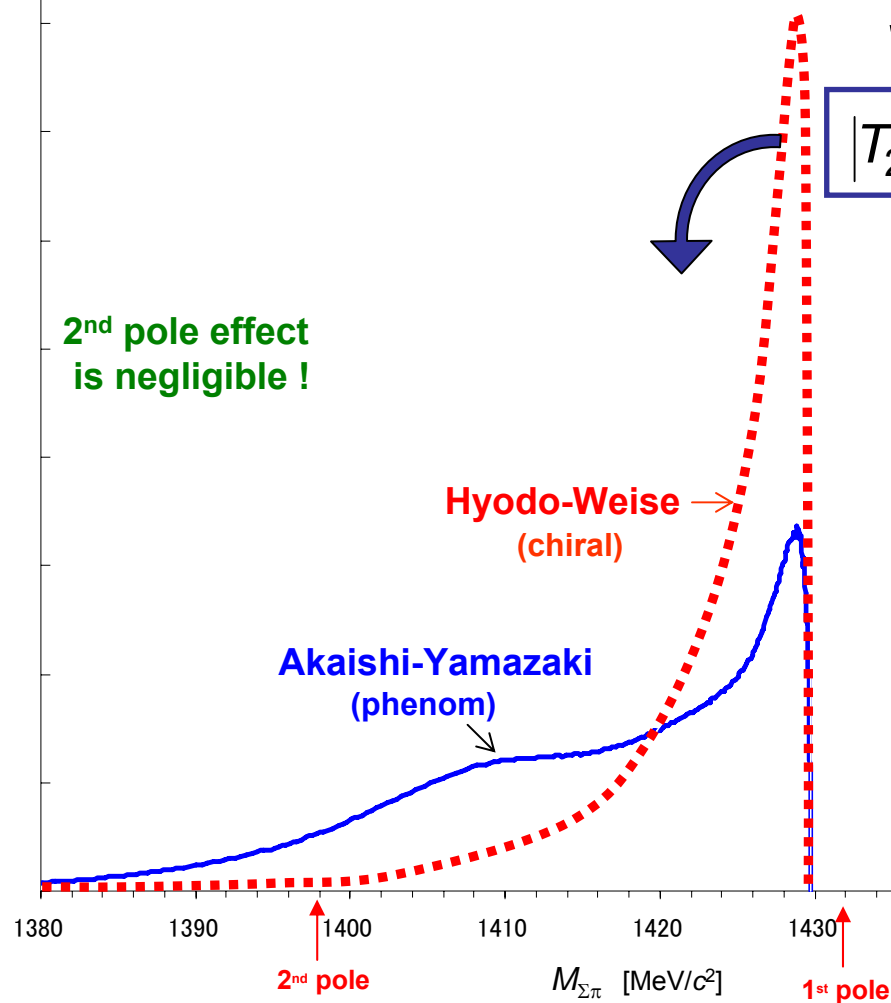
Momentum distribution of D

$$\left| F_0(k_n) \right|^2 \propto \left| \int_0^\infty j_0(k_n r) U_0(r) r dr \right|^2$$



# $\Sigma\pi$ invariant-mass spectrum

from stopped  $K^-$  on D



# Concluding remarks

The  $\Lambda^*$  resonance forms the basic structure of  $K\text{-}pp$ .

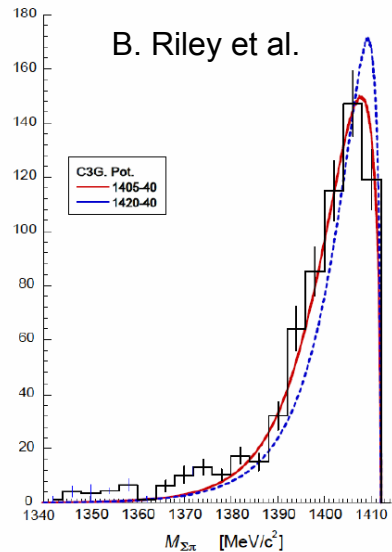
The observed  $\Lambda(1405)$  is of single-pole nature, since the 2<sup>nd</sup> pole (due to strongly energy-dependent interaction) is irrelevant to any peaked data.

It is virtually important to distinguish the mass of so-called  $\Lambda(1405)$ , 1405 MeV or 1420 MeV, by considering " $T_{21}/T_{22}$ " problem.

A conversion  $M_{\Sigma\pi}$  spectrum from stopped  $K^-$  absorption

supports the "1405 Ansatz".

Stopped  $K^-$  on  ${}^4\text{He}$



$$M_{\Lambda^*} = 1405.5^{+1.4}_{-1.0} \text{ MeV and } \Gamma = 25.6^{+4}_{-3} \text{ MeV}$$

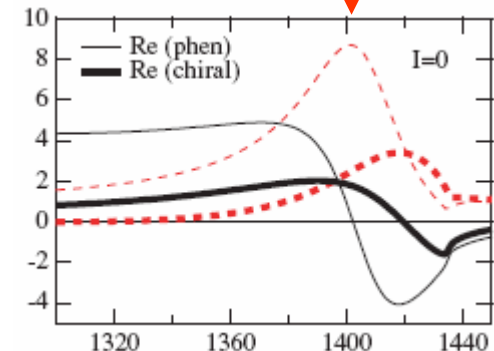
J. Esmaili et al., arXiv:0906.0505

$$|T_{21}|^2 \text{Im}G_2 = \text{Im}T_{11}$$

Stopped  $K^-$  on  $D$

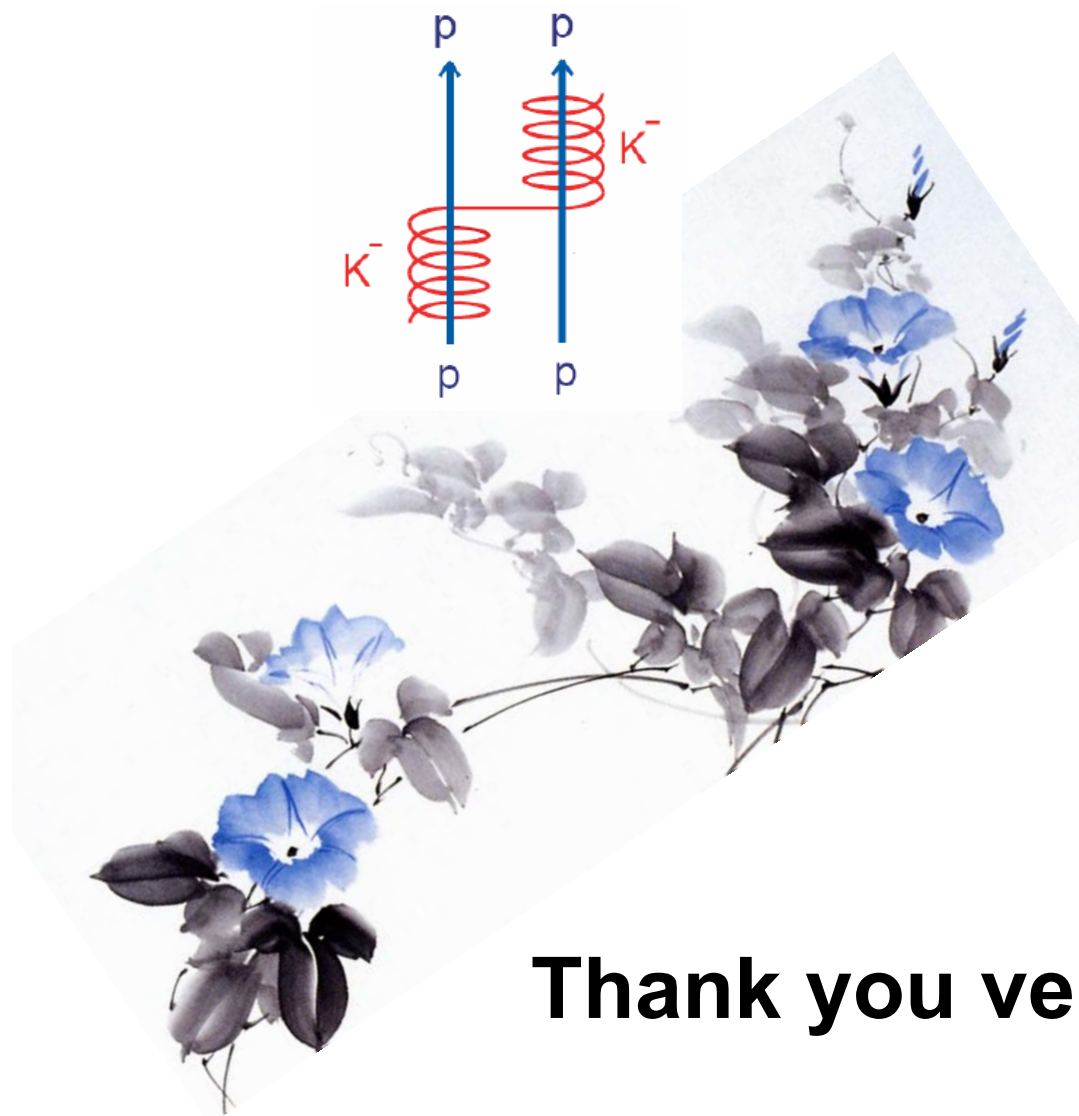
would provide a decisive datum.

T. Suzuki et al., on 15 Oct.



## Acknowledgments

M. Kawai, O. Morimatsu,  
K.S. Myint, M. Obu, M. Wada



**Thank you very much!**

# Moving "decaying-state" pole

Y. Akaishi, Khin Swe Myint & T. Yamazaki, Proc. Jpn Acad. B 84 (2008) 264

Reference spectra

$$|T(z; V(Z^{\text{ref}}))|^2$$

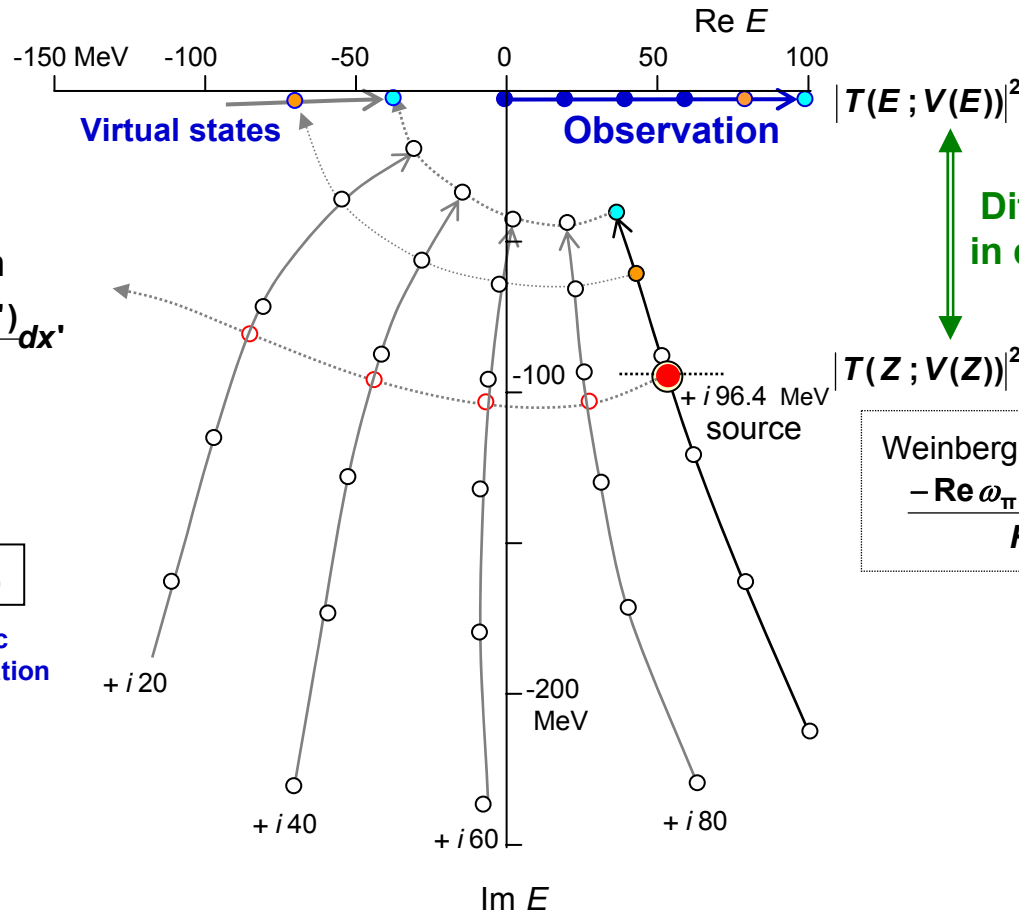
$$\omega_\pi = m_\pi + Z^{\text{ref}}$$

Observed spectrum

$$|T(z; V(E^{\text{obs}}))|^2$$

at  $z = E^{\text{obs}}$

$\Sigma\pi$  single channel



Dispersion relation

$$\text{Re } f(x) = \frac{2}{\pi} P \int_0^\infty \frac{x' \text{Im } f(x')}{x'^2 - x^2} dx'$$

Cross section

Optical theorem

$$\text{Im } f(E) \Rightarrow \text{Re } f(E)$$

Analytic continuation

$f(z)$

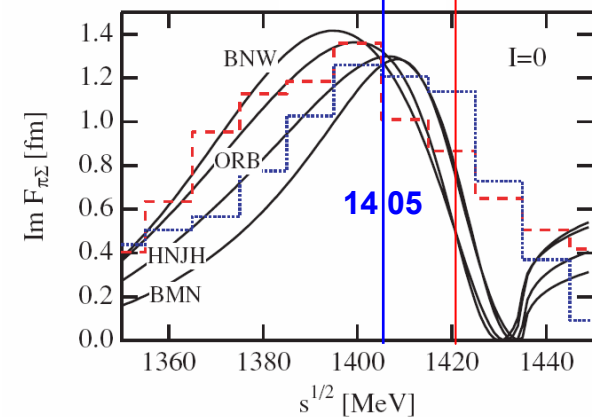
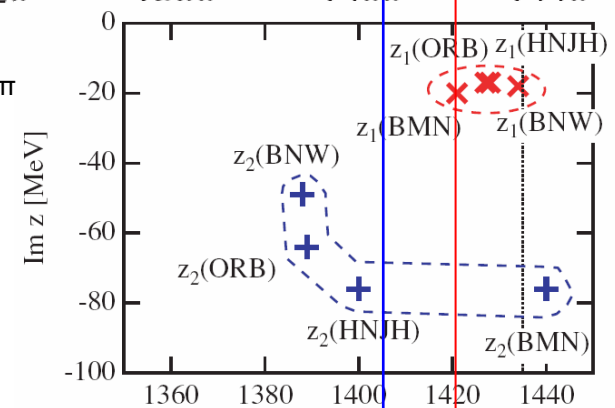
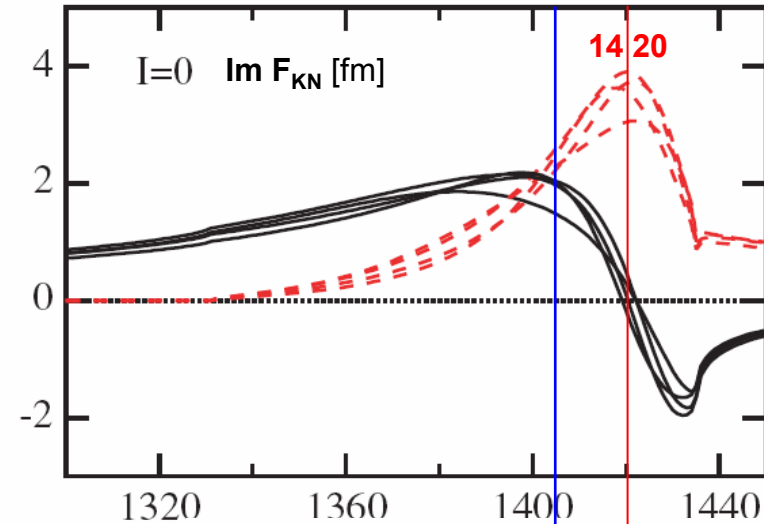
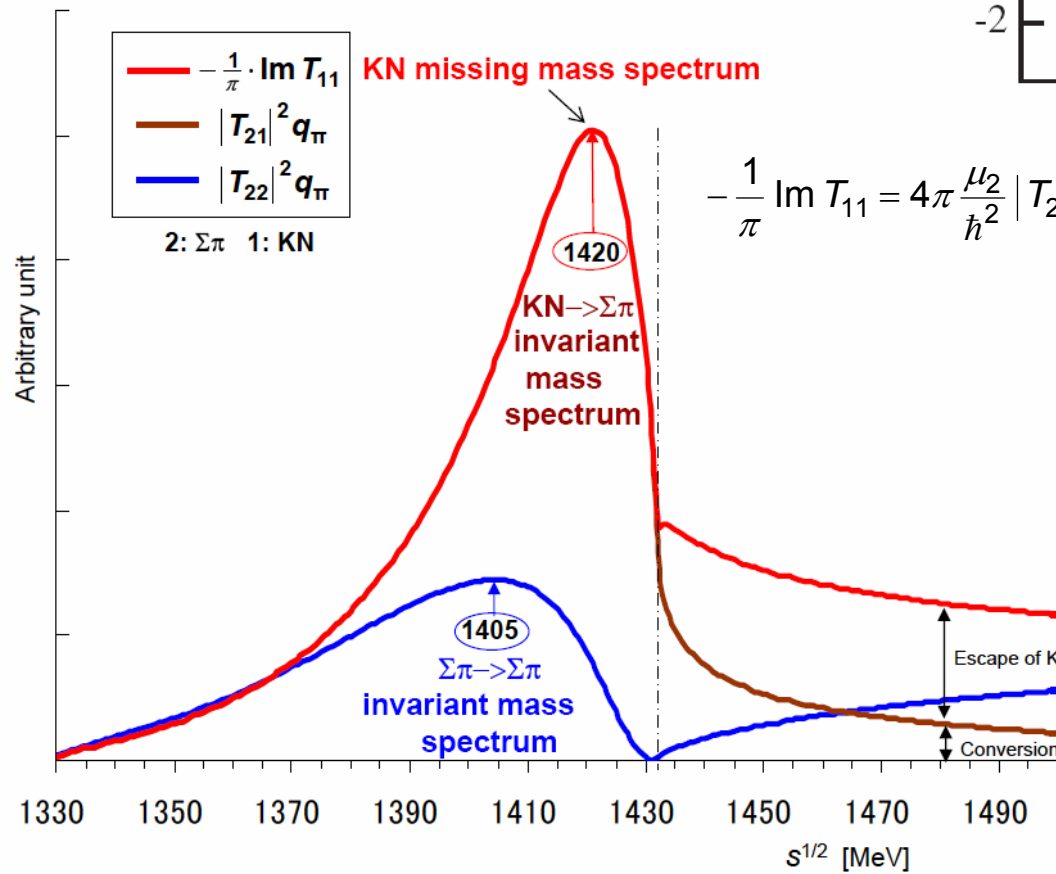
Pole

Weinberg-Tomozawa

$$\frac{-\text{Re } \omega_\pi - i \text{Im } \omega_\pi}{F_\pi^2}$$

# Observables of KN- $\Sigma\pi$ system

Hyodo-Weise's chiral SU(3) dynamics



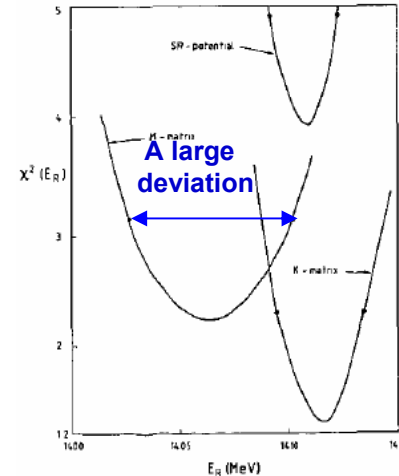
# The PDG value of $\Lambda(1405)$

Mass	$1406.5 \pm 4.0$ MeV
Width	$50 \pm 2$ MeV

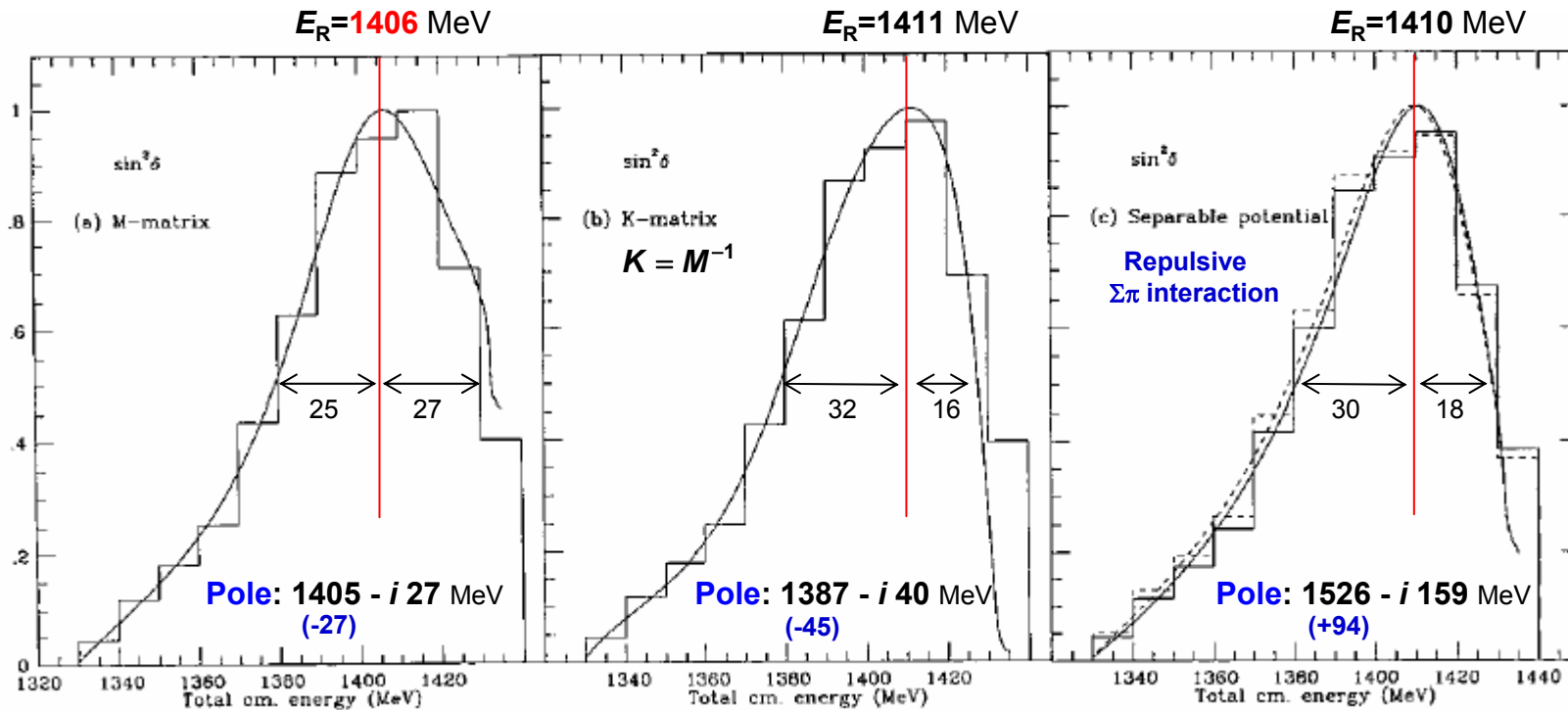
R.H. Dalitz and A. Deloff, J. Phys. G 17 (1991) 289

$$\sigma_{\text{tot}} = \frac{4\pi}{k^2} \sin^2 \delta, \quad \sin^2 \delta_R = \frac{\Gamma^2 / 4}{(E - E_R)^2 + \Gamma^2 / 4}$$

$$\delta_R = \frac{\pi}{2} \text{ at } E_R$$



Constraint :  
 $a = -1.54 + i 0.74$  fm

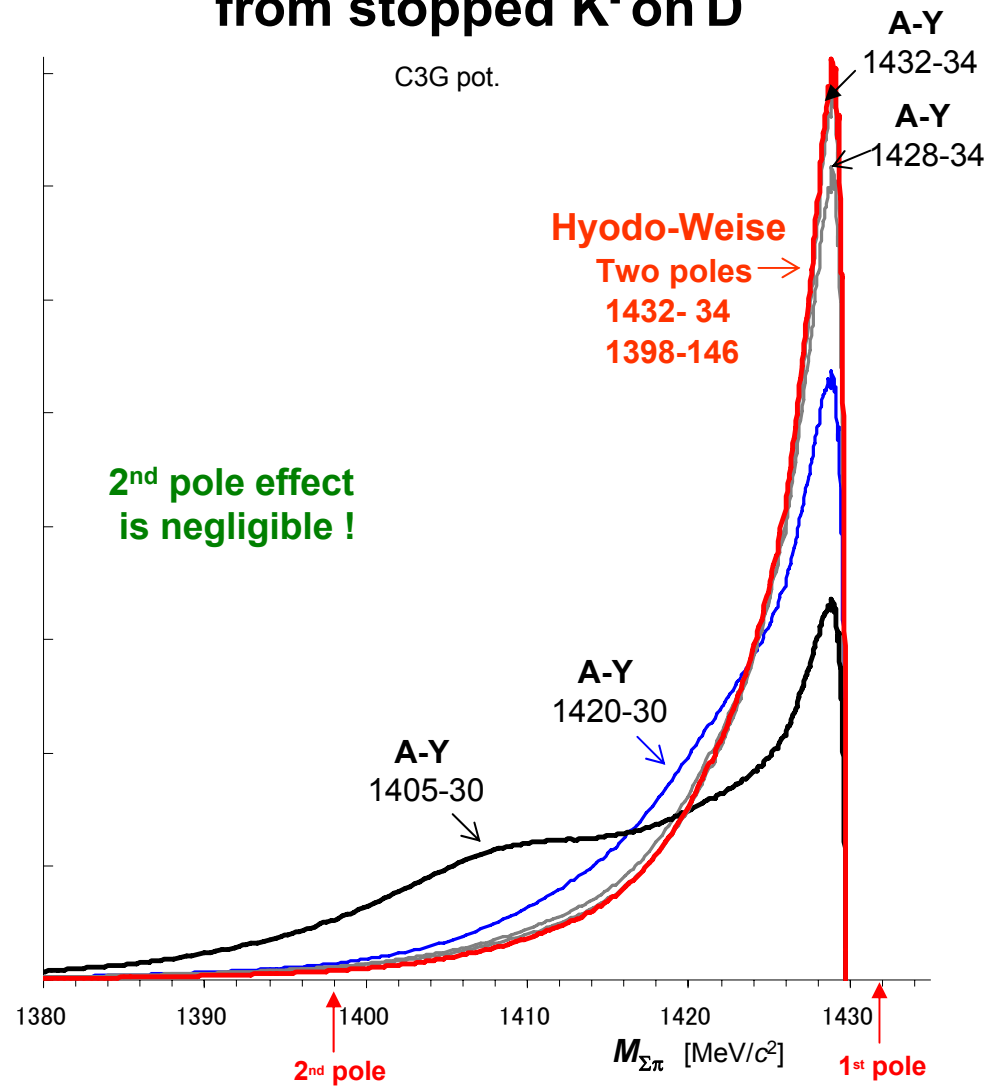


Dalitz's strong preference!

# Two-pole form of $\Lambda(1405)$

T. Hyodo and W. Weise, Phys. Rev. C 77 (2008) 035204

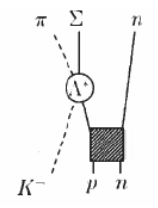
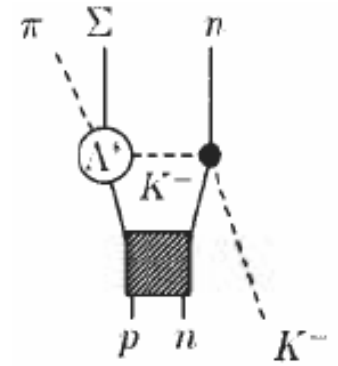
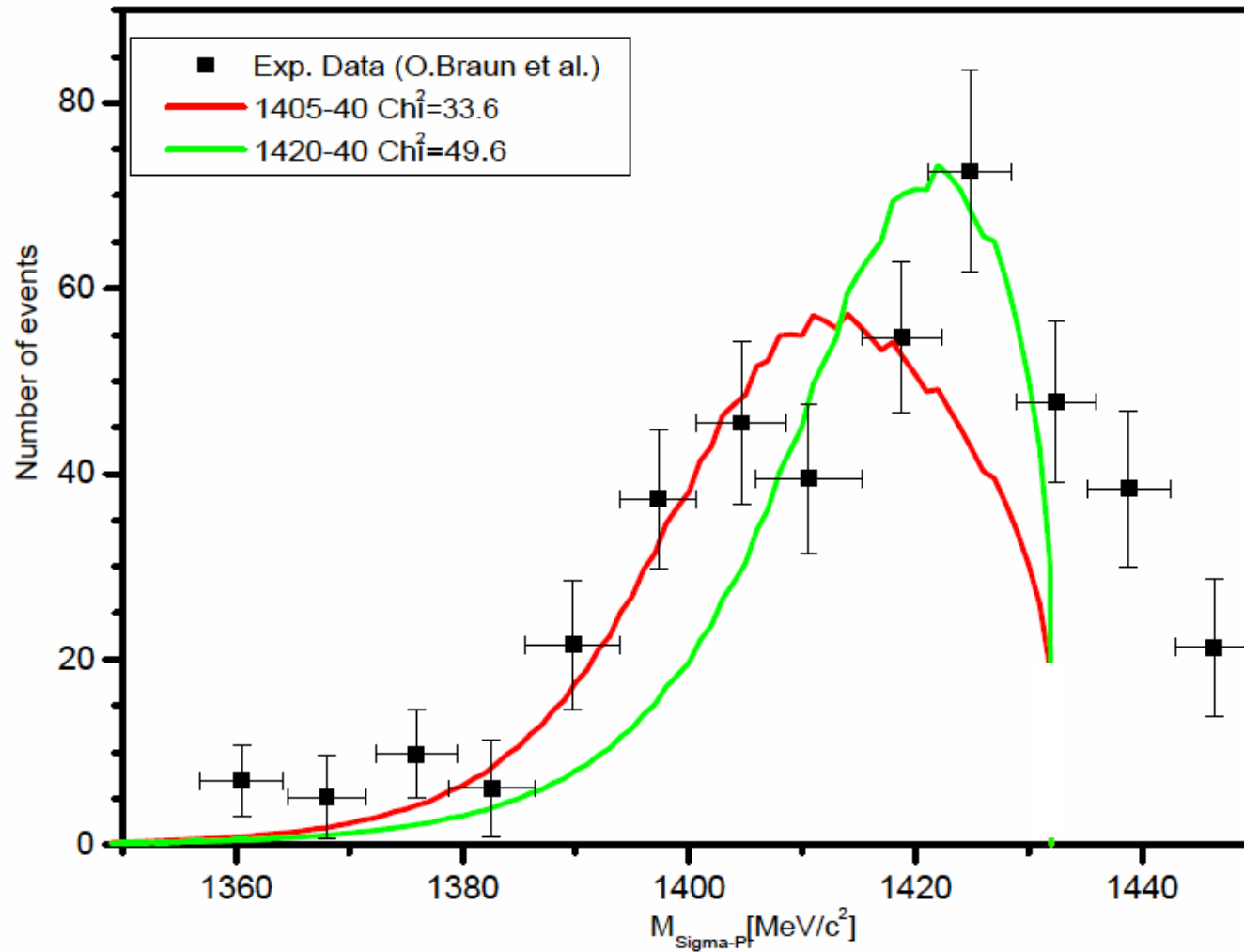
$\Sigma\pi$  invariant-mass spectrum  
from stopped  $K^-$  on D



# $K^-d \rightarrow \Sigma^- \pi^+ n$

$P_K = 760 \text{ MeV}/c$

O. Braun et al.,  
Nucl. Phys. B129 (1977) 1



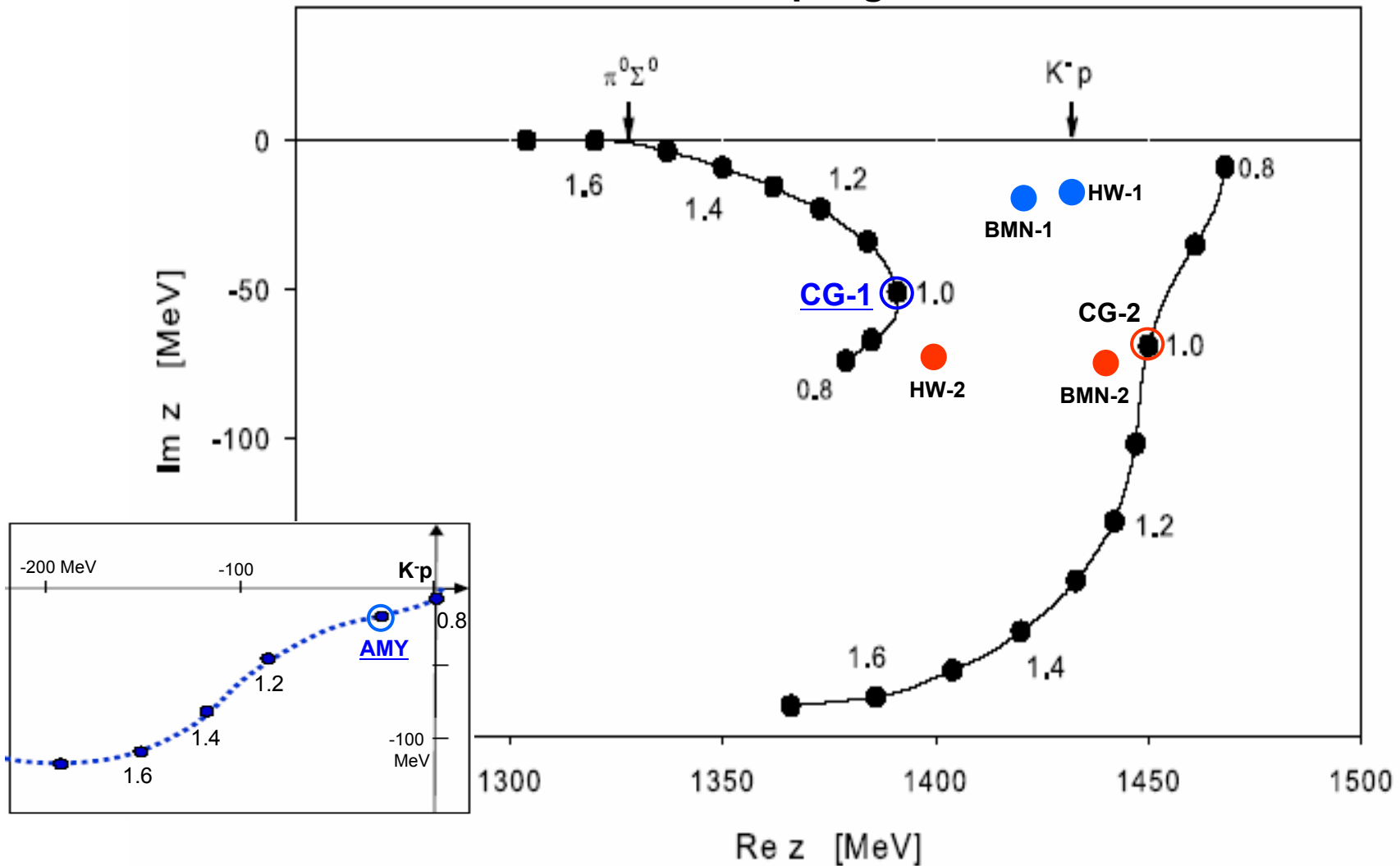
# Trajectories of Gamow poles

A. Cieply & A. Gal, arXiv:0809.0422v1

1<sup>st</sup> : 1450-69*i*

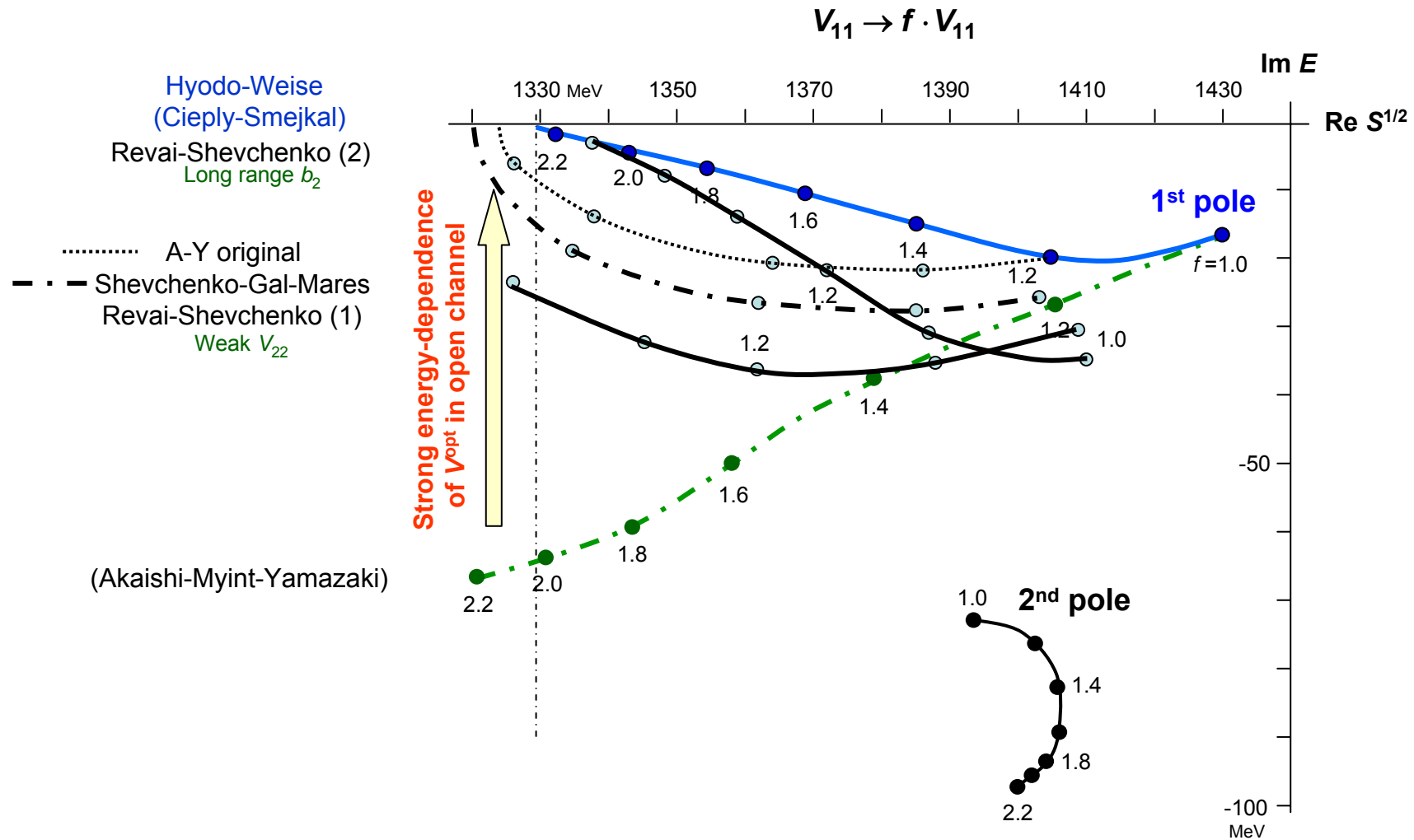
2<sup>nd</sup> : 1391-51*i*

KN coupling scaled



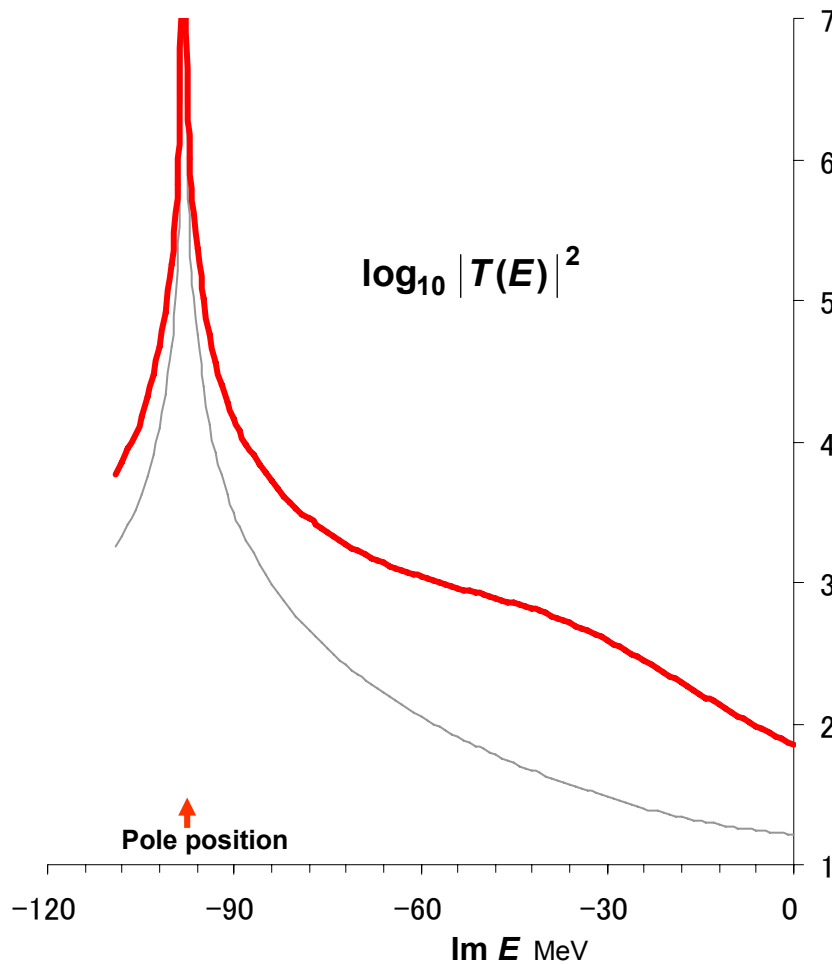
# Trajectories of Gamow poles

Hyodo-Weise's two-channel model



# Effect of Revai-Shevchenko's 2<sup>nd</sup> pole

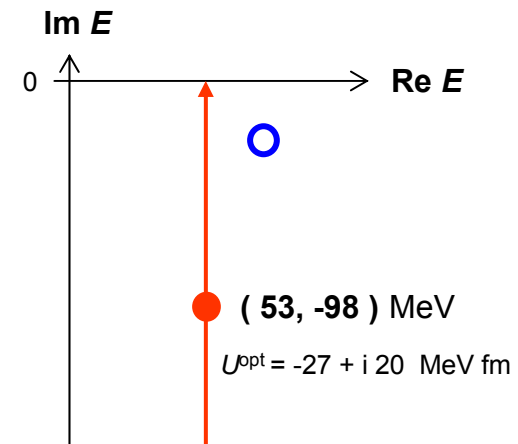
$\Sigma\pi$  single channel



Optical potential

$$s_{22}^{\text{opt}}(E) = s_{22} - s_{21} \frac{\Lambda^2}{(\Lambda - i\kappa_1)^2 + s_{11}\Lambda^2} s_{12}$$

$$E = \frac{\hbar^2}{2\mu_2} \kappa_2^2 = \frac{\hbar^2}{2\mu_1} \kappa_1^2 + Q$$



$$\langle k' | v_{ij} | k \rangle = \tilde{g}_i(k') U_{ij} \tilde{g}_j(k),$$

$$\tilde{g}_1(k) = \frac{1}{\beta_1^2 + k^2}, \quad \tilde{g}_2(k) = \frac{1}{\beta_2^2 + k^2} + \frac{s \beta_2^2}{(\beta_2^2 + k^2)^2}$$

$$\beta_1 = 631_{\text{MeV}} / \hbar c$$

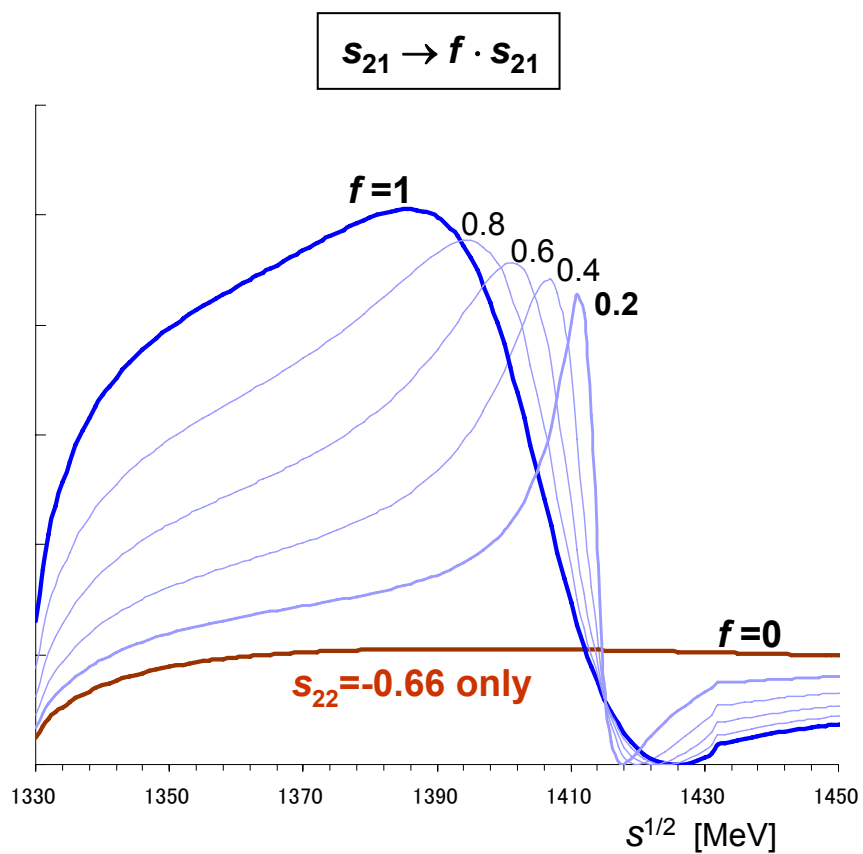
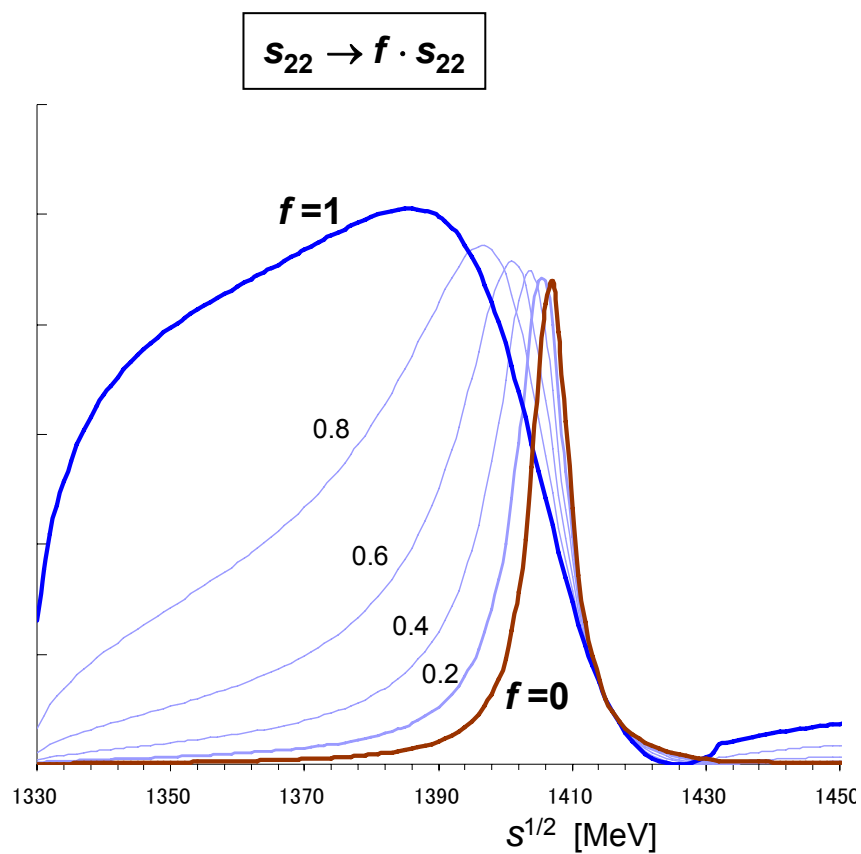
$$\beta_2 = 197_{\text{MeV}} / \hbar c$$



# $\Sigma\pi$ invariant mass spectrum

$$|T_{\Sigma\pi,\Sigma\pi}(\sqrt{s})|^2 \bar{q}$$

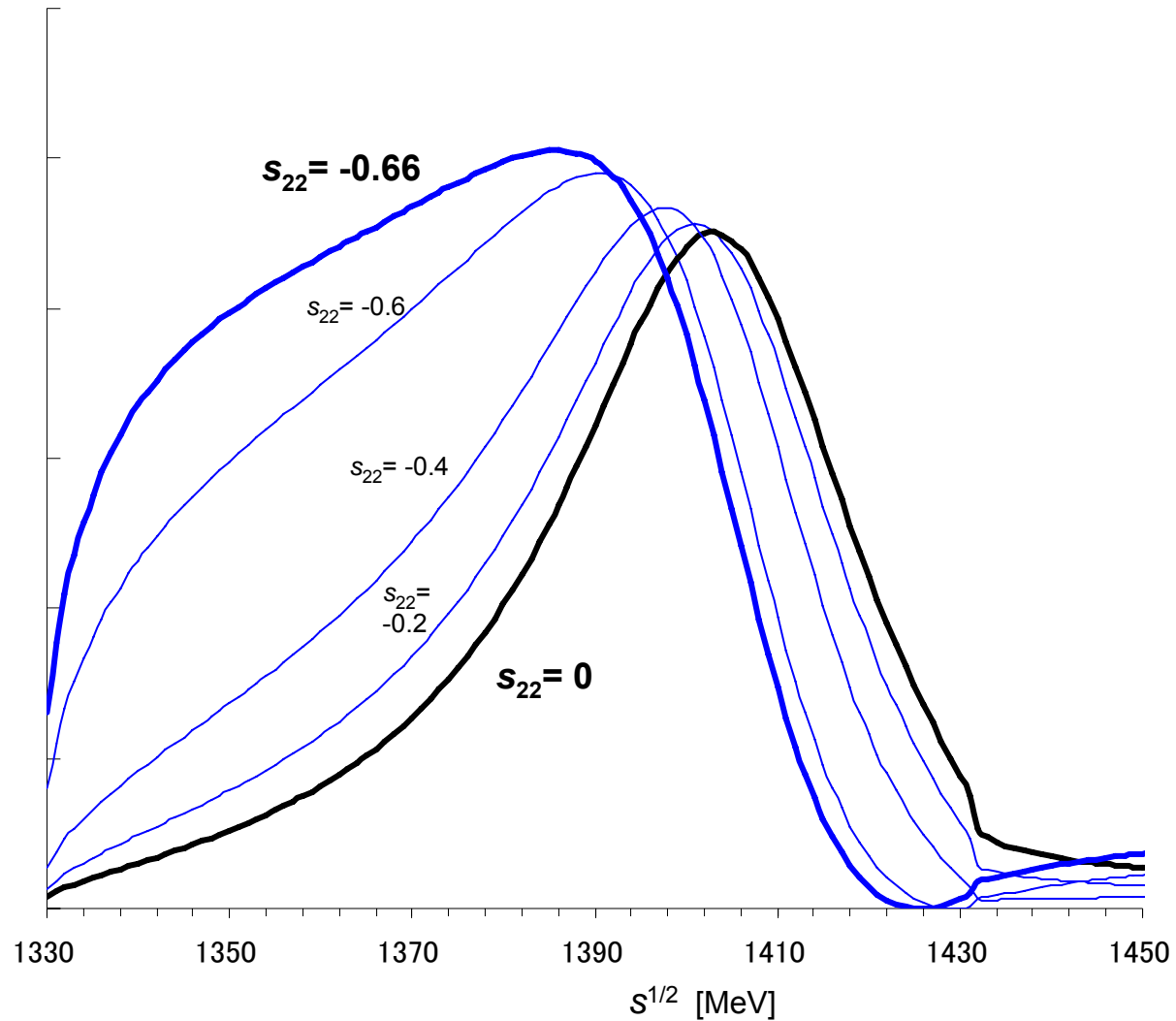
AMY interaction  
 $m_B c^2 = 770$  MeV



# $\Sigma\pi$ invariant mass spectrum

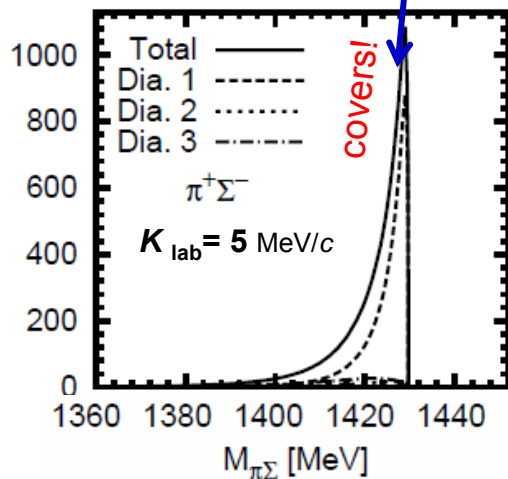
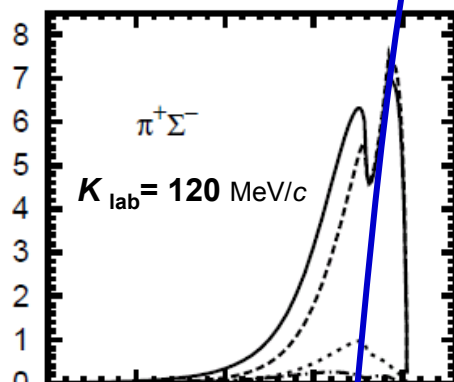
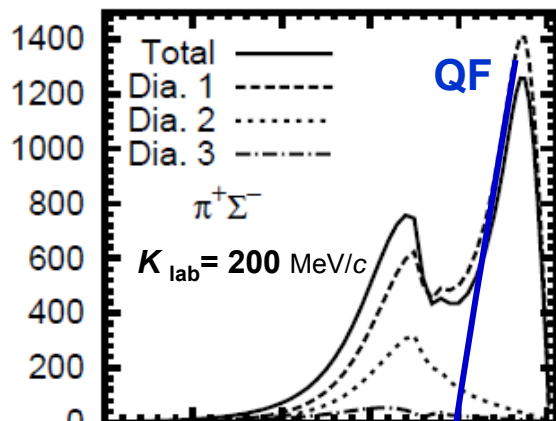
$$|T_{\Sigma\pi,\Sigma\pi}(\sqrt{s})|^2 \bar{q}$$

$m_B = 770 \text{ MeV}$



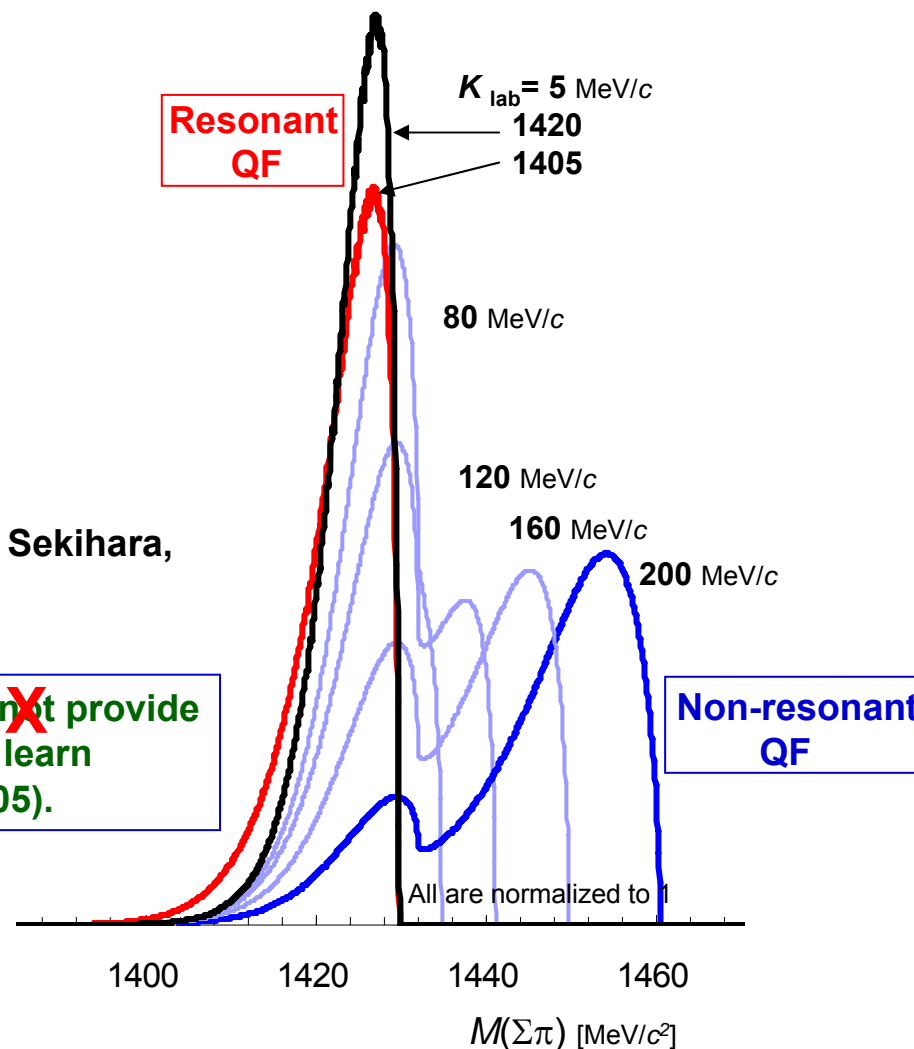
# Low-momentum K<sup>+</sup>D reaction

Spectator process dominates!



D. Jido, E. Oset & T. Sekihara,  
Eur. Phys. J.

Stopped kaon does **not** provide  
a good set up to learn  
about  $\Lambda(1405)$ .



cf. P. Kienle, Y. Akaishi & T. Yamazaki, Phys. Lett. B 632 (2006) 187  
 Dalitz plot for spectator process

O. Braun et al., Nucl. Phys. B 124 (1977) 45

