Chiral model for $\bar{K}N$ interactions

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Motivation

- meson-baryon dynamics at low energies is a testing ground of the CHPT incorporating the QCD symmetries in it's nonpertubative region
- new DEAR experimental data on the 1s energy level characteristics in the kaonic hydrogen atom (how they fit in our $\bar{K}N$ picture?); more precise data on kaonic hydrogen and deuterium from SIDDHARTA expected soon
- $\bar{K}N$ interaction is related to the properties of kaons in the nuclear medium (deep or shallow K^--nuclear optical potential?, $\bar{K}NN$ bound states?)

$\bar{K}N$: strongly interacting multichannel system with an s-wave resonance, the $\Lambda(1405)$, just below the K^-p threshold $=>$ coupled-channels techniques are useful

OUR AIM: simultaneous description of the K-atomic and low energy K^-p data to fix model parameters, then the model is used to study meson-baryon resonances and the behaviour of the K^-n amplitude

Chiral model for $\bar{K}N$

We employ the effective chiral model of Kaiser, Siegel and Weise (1995) with the s-wave meson-baryon lagrangian up to the second order:

$$
\begin{aligned}
\mathcal{L}_{\phi B} &= \mathcal{L}_{\phi B}^{(1)} + \mathcal{L}_{\phi B}^{(2)} \\
\mathcal{L}_{\phi B}^{(1)} &= i\langle \bar{B}\gamma_\mu [D^\mu, B]\rangle - M_0\langle \bar{B}B\rangle - \frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle - \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle \\
\mathcal{L}_{\phi B}^{(2)} &= b_D\langle \bar{B}\{\chi_+, B\}\rangle + b_F\langle \bar{B}[\chi_+, B]\rangle + b_0\langle \bar{B}B\rangle\langle \chi_+\rangle \\
&+ d_D\langle \bar{B}\{(u^2 + (v\cdot u)^2), B\}\rangle + d_F\langle \bar{B}[(u^2 + (v\cdot u)^2), B]\rangle \\
&+ d_0\langle \bar{B}B\rangle\langle u^2 + (v\cdot u)^2\rangle + d_2\langle \bar{B}(u_\mu B u^\mu + (v\cdot u)B(v\cdot u))\rangle \\
&+ d_1(\langle \bar{B}u_\mu\rangle\langle u^\mu B\rangle + \langle \bar{B}(v\cdot u)\rangle\langle (v\cdot u)B\rangle)
\end{aligned}
$$

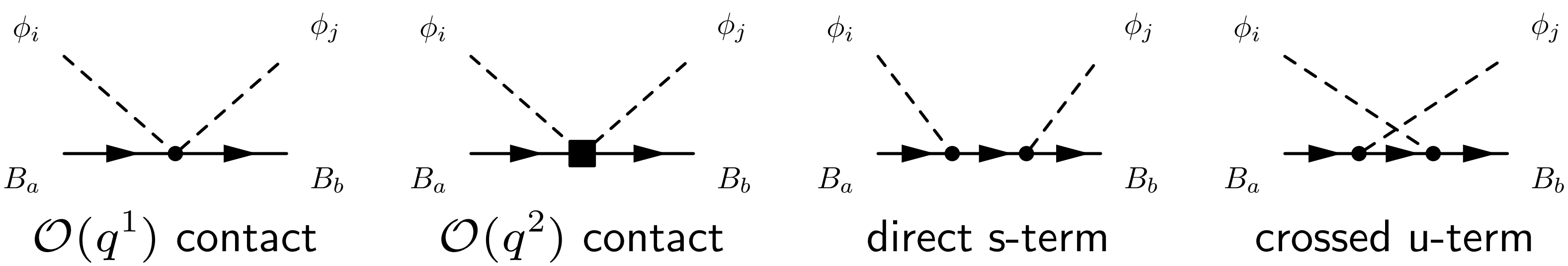
$\langle \ldots \rangle$ denotes the trace in flavor space flavor-matrices Φ and B of the lightest meson and baryon octets:

$$\phi = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix} \quad B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

10 coupled $Q = 0$ channels:	$\pi^0\Lambda$	$\sim$ 1250 MeV
	$\pi^0\Sigma^0$, $\pi^-\Sigma^+$, $\pi^+\Sigma^-$	$\sim$ 1330 MeV
	K^-p, $\bar{K}^0 n$	$\sim$ 1430 MeV
	$\eta\Lambda$, $\eta\Sigma^0$	$\sim$ 1700 MeV
	$K^0\Xi^0$, $K^+\Xi^-$	$\sim$ 1800 MeV
6 coupled $Q = -1$ channels:	$\pi^-\Lambda$	$\sim$ 1250 MeV
	$\pi^-\Sigma^0$, $\pi^0\Sigma^-$	$\sim$ 1330 MeV
	K^-n	$\sim$ 1430 MeV
	$\eta\Sigma^-$	$\sim$ 1700 MeV
	$K^0\Xi^-$	$\sim$ 1800 MeV

Parameters: f - pseudoscalar decay constant
M_0 - baryon octet mass
$D \simeq 3/4$, $F \simeq 1/2$ - axial vector couplings, $g_A = F + D$
b_0, b_D, b_F, five d's - second order couplings

Schematic picture (taken from Borasoy, Nissler, Weise - 2005):



Problem: χPT is not applicable in the resonance region!

Solution: effective separable potentials constructed to match the chiral ϕB amplitudes up to $\mathcal{O}(q^2)$, the loop series solved exactly using the Lippmann-Schwinger (or Bethe-Salpeter) equation the separable potential:

$$V_{ij}(k,k') = \sqrt{\frac{1}{2\omega_i}\frac{M_i}{E_i}}\, g_i(k)\, \frac{C_{ij}}{f^2}\, g_j(k')\, \sqrt{\frac{1}{2\omega_j}\frac{M_j}{E_j}}, \quad g_j(k) = \frac{1}{1+(k/\alpha_j)^2}$$

kinematical factors guarantie a proper relativistic flux normalization with ω_i, M_i and E_i denoting the meson energy, the baryon mass and energy in the c.m. system of channel i

In the Born approximation the potentials $V_{ij}(k,k')$ give the same (up to $\mathcal{O}(q^2)$) s-wave scattering lengths as are those derived from the chiral lagrangian.

our approach differs from the more popular on-shell scheme based on the Bethe-Salpeter equation and the unitarity relation for the inverse of the T-matrix

advantage: inverse range radii have a better physical meaning than the subtraction constants used in the other scheme and the formfactors $g_j(k)$ account naturally for the off-shell effects coupling matrix C_{ij} is determined by chiral SU(3) symmetry and includes terms up to second order in the meson c.m. kinetic energies

$$\begin{aligned} C_{ij} &= C_{ij}^{(WT)}\,\frac{E_i' + E_j'}{2} + C_{ij}^{(mm)}(m_i^2 + m_j^2) + C_{ij}^{\chi b}(m_K^2 - m_\pi^2) + C_{ij}^{(EE)}\,E_i E_j + \\ &+ C_{ij}^{(s)}\frac{E_i E_j}{2M_0} + C_{ij}^{(u)}\frac{1}{3M_0}\left(2m_i^2 + 2m_j^2 + \frac{m_i^2 m_j^2}{E_i E_j} - \frac{7}{2}E_i E_j\right) \end{aligned}$$

similar coefficients derived by Borasoy, Nissler, Weise (2005)

the chiral lagrangian parameters contribute to the coefficients $C_{ij}^{(\dots)}$

example:

$$C_{K^-p,K^-p} = -E_K' + 4m_K^2(b_D + b_0) - E_K^2(2d_D + 2d_0 + d_1) + (F^2 + \frac{D^2}{3})\frac{E_K^2}{2M_0}$$

$\bar{K}N$ data fits

Threshold branching ratios:

$$\gamma = \frac{\sigma(K^-p \to \pi^+\Sigma^-)}{\sigma(K^-p \to \pi^-\Sigma^+)} = 2.36 \pm 0.04 \ ,$$

$$R_c = \frac{\sigma(K^-p \to \text{charged particles})}{\sigma(K^-p \to \text{all})} = 0.664 \pm 0.011 \ ,$$

$$R_n = \frac{\sigma(K^-p \to \pi^0\Lambda)}{\sigma(K^-p \to \text{all neutral states})} = 0.189 \pm 0.015.$$

K^-p cross sections to six different meson-baryon final states:

at the $p_{LAB} = 110$ MeV for the K^-p, $\bar{K}^0n$, $\pi^+\Sigma^-$, $\pi^-\Sigma^+$
at the $p_{LAB} = 200$ MeV for the above channels plus $\pi^0\Lambda$, $\pi^0\Sigma^0$

$\pi\Sigma$ mass distribution:

$$dN_{\pi\Sigma}/dE \sim \left| T_{\pi\Sigma,\pi\Sigma}(I=0) + r_{KN/\pi\Sigma}\, T_{\pi\Sigma,\bar{K}N}(I=0) \right|^2 p_{\pi\Sigma}$$

we fit only the peak position at $\sqrt{s} \sim 1395$ (± 5 MeV)
one more parameter $r_{KN/\pi\Sigma}$ - relates contribution of $\bar{K}N$ and $\pi\Sigma$ states to the resonance

Kaonic hydrogen characteristics:

Recent (2005) measurement by the DEAR collaboration at DAΦNE:

strong interaction energy shift $\Delta E_N(1s) = 193 \pm 37(stat) \pm 6(syst.)$ eV
the decay width $\Gamma(1s) = 249 \pm 111(stat.) \pm 39(syst.)$ eV

our approach - exact solution of the K^-p bound state problem using the Vincent and Phatak method developed in 1974 for the scattering in one channel to deal with the Coulomb singularity in the momentum space, later modified for bound states and multiple channels (A.C. and R.Mach in 1990's)

We have too many parameters. Let's fix some of them:

- D and F were determined in the analysis of semileptonic hyperon decays,
  $D = 0.80$, $F = 0.46$ $\qquad (g_A = F + D = 1.26)$
- b_D and b_F fixed to satisfy the approximate formulas for the baryon mass splittings:

$$m_\Sigma - m_\Lambda = \frac{16}{3} b_D (m_K^2 - m_\pi^2), \qquad m_\Xi - m_N = 8 b_F (m_\pi^2 - m_K^2)$$

- similarly, b_0 and M_0 are determined from the relations

$$\sigma_{\pi N} = -2m_\pi^2(2b_0 + b_D + b_F), \quad m_p = M_0 - 4m_K^2(b_0 + b_D - b_F) - 2m_\pi^2(b_0 + 2b_F)$$

  and we consider $\sigma_{\pi N}$ from 20 to 50 MeV
- Cayley identity reduces the number of d's from 5 to 4 (we set $d_2 = 0$)

11 parameters (the couplings f, d_D, d_F, d_0, d_1, five inverse ranges α_{KN}, $\alpha_{\pi\Lambda}$, $\alpha_{\pi\Sigma}$, $\alpha_{\eta\Lambda/\Sigma}$, $\alpha_{K\Xi}$ and the $\pi\Sigma$ distribution ratio $r_{KN/\pi\Sigma}$) fitted to 16 data points

The fitted $\bar{K}N$ threshold data

$\sigma_{\pi N}$ [MeV]	χ^2/N	ΔE_N [eV]	Γ [eV]	γ	R_c	R_n
20	1.33	214	718	2.368	0.653	0.189
30	1.29	260	692	2.366	0.655	0.188
40	1.35	195	763	2.370	0.654	0.191
50	1.37	289	664	2.366	0.658	0.192
exp	-	193(43)	249(150)	2.36(4)	0.664(11)	0.189(15)

- comparable fits for all $\sigma_{\pi N}$ choices
- reasonable description of the energy shift ΔE_N(1s), decay width Γ(1s) three standard deviations off the DEAR value
- perfect description of the $K^- p$ branching ratios

Inverse range parameters α_j (in MeV):

$\sigma_{\pi N}$ [MeV]	$\alpha_{\pi\Lambda}$	$\alpha_{\pi\Sigma}$	α_{KN}	$\alpha_{\eta\Lambda/\Sigma}$	$\alpha_{K\Xi}$
20	226	579	625	917	260
30	291	601	639	568	151
40	219	640	638	936	226
50	345	600	608	507	152
27 [95KSW]	300	450	760	-	-

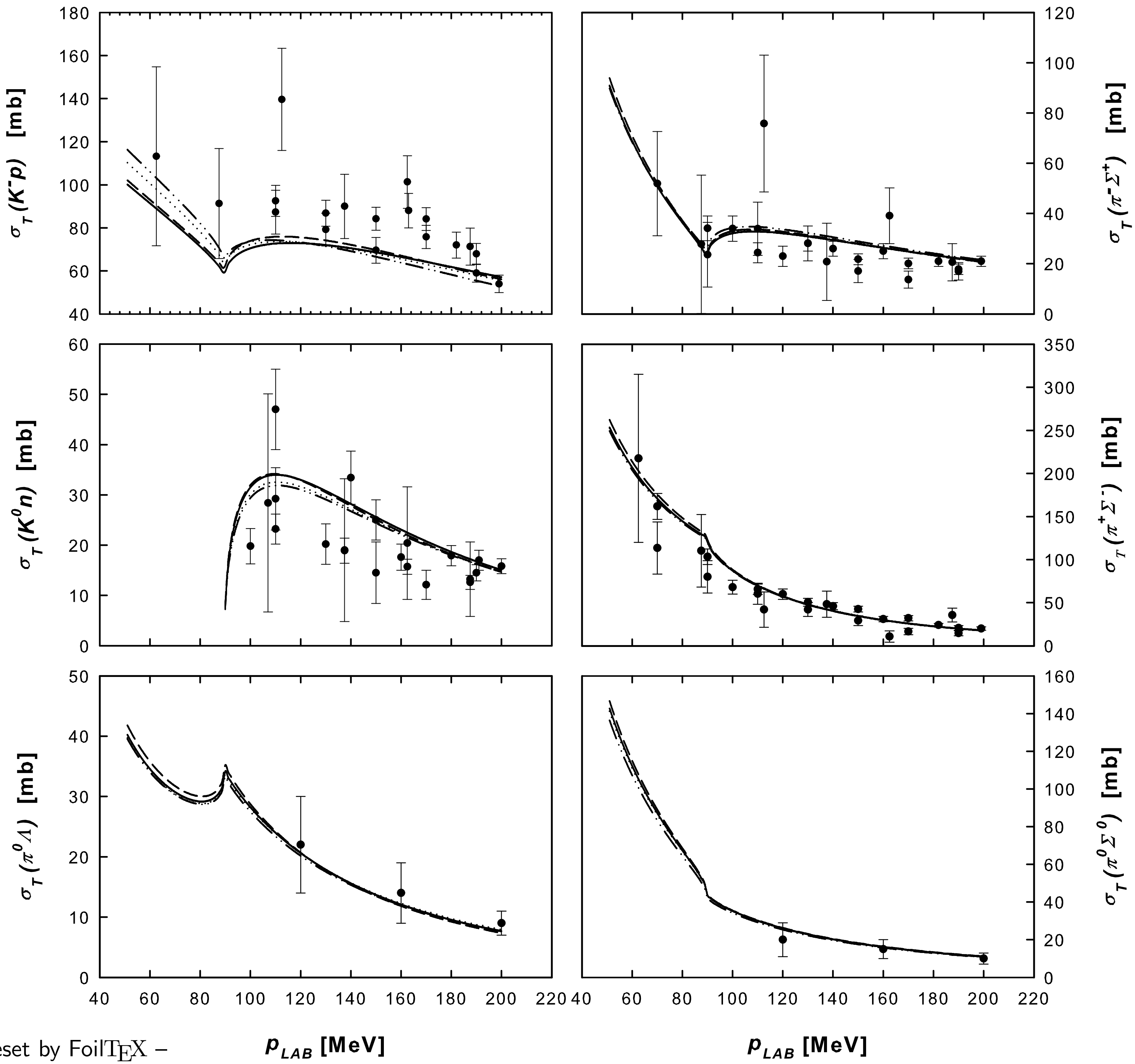
Chiral lagrangian parameters (d's in $1/$GeV):

$\sigma_{\pi N}$ [MeV]	M_0 [MeV]	$a^+_{\pi N}$ $[m_\pi^{-1}]$	f [MeV]	d_0	d_D	d_F	d_1
20	997	-0.009	111.0	-0.108	-0.446	-0.834	0.540
30	864	-0.001	109.1	-0.450	0.026	-0.601	0.235
40	729	-0.007	114.5	-0.492	-0.635	-0.788	0.616
50	594	0.002	107.6	-1.043	0.229	-0.478	0.161
27 [95KSW]	910	-0.002	94.5	-0.71	0.38	-0.43	-0.34

nice reproduction of the πN isospin-even scattering length

$$a^+_{0+} = -(0.25 \pm 0.49) \cdot 10^{-2}\, m_\pi^{-1}$$

$f \approx 110$ MeV in all our fits



$\sigma_T(K^-p)$ [mb]
$\sigma_T(\pi^-\Sigma^+)$ [mb]
$\sigma_T(\bar{K}^0n)$ [mb]
$\sigma_T(\pi^+\Sigma^-)$ [mb]
$\sigma_T(\pi^0\Lambda)$ [mb]
$\sigma_T(\pi^0\Sigma^0)$ [mb]
p_{LAB} [MeV]
p_{LAB} [MeV]

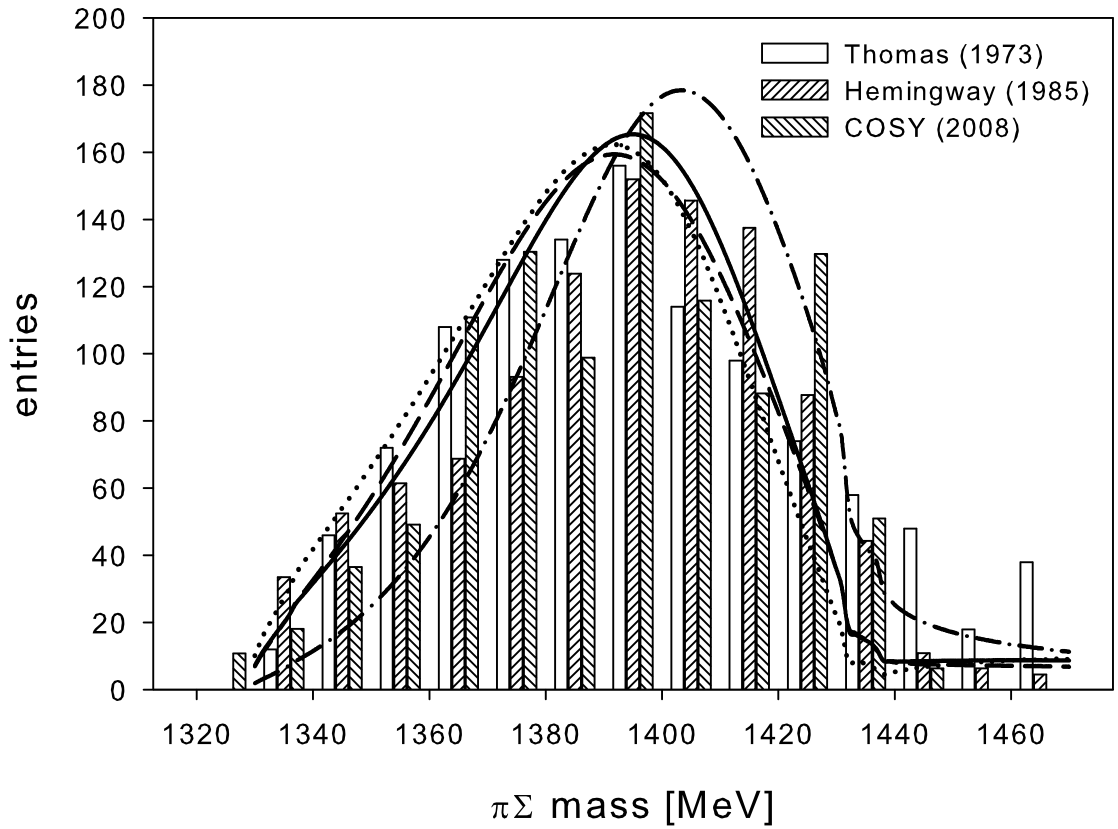
$\Lambda(1405)$ **resonance**

- four star resonance discovered in the $\pi\Sigma$ mass spectrum in the 1960's, its properties are known for a long time
- however, the dynamics is not fully understood - qqq baryon?, $\bar{K}N$ molecule?, $q^4\bar{q}$ pentaquark?
- the resonance is dynamically generated by the multiple channel chiral dynamics but there are two related poles in the respective Rieman sheet of the complex momentum plane

$\Lambda(1405) =$ two overlaping dynamical resonances?

- couplings of the poles to $\bar{K}N$ and $\pi\Sigma$ are different
- decay spectrum should depend on production mechanismus
- current experimental measurements of the $\pi\Sigma$ mass spectrum in various reactions are not conclusive

$\pi\Sigma$ mass distribution: comparison with results taken from three "compatible" experiments:



The $\pi\Sigma$ mass spectrum observed in the $K^- p \longrightarrow \pi^0\pi^0\Sigma^0$ reaction (Prakhov et al., Crystall Ball Collaboration) shows a peak at 1420 MeV (instead of 1395 MeV) but the analysis of the experimental results has been put in question.

The positions of the poles related to $\Lambda(1405)$ is also not well established due to large sensitivity to the model parameters and to other model specifics.

Table 1: The complex energies of the poles relevant to the $I = 0$ resonance.

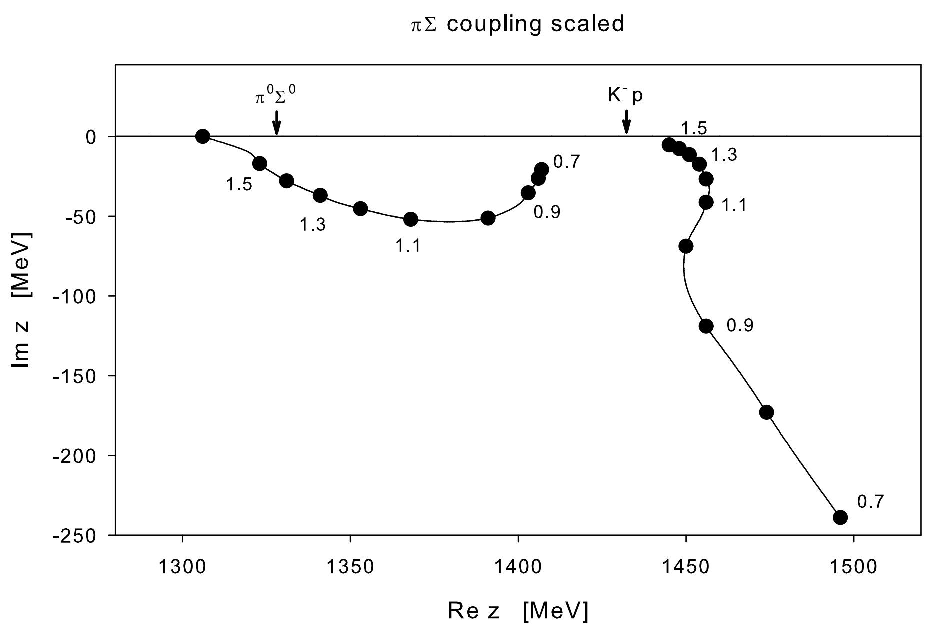
$\sigma_{\pi N}$ [MeV]	$r_{KN/\pi\Sigma}$	Re z_1 [MeV]	Im z_1 [MeV]	Re z_2 [MeV]	Im z_2 [MeV]
20	1.28	1395	-49	1456	-77
30	1.32	1398	-51	1441	-76
40	0.37	1401	-41	1519	-112
50	0.54	1406	-39	1436	-138

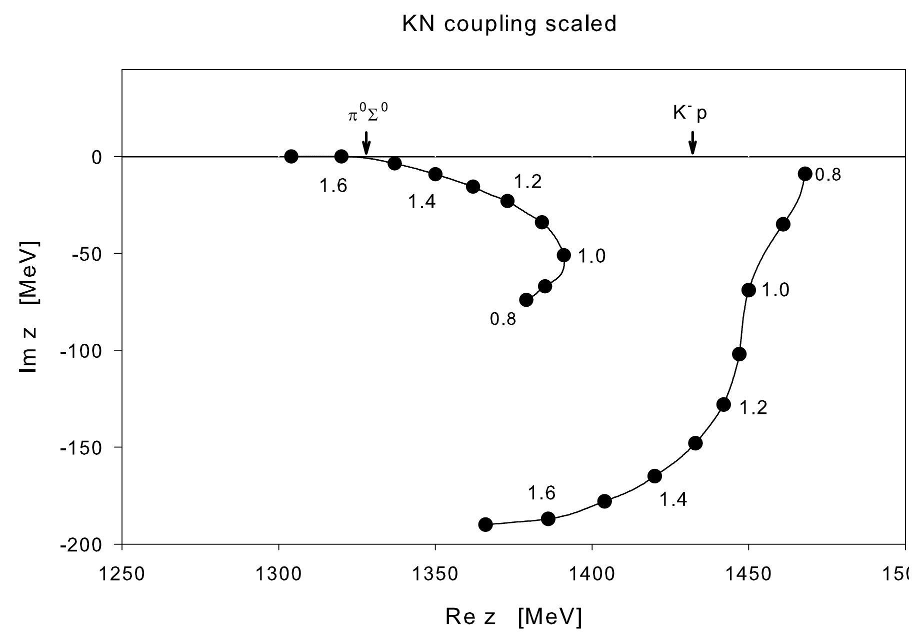
We do not see the poles as close to the real axis as other authors do !!!

Most likely explanation: NLO q^2 corrections in the chiral Lagrangian

only WT term: $\chi^2/N = 3.1$, $z_1 = (1368 - \mathrm{i}\, 42)$ MeV, $z_2 = (1441 - \mathrm{i}\, 22)$ MeV

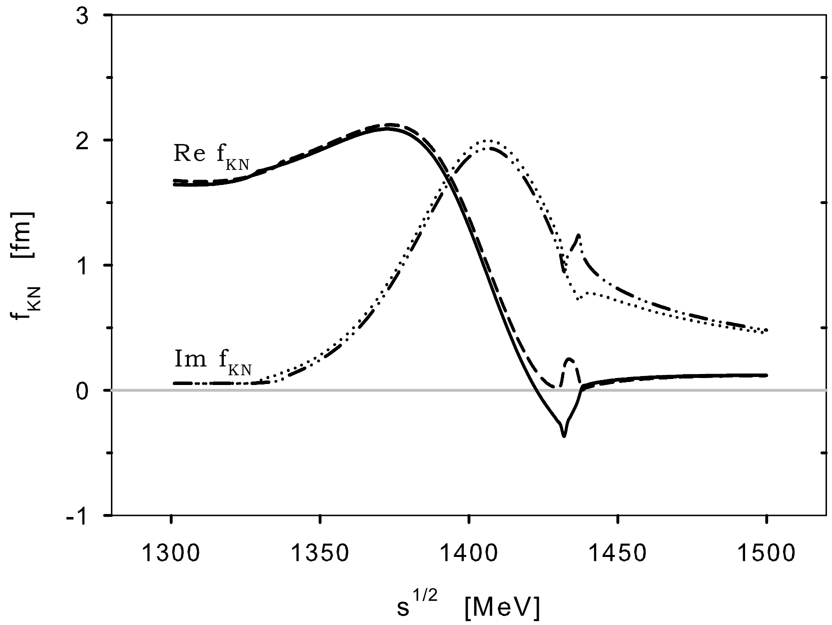
Pole movements in the complex energy plane upon scaling the couplings of the $\pi\Sigma$ and $\bar{K}N$ interactions (A.C., A.Gal - arXiv comment, 2008)



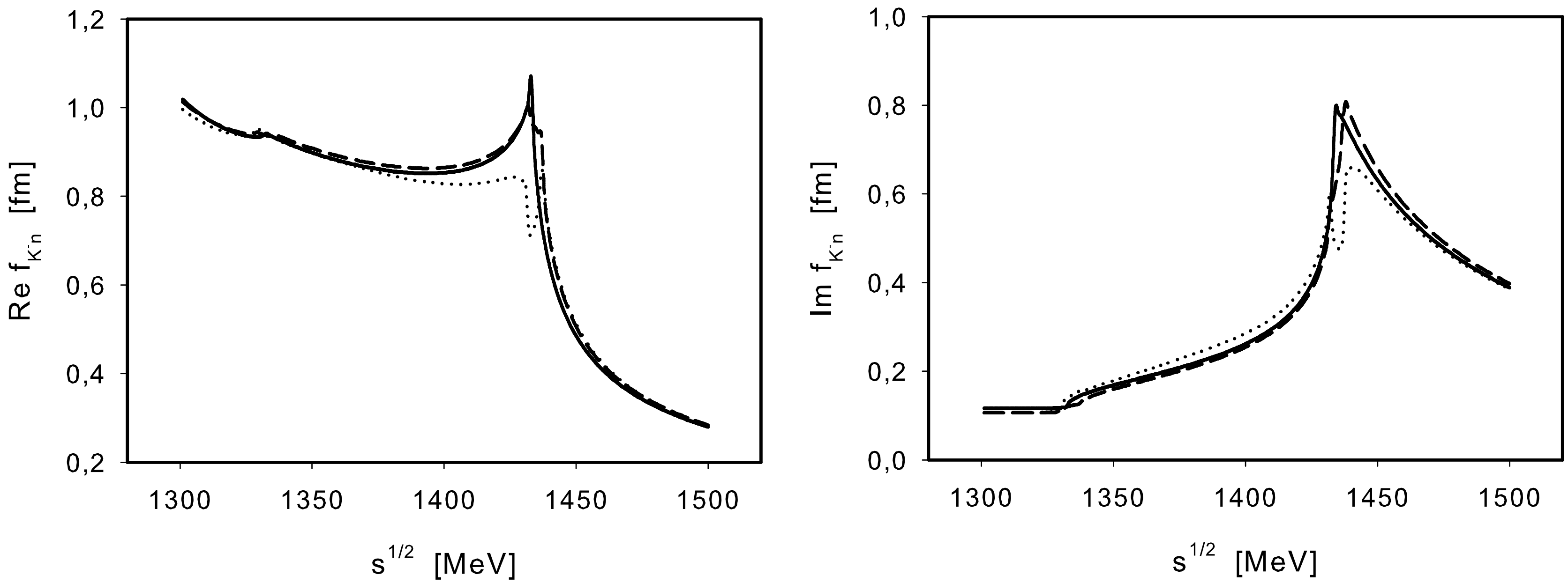


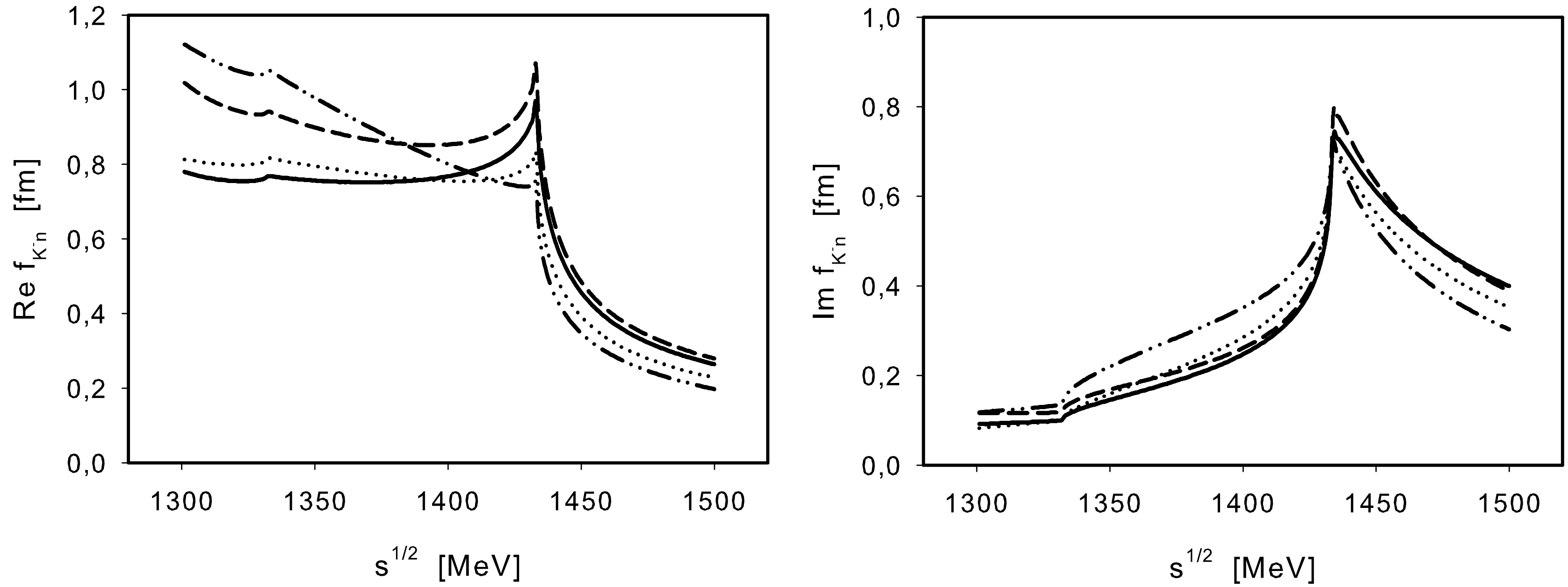
K^-n amplitude

it is a bit tricky to relate the K^-n amplitude to the isoscalar and isovector $\bar{K}N$ amplitudes obtained in the K^-p sector
physical masses $=>$ different $\bar{K}N$ thresholds and violation of the isospin symmetry $=>$ differences in the threshold region for the f_{K^-p,K^-p} and $f_{\bar{K}^0n,\bar{K}^0n}$ transition amplitudes



full line	direct calculation of the $K^- n$ amplitude
dotted line	$f_{K^- n, K^- n} = f_{K^- p, K^- p} - f_{\bar{K}^0 n, K^- p}$
dashed line	$f_{K^- n, K^- n} = (f_{K^- p, K^- p} + f_{\bar{K}^0 n, \bar{K}^0 n})/2 - f_{\bar{K}^0 n, K^- p}$

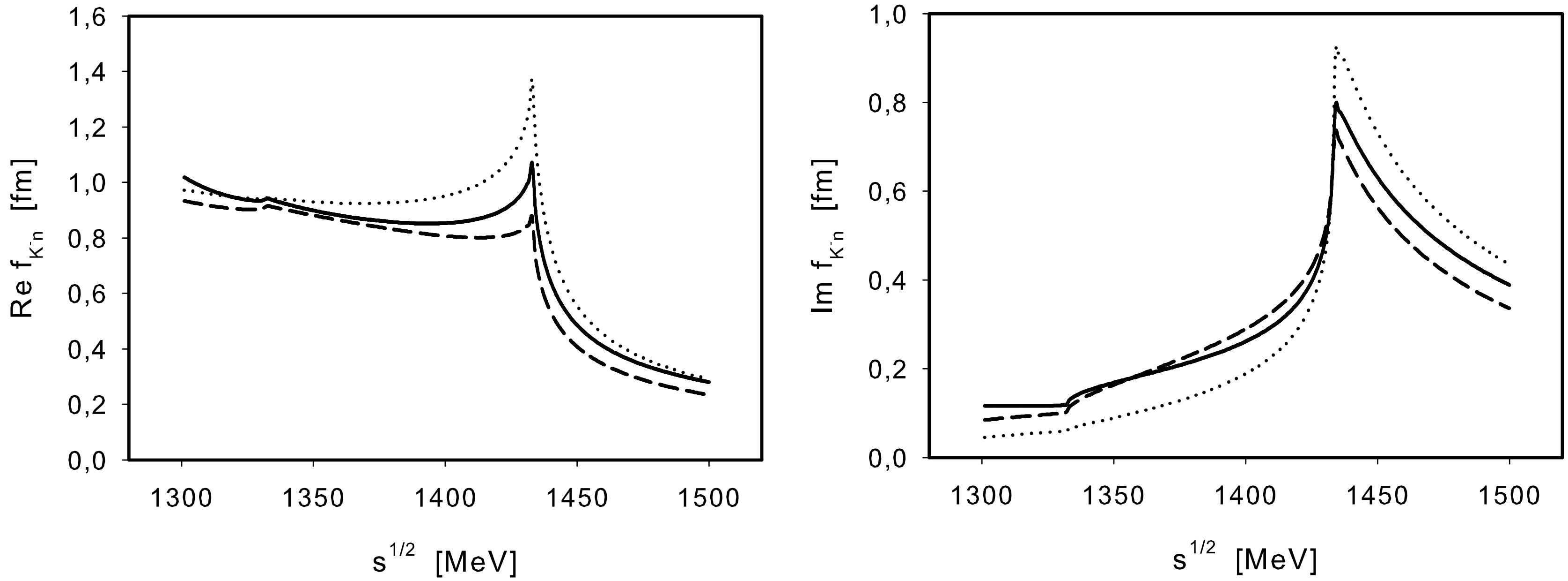




model dependence of the $K^- n$ amplitude:

- significant differences for various parameter sets
- the behavior of the amplitutude below and at the threshold differs (to some extend) from that reported by other authors (Borassoy, Niessler, Weise in 2005, Hyodo and Weise in 2008)

more on model dependence of the K^-n amplitude:



full line	direct calculation of the K^-n amplitude
dotted line	the crossed u-term excluded
dashed line	another χ^2 minimum (better χ^2/N but unphysical values of $\alpha_{K\Xi}$)

Summary

- the exact solution to the K^-p bound state problem is available and should be used instead of the approximate Deser-Trueman formula (or its improved version)
- the simultaneous fits to the kaonic hydrogen and the low energy $\bar{K}N$ data are satisfactory [though, the computed absorption width $\Gamma(1s)$ is too large]
- our model also describes well the πN scattering length a_{0+} and the $\pi\Sigma$ mass spectrum
- the position of poles related the $\Lambda(1405)$ resonance vary depending on particular parameter set and may drift too far from the real axis to affect the physical observables
- the threshold value of the K^-n amplitude is (to some 30%) model dependent and isospin relations are not sufficiently precise in the threshold region (important for the kaonic deuterium)