

# Many facets of the kaonic atoms ‘puzzle’

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# Outline

- The role of phenomenology
- Deep *vs.* shallow potentials: the evidence
- Radial sensitivity from functional derivatives
- Detailed comparisons with antiprotons:  
    global analysis from atoms up to 600 MeV/c
- Summary and conclusions

## Phenomenological analyses of data:

- handle large sets of data
- Could identify characteristic quantities
- serve as intermediaries between ‘genuine’ theories and experiment (e.g. in reproducing the characteristic quantities)

Tools of the trade: variants of an optical potential.

When analyzing several nuclear species together one must have some model for the nuclear geometry, e.g. **make the potential a functional of the nuclear density.**

The simplest class of optical potentials  $V_{\text{opt}}$  is the generic  $t\rho(r)$  potential:

$$2\mu V_{\text{opt}}(r) = -4\pi\left(1 + \frac{A-1}{A}\frac{\mu}{M}\right)\{b_0[\rho_n(r) + \rho_p(r)] + b_1[\rho_n(r) - \rho_p(r)]\}$$

$\rho_n$  and  $\rho_p$  are the neutron and proton density distributions normalized to the number of neutrons  $N$  and number of protons  $Z$ , respectively,  $M$  is the mass of the nucleon.

Global fits to kaonic atoms data (65 points)

model	$\chi^2$	$-\text{Re}V(0)$ (MeV)	$-\text{Im}V(0)$ (MeV)
$t\rho$	130	81( $\pm 10\%$ )	122( $\pm 5\%$ )
$t(\rho)\rho$	85	180( $\pm 3.5\%$ )	82( $\pm 8\%$ )
chiral *	266	33	45
chiral **	120	42	62

\*Ramos & Oset, NPA **671** (2000) 481

\*\* I=1 adjusted by +50% and +63% for Re and Im, respectively

From  $^{12}\text{C}(K^-,n)$  and  $^{12}\text{C}(K^-,p)$  at 1 GeV/c

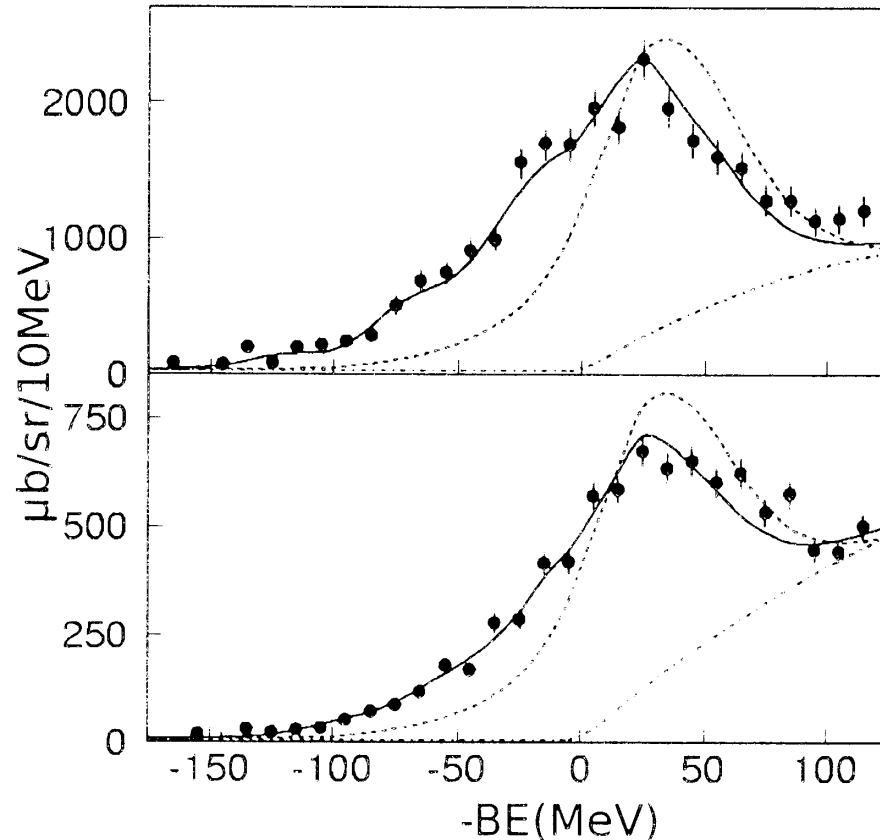


Fig. 1. Missing mass spectra of the  $^{12}\text{C}(K^-,n)$  reaction (upper) and  $^{12}\text{C}(K^-,p)$  reaction (lower). The solid curves represent the calculated best fit spectra for potentials with  $\text{Re}(V)=-190$  MeV and  $\text{Im}(V)=-40$  MeV (upper) and  $\text{Re}(V)=-160$  MeV  $\text{Im}(V)=-50$  MeV (lower). The dotted curves represent the calculated spectra for  $\text{Re}(V)=-60$  MeV and  $\text{Im}(V)=-60$  MeV. The dot-dashed curves represent a background process (see main text).

T. Kishimoto *et al.* Prog. Theor. Phys. **118** (2007) 181

The ‘intermediate status’ of  $\bar{K}$  concerning penetration into nuclei: mean-free-path  $\lambda \simeq 1/\rho\sigma_T$  where  $\rho$  is density ( $0.16 \text{ fm}^{-3}$ ),  $\sigma_T$  is total cross section.

particle	$\sigma_T$ (mb)	$\lambda$ (fm)
$\pi$	$\simeq 7$	9
$\bar{K}$	$\simeq 50$	1.2
$\bar{p}$	$\simeq 1000$	0.06

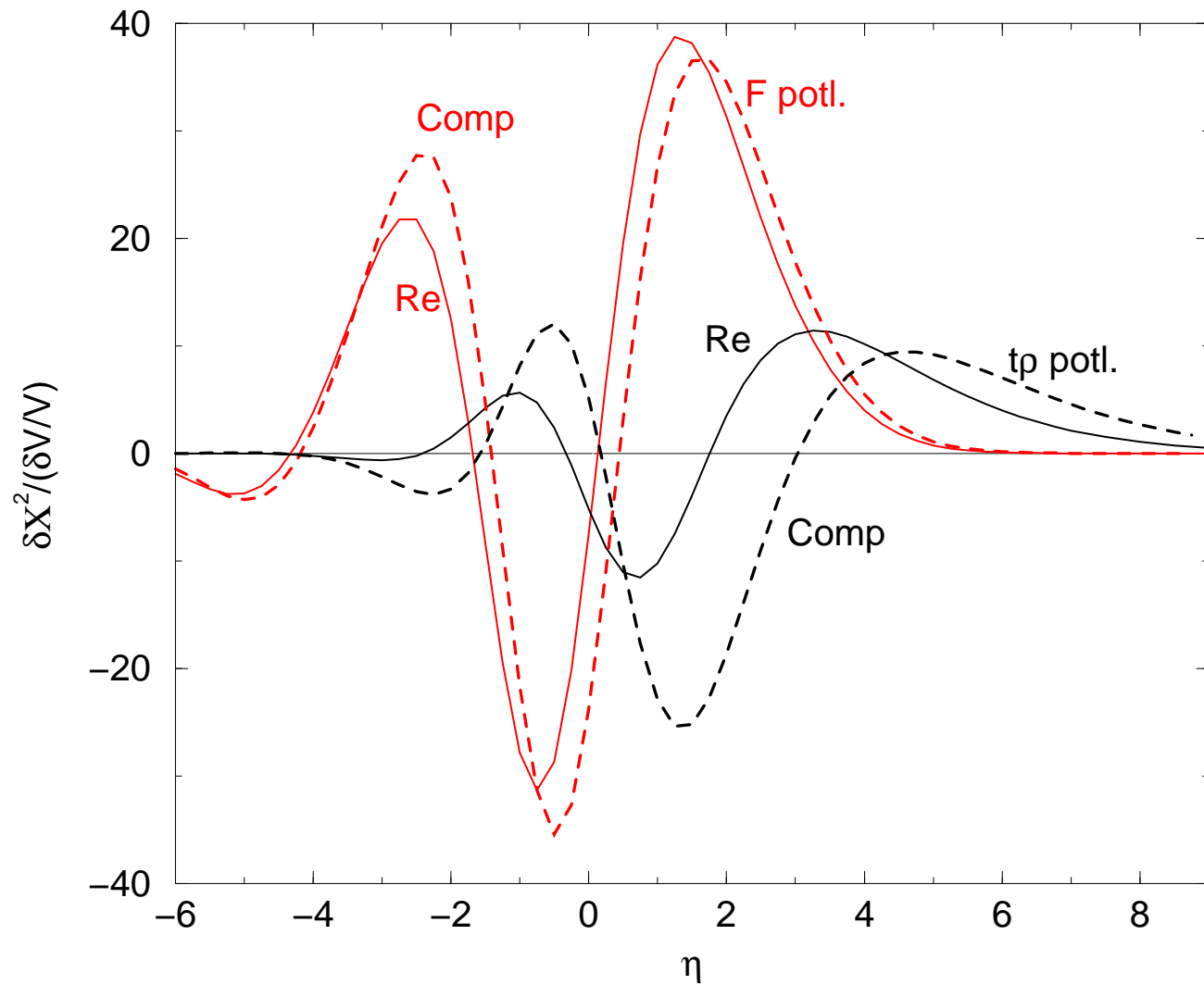
For better understanding study in parallel  $\bar{K}$  and  $\bar{p}$ .

## The functional derivative method

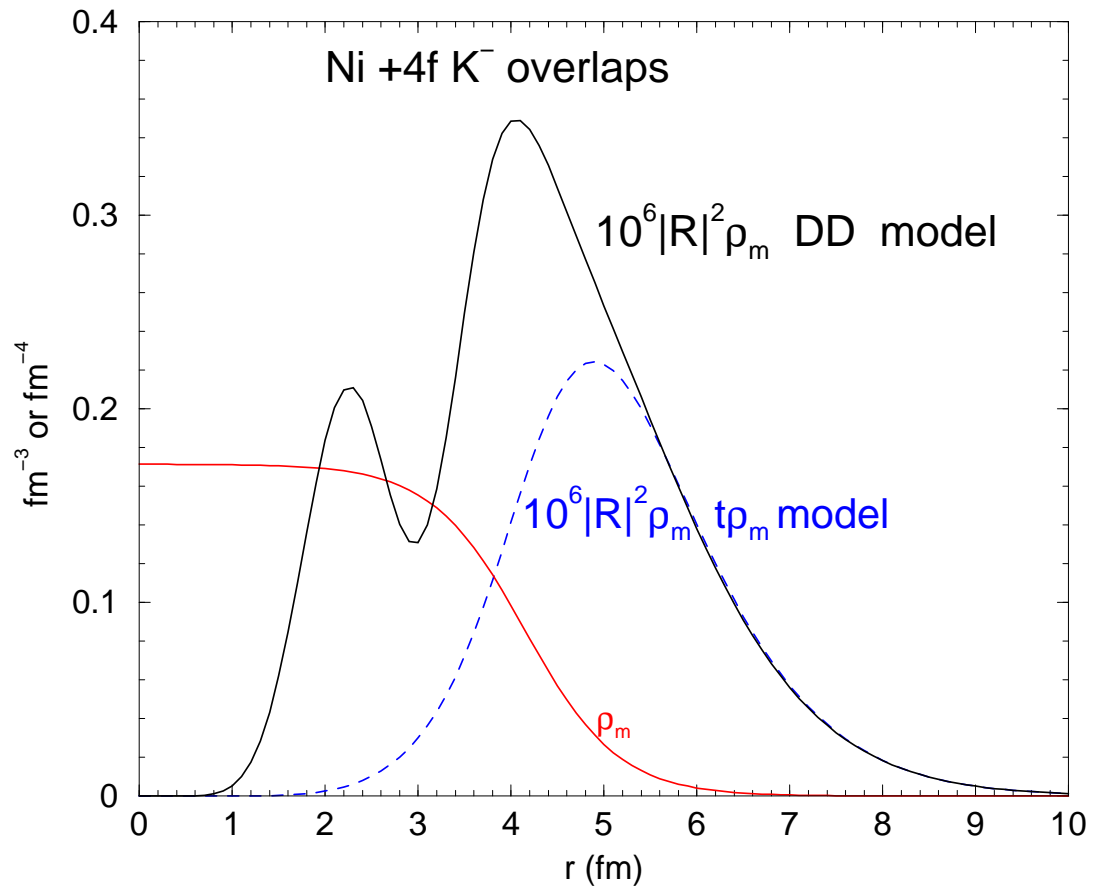
N. Barnea, E. Friedman, Phys. Rev. C **75** (2007)  
022202(R)

Define the radial position parameter *globaly* using as reference the known charge distribution for each nuclear species in the data base. The radial position  $r$  is then defined as  $r = R_c + \eta a_c$

where  $R_c$  and  $a_c$  are the radius and diffuseness parameters, respectively, of a two-parameter Fermi charge distribution. The value of  $\chi^2$  becomes a functional of a global optical potential  $V(\eta)$ .

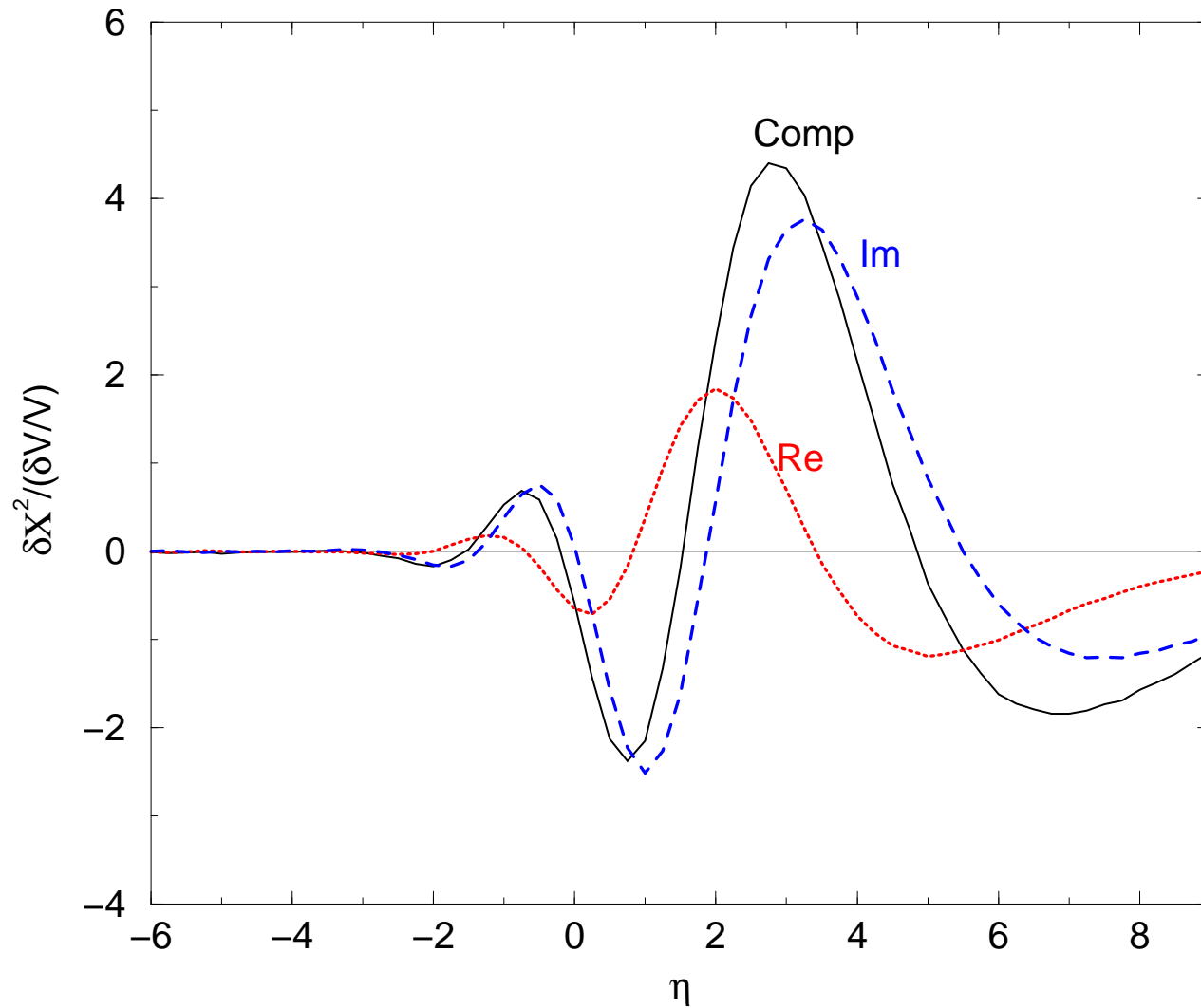


Functional derivatives for kaonic atoms  $\chi^2$

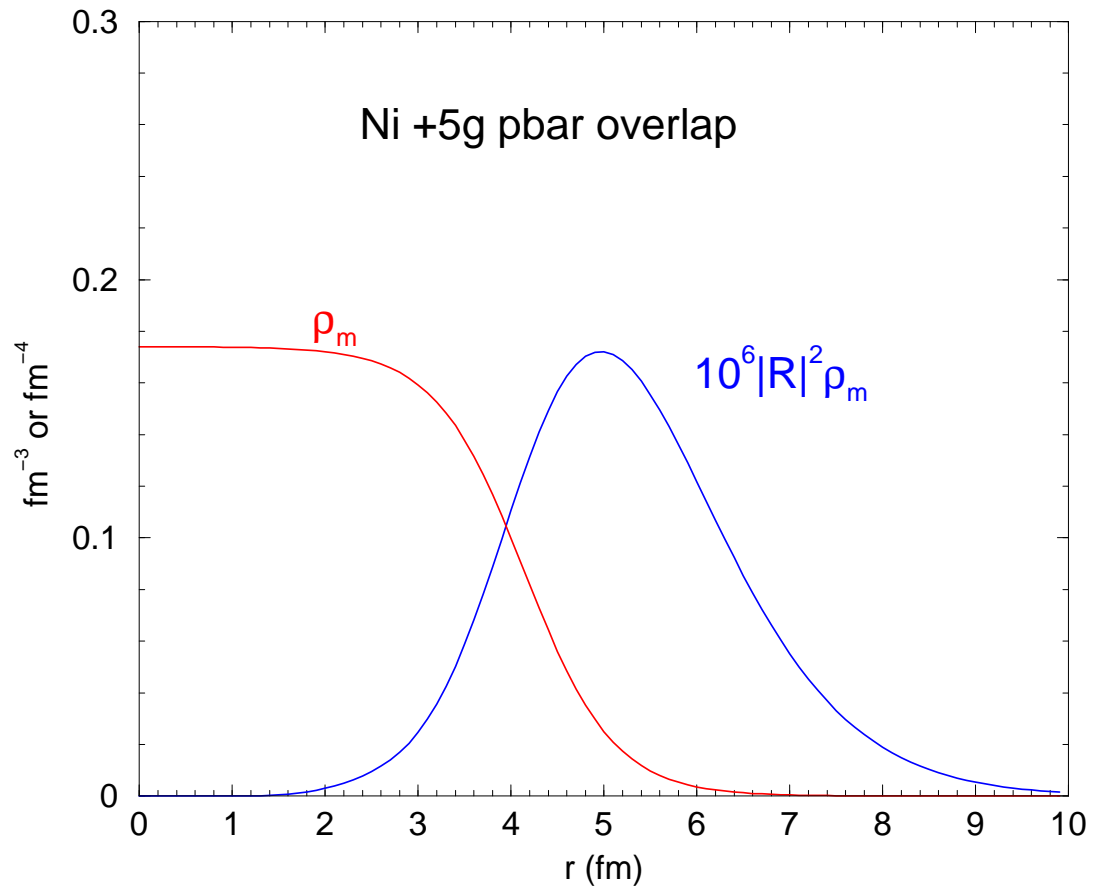


Overlap of  $K^-$  atomic density with the nuclear density.

$R_B = 31.5$  fm.



Functional derivatives for antiprotonic atoms  $\chi^2$



Overlap of  $\bar{p}$  atomic density with the nuclear density.

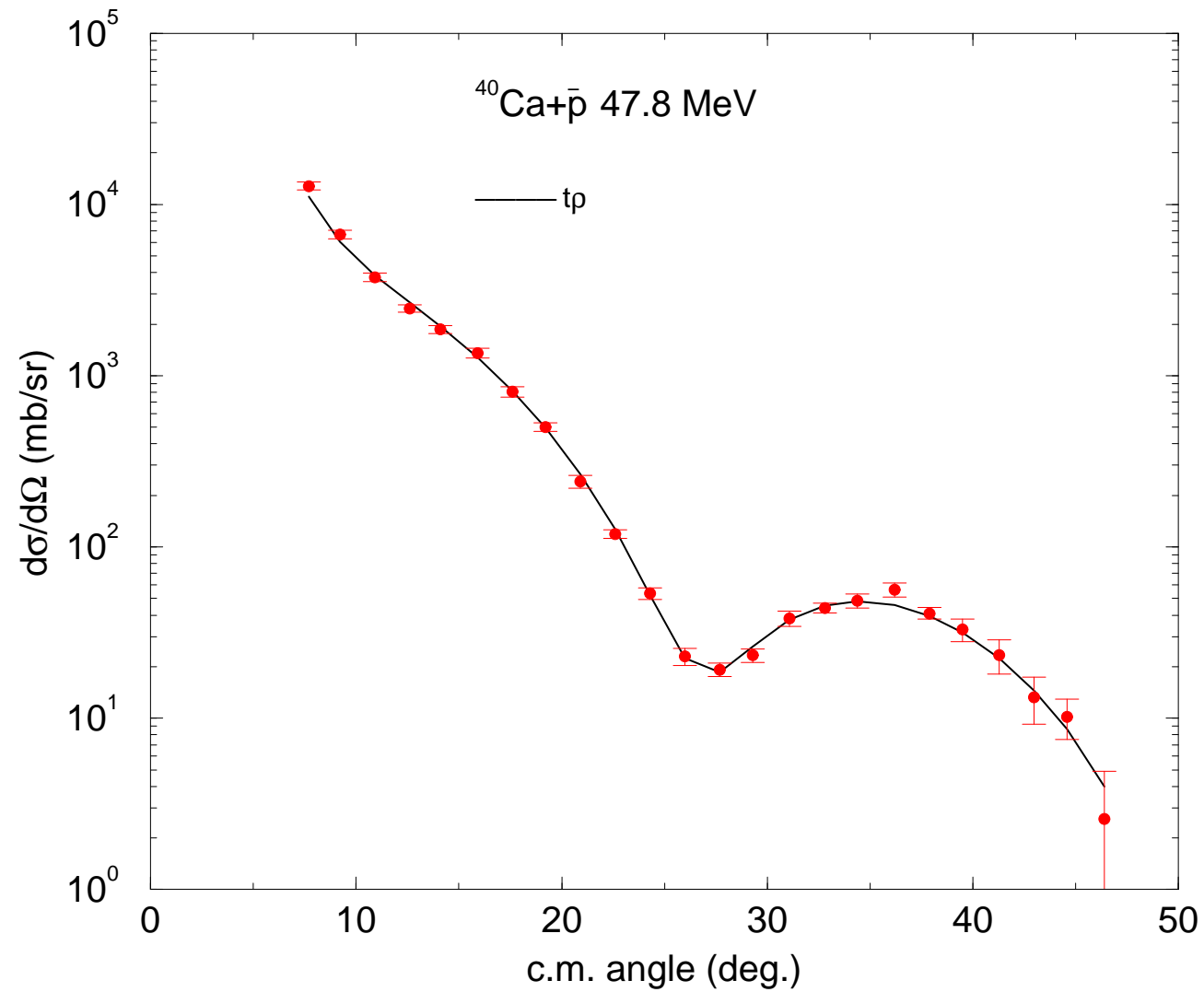
$R_B = 26.1$  fm.

## A brief history of low-energy $\bar{p}$ experiments

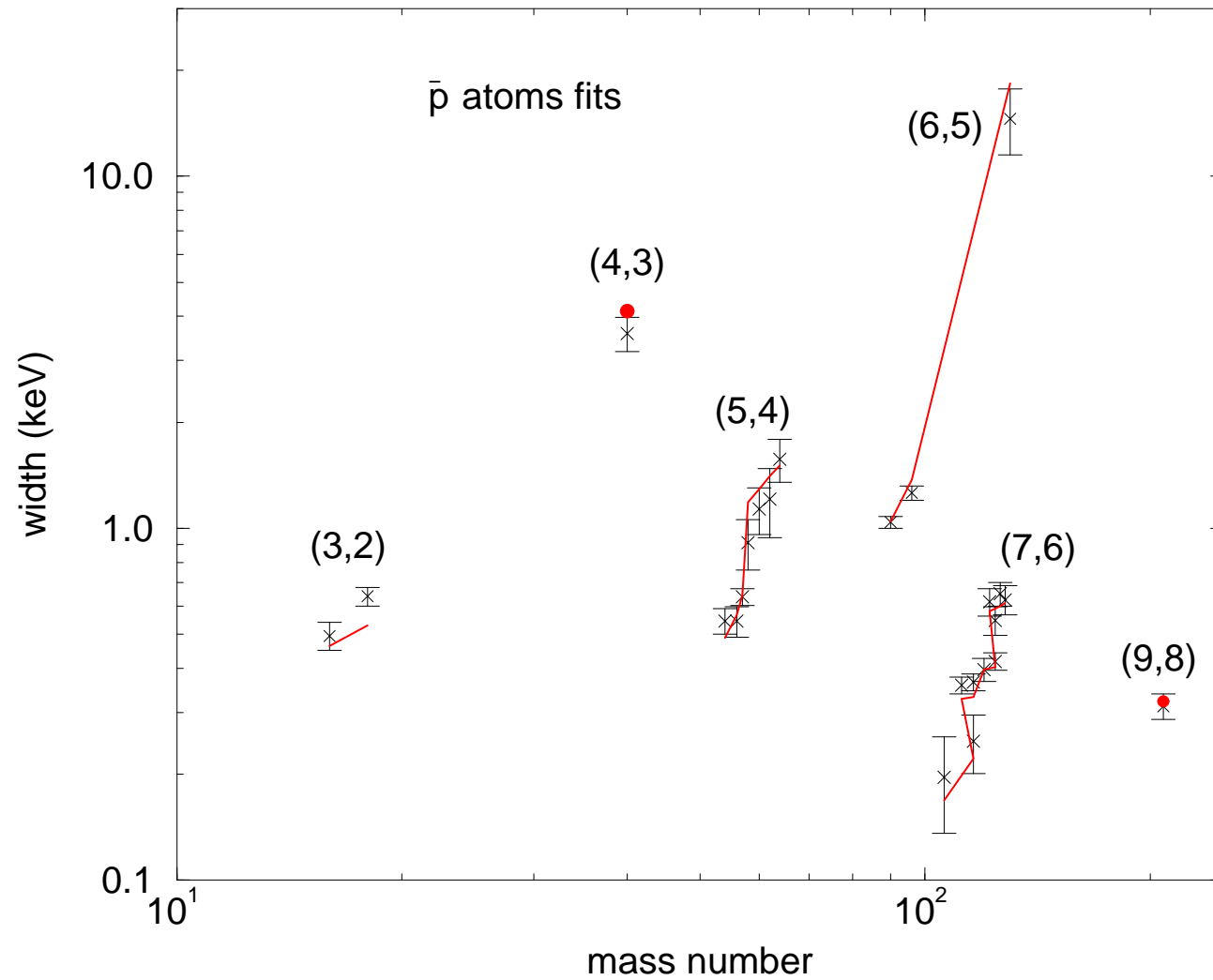
High quality data available from the LEAR facility in mid 1980's.

Analyses of elastic scattering followed soon after.

Results of **very large scale** measurements of  $\bar{p}$  atoms published only around 2000  
(the PS209 collaboration.)



Example of a fit to elastic scattering data



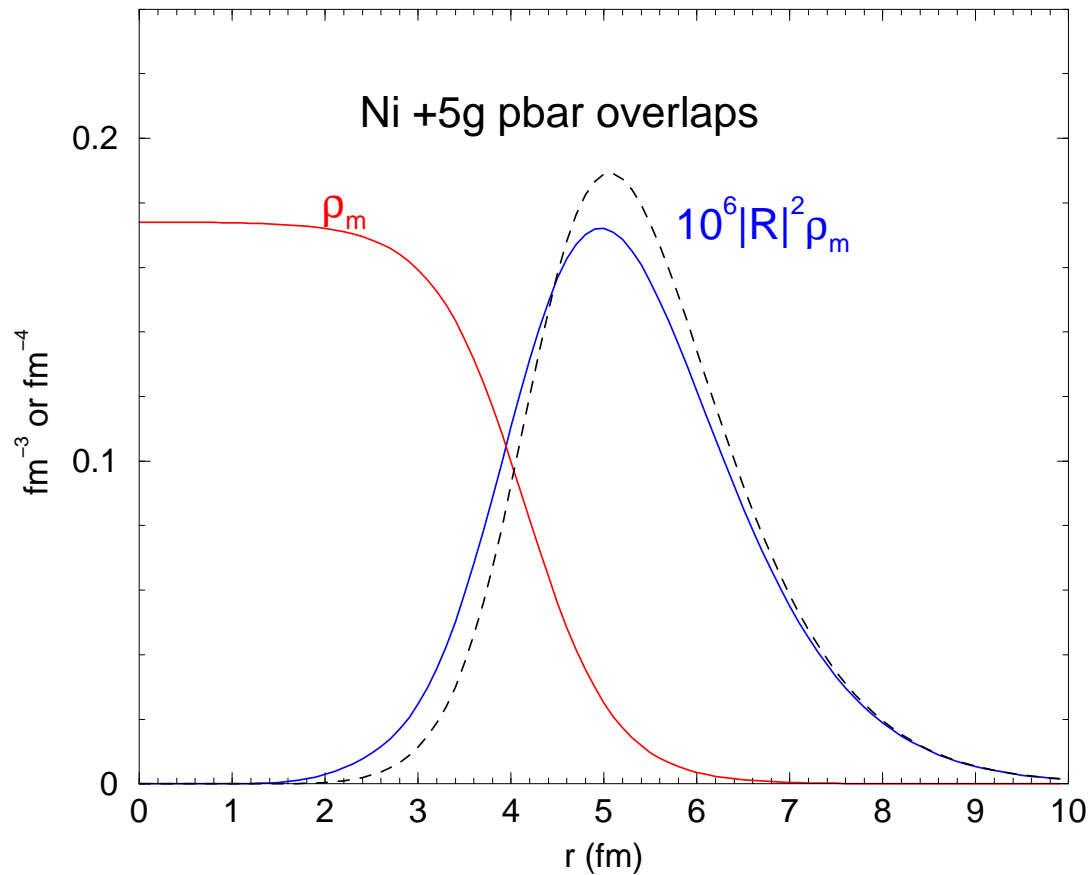
Example of a **global** fit to  $\bar{p}$  atoms data.

Use ‘finite-range’ (FR) interaction by folding density with a Yukawa potential  $e^{-y}/y$  with

$$y = \frac{|\mathbf{r}' - \mathbf{r}|}{a}. \quad (ms \text{ radius} = 6a^2)$$

Results for  $\bar{p}$  atoms

$\chi^2$ for 90 points	Re <i>b</i> (fm)	Im <i>b</i> (fm)	$a^R$ (fm)	$a^I$ (fm)
196.1	$1.36 \pm 0.67$	$1.28 \pm 0.63$	$0.06 \pm 1.8$	$0.54 \pm 0.17$
197.4	$0.91 \pm 0.25$	$1.63 \pm 0.39$	$= a^I$	$0.48 \pm 0.09$
201.1	$0.64 \pm 0.07$	$1.24 \pm 0.04$	<u>0.58</u>	<u>0.58</u>



Solid curve: ZR real, FR imaginary potential.

Dashed curve: common FR imposed by scattering data.

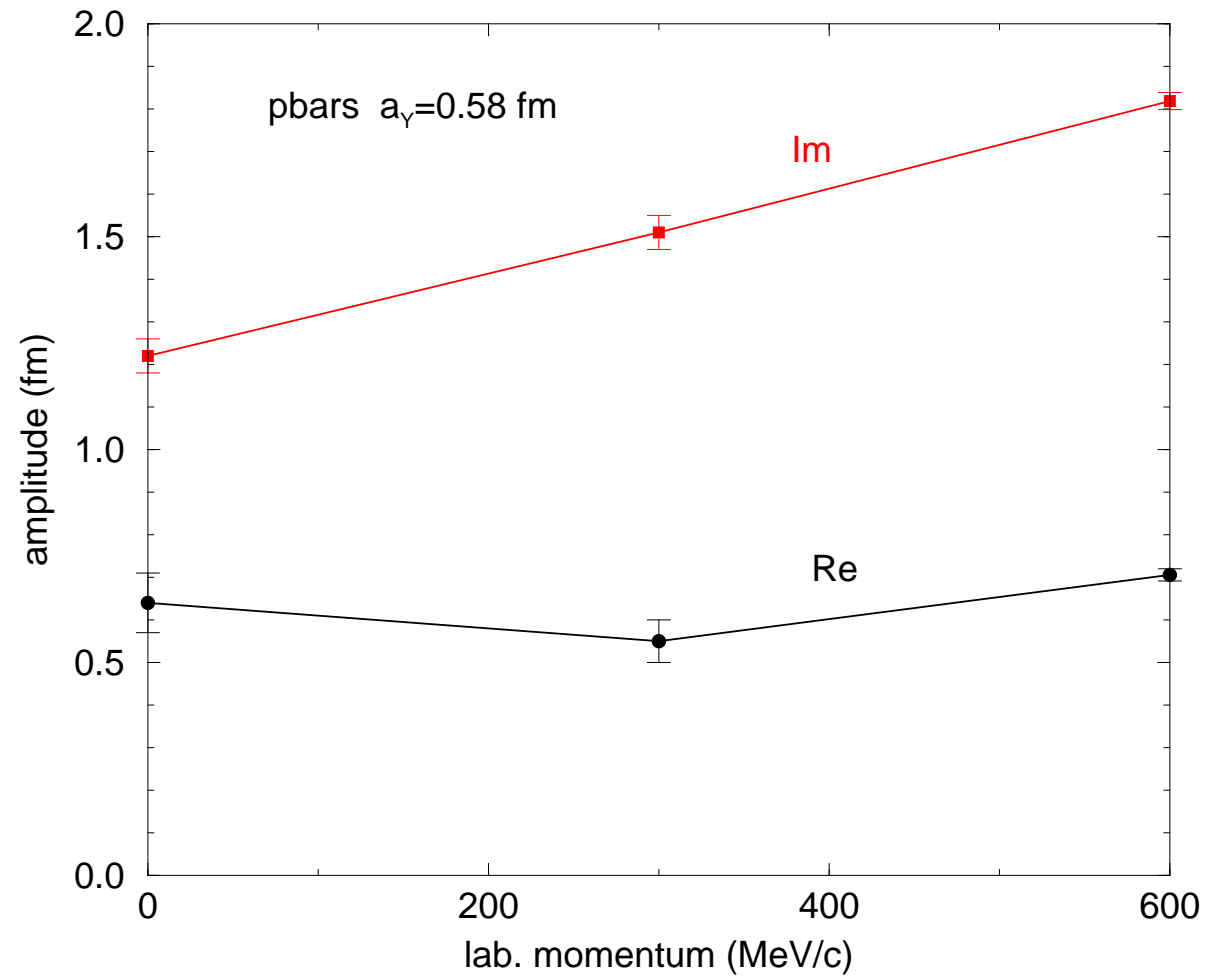
From atoms to scattering: seek support from **global** analyses of  $\bar{p}$  scattering:

300 MeV/c ( $^{12}\text{C}$ ,  $^{40}\text{Ca}$ ,  $^{208}\text{Pb}$ )

$\chi^2$ for 88 points	$\text{Re}b$ (fm)	$\text{Im}b$ (fm)	$a$ (fm)
187.2	$0.51 \pm 0.05$	$1.44 \pm 0.06$	$0.616 \pm 0.026$
191.6	$0.55 \pm 0.05$	$1.51 \pm 0.04$	<u>0.58</u>

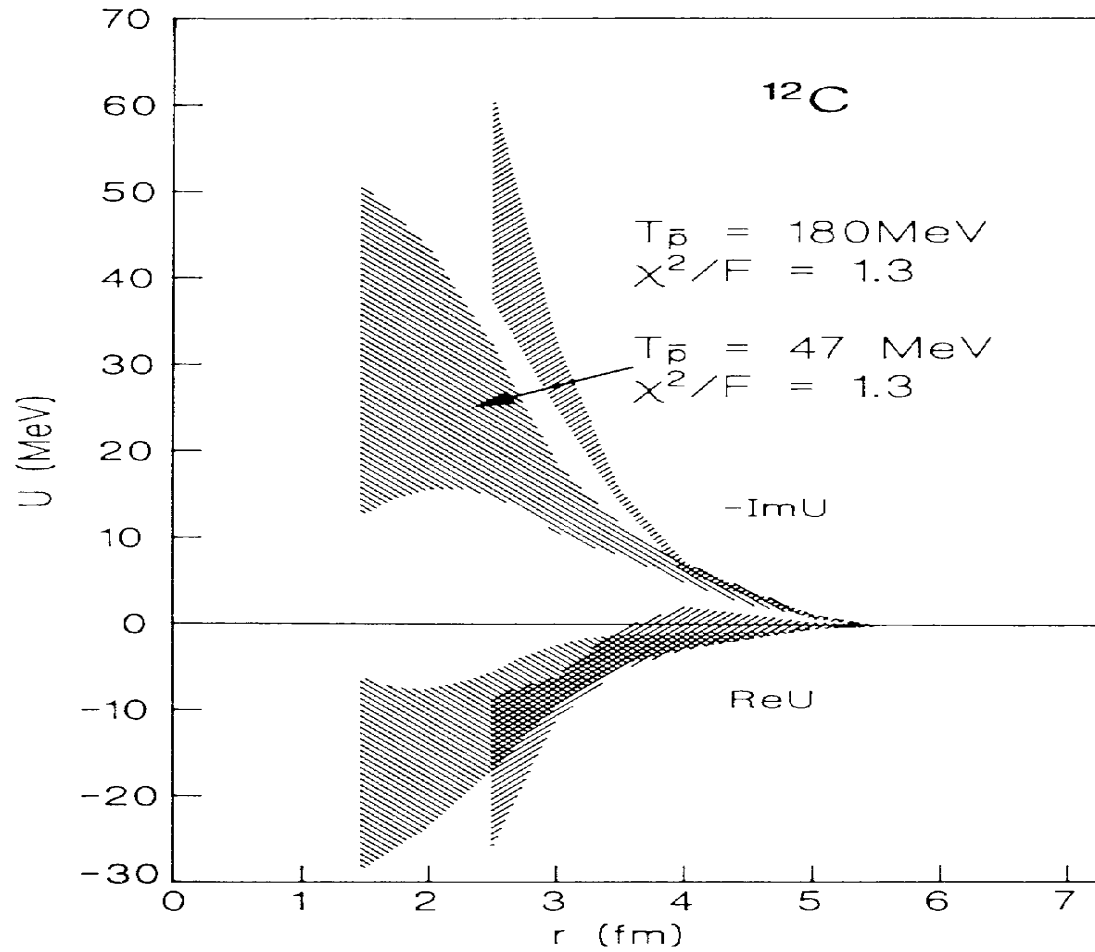
600 MeV/c ( $^{12}\text{C}$ ,  $^{16,18}\text{O}$ ,  $^{40}\text{Ca}$ ,  $^{208}\text{Pb}$ )

$\chi^2$ for 191 points	$\text{Re}b$ (fm)	$\text{Im}b$ (fm)	$a$ (fm)
507	$0.696 \pm 0.016$	$1.849 \pm 0.024$	$0.564 \pm 0.008$
518	$0.706 \pm 0.015$	$1.823 \pm 0.021$	<u>0.58</u>



From  $\bar{p}$  atoms to  $\bar{p}$  scattering  
 (with fixed  $a_Y = 0.58$  fm,  $rms$  radius = 1.42 fm)

Fourier-Bessel fits, Friedman + Lichtenstadt, 1986:  
no information about the interior.



What do we know about the potential in the interior?

Elastic scattering of 47.8 MeV  $\bar{p}$  on  $^{40}\text{Ca}$

model	$\chi^2$ for 24 points	$-V_R(0)$ (MeV)	$-V_I(0)$ (MeV)
$t\rho$	23.9	43.1	73.5
$t\rho'$	23.3	2.2	2.7

$\bar{p}$  totally insensitive to the nuclear interior, as expected.

**$\bar{p}$  atoms:** global fits to  $\bar{p}$  X-ray data covering the range of Mg to Pb, a total of 84 data points. The parameters  $a_Y^R$  and  $a_Y^I$  are the range parameters of Yukawa folding for the real and imaginary potentials, respectively.

model	$\chi^2$	Reb(fm)	Imb(fm)	$a_Y^R$ (fm)	$a_Y^I$ (fm)
$t\rho$	189.3	$2.33\pm 0.15$	$3.11\pm 0.15$	-	-
$t\rho$	183.9	$1.33\pm 0.49$	$1.49\pm 0.51$	<u>0.2</u>	$0.48\pm 0.13$
$t\rho'$	193.7	$0.66\pm 0.04$	$0.85\pm 0.04$	-	-
$t\rho'$	183.9	$0.58\pm 0.06$	$0.55\pm 0.11$	<u>0.1</u>	$0.33\pm 0.07$

Return to global fits to **kaonic atoms** data (65 points)

model	$\chi^2$	$-\text{Re}V(0)$ (MeV)	$-\text{Im}V(0)$ (MeV)
$t\rho$	130	81	122
$t\rho'$	168	–	–
$t(\rho)\rho$	85	180	82

In contrast to  $\bar{p}$  atoms, no comparable fit was found for kaonic atoms with the derivative shape.

**Conclude that for the deep potential solution kaonic atoms are sensitive to internal regions of nuclei.**

## Exotic atoms as a source of information on neutron densities

Various *mean-field* calculations show linear dependence of differences between rms radii on  $(N - Z)/A$ :

$$r_n - r_p = \gamma \frac{N - Z}{A} + \delta .$$

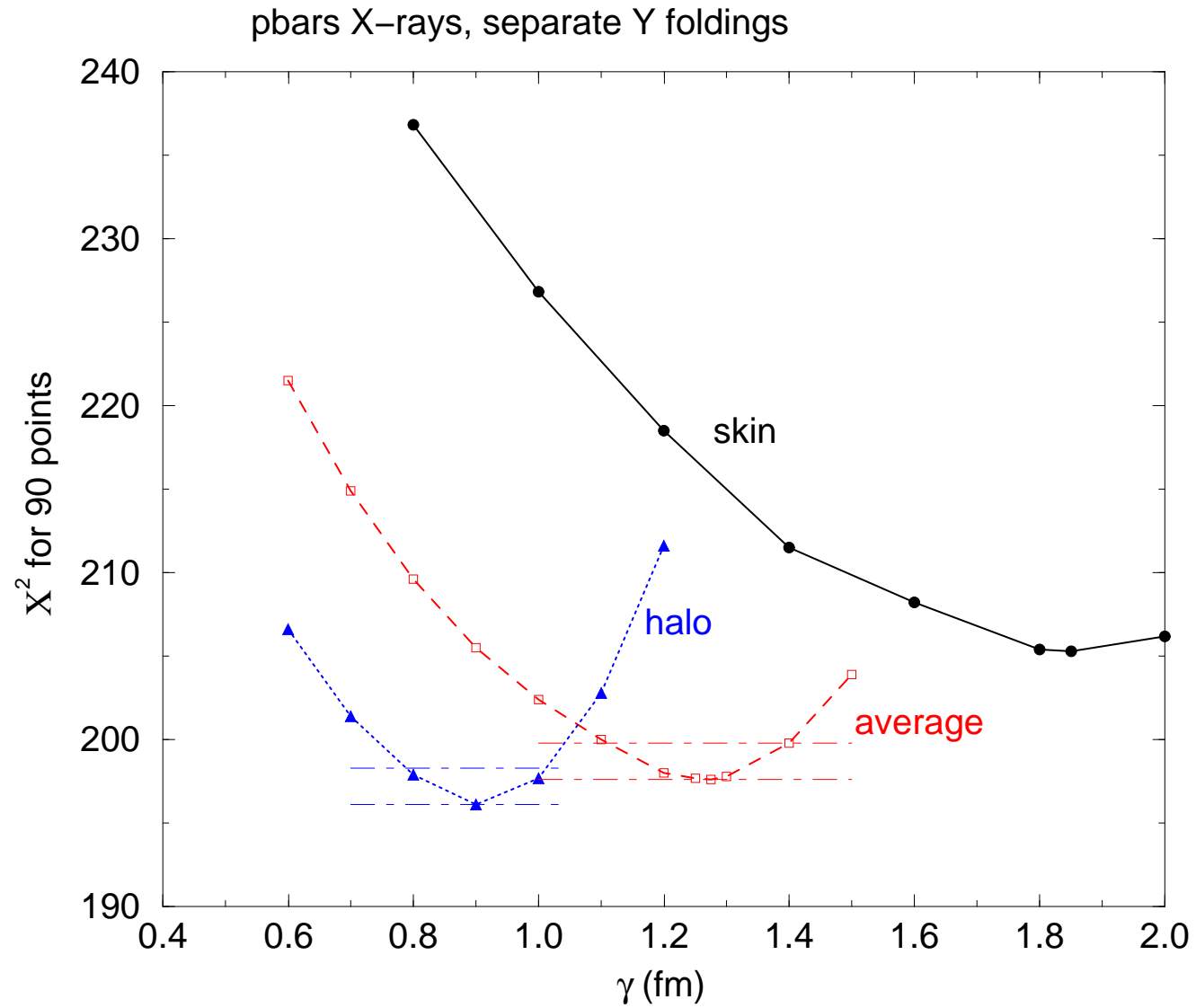
Use 2pF densities

$$\rho_{n,p}(r) = \frac{\rho_{0n,0p}}{1 + \exp((r - R_{n,p})/a_{n,p})} ,$$

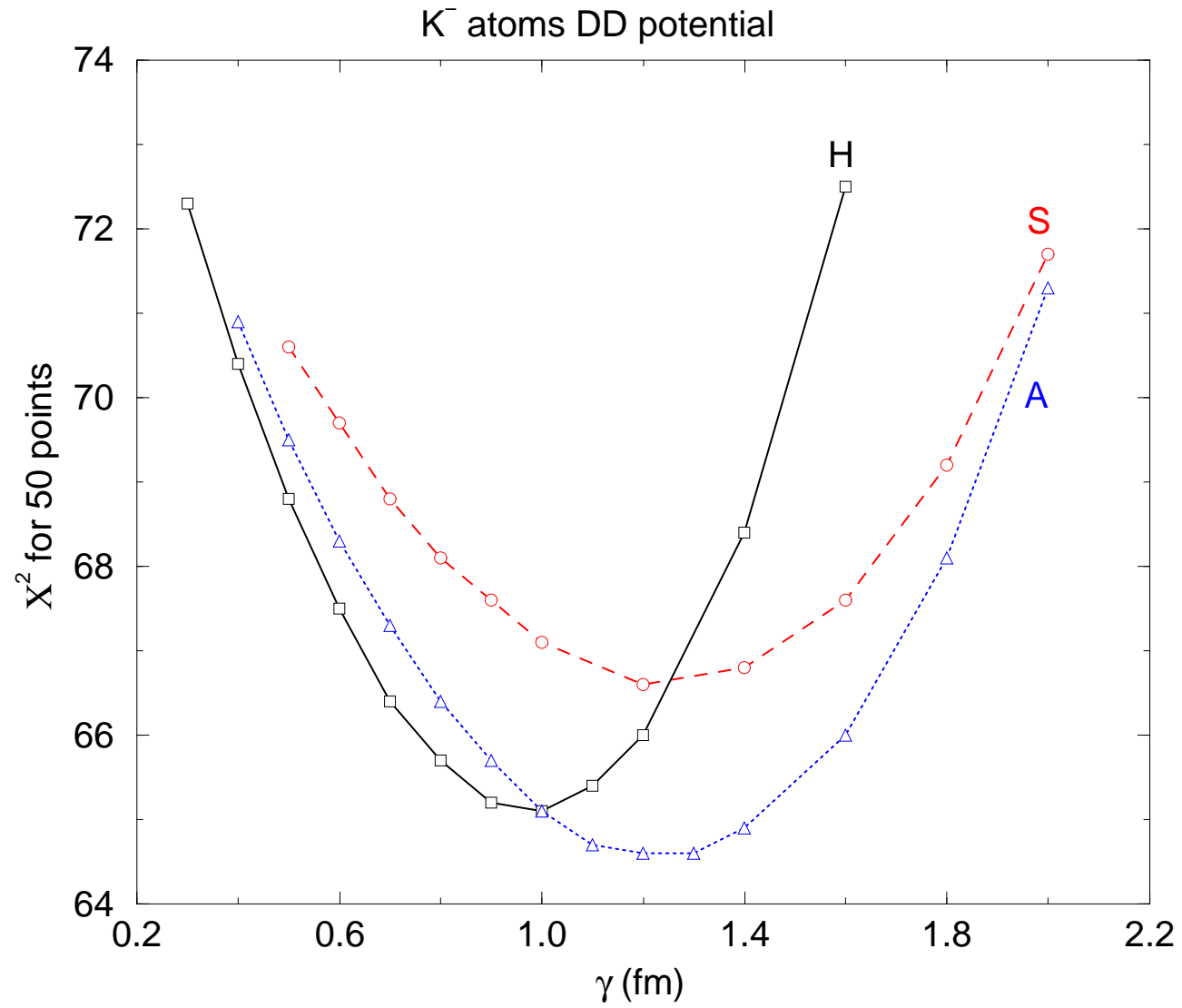
Different shapes for  $\rho_n$  are ‘skin’ ( $a_n = a_p$ ), ‘halo’ ( $R_n = R_p$ ) and their average.

Fits to exotic atoms data *vs* neutron radius parameter  $\gamma$ .

Global fits to 90 data points for antiprotonic atoms.



Global fits to 50 data points for kaonic atoms.



## Summary

- Kaonic atoms and  $^{12}\text{C}(K^-, \text{N})$  reactions favour deep real  $K^-$ -nucleus potential.
- Functional-derivative analysis shows:
  - Sensitivity to interior  $\rho$  for ‘deep’ potentials.
  - No such sensitivity for ‘shallow’ potentials.
- Global analyses of  $\bar{p}$  atoms and scattering establish insensitivity to interior  $\rho$ .
- If the  $K^-$  potential is shallow, we do not know it.
- If the  $K^-$  potential is deep, we know it rather reliably.
- Cannot dismiss the ‘deep’ potential for  $K^-$  atoms.

The variation of  $\chi^2$  due to a small change in  $\eta$  is

$$d\chi^2 = \int d\eta \frac{\delta\chi^2}{\delta V(\eta)} \delta V(\eta) ,$$

where

$$\frac{\delta\chi^2[V(\eta)]}{\delta V(\eta')} =$$

$$\lim_{\sigma \rightarrow 0} \lim_{\epsilon_V \rightarrow 0} \frac{\chi^2[V(\eta) + \epsilon_V \delta_\sigma(\eta - \eta')] - \chi^2[V(\eta)]}{\epsilon_V}$$

is the functional derivatives (FD) of  $\chi^2[V]$ .

The FD can be approximated by

$$\approx \frac{1}{V(\eta')} \frac{\chi^2[V(\eta)(1 + \epsilon\delta_\sigma(\eta - \eta'))] - \chi^2[V(\eta)]}{\epsilon}.$$

The limit  $\epsilon \rightarrow 0$  is obtained numerically for several values of  $\sigma$  and then extrapolated to  $\sigma = 0$ .

In practice the calculation of the FD was carried out by multiplying the best fit potential by a factor

$$f = 1 + \epsilon\delta_\sigma(\eta - \eta') \quad (1)$$

using a normalized Gaussian with a range parameter  $\sigma$  for the smeared  $\delta$ -function,

$$\delta_\sigma(\eta - \eta') = \frac{1}{\sqrt{2\pi}\sigma} e^{-(\eta - \eta')^2 / 2\sigma^2}. \quad (2)$$