On Molecule model for kaonic nuclear cluster K^-pp

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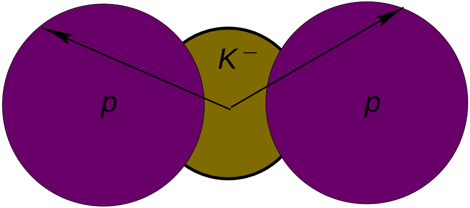
Kaonic nuclear cluster K^-pp with quantum numbers $I(J^P) = \frac{1}{2}(0^-)$. Theoretical approaches



- Potential model: Akaishi, Yamazaki, Wycech: $B_{{}^2_{\bar{K}}\mathrm{H}} \approx 50\,\mathrm{MeV}$, $\Gamma^{(\pi)}_{{}^2_{\bar{K}}\mathrm{H}} \approx 60\,\mathrm{MeV}$,
- Chiral dynamics: Weise *et al.*: $B_{{}^2_{\bar{K}}\mathrm{H}} = 20(3)\,\mathrm{MeV}$, $\Gamma^{(\pi)}_{{}^2_{\bar{K}}\mathrm{H}} = (40-70)\,\mathrm{MeV}$
- Faddeev's equation: Gal *et al.*, Ikeda, Sato: $B_{{}^2_{\bar{K}}\mathrm{H}} \sim (55-80)\,\mathrm{MeV}$, $\Gamma^{(\pi)}_{{}^2_{\bar{K}}\mathrm{H}} \sim (75-110)\,\mathrm{MeV}$

Kaonic nuclear cluster K^-pp with quantum numbers $I(J^P) = \frac{1}{2}(0^-)$. Experimental data

- M. Agnello *et al.* (FINUDA): PRL **94**, 212303 (2005)
  $B_{{}^2_{\bar K}\mathrm{H}} = 115(7)\,\mathrm{MeV}$, $\Gamma_{{}^2_{\bar K}\mathrm{H}} = 67(14)\,\mathrm{MeV}$
  These data have been obtained from stopped K^- meson reactions on ${}^6\mathrm{Li}$, ${}^7\mathrm{Li}$ and ${}^{12}\mathrm{C}$ by measuring the invariant-mass spectrum of $\Lambda^0 p$ pairs in the final state.
  These data have been heavily criticised by Oset *et al.*
- T. Yamazaki *et al.* (DISTO), arXiv: 0810.5182 [nucl-ex]
  $B_{{}^2_{\bar K}\mathrm{H}} = 105(2)\,\mathrm{MeV}$, $\Gamma_{{}^2_{\bar K}\mathrm{H}} = 118(8)\,\mathrm{MeV}$
  These data have been obtained from the $pp \to K^+\Lambda^0 p$ reaction at the laboratory momentum of the incident proton $p_{\ell ab} = 2.85\,\mathrm{GeV}$ by measuring the $p\pi^-$ invariant-mass spectrum and the pK^+ missing-mass spectrum.
  These data have not been yet criticised, isn't it?

Molecule model for kaonic nuclear cluster K^-pp with quantum numbers $I(J^P) = \frac{1}{2}(0^-)$: $(K^-p)_{I=0} \otimes p = {}^2_{\bar K}\mathrm{H}$

Wave function of $(K^-p)_{I=0} = {}^1_{\bar K}\mathrm{H}$

$$|{}^1_{\bar K}\mathrm{H}(\vec k, \sigma)\rangle = \sqrt{2E_{{}^1_{\bar K}\mathrm{H}}(\vec k\,)} \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{(2\pi)^3\delta^{(3)}(\vec k - \vec k_1 - \vec k_2)}{\sqrt{2E_K(\vec k_1)2E_p(\vec k_2)}}$$

$$\times\, \Phi_{\Lambda^*}\Big(\frac{m_p\vec k_1 - m_K\,\vec k_2}{m_K + m_p}\Big)\, c^{\dagger}_{K^-}(\vec k_1)\, a^{\dagger}_p(\vec k_2, \sigma)|0\rangle$$

Harmonic oscillator wave function of $\Phi_{{}^1_{\bar K}\mathrm{H}}(\vec q\,)$

$$\Phi_{\Lambda^*}(\vec q\,) = \Big(\frac{4\pi}{\mu_{\Lambda^*}\Omega_{\Lambda^*}}\Big)^{3/4} \exp\Big(-\frac{\vec q^{\,2}}{2\mu_{\Lambda^*}\Omega_{\Lambda^*}}\Big)$$

$$\mu_{\Lambda^*} = \frac{m_K m_p}{m_K + m_p} = 324\,\mathrm{MeV}$$

Wave function of $p \otimes (K^-p)_{I=0} = {}^2_K\mathrm{H}$

$$|{}^2_K\mathrm{H}(\vec{k}\,)\rangle =$$

$$= \sqrt{2E_{{}^2_K\mathrm{H}}(\vec{k}\,)} \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\,(2\pi)^3\delta^{(3)}(\vec{k} - \vec{k}_1 - \vec{k}_2 - \vec{k}_3)$$

$$\times \frac{\Phi_{{}^2_K\mathrm{H}}(\vec{k}, \vec{k}_1, \vec{k}_2, \vec{k}_3)}{\sqrt{2E_p(k_1)2E_p(k_2)2E_K(k_3)}}\, a^\dagger_p(\vec{k}_1, +\frac{1}{2})a^\dagger_p(\vec{k}_2, -\frac{1}{2})c^\dagger_{K^-}(\vec{k}_3)|0\rangle$$

Molecule model for kaonic nuclear cluster K^-pp with quantum numbers $I(J^P) = \frac{1}{2}(0^-)$: $p \otimes (K^-p)_{I=0} = {}^{2}_{\bar K}\mathrm{H}$

Harmonic oscillator wave function of $\Phi_{{}^{2}_{\bar K}\mathrm{H}}(\vec{k}, \vec{k}_1, \vec{k}_2, \vec{k}_3)$

$$\Phi_{{}^{2}_{\bar K}\mathrm{H}}(\vec{k}, \vec{k}_1, \vec{k}_2, \vec{k}_3) =$$

$$= \Phi_{\Lambda^* p}\Big(\frac{(m_K + m_p)\,\vec{k}_1 - m_p\,(\vec{k}_2 + \vec{k}_3)}{m_K + 2m_p}\Big)\,\Phi_{\Lambda^*}\Big(\frac{m_K \vec{k}_2 - m_p \vec{k}_3}{m_K + m_p}\Big)$$

$$\Phi_{\Lambda^* p}(\vec{q}\,) = \Big(\frac{4\pi}{\mu_{\Lambda^* p}\,\Omega_{\Lambda^* p}}\Big)^{3/4} \exp\Big(-\frac{\vec{q}^{\,2}}{2\mu_{\Lambda^* p}\,\Omega_{\Lambda^* p}}\Big)$$

$$\mu_{\Lambda^*} = \frac{m_K m_p}{m_K + m_p} = 324\,\mathrm{MeV} \qquad \mu_{\Lambda^* p} = \frac{m_p(m_K + m_p)}{m_K + 2m_p} = 568\,\mathrm{MeV}$$

Democracy of nuclear forces

$$\mu_{\Lambda^* p}\Omega^2_{\Lambda^* p} = \mu_{\Lambda^*}\Omega^2_{\Lambda^*}$$

Width of kaonic nuclear cluster ${}^{n}_{\bar{K}}\mathrm{H}$ for $n = 1, 2, \ldots,$ where n is the number of nucleons

Quantum field theoretic definition of total width of ${}^{n}_{\bar{K}}\mathrm{H} \to X$ decays

$$\Gamma_{{}^{n}_{\bar{K}}\mathrm{H}} = \frac{1}{2M_{{}^{n}_{\bar{K}}\mathrm{H}}} \sum_{X} (2\pi)^4\, \delta^{(4)}(k_X - k_{{}^{n}_{\bar{K}}\mathrm{H}}) |\langle X|\mathbb{T}|{}^{n}_{\bar{K}}\mathrm{H}(\vec{0}\,)\rangle|^2$$

Unitarity condition and definition of total width of ${}^{n}_{\bar{K}}\mathrm{H} \to X$ decays: $\mathbb{T} - \mathbb{T}^{\dagger} = i\mathbb{T}\,\mathbb{T}^{\dagger}$

$$\Gamma_{{}^{n}_{\bar{K}}\mathrm{H}} = \frac{1}{2M_{{}^{n}_{\bar{K}}\mathrm{H}}} \sum_{X} (2\pi)^4\, \delta^{(4)}(k_X - k_{{}^{n}_{\bar{K}}\mathrm{H}}) |\langle X|\mathbb{T}|{}^{n}_{\bar{K}}\mathrm{H}(\vec{0}\,)\rangle|^2 =$$

$$= \lim_{V,T\to\infty} \frac{2\mathrm{Im}\langle {}^{n}_{\bar{K}}\mathrm{H}(\vec{0}\,)|\mathbb{T}|{}^{n}_{\bar{K}}\mathrm{H}(\vec{0}\,)\rangle}{2M_{{}^{n}_{\bar{K}}\mathrm{H}}VT}$$

“Binding energy” and width of kaonic nuclear cluster ${}^{n}_{\bar{K}}\mathrm{H}$ for $n = 1, 2, \ldots$, where n is the number of nucleons

“Binding energy” $B_{{}^{n}_{\bar{K}}\mathrm{H}} = -\,\epsilon_{{}^{n}_{\bar{K}}\mathrm{H}}$ **and width** $\Gamma_{{}^{n}_{\bar{K}}\mathrm{H}}$ **of** ${}^{n}_{\bar{K}}\mathrm{H}$

$$B_{{}^{n}_{\bar{K}}\mathrm{H}} + i\,\frac{\Gamma_{{}^{n}_{\bar{K}}\mathrm{H}}}{2} = \lim_{V,T\to\infty}\frac{\langle {}^{n}_{\bar{K}}\mathrm{H}(\vec{0}\,)|\mathbb{T}|{}^{n}_{\bar{K}}\mathrm{H}(\vec{0}\,)\rangle}{2M_{{}^{n}_{\bar{K}}\mathrm{H}}VT}$$

Comment on “binding energy” $B_{{}^{n}_{\bar{K}}\mathrm{H}}$

“Binding energy” $B_{{}^{n}_{\bar{K}}\mathrm{H}} \longrightarrow$ “Coalescence energy” $B_{{}^{n}_{\bar{K}}\mathrm{H}}$

Technique for calculation of matrix elements $\langle {}^{n}_{K}\mathrm{H}(\vec{0}\,)|\mathbb{T}|{}^{n}_{K}\mathrm{H}(\vec{0}\,)\rangle$ of $\mathbb{T}$–matrix

- Chiral Lagrangian with derivative meson–baryon couplings invariant under $SU(3) \times SU(3)$ chiral symmetry

$$\mathcal{L}[B(x), P(x)]_{\rm int} = \langle \bar{B}(x) i\gamma^{\mu}[s_{\mu}(x), B(x)]\rangle$$

$$-g_A \langle (1-\alpha_D)\,\bar{B}(x)\gamma^{\mu}[p_{\mu}(x), B(x)] + \alpha_D\,\bar{B}(x)\gamma^{\mu}\{p_{\mu}(x), B(x)\}\rangle$$

$$s_{\mu}(x) = \frac{1}{2}[U^{\dagger}(x), \partial_{\mu}U(x)] \quad p_{\mu}(x) = \frac{1}{2i}\{U^{\dagger}(x), \partial_{\mu}U(x)\}$$

$$U^2(x) = e^{\sqrt{2}\, i\gamma^5 P(x)/F_{\pi}}$$

- Heavy–baryon approximation equivalent to large N_C expansion in multicolour QCD with $SU(N_C)$ for $N_C \to \infty$

Chiral Lagrangians

$$\mathcal{L}^{\mathrm{WT}}_{(K^-p)_{I=0}\to(K^-p)_{I=0}}(x) =$$

$$= \frac{3}{4}\frac{i}{F^2_\pi}\left[\bar p(x)\gamma^\mu p(x)\right]\left(K^{-\dagger}(x)\partial_\mu K^-(x) - \partial_\mu K^{-\dagger}(x)K^-(x)\right)$$

$$\mathcal{L}^{\mathrm{WT}}_{(K^-p)_{I=0}\to(\Sigma\pi)_{I=0}}(x) =$$

$$= \frac{1}{4}\sqrt{\frac{3}{2}}\frac{i}{F^2_\pi}\left[\bar{\vec\Sigma}(x)\gamma^\mu p(x)\right]\cdot\left(\vec\pi(x)\partial_\mu K^-(x) - \partial_\mu\vec\pi(x)K^-(x)\right)$$

“Binding energy” and width

$$B_{{}^{1}_{\bar K}\mathrm{H}} = \frac{3}{4}\frac{1}{F^2_\pi}\left(\frac{\mu_{\Lambda^*}\Omega_{\Lambda^*}}{\pi}\right)^{3/2} \qquad \Gamma_{{}^{1}_{\bar K}\mathrm{H}} = \frac{3}{8\pi}\frac{m_K m_\Sigma}{M_{{}^{1}_{\bar K}\mathrm{H}}}\frac{k_{\Sigma\pi}}{F^2_\pi}B_{{}^{1}_{\bar K}\mathrm{H}}$$

Numerical values of “binding energy” and width

- $\Omega_{\Lambda^*} = 50\,\mathrm{MeV}$: $B_{{}^{1}_{\bar K}\mathrm{H}} = 32\,\mathrm{MeV}$ $\qquad \Gamma_{{}^{1}_{\bar K}\mathrm{H}} = 27\,\mathrm{MeV}$

$$B_{{}^{2}_{\bar{K}}\mathrm{H}} = \lim_{V,T\to\infty} \frac{\mathrm{Re}\,\langle {}^{2}_{\bar{K}}\mathrm{H}(\vec{0}\,)|\mathbb{T}|{}^{2}_{\bar{K}}\mathrm{H}(\vec{0}\,)\rangle}{2M_{{}^{2}_{\bar{K}}\mathrm{H}}VT} =$$

$$= \frac{1}{2M_{{}^{2}_{\bar{K}}\mathrm{H}}}\langle {}^{2}_{\bar{K}}\mathrm{H}(\vec{0}\,)|\mathcal{L}^{\mathrm{WT}}_{(K^-p)_{I=0}\to(K^-p)_{I=0}}(0)|{}^{2}_{\bar{K}}\mathrm{H}(\vec{0}\,)\rangle =$$

$$= \int \frac{d^3k\,'_1}{(2\pi)^3}\frac{d^3k\,'_2}{(2\pi)^3}\frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}$$

$$\times \frac{\Phi^*_{\Lambda^*}(\vec{k}\,'_2 + \frac{m_p}{m_K + m_p}\vec{k}\,'_1)\Phi^*_{\Lambda^* p}(\vec{k}\,'_1)\Phi_{\Lambda^*}(\vec{k}_2 + \frac{m_p}{m_K + m_p}\vec{k}_1)\Phi_{\Lambda^* p}(\vec{k}_1)}{\sqrt{2E_p(\vec{k}\,'_1)2E_p(\vec{k}\,'_2)2E_K(\vec{k}\,'_1 + \vec{k}\,'_2)2E_p(\vec{k}_1)2E_p(\vec{k}_2)2E_K(\vec{k}_1 + \vec{k}_2)}}$$

$$\times \langle 0|a_p(\vec{k}\,'_2, -\frac{1}{2})\,a_p(\vec{k}\,'_1, +\frac{1}{2})\,c_{K^-}(-\vec{k}\,'_1 - \vec{k}\,'_2)\,\mathcal{L}^{\mathrm{WT}}_{(K^-p)_{I=0}\to(K^-p)_{I=0}}(0)$$

$$\times\, c^{\dagger}_{K^-}(-\vec{k}_1 - \vec{k}_2)\,a^{\dagger}_p(\vec{k}_1, +\frac{1}{2})\,a^{\dagger}_p(\vec{k}_2, -\frac{1}{2})|0\rangle$$

Analytical expression of $B_{{}^{2}_{\bar{K}}\mathrm{H}}$

$$B_{{}^{2}_{\bar{K}}\mathrm{H}} =$$

$$= \frac{3}{4}\frac{1}{F^2_\pi}\Big(\frac{\mu_{\Lambda^*}\Omega_{\Lambda^*}}{\pi}\Big)^{3/2} + \frac{3}{4}\frac{1}{F^2_\pi}\Big(\frac{\mu_{\Lambda^* p}\Omega_{\Lambda^* p}}{\pi}\Big)^{3/2}\frac{1}{\Big(1+\dfrac{\mu_{\Lambda^* p}\Omega_{\Lambda^* p}}{\mu_{\Lambda^*}\Omega_{\Lambda^*}}\,\dfrac{\mu^2_{\Lambda^*}}{m^2_K}\Big)^{3/2}} =$$

$$= 58\,\mathrm{MeV}$$

Numerical value $B_{{}^{2}_{\bar{K}}\mathrm{H}} = 58\,\mathrm{MeV}$ is calculated

for $\Omega_{\Lambda^*} = 50\,\mathrm{MeV}$ and $\Omega_{\Lambda^* p} = \Omega_{\Lambda^*}\sqrt{\mu_{\Lambda^*}/\mu_{\Lambda^* p}} = 38\,\mathrm{MeV}$

Decay mode ${}^{2}_{\bar{K}}\mathrm{H} \to p\Lambda^0$

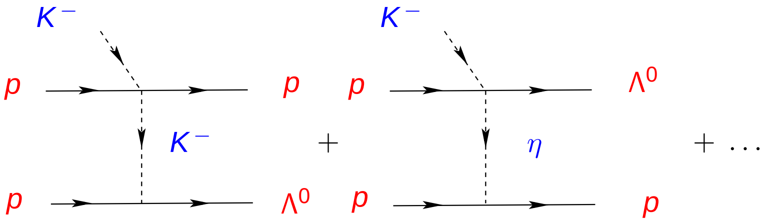


Figure: The Feynman diagrams of the amplitude of the reaction $K^- pp \to p\Lambda^0$ in the Molecule model of the KNC ${}^{2}_{\bar{K}}\mathrm{H}$

Amplitude of decay mode ${}^{2}_{K}\mathrm{H} \to p\Lambda^0$

$$M({}^{2}_{K}\mathrm{H} \to p\Lambda^0) =$$

$$= i\,\sqrt{2M_{{}^{2}_{\bar{K}}\mathrm{H}}}\,\Psi_{{}^{2}_{\bar{K}}\mathrm{H}}(0)\,\frac{m_K\,(M_{{}^{2}_{\bar{K}}\mathrm{H}} + m_{\Lambda^0} + m_p)}{\sqrt{(E_{\Lambda^0} + m_{\Lambda^0})(E_p + m_p)2m_K}}$$

$$\times\,\frac{3}{4}\,\frac{g_A m_p}{F^3_\pi}\Big[\frac{3-2\alpha_D}{\sqrt{3}}\,\frac{f^{p\Lambda^0}_{K^-}(\Omega_{\Lambda^*}, |\vec{k}_{p\Lambda^0}|)}{m^2_K} - \frac{3-4\alpha_D}{\sqrt{6}}\,\frac{f^{p\Lambda^0}_{\eta}(\Omega_{\Lambda^*}, |\vec{k}_{p\Lambda^0}|)}{m^2_\eta}\Big]$$

$$\times\,[\varphi^{\dagger}_{\Lambda^0}(\vec{\sigma}\cdot\vec{k}_{p\Lambda^0})\chi_p]$$

Width of decay mode ${}^{2}_{K}\mathrm{H} \to p\Lambda^0$

$$\Gamma({}^{2}_{K}\mathrm{H} \to p\Lambda^0) = \frac{1}{4\pi}\sum_{\sigma_{\Lambda^0},\sigma_p = \pm\frac{1}{2}} |M({}^{2}_{K}\mathrm{H} \to p\Lambda^0)|^2\,\frac{|\vec{k}_{p\Lambda^0}|}{M_{{}^{2}_{\bar{K}}\mathrm{H}}} = 11\,\mathrm{MeV}$$

Amplitude of decay mode ${}^{2}_{\bar K}\mathrm{H} \to n\Sigma^+$

$$M({}^{2}_{\bar K}\mathrm{H} \to n\Sigma^+) =$$

$$= i\,\sqrt{2M_{{}^{2}_{\bar K}\mathrm{H}}}\;\Psi_{{}^{2}_{\bar K}\mathrm{H}}(0)\,\frac{m_K\,(M_{{}^{2}_{\bar K}\mathrm{H}} + m_{\Sigma^+} + m_n)}{\sqrt{(E_{\Sigma^+} + m_{\Sigma^+})(E_n + m_n)2m_K}}$$

$$\times\,\frac{3}{4}\,\frac{g_A m_p}{F^3_\pi}\Big[\frac{2\alpha_D - 1}{\sqrt{2}}\,\frac{f^{n\Sigma^+}_{\bar K^0}(\Omega_{\Lambda^*}, |\vec{k}_{n\Sigma^+}|)}{m^2_K} - \frac{1}{\sqrt{3}}\,\frac{f^{n\Sigma^+}_{\pi^-}(\Omega_{\Lambda^*}, |\vec{k}_{n\Sigma^+}|)}{m^2_\pi}\Big]$$

$$\times\,[\varphi^\dagger_{\Sigma^+}(\vec{\sigma}\cdot\vec{k}_{n\Sigma^+})\chi_n]$$

Width of decay mode ${}^{2}_{\bar K}\mathrm{H} \to n\Sigma^+$

$$\Gamma({}^{2}_{\bar K}\mathrm{H} \to n\Sigma^+) = \frac{1}{4\pi}\sum_{\sigma_{\Sigma^+},\sigma_n = \pm\frac{1}{2}} |M({}^{2}_{\bar K}\mathrm{H} \to n\Sigma^+)|^2\,\frac{|\vec{k}_{n\Sigma^+}|}{M_{{}^{2}_{\bar K}\mathrm{H}}} = 11\,\mathrm{MeV}$$

Amplitude of decay mode ${}^{2}_{\bar K}\mathrm{H} \to p\Sigma^0$

$$M({}^{2}_{\bar K}\mathrm{H} \to p\Sigma^0) =$$

$$= i\,\sqrt{2M_{{}^{2}_{\bar K}\mathrm{H}}}\;\Psi_{{}^{2}_{\bar K}\mathrm{H}}(0)\,\frac{m_K\,(M_{{}^{2}_{\bar K}\mathrm{H}} + m_{\Sigma^+} + m_n)}{\sqrt{(E_{\Sigma^0} + m_{\Sigma^0})(E_p + m_p)2m_K}}$$

$$\times \frac{3}{4}\,\frac{g_A m_p}{F^3_\pi}\Big[(2\alpha_D - 1)\,\frac{f^{p\Sigma^0}_{\bar K^0}(\Omega_{\Lambda^*}, |\vec k_{p\Sigma^0}|)}{m^2_K} - \frac{1}{\sqrt{6}}\,\frac{f^{p\Sigma^0}_{\pi^0}(\Omega_{\Lambda^*}, |\vec k_{p\Sigma^0}|)}{m^2_\pi}\Big]$$

$$\times\,[\varphi^{\dagger}_{\Sigma^0}(\vec\sigma\cdot\vec k_{p\Sigma^0})\chi_p]$$

Width of decay mode ${}^{2}_{\bar K}\mathrm{H} \to p\Sigma^0$

$$\Gamma({}^{2}_{\bar K}\mathrm{H} \to p\Sigma^0) = \frac{1}{4\pi}\sum_{\sigma_{\Sigma^0},\sigma_p = \pm\frac{1}{2}} |M({}^{2}_{\bar K}\mathrm{H} \to p\Sigma^0)|^2\,\frac{|\vec k_{p\Sigma^0}|}{M_{{}^{2}_{\bar K}\mathrm{H}}} = 3\,\mathrm{MeV}$$

Meson propagators $f_M^{NY}(\Omega_{\Lambda^*}, q)$

$$f_M^{NY}(\Omega_{\Lambda^*}, q) = \frac{1}{2}\frac{1}{\Psi_{\bar{K}^2\mathrm{H}}(0)}\int \frac{d^3k}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}\Phi_{\Lambda^*}(\vec{k}\,)\Phi_{\Lambda^* p}(\vec{p}\,)$$

$$\times\Big(\frac{m_M^2}{m_M^2(q^2) + (\vec{q} + \vec{p}\,)^2} + \frac{m_M^2}{m_M^2(q^2) + \Big(\vec{q} + \dfrac{m_p}{m_p + m_K}\vec{p} + \vec{k}\Big)^2}\Big) =$$

$$= \frac{1}{2}\int_0^\infty \frac{dt\, m_M^2}{(1 + 2\mu_{\Lambda^* p}\Omega_{\Lambda^* p} t)^{3/2}} \exp\Big(-m_M^2(q^2)t - \frac{q^2 t}{1 + 2\mu_{\Lambda^* p}\Omega_{\Lambda^* p} t}\Big)$$

$$+\frac{1}{2}\int_0^\infty \frac{dt\, m_M^2}{\Big(1 + \Big(2\mu_{\Lambda^*}\Omega_{\Lambda^*} + 2\dfrac{\mu_{\Lambda^*}^2}{m_K^2}\mu_{\Lambda^* p}\Omega_{\Lambda^* p}\Big)t\Big)^{3/2}}$$

$$\times \exp\Big(- m_M^2(q^2)t - \frac{q^2 t}{1 + \Big(2\mu_{\Lambda^*}\Omega_{\Lambda^*} + 2\dfrac{\mu_{\Lambda^*}^2}{m_K^2}\mu_{\Lambda^* p}\Omega_{\Lambda^* p}\Big)t}\Big)$$

$$m_K^2(q^2) = m_K^2 - (E_Y(q^2) - m_N) \quad m_{\eta,\pi}^2 = m_{\eta,\pi}^2 - (E_N(q^2) - m_N)$$

Total width of non–pionic decay modes ${}^{2}_{\bar{K}}\mathrm{H} \to NY$

$$\Gamma^{(\not\pi)}_{{}^{2}_{\bar{K}}\mathrm{H}} = 25\,\mathrm{MeV}$$

Estimate of total width of non–pionic decay modes ${}^{2}_{\bar{K}}\mathrm{H} \to NY$ by Akaishi and Yamazaki

$$\Gamma^{(\not\pi)}_{{}^{2}_{\bar{K}}\mathrm{H}} \approx 12\,\mathrm{MeV}$$

Pionic decay mode ${}^{2}_{\bar{K}}\mathrm{H} \to p\Sigma\pi$

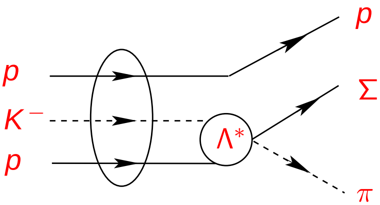


Figure: The Feynman diagram of the pionic decay mode ${}^{2}_{\bar{K}}\mathrm{H} \to p\Sigma\pi$ of the KNC ${}^{2}_{\bar{K}}\mathrm{H}$ with the structure $p \otimes (K^-p)_{I=0}$

Amplitude of pionic decay mode ${}^{2}_{\bar{K}}\mathrm{H} \to p\Sigma\pi$

$$M({}^{2}_{\bar{K}}\mathrm{H} \to p\Sigma\pi) =$$

$$= \frac{1}{F_\pi^2} \frac{\sqrt{6M_{{}^{2}_{\bar{K}}\mathrm{H}} M^2_{{}^{1}_{\bar{K}}\mathrm{H}} m_p m_\Sigma m_K}}{B_{{}^{2}_{\bar{K}}\mathrm{H}} - B_{{}^{1}_{\bar{K}}\mathrm{H}}} \int \frac{d^3k}{(2\pi)^3} \Phi_{\Lambda^*}(\vec{k}\,)$$

$$\times \Big\{ \delta_{\sigma_p,+\frac{1}{2}} \delta_{\sigma_\Sigma,-\frac{1}{2}} \Phi_{\Lambda^* p}(\vec{k}_p) - \delta_{\sigma_p,-\frac{1}{2}} \delta_{\sigma_\Sigma,+\frac{1}{2}}$$

$$\int \frac{d^3q}{(2\pi)^3} \Phi^*_{\Lambda^*}\Big(\vec{q} + \frac{m_p}{m_p + m_K}\vec{k}_p\Big) \Phi_{\Lambda^*}\Big(\vec{k}_p + \frac{m_p}{m_p + m_K}\vec{q}\,\Big) \Phi_{\Lambda^* p}(\vec{q}\,)\Big\}$$

Width of pionic decay mode ${}^{2}_{\bar K}\mathrm{H} \to p\Sigma\pi$

$$\Gamma^{(\pi)}_{{}^{2}_{\bar K}\mathrm{H}} = |\Psi_{\Lambda^*}(0)|^2 |\,\Phi_{\Lambda^* p}(0)|^2 \,\frac{3M^2_{{}^{2}_{\bar K}\mathrm{H}} M^2_{{}^{1}_{\bar K}\mathrm{H}} m_\Sigma m_K}{(B_{{}^{2}_{\bar K}\mathrm{H}} - B_{{}^{1}_{\bar K}\mathrm{H}})^2 F^4_\pi}\, f_{p\Sigma\pi}(\Omega_{\Lambda^*}) =$$

$$= 2.0 \times 10^9\, f_{p\Sigma\pi}(\Omega_{\Lambda^*}) = 56\,\mathrm{MeV}$$

Phase volume contribution

$$f_{p\Sigma\pi}(\Omega_{\Lambda^*}) =$$

$$= \frac{1}{128\pi^3 M^4_{{}^{2}_{\bar K}\mathrm{H}}} \int_{(m_\Sigma + m_\pi)^2}^{(M_{{}^{2}_{\bar K}\mathrm{H}} - m_p)^2} \frac{ds}{s} \sqrt{(s - (m_\Sigma + m_\pi)^2)(s - (m_\Sigma - m_\pi)^2}$$

$$\times \sqrt{(M_{{}^{2}_{\bar K}\mathrm{H}} + m_p)^2 - s)(M_{{}^{2}_{\bar K}\mathrm{H}} - m_p)^2 - s}\, \Big\{\mathrm{R.L.\&C.F.}\Big\} = 2.8 \times 10^{-8}$$

Total width of kaonic nuclear cluster ${}^{2}_{\bar{K}}\mathrm{H}$

$$\Gamma_{{}^{2}_{\bar{K}}\mathrm{H}} = \Gamma^{(\not{\pi})}_{{}^{2}_{\bar{K}}\mathrm{H}} + \Gamma^{(\pi)}_{{}^{2}_{\bar{K}}\mathrm{H}} = (25 + 56)\,\mathrm{MeV} = 81\,\mathrm{MeV}$$

Theoretical analysis of kaonic nuclear cluster ${}^{2}_{\bar{K}}\mathrm{H}$, observed by Yamazaki *et al.* (DISTO), arXiv: 0810.5182 [nucl-ex]

Experimental data
by Yamazaki *et al.* **(DISTO), arXiv: 0810.5182 [nucl-ex]**

$$B^{(\exp)}_{{}^{2}_{\bar{K}}\mathrm{H}} = 105(2)\,\mathrm{MeV} \qquad \Gamma^{(\exp)}_{{}^{2}_{\bar{K}}\mathrm{H}} = 118(8)\,\mathrm{MeV}$$

Peculiarity of KNC ${}^{2}_{\bar{K}}\mathrm{H}$, observed
by Yamazaki *et al.* **(DISTO), arXiv: 0810.5182 [nucl-ex]**

Only non–pionic decay modes ${}^{2}_{\bar{K}}\mathrm{H} \to NY$ are allowed

Molecule model for kaonic nuclear cluster ${}^{2}_{\bar{K}}\mathrm{H}$, observed by Yamazaki *et al.* (DISTO), arXiv: 0810.5182 [nucl-ex]

Experimental data
by Yamazaki *et al.* (DISTO), arXiv: 0810.5182 [nucl-ex]

$$B^{(\exp)}_{{}^{2}_{\bar{K}}\mathrm{H}} = 105(2)\,\mathrm{MeV} \qquad \Gamma^{(\exp)}_{{}^{2}_{\bar{K}}\mathrm{H}} = 118(8)\,\mathrm{MeV}$$

Theoretical "binding energy" and width of KNC ${}^{2}_{\bar{K}}\mathrm{H}$, observed by Yamazaki *et al.* (DISTO), arXiv: 0810.5182 [nucl-ex] for model parameter $\Omega_{\Lambda^*} = 85\,\mathrm{MeV}$

$$B_{{}^{2}_{\bar{K}}\mathrm{H}} = 109\,\mathrm{MeV}$$

$$\Gamma_{{}^{2}_{\bar{K}}\mathrm{H}} = \Gamma_{p\Lambda^0} + \Gamma_{n\Sigma^+} + \Gamma_{p\Sigma^0} = (45 + 48 + 15)\,\mathrm{MeV} = 108\,\mathrm{MeV}$$

KNC ${}^{1}_{\bar{K}}\mathrm{H}$ in Molecule model for kaonic nuclear cluster ${}^{2}_{\bar{K}}\mathrm{H}$, observed by Yamazaki *et al.* (DISTO), arXiv: 0810.5182 [nucl-ex]

Properties of KNC ${}^{1}_{\bar{K}}\mathrm{H}$

$$B_{{}^{1}_{\bar{K}}\mathrm{H}} = 72\,\mathrm{MeV} \quad \Gamma_{{}^{1}_{\bar{K}}\mathrm{H}} = 39\,\mathrm{MeV} \quad M_{{}^{1}_{\bar{K}}\mathrm{H}} = 1362\,\mathrm{MeV}$$

Possible interpretation of ${}^{1}_{\bar{K}}\mathrm{H}$

- KNC ${}^{1}_{\bar{K}}\mathrm{H}$ is one of two resonances, producing the observed $\Lambda^0(1405)$ resonance
- $M_{{}^{1}_{\bar{K}}\mathrm{H}} = (1362 + \delta M^{(\pi\Sigma)}_{{}^{1}_{\bar{K}}\mathrm{H}})\,\mathrm{MeV}$

Summary

We have shown that

- Molecule model of kaonic nuclear clusters describes well the properties of kaonic nuclear clusters ${}^{1}_{K}\mathrm{H}$ and ${}^{2}_{K}\mathrm{H}$, predicted by Akaishi and Yamazaki
- Molecule model of kaonic nuclear clusters describes the experimental data by Yamazaki *et al.* (DISTO), arXiv: 0810.5182 [nucl-ex] on the "binding energy" and width of kaonic nuclear cluster ${}^{2}_{K}\mathrm{H}$

Perspective

- Molecule model of kaonic nuclear clusters can be extended on the description of kaonic nuclear clusters ${}^{n}_{K}\mathrm{H}$ and ${}^{n}_{K\bar{K}}\mathrm{H}$ with $S = -1$ and $S = -2$ and n nucleons

The results, expounded in this talk, have been obtained in Collaboration with

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Thank You for Attention