Λ-hypernuclear production in $(K^-_{\mathrm{stop}}, \pi)$ reactions

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Hadronic Atoms and Kaonic Nuclei -
solved puzzles, open problems and future challenges
in theory and experiment

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Introduction

Hypernucleus - a system containing nucleons (p,n) and at least one hyperon (Λ, Σ, ...)

- strangeness, Pauli exclusion principle, nuclear models, baryon-baryon interaction, weak decays

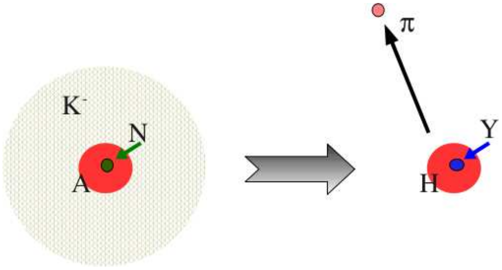
Production - ($K^-_{\rm stop}$, π) reaction, K^- captured at an atomic orbit

- study of K^--nucleus potential (deep x shallow)
- previous calculations unsatisfactory

Experimental study - CERN, KEK, BNL, Frascati (DAΦNE)

Reaction

$$K^- + \mathrm{A} \rightarrow \pi + \mathrm{H}$$



Description - Distorted Wave Impulse Approximation

- reaction takes place on one nucleon
  (process K^--nucleus $\rightarrow$ process K^--nucleon)
- nucleus generates potential that affects K^- (π)
  in initial (final) state

Formalism

T-matrix element[a] $$T_{\rm if}(\mathbf{q}_{\rm f}) = t_{\rm if}(\mathbf{q}_{\rm f}) \int {\rm d}^3\mathbf{r}\ \chi^*_{q_{\rm f}}(\mathbf{r})\ \rho_{\rm if}(\mathbf{r})\ \Psi(\mathbf{r})$$

- $t_{\rm if}$ t-matrix element for elementary process
- $\chi^*_{q_{\rm f}}(\mathbf{r})$ π wave function
- $\rho_{\rm if}(\mathbf{r})$ transition density
- $\Psi(\mathbf{r})$ K^- wave function

Capture rate $$\Gamma_{\rm if} = 2\pi \int \delta(E_{\rm i} - E_{\rm f})\ \left\langle \left| T_{\rm if}(\mathbf{q}_{\rm f}) \right|^2 \right\rangle\ \frac{d^3 q_{\rm f}}{(2\pi)^3}$$

Capture rate per one stopped kaon $$R_{\rm if} = \frac{\Gamma_{\rm if}}{\Gamma_{\rm tot}}$$

[a] Gal A., Klieb L.: Phys. Rev. C **34**, 956 (1986)

Formalism

$R_{\rm if}$ can be simplified to a product of three terms

$$R_{\rm if} = \frac{q_{\rm f}\omega_{\rm f}}{\overline{q}_{\rm f}\overline{\omega}_{\rm f}} \cdot R(K^- N \to \pi Y) \cdot R_{\rm if}/Y$$

- $\frac{q_{\rm f}\omega_{\rm f}}{\overline{q}_{\rm f}\overline{\omega}_{\rm f}}$
  kinematic factor
- $R(K^- N \to \pi Y) = \frac{\overline{\gamma}(K^- N \to \pi Y)\widetilde{\rho}_N}{\overline{\gamma}(K^- p \to {\rm all})\widetilde{\rho}_p + \overline{\gamma}(K^- n \to {\rm all})\widetilde{\rho}_n}$
  elementary branching ratio
- $R_{if}/Y = \frac{\int \left\langle \left| \int {\rm d}^3 r \; \chi_{q_{\rm f}}^{(-)^*}(\mathbf{r}) \; \rho_{\rm if}(\mathbf{r}) \; \Psi(\mathbf{r}) \right|^2 \right\rangle \frac{d\Omega_{q_{\rm f}}}{4\pi}}{\widetilde{\rho}_N}$
  capture rate per hyperon

Elementary process

Elementary branching ratios: chiral model for meson-baryon interactions (previous works used elementary branching ratios derived from experiment)

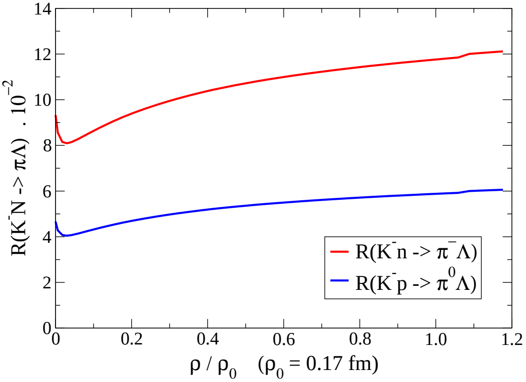
Multichannel Lippmann-Schwinger equation, effective separable potentials[b,c]

Nuclear medium: Pauli principle[d], K^- selfenergy[e]

[b] Kaiser N., Siegel P.B., Weise W.: Nucl. Phys. A **594**, 325 (1995).
[c] Cieplý A., Smejkal J.: Eur. Phys. J. A **34**, 237 (2007).
[d] Waas T., Kaiser N., Weise W.: Phys. Lett. B **365**, 12 (1996).
[e] Cieplý A., Friedman E., Gal A., Mareš J.: Nucl. Phys. A **696**, 173 (2001).

Elementary process

Branching ratios as function of ρ



branching ratio $.10^{-2}$	BR1 $\rho = \rho_0/2$	BR2 $\rho = 0$	BR3 $^{12}\mathrm{C}$	BR3 $^{16}\mathrm{O}$
$R(K^- n \to \pi^- \Lambda)$	10.72	9.33	8.7	7.7
$R(K^- p \to \pi^0 \Lambda)$	5.36	4.66	4.4	3.9

Capture rate per one hyperon

R_{if}/Y can be simplified analytically

$$R_{N\to Y}/Y = \frac{N(j_N)\sum_k (2k+1)(l_N 0 k 0|l_Y 0) N^{(k)}_{\gamma_Y \gamma_N}}{\int \mathrm{d}r\ \rho_N(r)\,|R_{nl}(r)|^2 \sum_l (2l+1)|\tilde{j}_l(r)|^2}$$

- numerator - overlap of wave functions
- denominator - effective nucleon density, effect of distortion of the π w.f. $\tilde{j}_l(r)$ considered {C} or neglected {N}

Input wave functions

Baryons (N,Λ) - Wood-Saxon potential

Standard π-nucleus optical potential (Ericson, Ericson)

- 3 sets of parameters - $(\pi_{\rm free})$, $(\pi_{\rm b})^{(f)}$, $(\pi_{\rm c})^{(g)}$

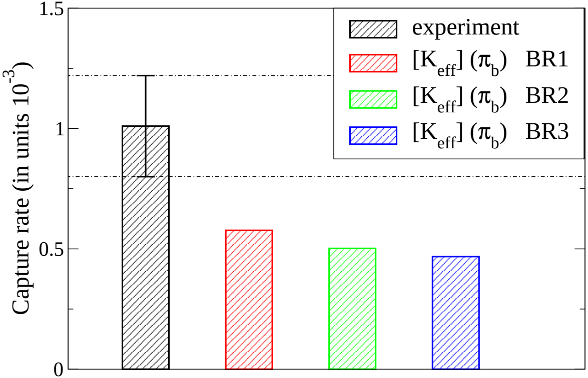
K^--nucleus optical potential

- $V^K_{opt}(r) \approx -a(r)\ \rho(r)$.
- 4 sets of parameters - $[K_{\rm coul}]$, $[K_\chi]\approx$50MeV, $[K_{\rm eff}]\approx$80MeV$^{(h)}$, $[K_{\rm DD}]\approx$190MeV$^{(h)}$

(f) Thiessen H. A. et al.: LAMPF Report no. LA-7607-PR (1978)
(g) Harvey C. J. et al.: LAMPF Report no. LA-UR-84-1732 (1984)
(h) Friedmann E., Gal A., Batty C.J.: Nucl. Phys. A **696**, 173 (1994)

Sensitivity of R_{if} to input data

Sensitivity to the elementary branching ratios

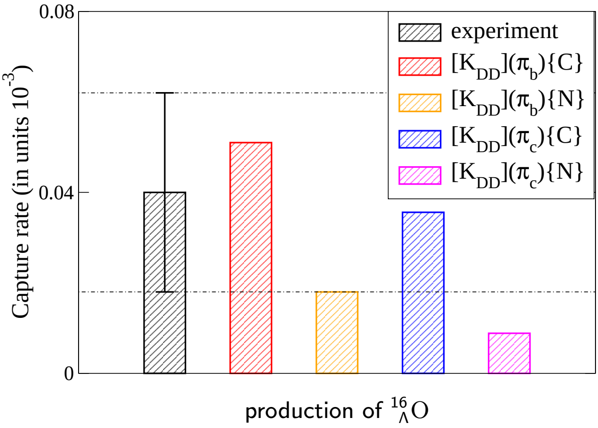


production of ${}^{12}_{\Lambda}\mathrm{C}$

- small differences $\longrightarrow$ BR1 used further

Sensitivity of R_{if} to input data

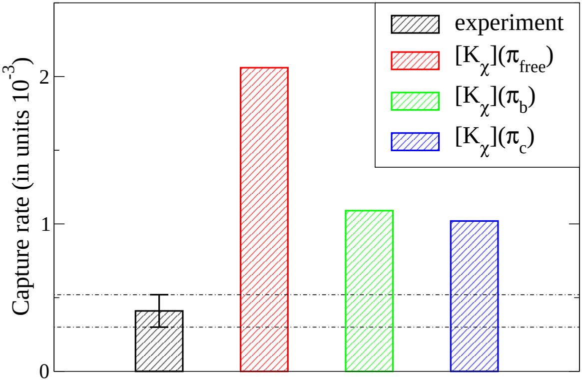
Sensitivity to the π distortion in effective nucleon density



- up to 5-times different $\rightarrow$ neglect is not justified

Sensitivity of R_{if} to input data

Sensitivity to the π wave function

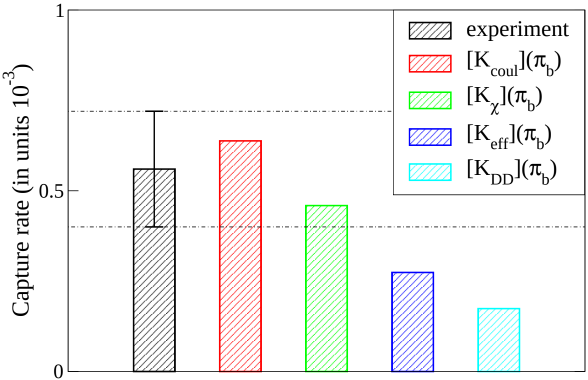


production of $^{7}_{\Lambda}$Li

- significant difference between free and distorted pion
- difference between (π_{b}) and (π_{b}) small

Sensitivity of R_{if} to input data

Sensitivity to the K^- wave function



production of $^{12}_{\Lambda}\mathrm{B}$

- significant differences
- decreasing function of the K^- potential depth

Results

Production of ${}^{7}_{\Lambda}\mathrm{Li}$, ${}^{9}_{\Lambda}\mathrm{Be}$, ${}^{12}_{\Lambda}\mathrm{B}$, ${}^{12}_{\Lambda}\mathrm{C}$, ${}^{13}_{\Lambda}\mathrm{C}$, ${}^{16}_{\Lambda}\mathrm{O}$ hypernuclei in the ground state ($1s_\Lambda$) calculated.

Experimental data (Li$^{(i)}$, Be$^{(i)}$, B$^{(j)}$, C$^{(i,k)}$, O$^{(i,l)}$) used to determine the best combination of potentials (χ^2 test)

χ^2 / N	$[K_{\mathrm{coul}}]$	$[K_\chi]$	$[K_{\mathrm{eff}}]$	$[K_{\mathrm{DD}}]$
(π_{free})	297.7	387.3	313.3	343.9
(π_{b})	58.5	29.2	9.3	6.3
(π_{c})	75.0	32.9	9.6	**5.7**

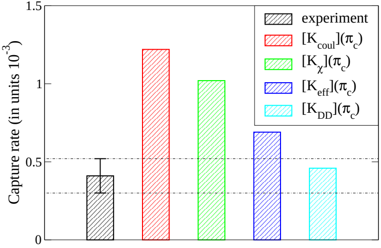
$^{(i)}$ G. Bonomi, HYP-X @ J-PARC (2009).

$^{(j)}$ Ahmed M.W., Cui X., Empl. A., et al.: Phys. Rev. C **68**, 64004-1 (2003).

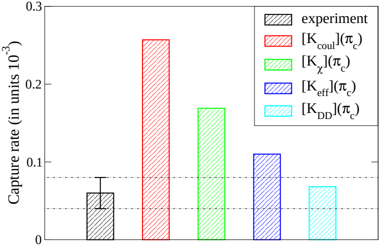
$^{(k)}$ M. Agnello et al., Phys. Lett. B **622**, 35 (2005).

$^{(l)}$ Tamura H., Hayano R.S., Outa H., Yamazaki T.: Prog. Theor. Phys. Suppl. **117**, 1 (1994).

Results



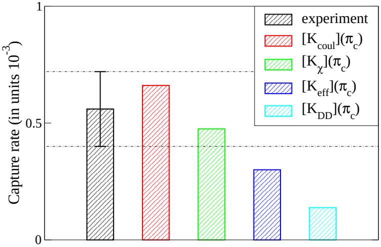
production of ${}^{7}_{\Lambda}\mathrm{Li}$



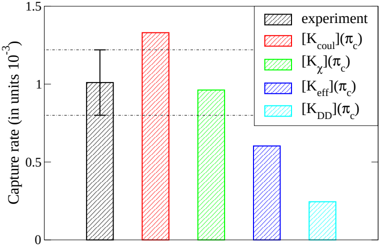
production of ${}^{13}_{\Lambda}\mathrm{C}$

- experiment : FINUDA 2009[(i)]
- potential $[K_{\mathrm{DD}}]$ best

Results



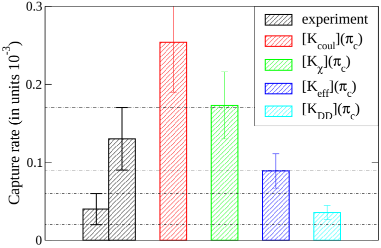
production of ${}^{12}_{\Lambda}\mathrm{B}$



production of ${}^{12}_{\Lambda}\mathrm{C}$

- experiment : BNL$^{(j)}$ FINUDA 2005$^{(k)}$
- potential $[K_\chi]$ best

Results



production of ${}^{16}_{\Lambda}$O

experiment:
left = FINUDA 2009[(i)]
right = KEK[(l)]

theoretical error : 25%

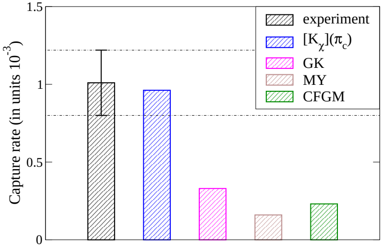
guess based on uncertain values of various input data and parameters

- for particular hypernucleus, we are able to find the combination of potentials within experimental error bars
- inconsistency in experimental data (FINUDA 2009)?

Results

Comparison with previous calculations

- GK - Gal, Klieb, Phys. Rev. C, 1986 [a]
- MY - Matsuyama, Yazaki, Nucl. Phys. A, 1988 [m]
- CFGM - Cieplý et al., Nucl. Phys. A, 2001 [e]



production of $^{12}_{\Lambda}\mathrm{C}$

[m] Matsuyama A., Yazaki K.: Nucl. Phys. A **477**, 6 (1988).

Conclusion

Calculation of $(K^-_{\rm stop}, \pi)$ Λ-hypernuclear production

- significant sensitivity to the wave functions of K^- ($\leq$ 200%)
- capture rate is a decreasing function of the K^--nucleus potential depth
- π distortion in the effective nucleon density is crucial ($\leq$ 500%)
- difference between various distorted π wave functions smaller ($\leq$ 50%) than between free and distorted π w.f. ($\leq$ 300%)
- small effect of elementary branching ratios ($\leq$ 25%)

Better agreement with experiment than previous calculations