



The CGC approach to hydro initial conditions

ECT*, September 2009

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_j

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

François Gelis
IPhT, CEA/Saclay



1 Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

2 Event averaged initial condition

Reminder on hydrodynamics
CGC prediction for $\langle T^{\mu\nu} \rangle$

3 Initial correlations and fluctuations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_t

4 Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

5 Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_t

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

Recombination cross-section :

$$\sigma_{gg \rightarrow g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if $\rho \sigma_{gg \rightarrow g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s x G_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} \frac{1}{x^{0.3}}$$



Gluon saturation

Saturation, rescatterings

Heavy Ion Collisions

Color Glass Condensate

Event averaged initial condition

Reminder on hydro

Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?

Sources of correlations

Correlations in τ_j and \vec{x}_i

Glasma instabilities

Unstable fluctuations

Anomalous transport

Resummation

Open issues

State of the art

Limitations to keep in mind

Instabilities, thermalization

Initial time (in)dependence

Summary

Gluon saturation

Saturation, rescatterings

Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_i

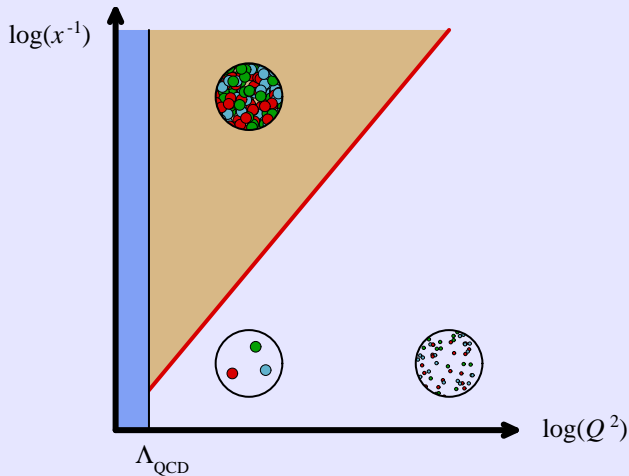
Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



Gluon saturation

Saturation, rescatterings

Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_i

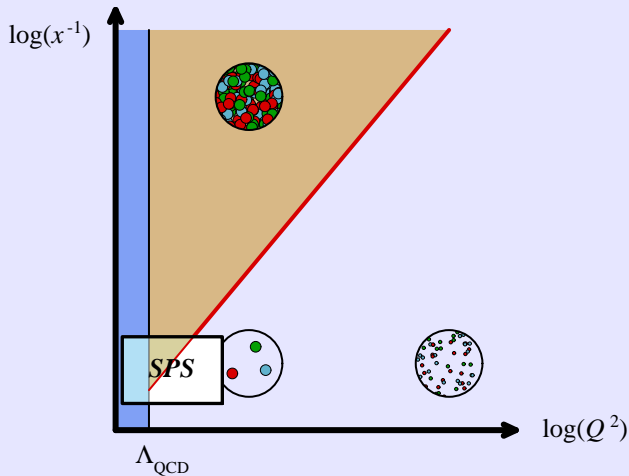
Glasma instabilities

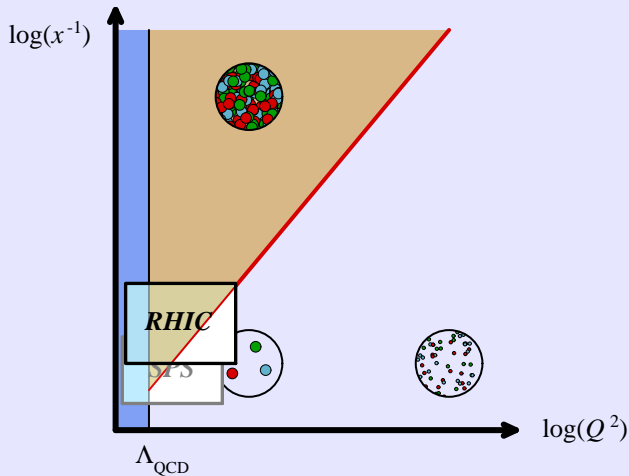
Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary





Gluon saturation

Saturation, rescatterings

Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_T

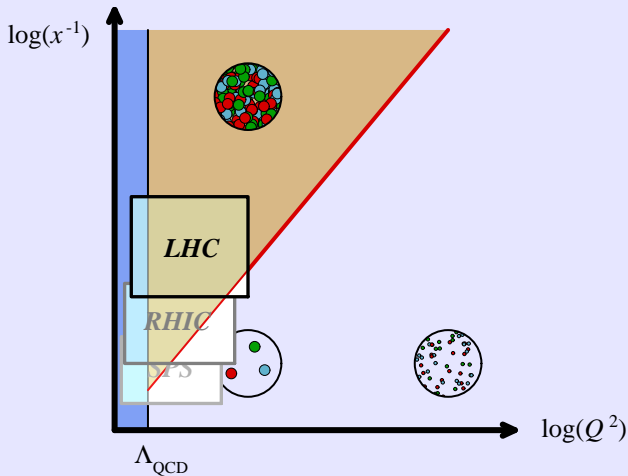
Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



Gluon saturation

Saturation, rescatterings

Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_i

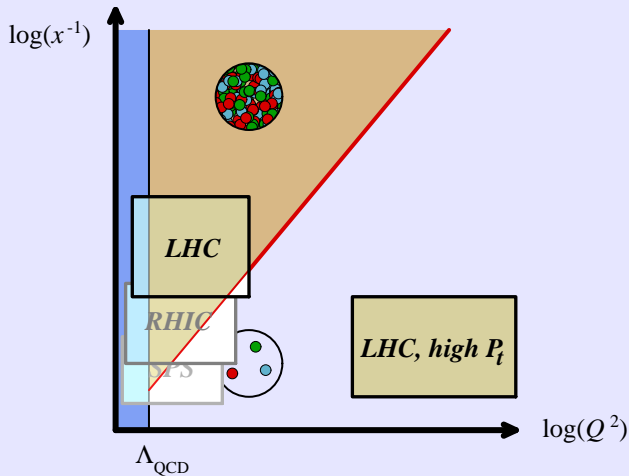
Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



Gluon saturation

Saturation, rescatterings

Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_t

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

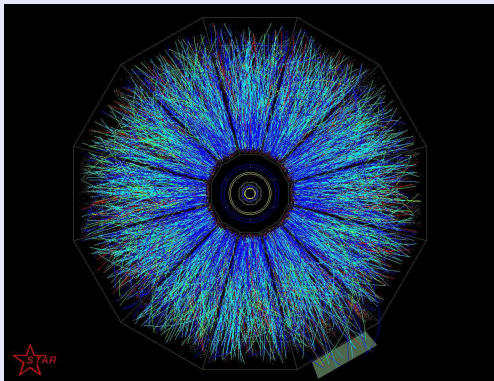
Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



Nucleus-Nucleus collision at RHIC



Gluon saturation

Saturation, rescatterings

Heavy Ion Collisions

Color Glass Condensate

Event averaged initial condition

Reminder on hydro

Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?

Sources of correlations

Correlations in η and \vec{x}_\perp

Glasma instabilities

Unstable fluctuations

Anomalous transport

Resummation

Open issues

State of the art

Limitations to keep in mind

Instabilities, thermalization

Initial time (in)dependence

Summary

- 99% of the multiplicity below $p_\perp \sim 2 \text{ GeV}$
- Q_s^2 might be as large as 10 GeV^2 at the LHC ($\sqrt{s} = 5.5 \text{ TeV}$)
 - ▷ saturation expected to play an important role



McLerran, Venugopalan (1994)

- The fast partons (large $x > x_0$) are frozen by time dilation
▷ described as **static color sources** on the light-cone :

$$J^\mu = \delta^{\mu+} \rho(x^-, \vec{x}_\perp) \quad (x^- \equiv (t - z)/\sqrt{2})$$

Note: $\rho(x^-, \vec{x}_\perp) \propto \delta(x^-)$

- Slow partons (small $x < x_0$) are not static over the time-scales of the collision process
▷ must be treated as the usual gauge fields
▷ coupled to the current J^μ by a term : $J^\mu A_\mu$
- The color sources ρ are **random**, with a **distribution** $W_Y[\rho]$
($Y \equiv \ln(1/x_0)$) is the rapidity separating “slow” and “fast”)

Gluon saturation

Saturation, rescatterings

Heavy Ion Collisions

Color Glass Condensate

Event averaged initial condition

Reminder on hydro

Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?

Sources of correlations

Correlations in η and \vec{x}_\perp

Glasma instabilities

Unstable fluctuations

Anomalous transport

Resummation

Open issues

State of the art

Limitations to keep in mind

Instabilities, thermalization

Initial time (in)dependence

Summary



Evolution equation (JIMWLK) :

$$\frac{\partial W_Y}{\partial Y} = \mathcal{H} W_Y$$
$$\mathcal{H} = \frac{1}{2} \int_{\vec{x}_\perp, \vec{y}_\perp} \frac{\delta}{\delta \tilde{\mathcal{A}}^+(\epsilon, \vec{y}_\perp)} \eta(\vec{x}_\perp, \vec{y}_\perp) \frac{\delta}{\delta \tilde{\mathcal{A}}^+(\epsilon, \vec{x}_\perp)}$$

where $-\partial_\perp^2 \tilde{\mathcal{A}}^+(\epsilon, \vec{x}_\perp) = \rho(\epsilon, \vec{x}_\perp)$

- $\eta(\vec{x}_\perp, \vec{y}_\perp)$ is a non-linear functional of ρ
- This evolution equation resums all the powers of $\alpha_s \ln(1/x)$ and of Q_s/ρ_\perp that arise in loop corrections
- This equation simplifies into the BFKL equation when the source ρ is small (one can expand η in powers of ρ)

Gluon saturation

Saturation, rescatterings

Heavy Ion Collisions

Color Glass Condensate

Event averaged initial condition

Reminder on hydro

Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?

Sources of correlations

Correlations in η and \vec{x}

Glasma instabilities

Unstable fluctuations

Anomalous transport

Resummation

Open issues

State of the art

Limitations to keep in mind

Instabilities, thermalization

Initial time (in)dependence

Summary



- For nucleus-nucleus collisions, there are two strong sources that contribute to the color current :

$$J^\mu \equiv \delta^{\mu+} \delta(x^-) \rho_1(\vec{x}_\perp) + \delta^{\mu-} \delta(x^+) \rho_2(\vec{x}_\perp)$$

Average over the sources ρ_1, ρ_2 :

$$\langle \mathcal{O} \rangle_Y = \int [D\rho_1] [D\rho_2] W_{Y_{\text{beam}}-Y}[\rho_1] W_{Y+Y_{\text{beam}}}[\rho_2] \mathcal{O}[\rho_1, \rho_2]$$

Gluon saturation

Saturation, rescatterings

Heavy Ion Collisions

Color Glass Condensate

Event averaged initial condition

Reminder on hydro

Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?

Sources of correlations

Correlations in η and \vec{x}_\perp

Glasma instabilities

Unstable fluctuations

Anomalous transport

Resummation

Open issues

State of the art

Limitations to keep in mind

Instabilities, thermalization

Initial time (in)dependence

Summary



1 Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

2 Event averaged initial condition

Reminder on hydrodynamics
CGC prediction for $\langle T^{\mu\nu} \rangle$

3 Initial correlations and fluctuations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_t

4 Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

5 Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_t

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

Reminder on hydro

Equations of hydrodynamics :

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu J_B^\mu = 0$$

Additional inputs :

$$p = f(\epsilon) \quad , \quad \eta, \zeta, \dots$$

- Required initial conditions :

$$T^{\mu\nu}(\tau = \tau_0, \eta, \vec{\mathbf{x}}_\perp), J_B^\mu(\tau = \tau_0, \eta, \vec{\mathbf{x}}_\perp)$$

Glouon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and $\vec{\mathbf{x}}_\perp$

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

CGC initial conditions (Leading Order)

- In the CGC framework, $\mathbf{J}_B^\mu = 0$
- In the saturation regime, $\rho_{1,2} \sim g^{-1}$, and we have the following expansion for $T^{\mu\nu}$:

$$T^{\mu\nu} = \frac{Q_s^4}{g^2} \left[c_0 + c_1 g^2 + c_2 g^4 + \dots \right]$$

- The Leading Order contribution is given by **classical fields**:

$$T_{LO}^{\mu\nu} \equiv c_0 \frac{Q_s^4}{g^2} = \frac{1}{4} g^{\mu\nu} \mathcal{F}^{\lambda\sigma} \mathcal{F}_{\lambda\sigma} - \mathcal{F}^{\mu\lambda} \mathcal{F}^\nu{}_\lambda$$

with $\underbrace{[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}]} = \mathbf{J}^\nu$, $\lim_{t \rightarrow -\infty} \mathcal{A}^\mu(t, \vec{x}) = 0$
Yang–Mills equation



Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_i

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

CGC initial conditions (Leading Log resummation)

- The previous power counting implicitly assumes that the coefficients c_n are numbers of order one. However, they contain (possibly large) logarithms of $1/x_{1,2}$:

$$\begin{aligned}
 c_1 &= d_{10} + d_{11} \ln\left(\frac{1}{x_{1,2}}\right) \\
 c_2 &= d_{20} + d_{21} \ln\left(\frac{1}{x_{1,2}}\right) + \underbrace{d_{22} \ln^2\left(\frac{1}{x_{1,2}}\right)}_{\text{Leading Log terms}}
 \end{aligned}$$

$$\langle T^{\mu\nu}(\tau, \eta, \vec{x}_\perp) \rangle_{\text{LLog}} = \int [D\rho_1 D\rho_2] W_{Y_1}[\rho_1] W_{Y_2}[\rho_2] \underbrace{T_{\text{LO}}^{\mu\nu}(\tau, \vec{x}_\perp)}_{\text{for fixed } \rho_{1,2}}$$

with $\partial_\gamma W = \mathcal{H}W$, $Y_1 = Y_{\text{beam}} - \eta$, $Y_2 = Y_{\text{beam}} + \eta$

(FG, Lappi, Venugopalan (2008))

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_\perp

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



Importance of factorization

- A factorization formula divides an observable into a **perturbatively calculable part** and a **non-perturbative part** describing the partonic content of nuclei :

$$\langle \mathcal{O} \rangle = \int W_1 \otimes W_2 \otimes \mathcal{O}_{\text{partonic}}$$

- QCD has no predictive power, unless :

- W does not depend on the observable
- W of one projectile does not depend on the second projectile

- Factorization is related to log resummations :

- Loop corrections generate corrections $\sim [g^2 \log(x^{-1})]^n$
- Universality requires that these logs are resummed by the same evolution equation (JIMWLK in the saturation regime)
- The summation of these logs drives the x dependence of W

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_i

Glasma instabilities

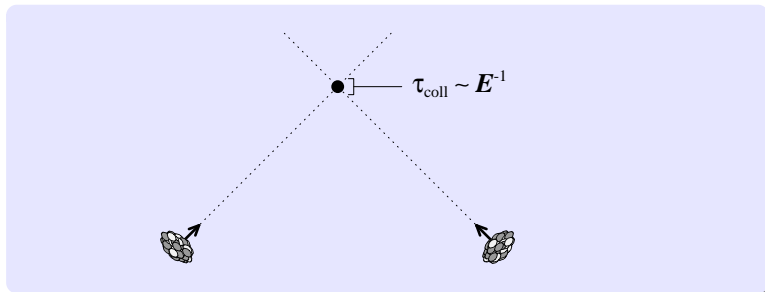
Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

Why factorization works: causality



- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro

Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?

Sources of correlations

Correlations in η and \vec{x}_T

Glasma instabilities

Unstable fluctuations

Anomalous transport

Resummation

Open issues

State of the art

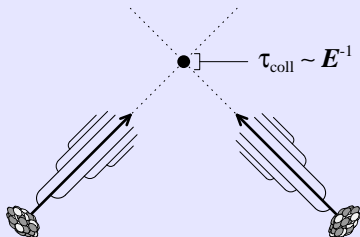
Limitations to keep in mind

Instabilities, thermalization

Initial time (in)dependence

Summary

Why factorization works: causality



- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
 - ▷ it must happen (long) before the collision



Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro

Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?

Sources of correlations

Correlations in η and \vec{x}_T

Glasma instabilities

Unstable fluctuations

Anomalous transport

Resummation

Open issues

State of the art

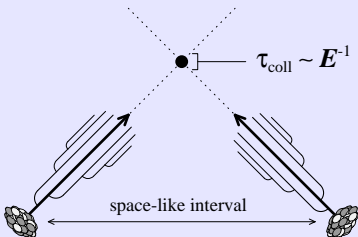
Limitations to keep in mind

Instabilities, thermalization

Initial time (in)dependence

Summary

Why factorization works: causality



- The duration of the collision is very short: $\tau_{\text{coll}} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
 - ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
 - ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro

Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_T

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



1 Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

2 Event averaged initial condition

Reminder on hydrodynamics
CGC prediction for $\langle T^{\mu\nu} \rangle$

3 Initial correlations and fluctuations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_t

4 Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

5 Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_t

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



Initial correlations matter for hydro

- The equations of hydrodynamics are **non-linear**. Therefore, running hydro for event averaged initial conditions is not the same as running hydro event-by-event, and averaging observables at the end :

$$\text{HYDRO} \left[\left\langle T_{\text{init}}^{\mu\nu} \right\rangle \right] \neq \left\langle \text{HYDRO} \left[T_{\text{init}}^{\mu\nu} \right] \right\rangle$$

- To do hydro event by event, one needs an event generator for $T^{\mu\nu}(\tau_0, \eta, \vec{\mathbf{x}}_{\perp})$
- To achieve this, it is not sufficient to know the average $\left\langle T^{\mu\nu}(\tau_0, \eta, \vec{\mathbf{x}}_{\perp}) \right\rangle$. We also need :

$$\left\langle T^{\mu_1\nu_1}(\tau_0, \eta_1, \vec{\mathbf{x}}_{1\perp}) T^{\mu_2\nu_2}(\tau_0, \eta_2, \vec{\mathbf{x}}_{2\perp}) \right\rangle$$

$$\left\langle T^{\mu_1\nu_1}(\tau_0, \eta_1, \vec{\mathbf{x}}_{1\perp}) T^{\mu_2\nu_2}(\tau_0, \eta_2, \vec{\mathbf{x}}_{2\perp}) T^{\mu_3\nu_3}(\tau_0, \eta_3, \vec{\mathbf{x}}_{3\perp}) \right\rangle$$

...

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?

Sources of correlations
Correlations in η and $\vec{\mathbf{x}}_{\perp}$

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

Initial rapidity correlations matter by themselves

Glauon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?

Sources of correlations
Correlations in η and \vec{x}_\perp

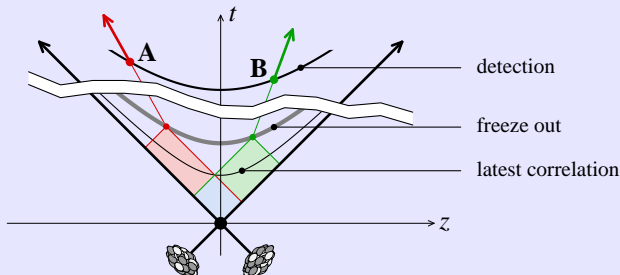
Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



Long range rapidity correlations are created early

From causality, the latest time at which a correlation between two particles can be created is :

$$t_{\text{correlation}} \leq t_{\text{freeze out}} e^{-\frac{1}{2}|y_A - y_B|}$$

With $t_{\text{freeze out}} = 10 \text{ fm}/c$, $|y_A - y_B| = 6$: $t_{\text{correlation}} \leq 0.5 \text{ fm}/c$

- Nucleons inside a nucleus are correlated by the binding nuclear forces
 - ▷ Monte-Carlo generator for the positions of the nucleons inside the nuclei (this information enters in the initial condition for the JIMWLK equation)
- The color sources inside the nucleons acquire correlations when they evolve to smaller values of x
 - ▷ two color sources that resulted from the splitting of a common ancestor are correlated. These correlations are included in the solution $W_\gamma[\rho]$ of the JIMWLK equation



Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?

Sources of correlations

Correlations in τ_j and \vec{x}_i

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

- The factorization valid for $\langle T^{\mu\nu} \rangle$ can be extended to multi-point correlations :

$$\begin{aligned} \langle T^{\mu_1\nu_1}(\tau, \eta_1, \vec{x}_{1\perp}) \cdots T^{\mu_n\nu_n}(\tau, \eta_n, \vec{x}_{n\perp}) \rangle_{\text{LLog}} &= \\ &= \int [D\rho_1 D\rho_2] W[\rho_1] W[\rho_2] \\ &\quad \times T_{\text{LO}}^{\mu_1\nu_1}(\tau, \vec{x}_{1\perp}) \cdots T_{\text{LO}}^{\mu_n\nu_n}(\tau, \vec{x}_{n\perp}) \end{aligned}$$

- Note: at Leading Log accuracy, all the rapidity correlations come from the evolution of the distributions $W[\rho_{1,2}]$
 - ▷ they are a property of the pre-collision initial state
- The most difficult part here is to solve the JIMWLK equation to obtain the evolved W 's

Glouon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations

Correlations in η and \vec{x}_\perp

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

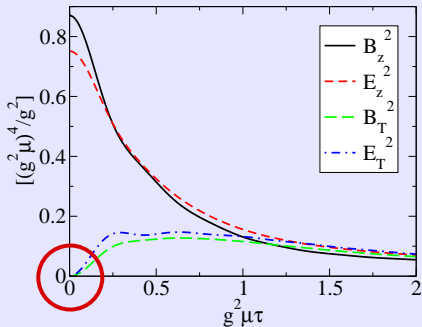
Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

Lappi, McLerran (2006)

- Immediately after the collision, the chromo- \vec{E} and \vec{B} fields are purely longitudinal and boost invariant :



Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations

Correlations in η and \vec{x}_T

Glasma instabilities

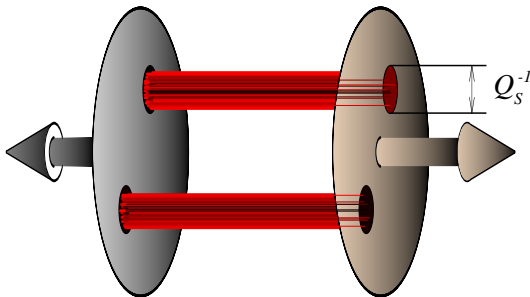
Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

- The initial chromo- \vec{E} and \vec{B} fields form longitudinal “flux tubes” extending between the projectiles:



- The color correlation length in the transverse plane is Q_s^{-1}
 - ▷ flux tubes of diameter Q_s^{-1} , filling up the transverse area
- The correlation length in the η direction is $\Delta\eta \sim \alpha_s^{-1}$



1 Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

2 Event averaged initial condition

Reminder on hydrodynamics
CGC prediction for $\langle T^{\mu\nu} \rangle$

3 Initial correlations and fluctuations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_t

4 Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

5 Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_t

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



Classical prediction for $T^{\mu\nu}$

- The $T^{\mu\nu}$ predicted from the classical color fields is far from equilibrium:
 - $T^\mu{}_\mu = 0$ (classical equations are scale invariant)
 - At $\tau = 0^+$, $T^{xx}, T^{yy} > 0$, while $T^{zz} < 0$:

$$\langle T^{\mu\nu}(\tau = 0^+) \rangle = \begin{pmatrix} \epsilon_0 & & & \\ & \epsilon_0 & & \\ & & \epsilon_0 & \\ & & & -\epsilon_0 \end{pmatrix}$$

- Using these initial conditions for hydro requires some “voodoo”... Implicitly, one assumes the existence of a mechanism that brings $T^{\mu\nu}$ closer to the equilibrium form
- Open issue: is there such a mechanism in the CGC framework?

Glouon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_i

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

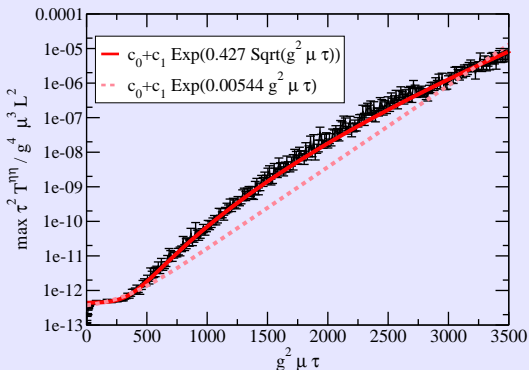
Summary



Unstable fluctuations

Romatschke, Venugopalan (2005)

- Rapidity dependent perturbations to the classical fields grow like $\exp(\sqrt{\mu\tau})$ ($\mu \sim Q_s$) until the non-linearities become important :



Glouon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_T

Glasma instabilities

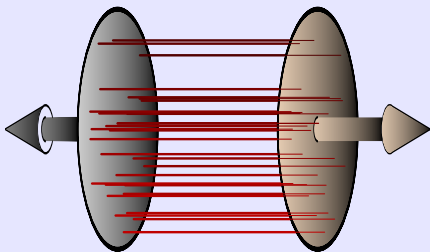
Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

- Leading order magnetic fields at $\tau = 0^+$:



- At $\tau = 0^+$, the classical chromo-electric and chromo-magnetic fields are longitudinal
- They are also boost invariant in the longitudinal direction (independent of η)

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_i

Glasma instabilities

Unstable fluctuations

Anomalous transport
Resummation

Open issues

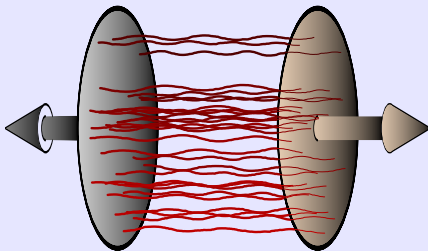
State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



Unstable fluctuations

- Leading order + quantum fluctuations at $\tau = 0^+$:



- Loop corrections bring quantum fluctuations in this picture
- In the weak coupling regime, they are small corrections

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_T

Glasma instabilities

Unstable fluctuations

Anomalous transport
Resummation

Open issues

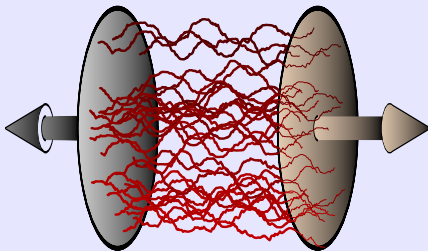
State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



Unstable fluctuations

- Effect of the instability :



- η -dependent perturbations grow quickly in time $\sim e^{\sqrt{Q_s \tau}}$
- Breakdown of the CGC approach at $\tau_{\max} \sim Q_s^{-1} \ln^2(g^{-2})$?
- Outcome : disordered configurations of color fields?

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_\perp

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

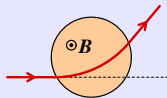
Summary

Asakawa, Bass, Muller (2006)

- Consider a domain of size Q_s^{-1} , in which the magnetic field is uniform and large, of order $B \sim Q_s^2/g$
- Let a particle of energy $E \sim Q_s$ go through this domain

$$\frac{d\vec{p}}{dt} = g \vec{v} \times \vec{B} \quad \Rightarrow \quad \dot{\theta} = \frac{gB}{E} \sim Q_s$$

time spent in the domain : $\delta\tau \sim Q_s^{-1}$



▷ The Lorentz force deflects its trajectory by an angle of order unity

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \bar{x}_i

Glasma instabilities

Unstable fluctuations

Anomalous transport

Resummation

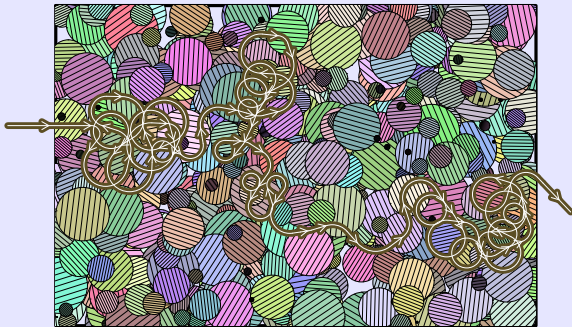
Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

Anomalous transport

- Consider now a region filled with such domains, with random orientations for the magnetic field in each domain



- ▷ In such a medium, the mean free path of a particle of energy Q_s is of order Q_s^{-1} , i.e. as low as permitted by the uncertainty principle ▷ fast thermalization?

Glouon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_i

Glasma instabilities

Unstable fluctuations

Anomalous transport

Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



Resummation of the unstable terms

- To go beyond the time τ_{\max} , one must resum all the **fastest growing instabilities** $\sim [g^2 e^{\sqrt{\mu\tau}}]^n$
- Summing both the large logs of $1/x_{1,2}$ and these unstable terms, we get (FG, Lappi, Venugopalan (2008)) :

$$\begin{aligned}
 \langle T^{\mu\nu}(\tau, \eta, \vec{x}_\perp) \rangle & \underset{\text{LLog resummed}}{=} \int [D\rho_1 D\rho_2] W_{Y_1}[\rho_1] W_{Y_2}[\rho_2] \\
 & \times \int [Da] F[a] T_{\text{LO}}^{\mu\nu}[\underbrace{\mathcal{A} + a}_{\text{initial field}}]
 \end{aligned}$$

- $F[a]$ is the spectrum of the initial color field fluctuations. It can be calculated analytically (in some approximation) (Fukushima, FG, McLerran (2006))

Gloun saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_\perp

Glasma instabilities

Unstable fluctuations
Anomalous transport

Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary



1 Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

2 Event averaged initial condition

Reminder on hydrodynamics
CGC prediction for $\langle T^{\mu\nu} \rangle$

3 Initial correlations and fluctuations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_t

4 Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

5 Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_t

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

- Solution of the **Yang-Mills equations** :
Krasnitz, Nara, Venugopalan (1999-2003)
Lappi (2003)
Romatschke, Venugopalan (2005) (in 3-dimensions)
- Solution of the **JIMWLK equation** for $W_Y[\rho]$:
Rummukainen, Weigert (2004) (not reproduced since then, very heavy computationnally)
- Solution of JIMWLK in the mean field approximation :
▷ **Balitsky-Kovchegov equation**
Many independent codes exist for solving the BK equation
- Average over the spectrum of initial fluctuations : work in progress, complicated by strong ultraviolet divergences



Glouon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_i

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art

Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary

Limitations to keep in mind

- All this is based so far on **Leading Log** calculations
 - ▷ as usual, the K -factor for going from LLog to NLog could be rather large. Maybe up to $K \sim 2$
 - ▷ some efforts are ongoing to extend the JIMWLK evolution equation to NLog (**Kovchegov, Weigert, Balitsky,...**), but that's only half of the story
- As with any evolution equation, **the initial condition** (for $W[\rho]$) **is not predicted by pQCD**
 - ▷ it must be constrained by data. eA collisions would be the best. pA collisions are the next best thing

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_j

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art

Limitations to keep in mind

Instabilities, thermalization
Initial time (in)dependence

Summary



Instabilities, fluctuations and thermalization

- Why is it difficult to resum the initial fluctuations ?
 - The spectrum of these fluctuations contains arbitrarily high momentum modes (origin : these fluctuations amount to loop corrections, in which the momentum that circulates has no upper bound)
 - The energy momentum tensor has dimension (momentum)⁴. It diverges like the power four of the hardest momentum in the fluctuation spectrum
- Strategy 1 : put a cutoff $\Lambda \sim Q_s$ by hand
- Strategy 2 : do all the calculations with a cutoff Λ . Try to perform the cancellation of the UV divergences numerically
 - ▷ delicate cancellations between large numbers
- Strategy 3 : work out the structure of the UV divergences analytically, in order to get finite analytical expressions, that are then evaluated numerically

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_i

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind

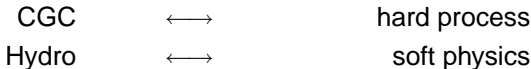
Instabilities, thermalization

Initial time (in)dependence

Summary

Initial time (in)dependence

- The CGC provides the value of $T^{\mu\nu}$ at some initial time τ_0
- Hydrodynamics is used to describe the system at $\tau \geq \tau_0$
- Some observable \mathcal{O} is measured at a late time τ_f
- The value of $\mathcal{O}(\tau_f)$ should be independent of the time τ_0 at which one switches from the CGC description to the hydro description (at least in a reasonable window)
- Analogy : τ_0 is similar to the factorization scale used in fragmentation functions



▷ Varying the factorization scale (i.e. τ_0) and checking the effect of this variation on the final result is a good way of assessing the uncertainty of the whole chain

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in τ_j and \vec{x}_j

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization

Initial time (in)dependence

Summary



Things that work right now: Leading Log framework

- Evolution equations for the distribution of sources (JIMWLK, BK)
- Initial $W[\rho]$ (some constraints from DIS and RHIC)
- $\langle T^{\mu\nu}(\tau, \eta_1, \vec{x}_{1\perp}) T^{\rho\sigma}(\tau, \eta_2, \vec{x}_{2\perp}) \cdots \rangle$

Things that need serious work

- Resummation of the unstable modes (\triangleright positive P_L ?)

Things that should be looked at eventually

- Independence w.r.t. the time at which one starts hydro
- Next to Leading Log

Gluon saturation

Saturation, rescatterings
Heavy Ion Collisions
Color Glass Condensate

Event averaged initial condition

Reminder on hydro
Prediction for $\langle T^{\mu\nu} \rangle$

Initial correlations

Why does it matter?
Sources of correlations
Correlations in η and \vec{x}_\perp

Glasma instabilities

Unstable fluctuations
Anomalous transport
Resummation

Open issues

State of the art
Limitations to keep in mind
Instabilities, thermalization
Initial time (in)dependence

Summary