Effect of fluctuations on v_4

Clément Gombeaud
IPhT-CEA/Saclay
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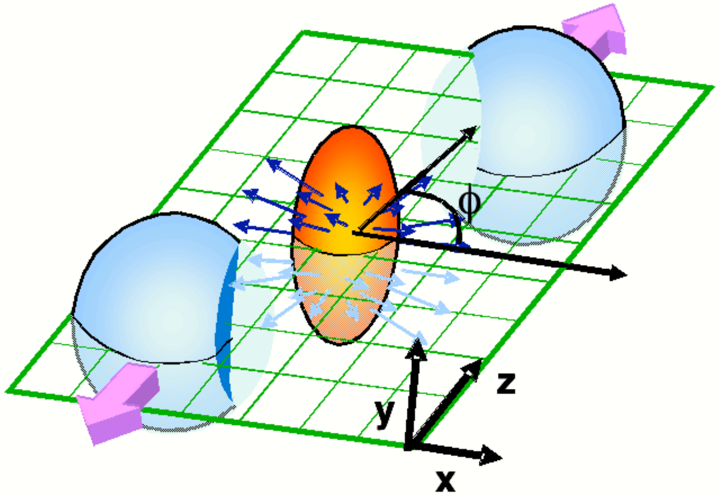
Outline

- Introduction: Anisotropic flow
- v_4 in hydrodynamics
- Flow Fluctuations
  - From eccentricity fluctuations
  - From elliptic flow analysis
- Effects of partial thermalization
- Comparison with data
- Open discussion
  - Toy model of flow fluctuations
- Conclusion

Anisotropic flow



$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \cdots$$

v_2 well understood

Values of v_2 observed at RHIC

→ Nearly perfect fluid

Centrality dependence of v_2

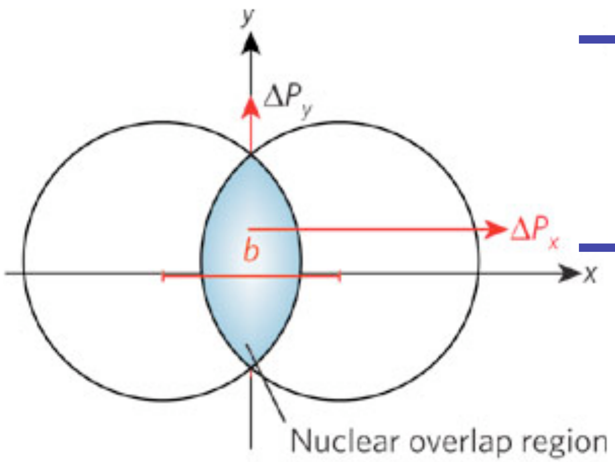
→ Not fully thermalized system

Drescher, Dumitru, CG, Ollitrault, Phys. Rev; C76: 024905, 2007

Values observed for v_4 not explained

Hydrodynamic predictions

Pressure gradient



→ Anisotropic fluid velocity distribution

$$u(\phi) = U\left(1 + 2V_2 \cos 2\phi + 2V_4 \cos 4\phi \cdots\right)$$

→ Anisotropic distribution of particles

$$\frac{dN}{p_t dp_t d\phi} \propto e^{-p \cdot u/T} = \exp\left(-\frac{m_t u_0(\phi) - p_t u(\phi)}{T}\right)$$

Expanding in Fourier series

$$v_2(p_t) = \frac{V_2 U}{T}(p_t - m_t v)$$

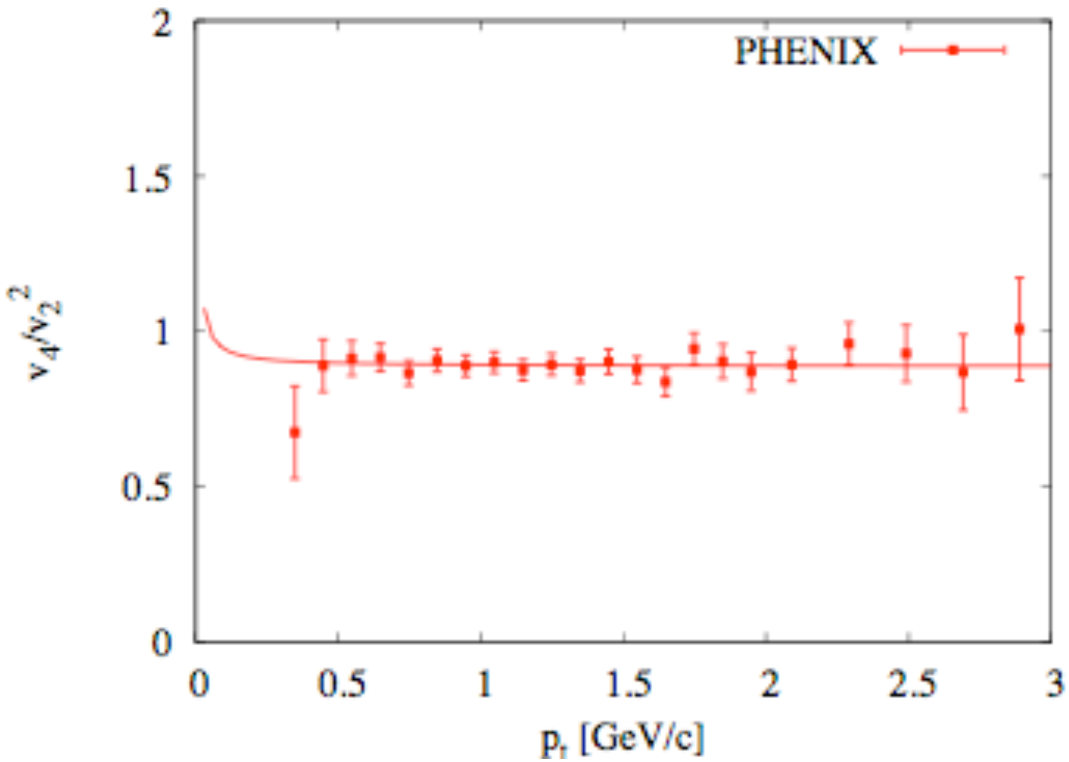
$$v_4(p_t) = \frac{1}{2} v_2(p_t)^2 + \frac{V_4 U}{T}(p_t - m_t v)$$

quadratic in p_t

linear in p_t

$v_4 = 0.5 v_2^2$ at high p_t

PHENIX Results



PHENIX data for charged pions

$$m_t = p_t$$

Au-Au $\sqrt{s} = 200\text{GeV}$

20-60% most central

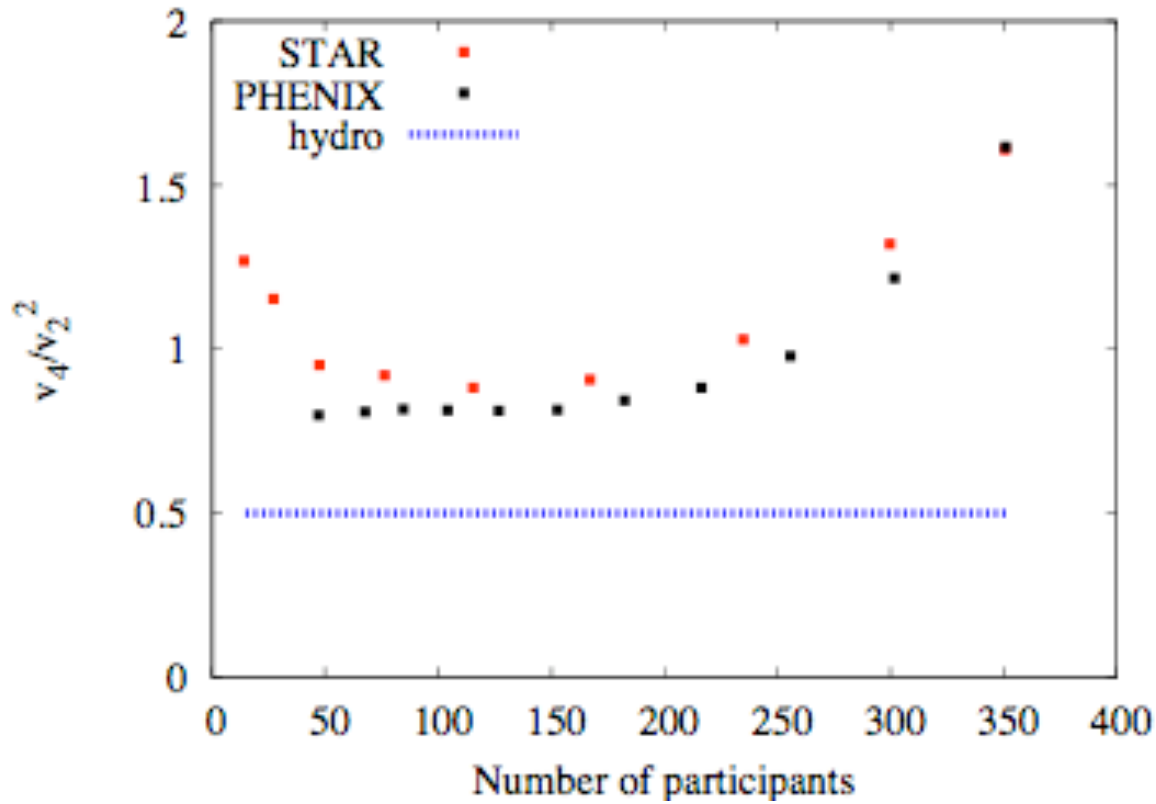
Line, fit using $$\frac{v_4(p_t)}{v_2(p_t)^2} = A + B\frac{\langle p_t \rangle}{p_t}$$

Fit formula motivated by hydro

In practice the constant term dominates even at relatively low p_t

Asymptotic value much above the hydro prescription

Data versus hydro



Au-Au

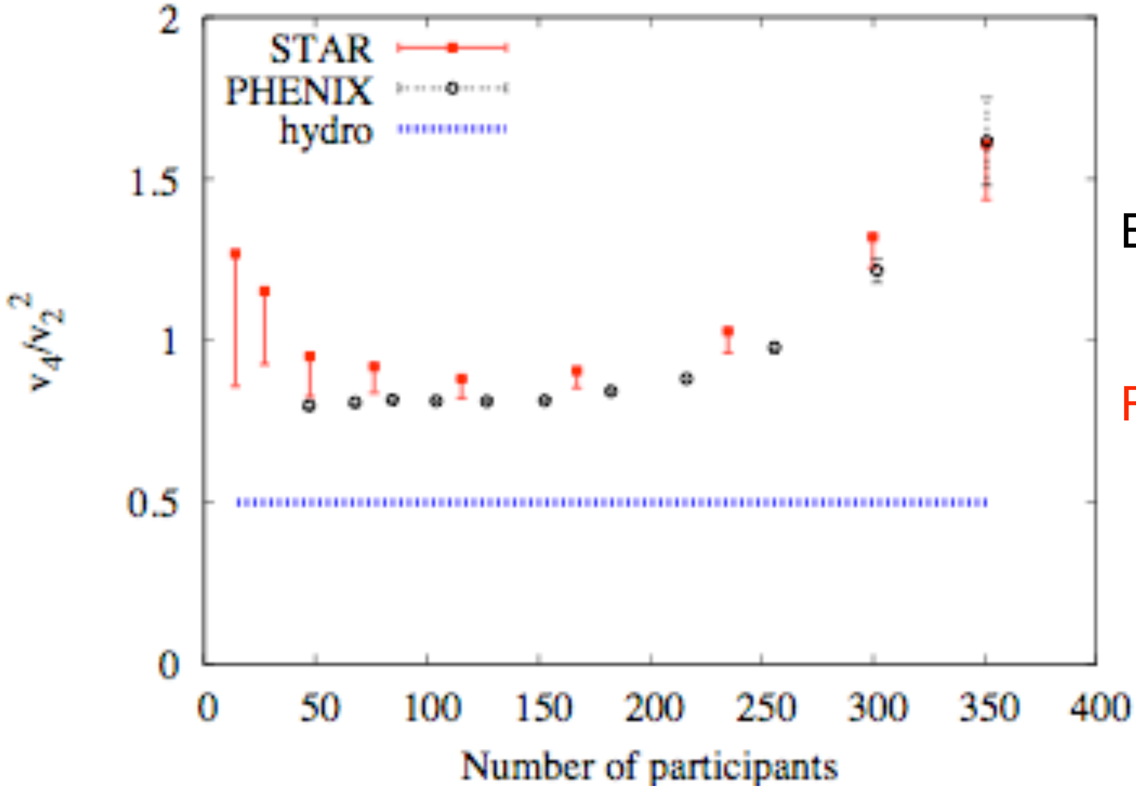
$$\sqrt{s} = 200\mathrm{GeV}$$

per nucleon

PHENIX: charged hadrons, p_t between 1 and 2.4 GeV
STAR: charged particles, p_t between 1 and 2.7 GeV

Data > hydro
Small discrepancy between STAR and PHENIX data

Experimental errors

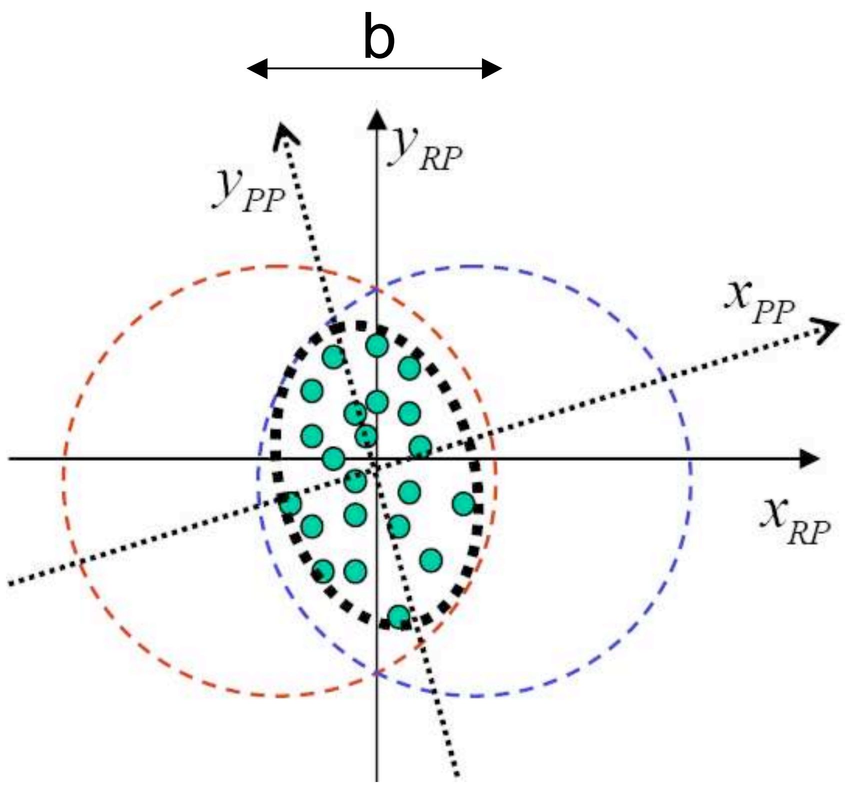


Black errors: statistical errors from PHENIX

Red errors: Order of magnitude of the non-flow effects on the measured v_4 (Our estimate)

Non-Flow effects on v_2 not included

Initial eccentricity



For each event:

-The Reaction Plane eccentricity (or standard eccentricity) is defined as

$$\epsilon_s(b) = \frac{\langle y_{RP} - x_{RP} \rangle}{\langle y_{RP} + x_{RP} \rangle}$$

-Distribution of participating nucleons defines the Participant plane eccentricity

$$\epsilon_{PP} = \frac{\langle y_{PP} - x_{PP} \rangle}{\langle y_{PP} + x_{PP} \rangle}$$

Flow fluctuations from initial eccentricity

v_2 created by initial eccentricity in the participant plane

From one event to another, ϵ_{PP} may fluctuate around $\epsilon_s(b)$

→ Standard model: 2 dimensional gaussian statistics

Voloshin & al Phys. Lett. B659, 537-541 (2008)

eccentricity fluctuates → v_2 and v_4 fluctuate

Why ε fluctuations change v_4/v_2^2

Experimentally, no direct measure of v_2 and v_4

v_2 and v_4 are measured via azimuthal correlations

v_2 from $\langle cos(2\phi_1 - 2\phi_2)\rangle = \langle (v_2)^2 \rangle$

v_4 from $\langle cos(4\phi_1 - 2\phi_2 - 2\phi_3)\rangle = \langle v_4 (v_2)^2 \rangle$

Experimentally measured

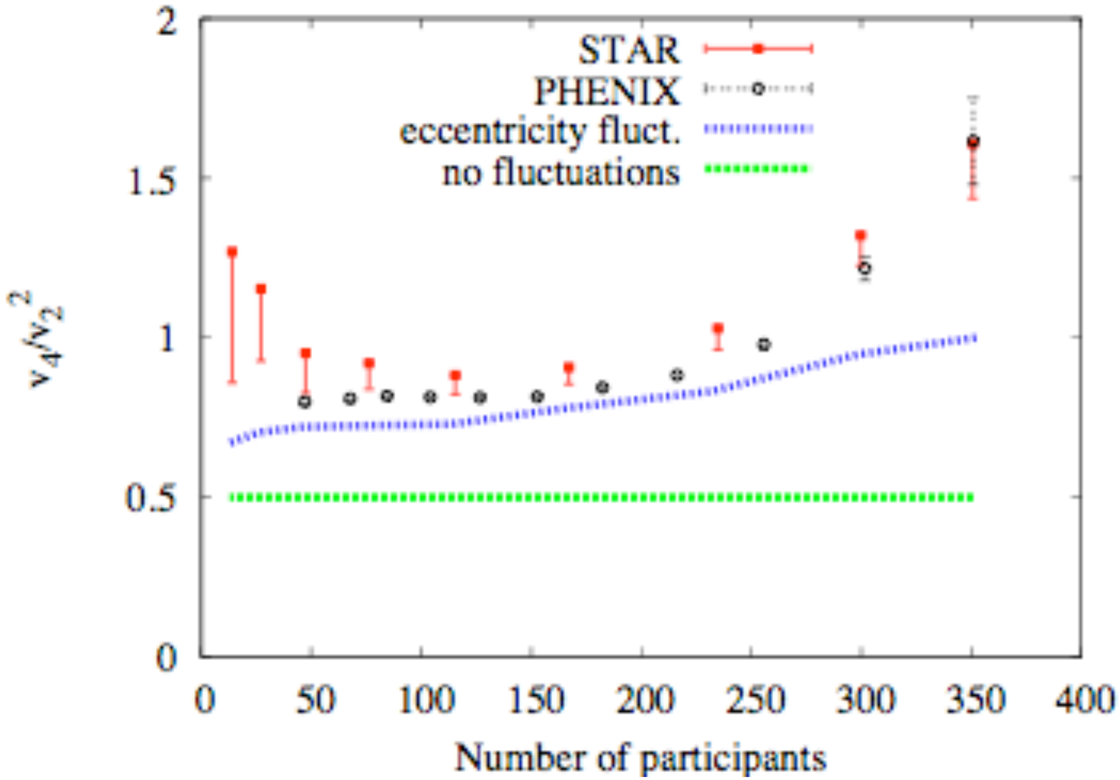
$$\frac{v_4}{v_2^2} = \frac{\langle v_4 (v_2)^2 \rangle}{\langle (v_2)^2 \rangle^2} = \frac{1}{2}\frac{\langle (v_2)^4 \rangle}{\langle (v_2)^2 \rangle^2} > \frac{1}{2}$$

fluctuations

hydro

Similar results obtained using Event Plane method

Data versus eccentricity fluctuations



Fluctuations explain most of the discrepancy between data and hydro

Can we extract fluctuations from data?

Fluctuations from v_2 analyses

Two different ways of extracting fluctuations from data

- PHOBOS method

B. Alver & al, nucl-ex/0608025v2
B. Alver & al, nucl-ex/0702036v1

- Difference between flow analysis methods (our method)

v_2 available from 2 and 4 particle cumulants

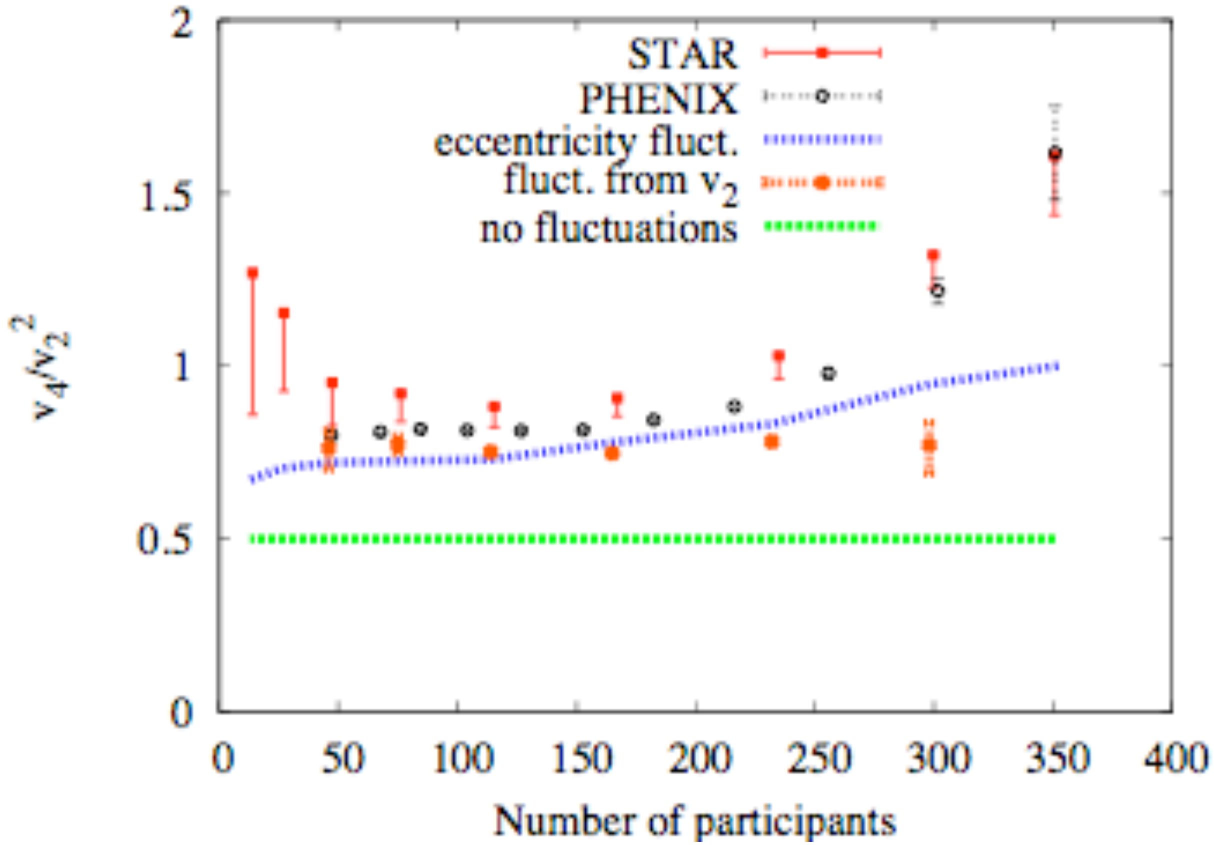
2-particles $$v_2\{2\}^2 = \langle (v_2)^2 \rangle$$

4-particles (STAR) $$v_2\{4\}^4 = 2\langle (v_2)^2\rangle^2 - \langle (v_2)^4 \rangle$$

Inverting these relations we obtain

$$\frac{v_4}{v_2^2} = \frac{1}{2}\frac{\langle (v_2)^4\rangle}{\langle (v_2)^2\rangle^2} = \frac{1}{2}(2 - (\frac{v_2\{4\}}{v_2\{2\}})^4)$$

Data versus v_2 fluctuations



Good agreement with eccentricity fluctuations for the mid-central region

Residual discrepancy between fluctuation models and v_4 data

Partial thermalization effects(1)

Hydro implies local thermalization ➡ $n_{coll} >> 1$

200GeV Au-Au @ RHIC ➡ $n_{coll} \simeq 3-5$

What is the effect on v_4/v_2^2?

Qualitatively $n_{coll} << 1$ ➡ $v_2 \simeq n_{coll}$, $v_4 \simeq n_{coll}$ ➡ $\frac{v_4}{v_2^2} \simeq \frac{1}{n_{coll}}$

R. S. Bhalerao & al Phys. Lett. B627:49-54 (2005)

Quantitatively: We use a numerical solution of the relativistic 2+1 d Boltzmann equation to extract the behavior of v_4/v_2^2.

System of massless particles with arbitrary mean free path (λ)

$$K = \frac{\lambda}{R} = \frac{1}{n_{coll}}$$

degree of thermalization

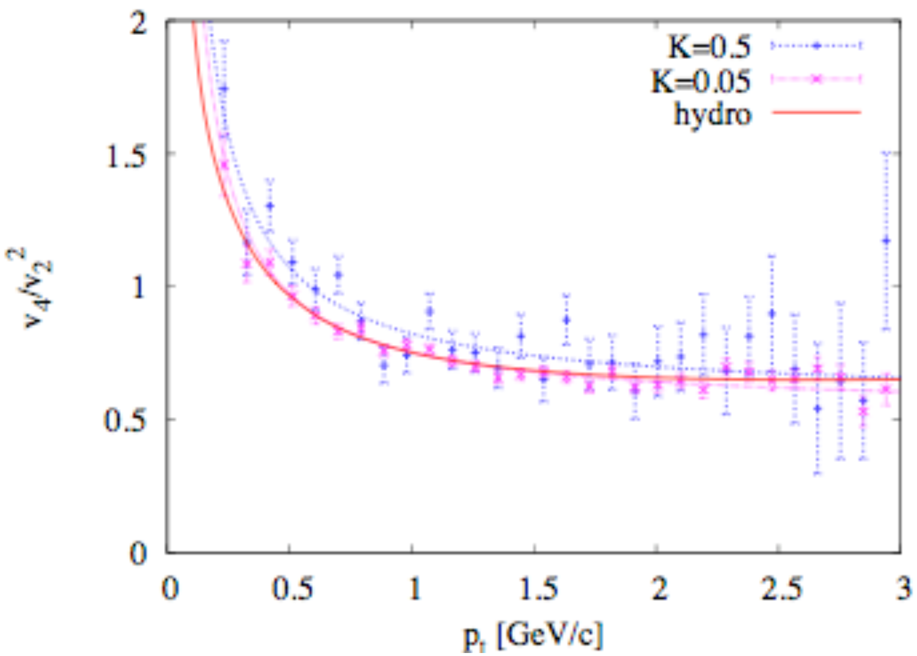
Partial thermalization effects (2)

Implementation and initial conditions

- Initial conditions based on a Monte-Carlo sampling
  - Gaussian density profile (~ Glauber)
    - ✓Aspect ratio $\frac{\sigma_Y}{\sigma_X} = \frac{3}{2}$
  - Dilute gas ⟶ 2-2 processes dominate
  - Thermal Boltzmann momentum distribution (with $T=n^{1/2}$)
    - → Allow comparison between transport and hydro simulations
  - Ideal gas EOS

Partial thermalization effects (3)

Transverse momentum dependence of v_4/v_2^2



For a given value of K

$$\frac{v_4(p_t)}{v_2(p_t)^2} = A + B\frac{\langle p_t \rangle}{p_t}$$

Fit formula motivated by hydro

- Small effect of the deviation from local equilibrium
- Transport with small K agrees with hydro
- As expected, increasing K leads to an increase of v_4/v_2^2

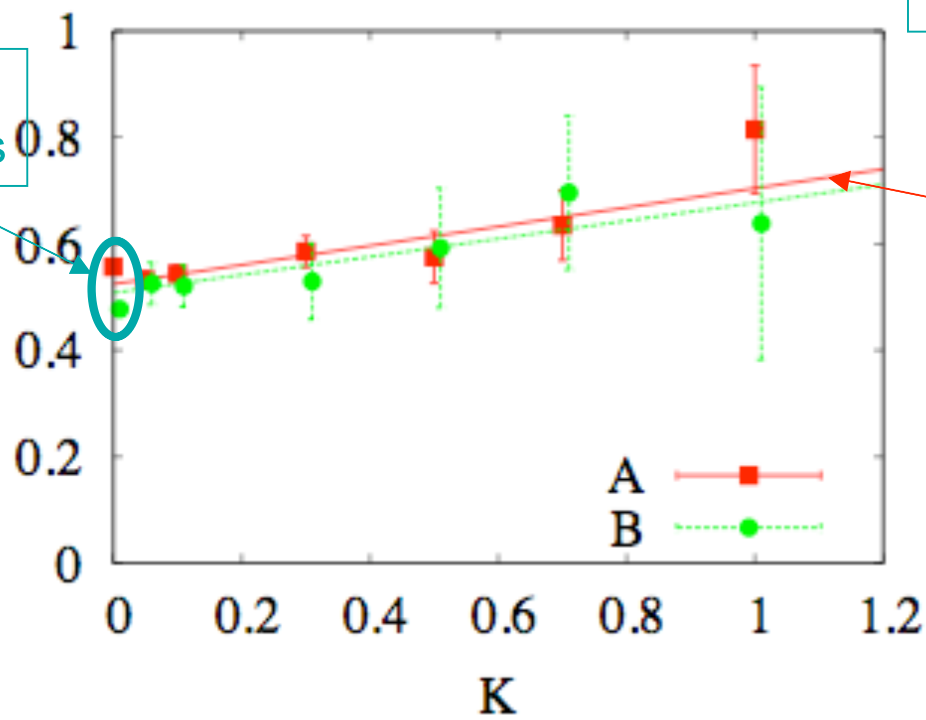
Partial thermalization effects (4)

K dependence of v_4/v_2^2

Assuming $$\frac{v_4(p_t)}{v_2(p_t)^2} = A + B\frac{\langle p_t \rangle}{p_t}$$ extract the dependence of A and B on K

Transport confirms hydro

Ideal Hydro simulation results



$$\frac{v_4}{v_2^2} = 0.52 + 0.18K$$

Scaling in $1/n_{coll}$ as expected from low density limit $n_{coll} << 1$

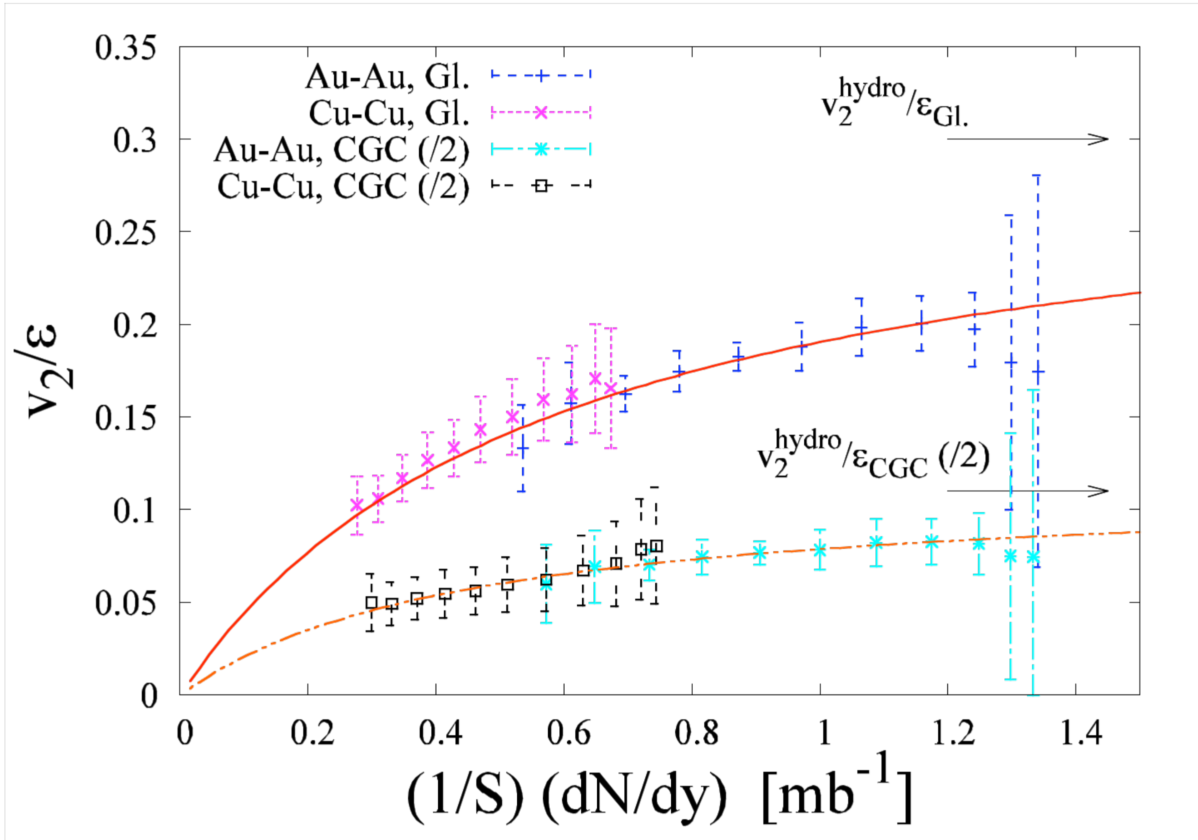
Effects of thermalization are small

Partial thermalization effects(5)

Relating K with measured quantities

$$\frac{v_2}{\epsilon} = A \frac{1}{1 + \frac{K}{K_0}}$$

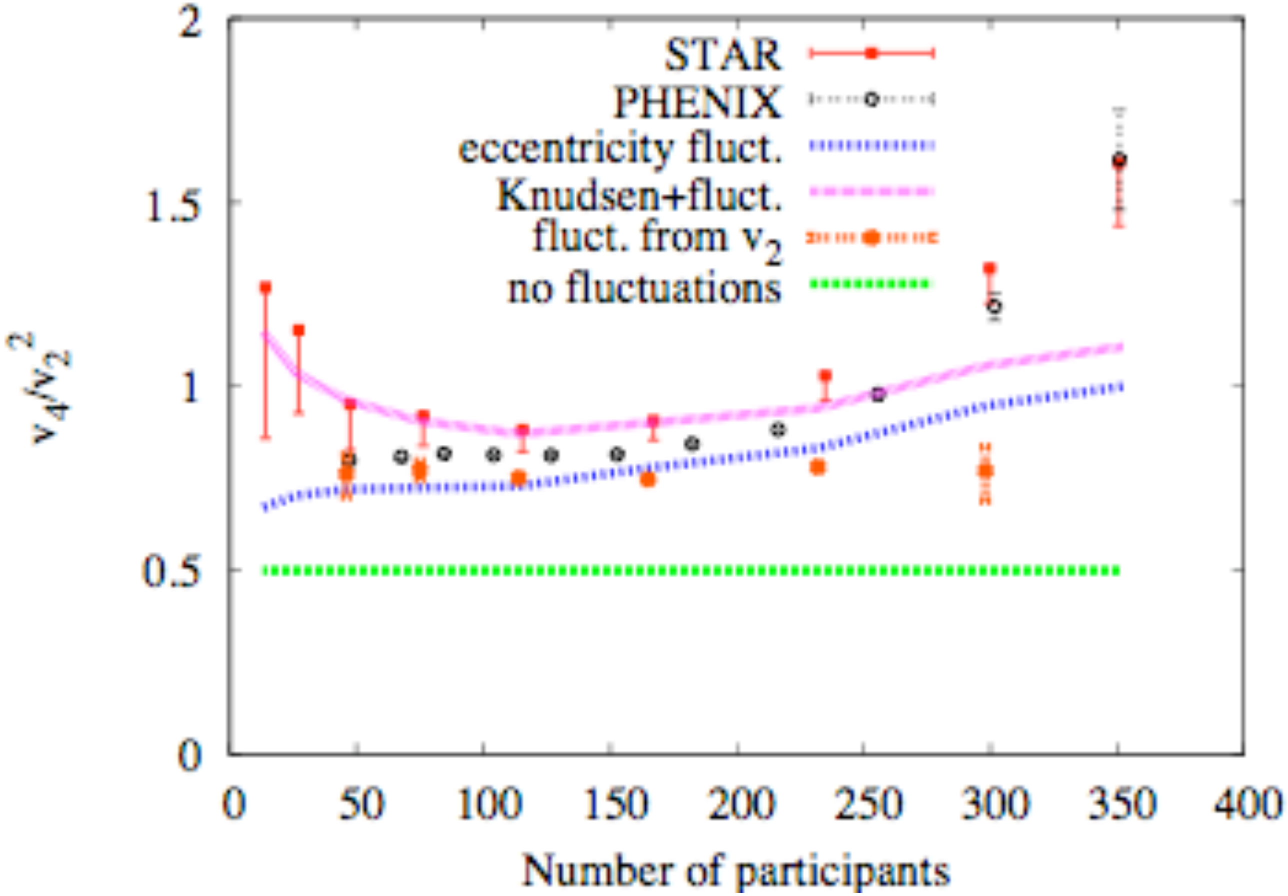
$$\frac{1}{K} \propto \alpha \frac{1}{S} \frac{dN}{dy}$$



Drescher, Dumitru, CG, Ollitrault, Phys. Rev; C76: 024905, 2007

α extracted from the centrality dependence of v_2

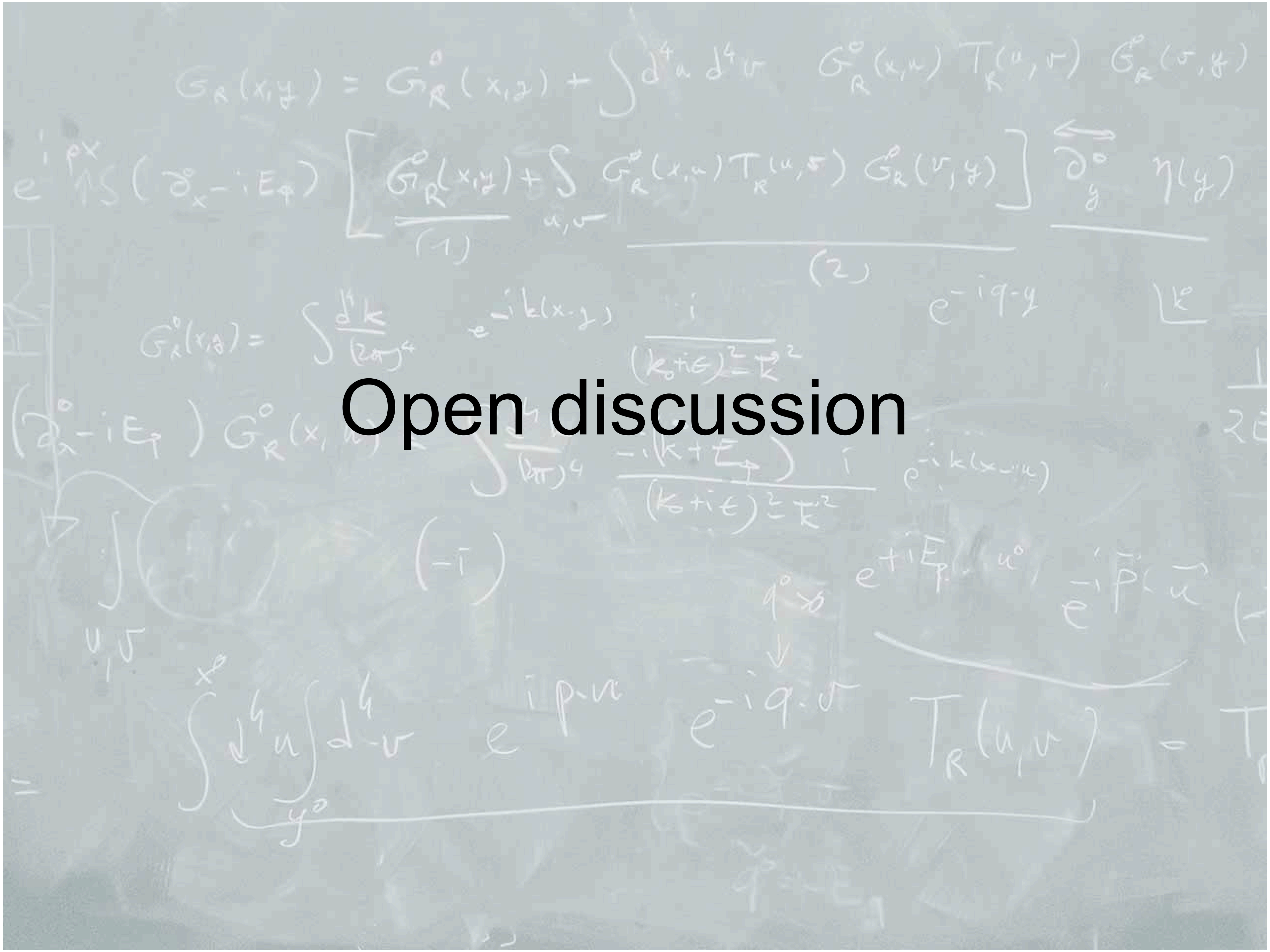
Comparison with data



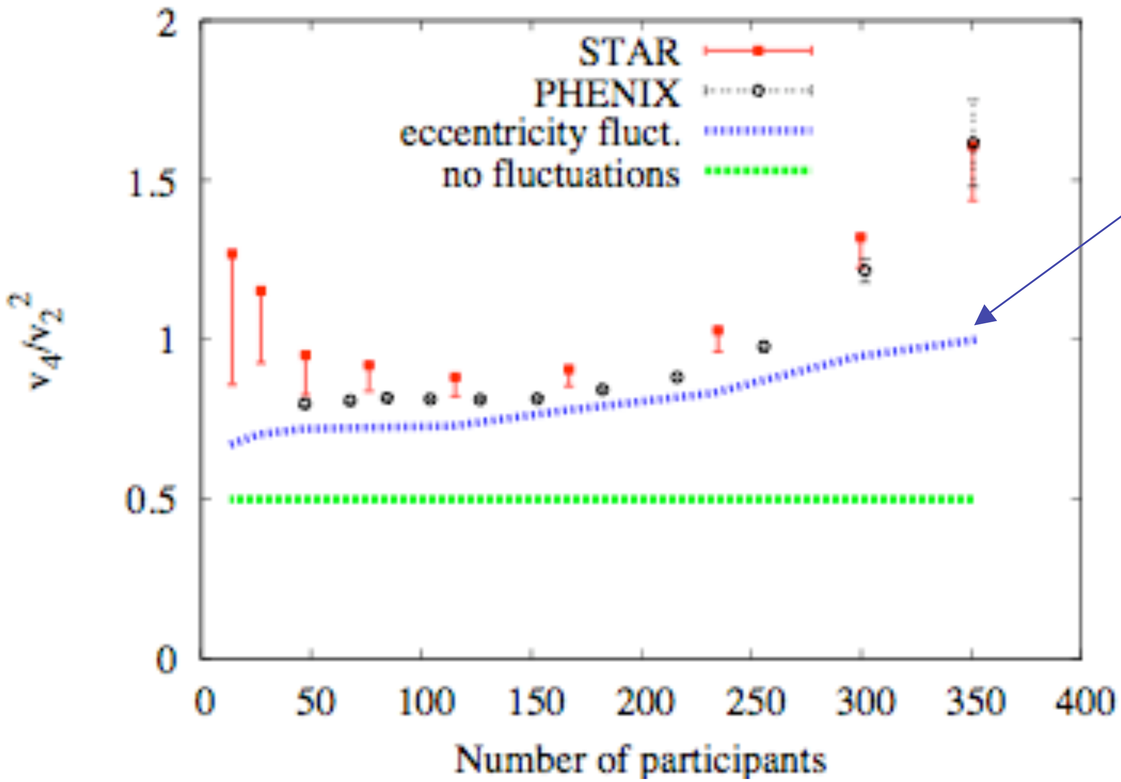
Hydro + fluctuations + partial thermalization explains data except for the most central collisions

CG and Ollitrault, arXiv:0907.4664v1

Open discussion



Problem of fluctuations model



Eccentricity fluctuation model

Central collisions $\frac{v_4}{v_2^2} = 1$

2 dimensional gaussian statistics

Data show $\frac{v_4}{v_2^2} = 1.5$ requires a different model for fluctuations

Toy model for fluctuations

Gaussian distribution of v_2 at fixed impact parameter

$$\frac{dN}{dv_2} = \frac{1}{\sigma_v\sqrt{2\pi}}\left(-\frac{(v_2-\kappa\epsilon_s(b))^2}{2\sigma_v^2}\right) \qquad \text{with} \qquad \sigma_v \propto \frac{k}{\sqrt{N_{part}}}$$

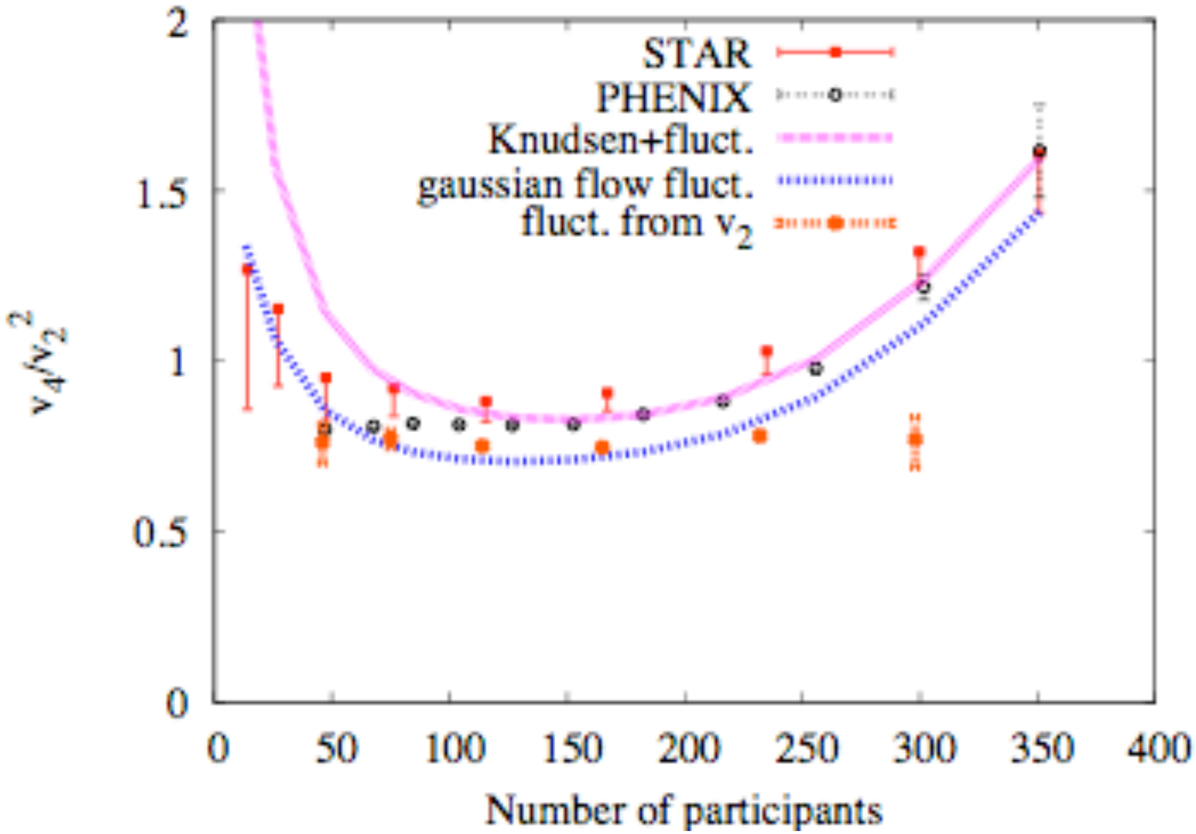
Parameters adjusted to match $v_2\{2\} - v_2\{4\}$

→ Agreement with previous results for mid-central region

1 dimensional gaussian statistics → $\frac{\langle v_2^4\rangle}{\langle v_2^2\rangle^2} = 3$ for central collisions

→ $\frac{v_4}{v_2^2} \simeq 1.5$

Comparison with data



Good match for the central and mid-central collisions

Limitations of the Toy Model

No underlying microscopic physical processes

More information needed

To compute the correct statistics for flow fluctuations

Measure of $v_2\{4\}$ for most central bins (not yet available for $N_{part} > 300$)

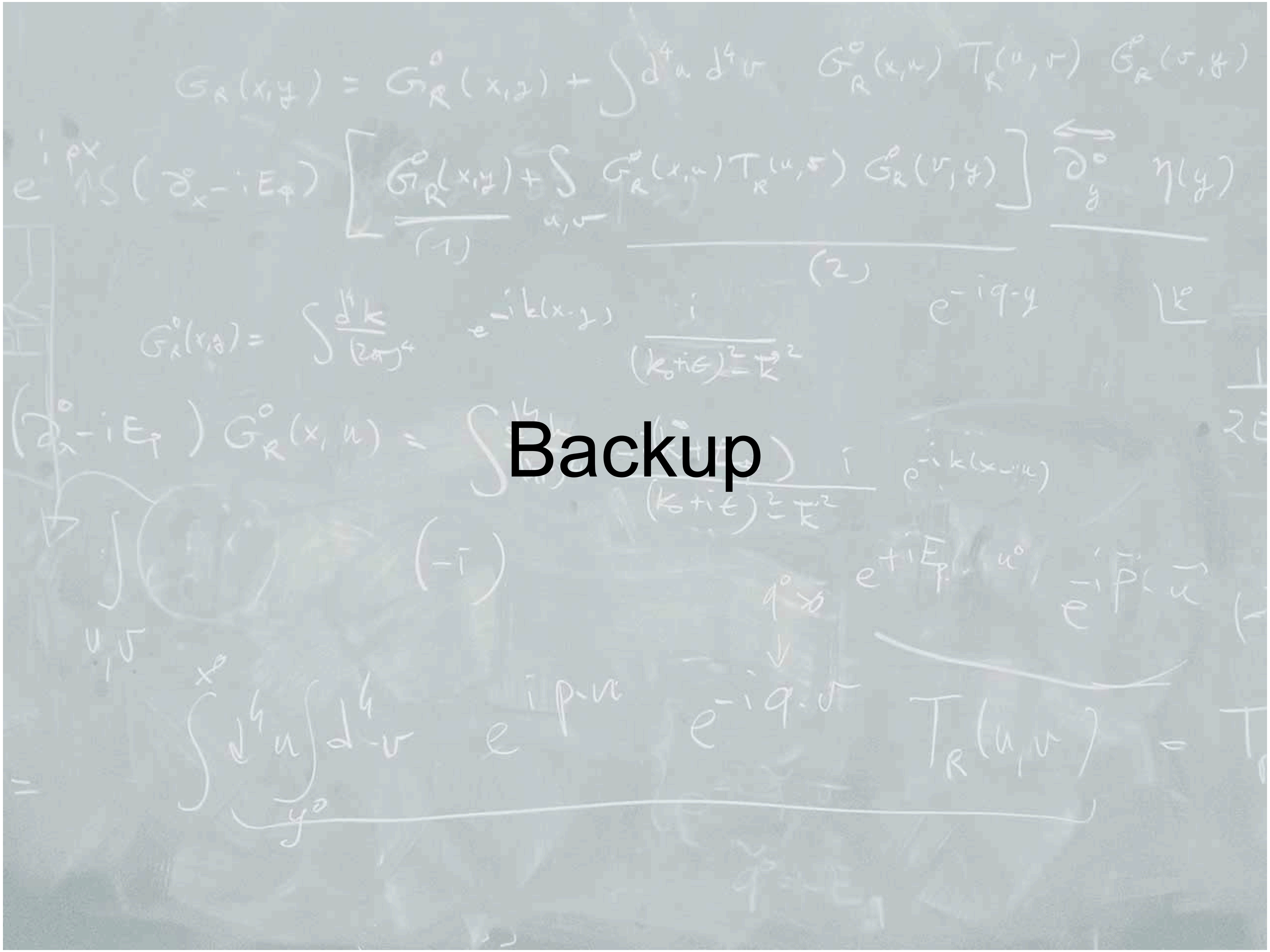
$$v_2\{4\}^4 = 2\langle(v_2)^2\rangle^2 - \langle(v_2)^4\rangle$$

May be negative if fluctuations are large enough

Other observables sensitive to the fluctuation statistics for central collisions

Conclusion

- v_4 is mainly induced from v_2
- Partial thermalization has a small effect on v_4/v_2^2
- Fluctuations+partial thermalization explain the observations except for the most central collisions
  - Need of a new model for flow fluctuations?



Backup

Flow fluctuations

$$\frac{v_4}{v_2^2} = \frac{\langle v_4 (v_2)^2 \rangle}{\langle (v_2)^2 \rangle^2} = \frac{1}{2} \frac{\langle (v_2)^4 \rangle}{\langle (v_2)^2 \rangle^2}$$

with $v_2 = \langle v_2 \rangle + \delta_v$ $\quad \langle \delta_v \rangle = 0$ $\quad \langle \delta_v^2 \rangle = \sigma_v^2$

Azimuthal correlations method

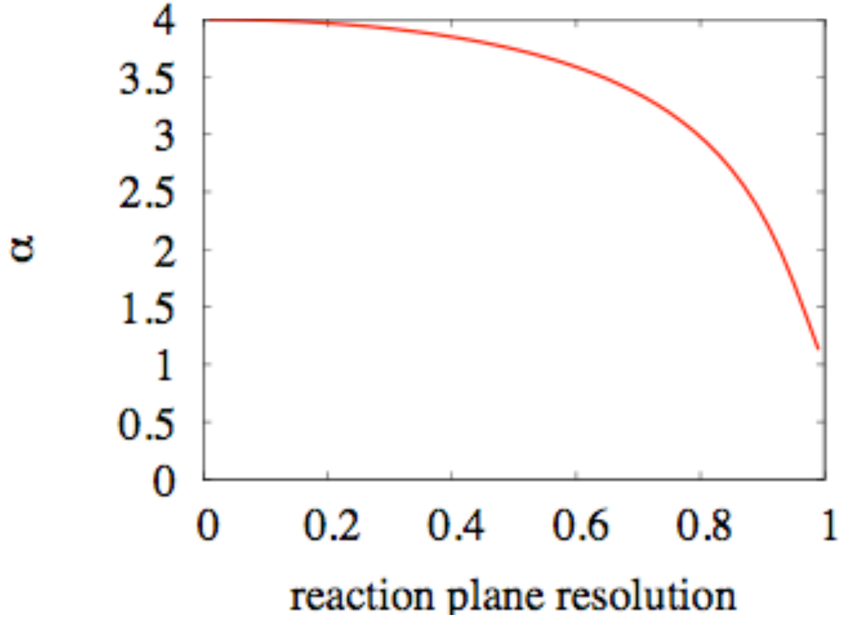
$$\frac{v_4\{3\}}{v_2\{2\}^2} = \frac{1}{2}\left(1 + 4\frac{\sigma_v^2}{\langle v_2 \rangle^2}\right)$$

In practice
resolution ≤ 0.74

$\alpha \simeq 4$

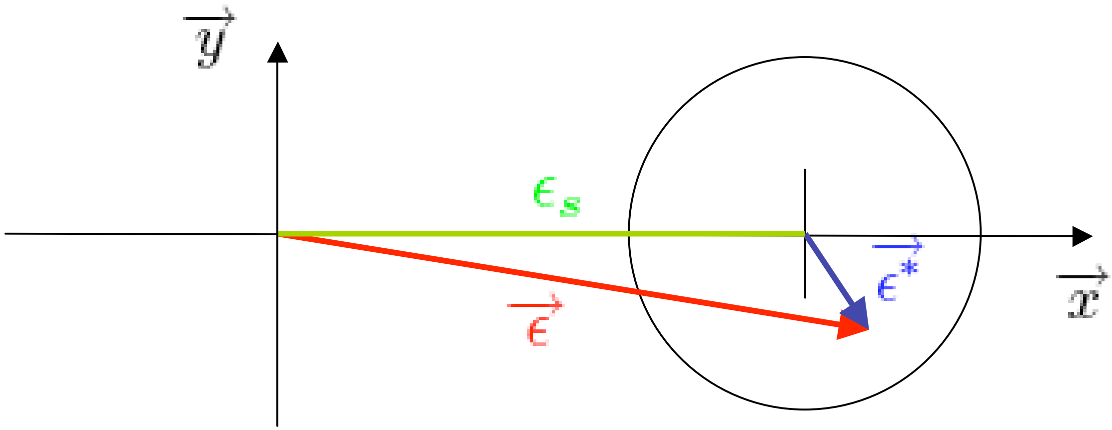
Event Plane method

$$\frac{v_4\{EP\}}{v_2\{EP\}^2} = \frac{1}{2}\left(1 + \alpha\frac{\sigma_v^2}{\langle v_2 \rangle^2}\right)$$



α depends on the reaction plane resolution

Gaussian model of eccentricity fluctuations



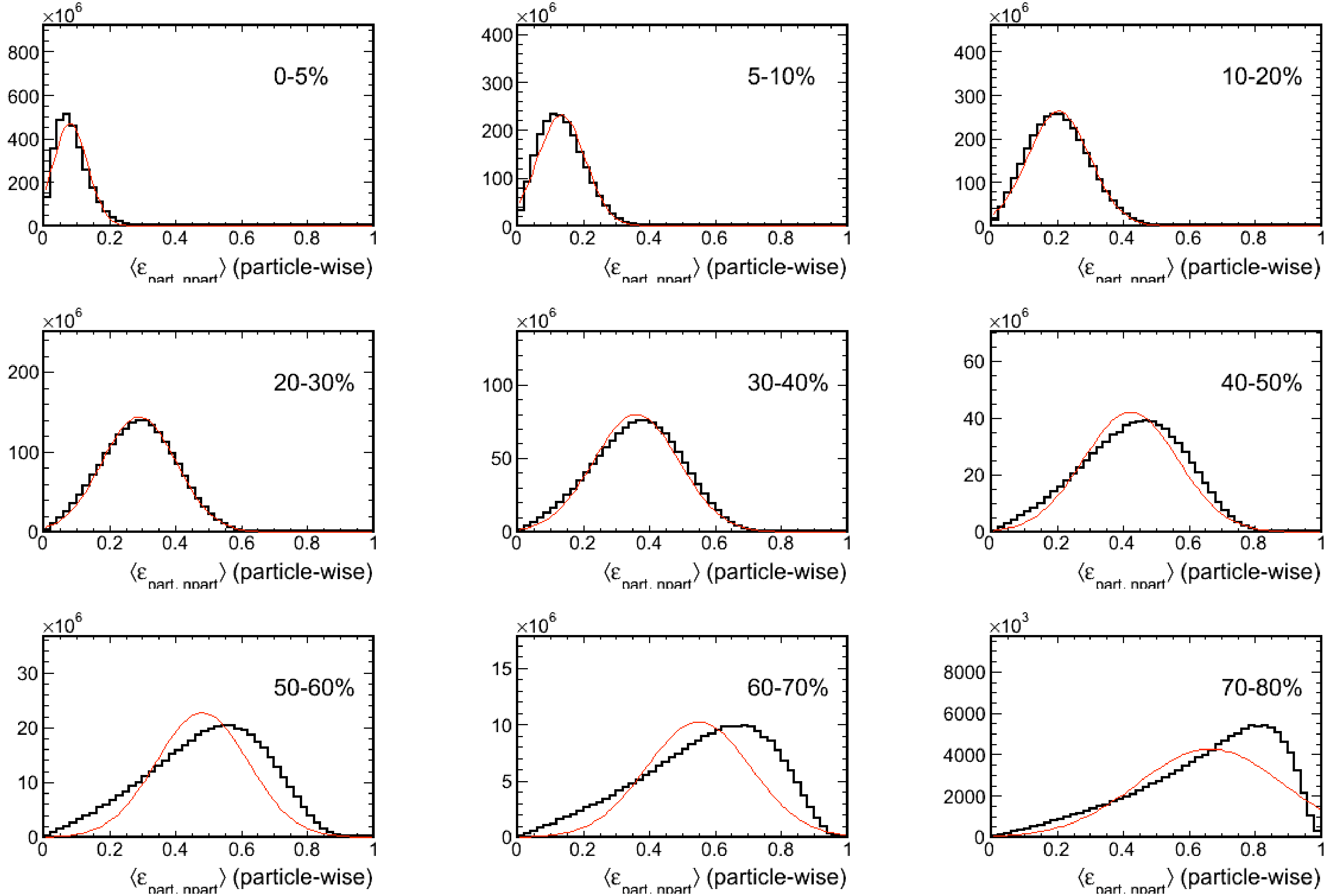
$$\epsilon = |\epsilon_s(b)\vec{e}_x + \vec{\epsilon}^*|$$ with $$\frac{dN}{d\epsilon_x^* d\epsilon_y^*} = \frac{1}{\pi\sigma_0^2}\exp\left(-\frac{\epsilon_x^{*2}+\epsilon_y^{*2}}{\sigma_0^2}\right)$$

Voloshin & al Phys. Lett. B659, 537-541 (2008)

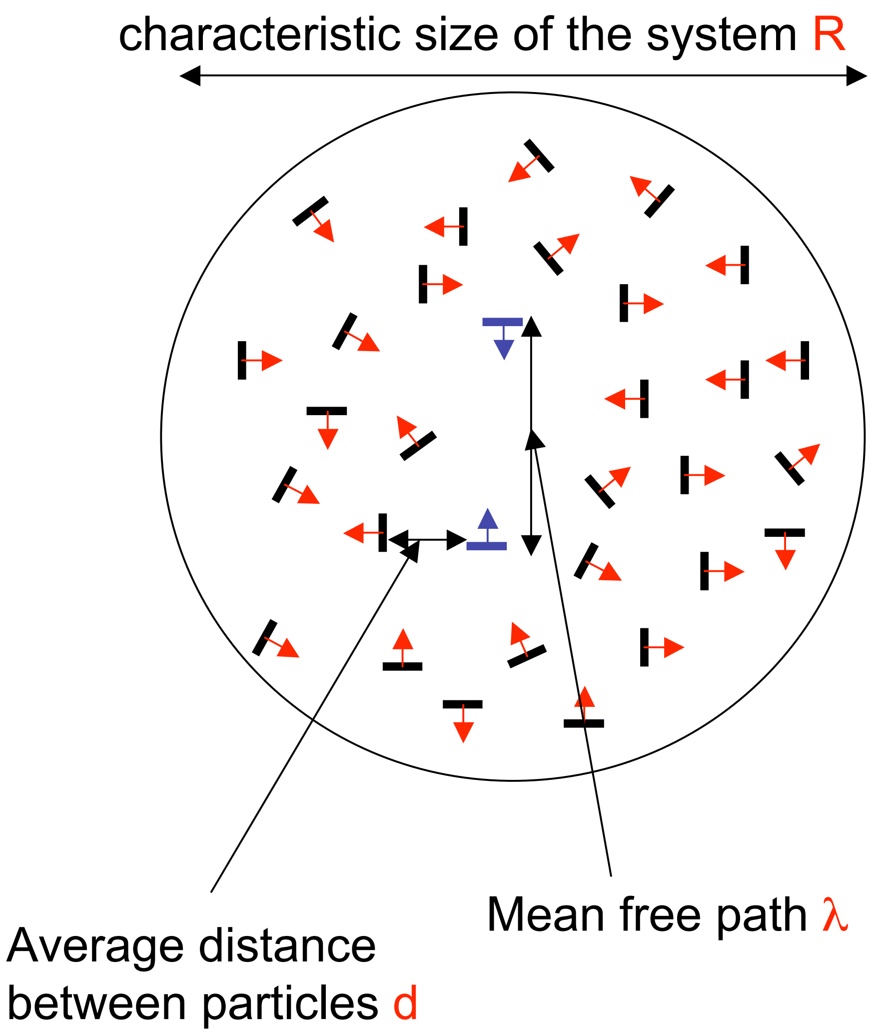
Fluctuations satisfy $$\frac{\langle\epsilon^4\rangle}{\langle\epsilon^2\rangle^2} = 2$$ for central collisions

Gaussian fit on MC glauber

figure is from Hiroshi Masui



Dimensionless quantities



We define 2 dimensionless quantities

- Dilution $D=d/\lambda$
- Knudsen $K=\lambda/R\sim 1/n_{coll}$

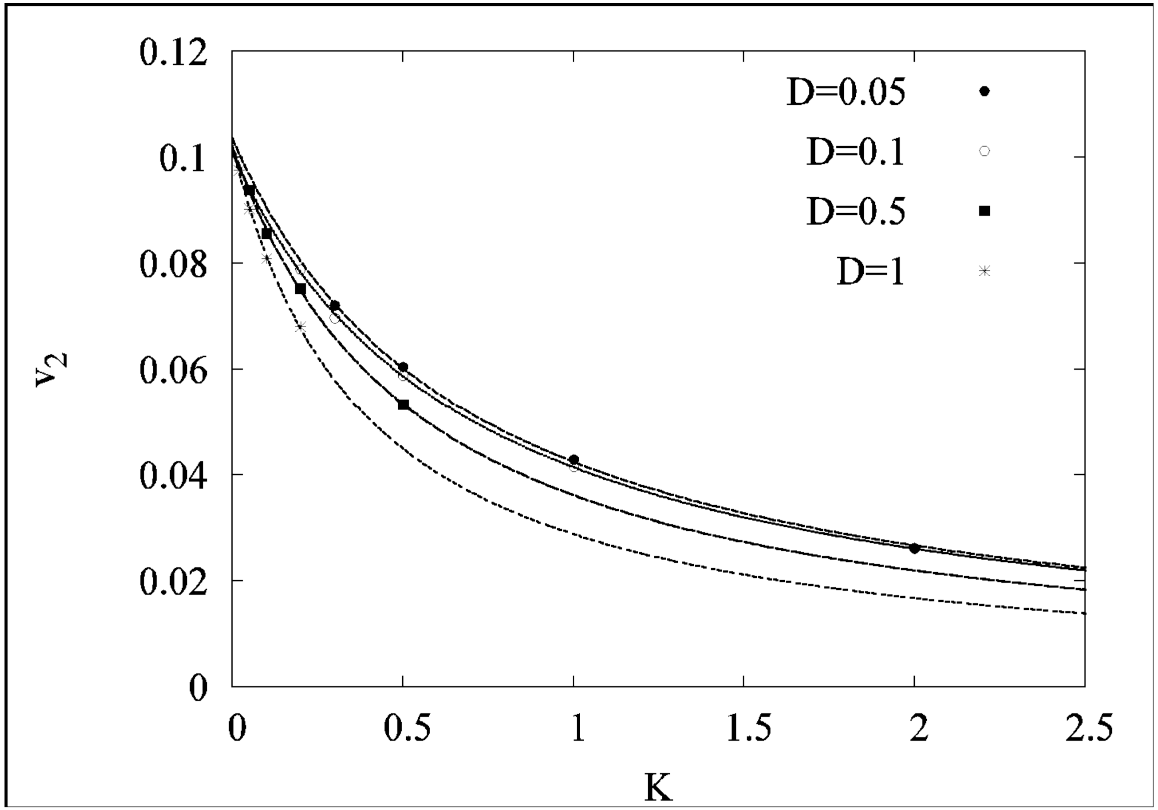
Boltzmann requires $D \ll 1$
Ideal hydro requires $K \ll 1$

Previous study of v_2 for Au-Au At RHIC gives

Central collisions ⇔ K=0.3

Drescher & al, Phys. Rev. C**76,** 024905 (2007)

Elliptic flow versus Kn



$v_2 = v_2^{hydro}/(1+1.4\ Kn)$

Smooth convergence to ideal hydro as $Kn \rightarrow 0$

Viscosity and partial thermalization

- Non relativistic case

$$\frac{\eta}{\rho} \approx \lambda \upsilon_{therm}$$

- Israel-Stewart corresponds to an expansion in power of Knudsen number