VISCOUS HYDRODYNAMIC PREDICTIONS FOR THE LHC

Matt Luzum
(in collaboration with Paul Romatschke)

Institut de Physique Théorique
CEA/Saclay, France

Flow and Dissipation in Ultrarelativistic Heavy Ion Collisions
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Goal

To make a prediction of the measured (integrated) elliptic flow for 5.5 TeV Pb+Pb collisions by:

1. taking best fit viscous hydrodynamic simulations from RHIC,
2. making appropriate modifications for LHC collisions,
3. running viscous hydrodynamic simulations to calculate v_2.

(arXiv:0901.4588)

Outline

1. Introduction/Setup
   - Elliptic Flow
   - Hydrodynamic Equations
   - Viscous Hydrodynamic Simulations
2. RHIC Analysis
3. LHC Predictions

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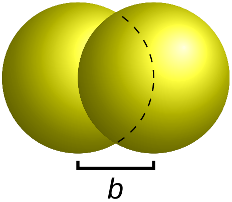
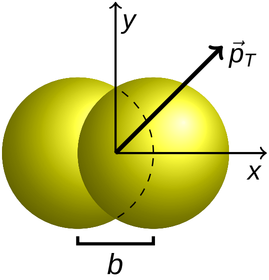
Elliptic Flow



Elliptic Flow

Single-particle momentum spectra:

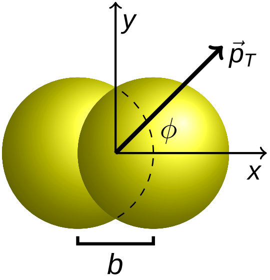
$$\frac{dN}{dY\, d^2p_T}(\vec{p}, b)$$



Elliptic Flow

Single-particle momentum spectra:

$$\frac{dN}{dY\, d^2p_T}(\vec{p}, b) = v_0(\vec{p}, b)\left[1 + \sum_n 2v_n(\vec{p}, b)\, \cos(n\,\phi)\right]$$

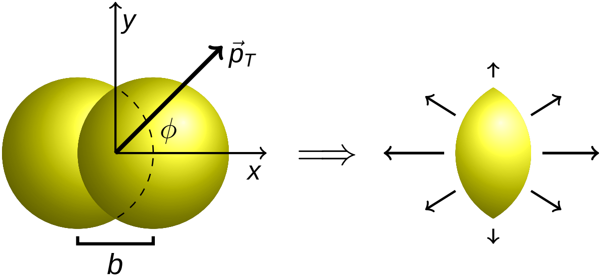


Elliptic Flow

Single-particle momentum spectra:

$$\frac{dN}{dY\, d^2p_T}(\vec{p}, b) = v_0(\vec{p}, b) \left[1 + \sum_n 2v_n(\vec{p}, b)\ \cos(n\,\phi)\right]$$

Elliptic flow: $v_2(\vec{p}, b) \equiv \langle \cos(2\,\phi) \rangle$



Ideal (Relativistic) Hydrodynamic Equations

- Assume isotropic energy-momentum tensor:

$$T^{\mu\nu} = T_0^{\mu\nu} = (\epsilon + p)\, u^\mu u^\nu - p\, g^{\mu\nu}$$

$$\Rightarrow T_{0_{rest}}^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

- Conservation equations:

$$\partial_\mu T^{\mu\nu} = 0$$

- Equation of State:

$$p = p(\epsilon)$$

- An additional relation for each additional conserved current (assumed unimportant for the following)

Viscous Hydrodynamics

- Add dissipative (viscous) effects—derivative expansion:

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu}$$

- To first order (Navier-Stokes):

$$\Pi^{\mu\nu} = \eta \nabla^{\langle\mu} u^{\nu\rangle} + \zeta \Delta^{\mu\nu} \nabla_\alpha u^\alpha$$

- Acausal signal propagation $\Rightarrow$ instabilities $\Rightarrow$ difficult to solve numerically.
- Can be fixed by adding second-order term(s).

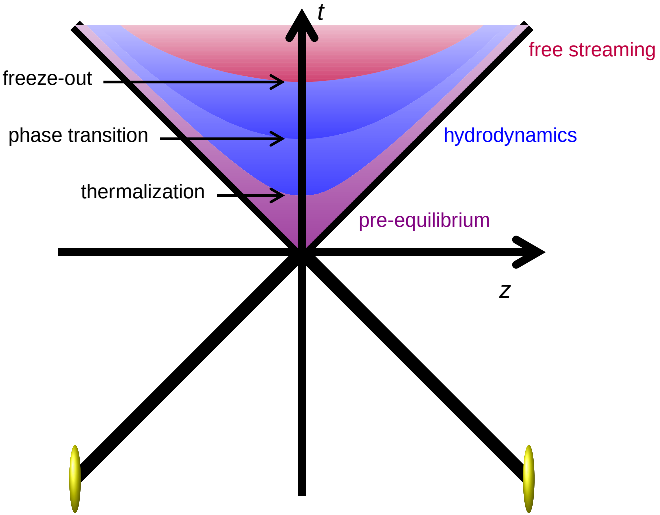
CAUSAL RELATIVISTIC VISCOUS HYDRODYNAMICS

- To start, set bulk viscosity to zero.
- $\Rightarrow$ most general form for a conformal fluid in flat space to second order [BRSSS]:

$$\Pi^{\mu\nu} = \eta \nabla^{\langle\mu} u^{\nu\rangle} - \tau_\pi \left[\Delta^\mu_\alpha \Delta^\nu_\beta D \Pi^{\alpha\beta} + \frac{4}{3} \Pi^{\mu\nu} (\nabla_\alpha u^\alpha) \right] - \frac{\lambda_1}{2\eta^2} \Pi^{\langle\mu}{}_\lambda \Pi^{\nu\rangle\lambda} + \frac{\lambda_2}{2\eta} \Pi^{\langle\mu}{}_\lambda \omega^{\nu\rangle\lambda} - \frac{\lambda_3}{2} \omega^{\langle\mu}{}_\lambda \omega^{\nu\rangle\lambda}$$

- (Simulations insensitive to second-order transport coefficients $\Rightarrow$ can isolate effect of shear viscosity η.)

Anatomy of a Heavy Ion Collision



THE REST OF OUR HYDRO MODEL

- "Realistic" QCD equation of state (Laine and Schröder '06)
  $\longrightarrow$ similar to more recent lattice QCD-derived equations of state
  $\longrightarrow$ no treatment of chemical non-equilibrium
- Glauber and Color Glass Condensate (fKLN) initial conditions
  $\longrightarrow$ optical model (no fluctuations)
  $\longrightarrow$ free parameters T_0, τ_0
- Cooper-Frye freeze out prescription (with resonance feeddown)
  $\longrightarrow$ free parameter T_f

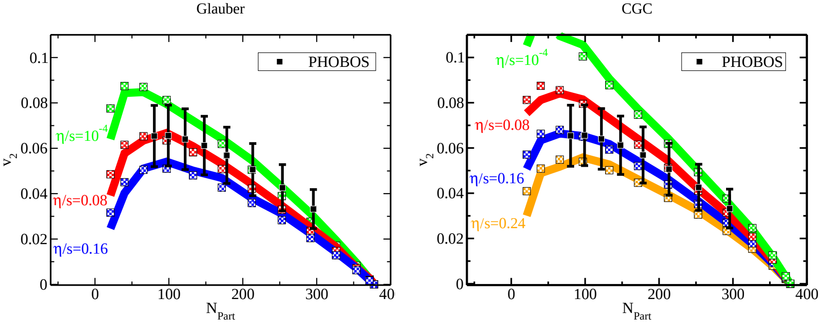
Outline

Analysis Procedure/Free Parameters

The procedure used is as follows:

1. Choose model for initial conditions (Glauber or CGC)
2. Choose value of η/s to study (set to a constant throughout the evolution)
3. Use muliplicity data to fix the energy density normalization (T_0) and thermalization time (τ_0)
4. Use $\langle p_T \rangle$ data to fix the freezeout temperature (T_F)
5. With all the parameters now fixed, compare v_2 to RHIC data

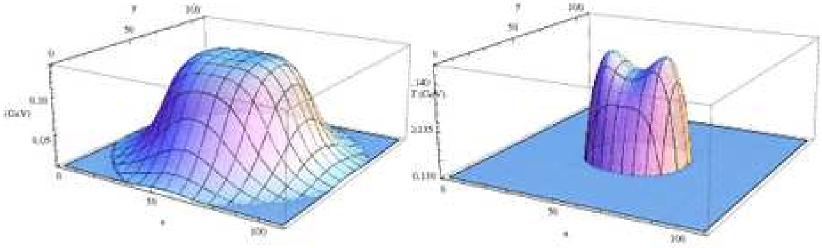
RHIC Results: Momentum Integrated v_2



Note that squares follow curves:

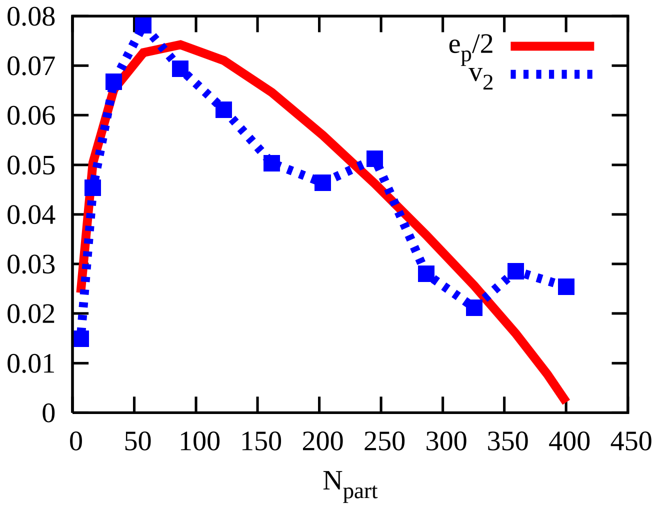
$$\frac{1}{2}e_p \equiv \frac{1}{2}\frac{\langle T^{xx} - T^{yy}\rangle}{\langle T^{xx} + T^{yy}\rangle} \simeq v_2$$

PROBLEMATIC FREEZE OUT



Temperature profile in the transverse plane just before complete freeze out of a Pb+Pb collision

v_2 vs. $e_p/2$


0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
50
100
150
200
250
300
350
400
450
N_part
e_p/2
v_2

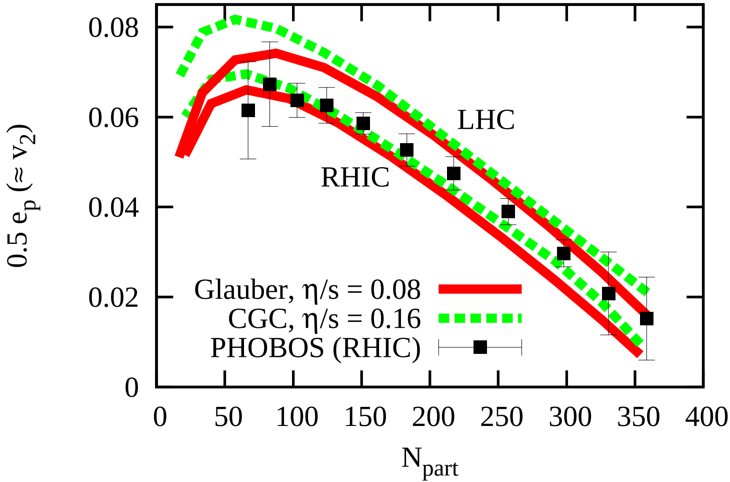
Outline

How to predict results at the LHC

Using the knowledge gained from RHIC, we can make a prediction for Pb-Pb collisions at top LHC energies.

1. Assume T_f, $\frac{\eta}{s}$, τ_0 do not change much $\Rightarrow$ use best RHIC values for each initial condition (Glauber and CGC)
2. Choose T_0 to match predicted multiplicity $\frac{dN_{ch}}{dY} \approx 1800$. (Can instead fix $\frac{dS}{dY} \simeq 7.85 \frac{dN_{ch}}{dY}$)
3. Make appropriate changes to the initial conditions (Pb instead of Au and increased collision energy)
4. $\Rightarrow v_2$ prediction!

Momentum Integrated v_2 at RHIC and LHC



Summary/Conclusions

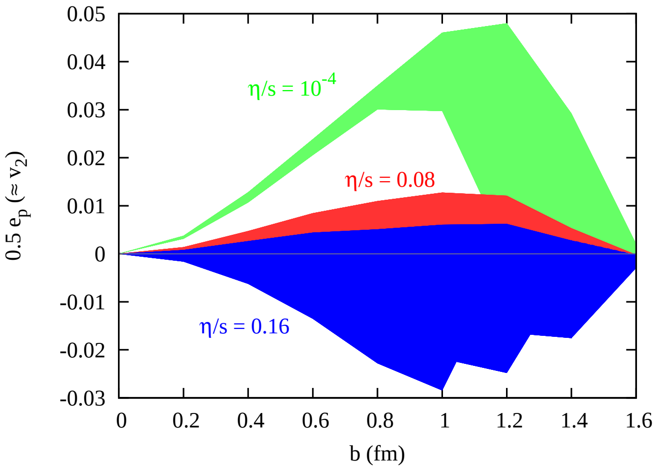
- Viscous hydrodynamic simulations were performed of top-energy heavy ion collisions at LHC
- Prediction: Integrated v_2 will be $\sim$ 10% larger than measured at RHIC

pp collisions at LHC

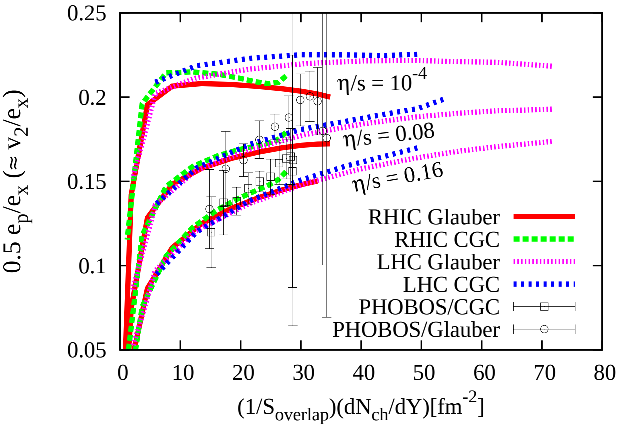
One can perform the same procedure for proton-proton collisions at LHC:

- Assume $dN_{ch}/dY \approx 6$ for "central" collisions
- Glauber initial conditions, using the charge density of the proton.

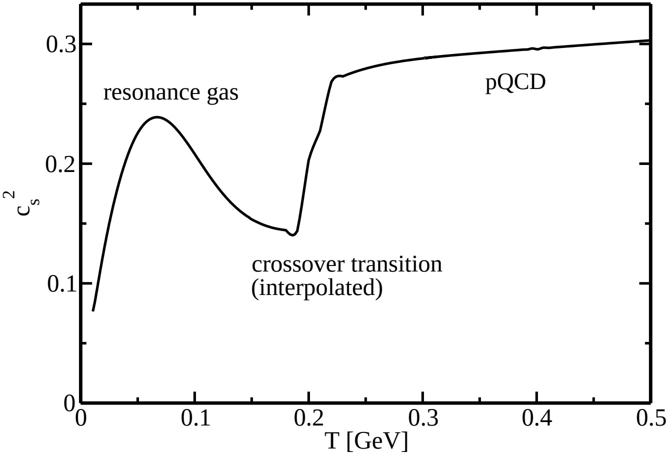
Integrated elliptic flow



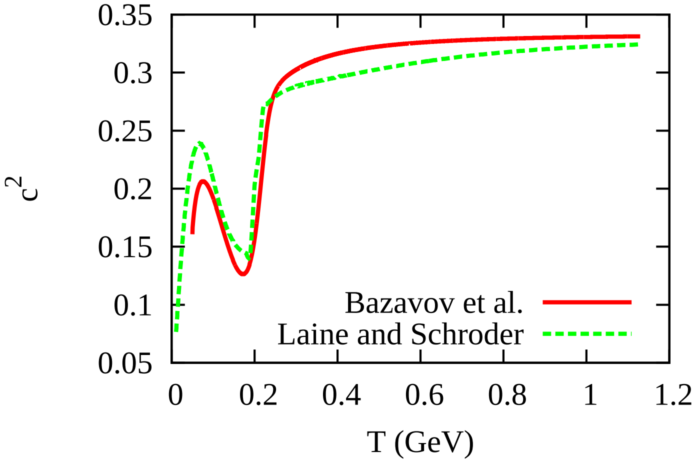
UNIVERSAL CURVES

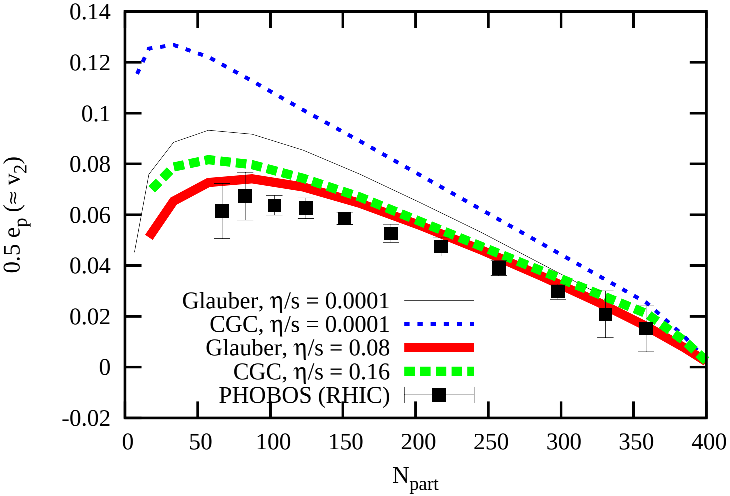


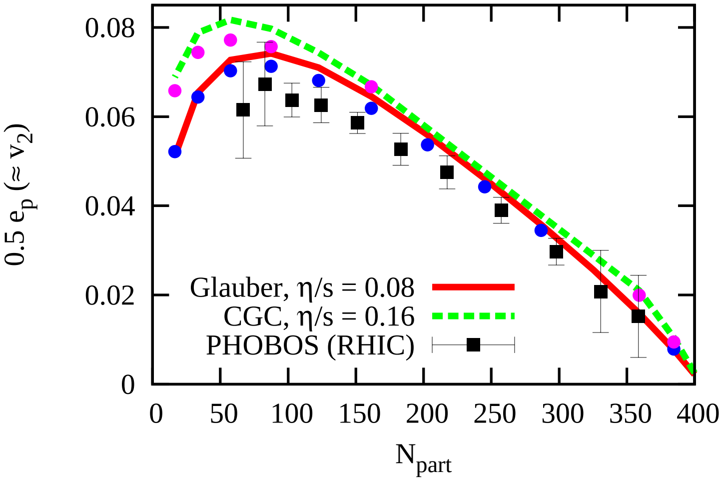
$$\left(S_{overlap} \equiv \pi \sqrt{\langle x^2 \rangle \langle y^2 \rangle} \right)$$



(Laine and Schröder)


0.35
0.3
0.25
0.2
0.15
0.1
0.05
c^2
0
0.2
0.4
0.6
0.8
1
1.2
T (GeV)
Bazavov et al.
Laine and Schroder


0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-0.02
0
50
100
150
200
250
300
350
400
$0.5\ e_p\ (\approx v_2)$
N_{part}
Glauber, η/s = 0.0001
CGC, η/s = 0.0001
Glauber, η/s = 0.08
CGC, η/s = 0.16
PHOBOS (RHIC)


0.5 e_p ($\approx v_2$)
0.08
0.06
0.04
0.02
0
0 50 100 150 200 250 300 350 400
N_{part}
Glauber, η/s = 0.08
CGC, η/s = 0.16
PHOBOS (RHIC)

Parameter Values

Beam	Initial cond.	$\frac{dN_{\mathrm{ch}}}{dY}$	T_i [GeV]	$\sqrt{s}$ [GeV]	τ_0 [fm/c]
Gold	Glauber	800	0.34	200	1
Gold	CGC	800	0.31	200	1
Lead	Glauber	1800	0.42	5500	1
Lead	CGC	1800	0.39	5500	1

Table: Central collision parameters used for the viscous hydrodynamics simulations ($T_f = 0.14$ GeV for all).

Notation

$$A_{\langle\mu}B_{\nu\rangle} \equiv \left(\Delta^{\alpha}_{\mu}\Delta^{\beta}_{\nu} + \Delta^{\alpha}_{\nu}\Delta^{\beta}_{\mu} - \frac{2}{3}\Delta^{\alpha\beta}\Delta_{\mu\nu}\right) A_{\alpha}B_{\beta}$$

$$\Delta^{\mu\nu} \equiv g^{\mu\nu} - u^{\mu}u^{\nu}$$

$$\nabla^{\mu} \equiv \Delta^{\mu\alpha}D_{\alpha}$$

$$D \equiv u_{\alpha}D^{\alpha}$$

$$\omega_{\mu\nu} \equiv \frac{1}{2}\left[\nabla_{\nu}u_{\mu} - \nabla_{\mu}u_{\nu}\right]$$

e.g. Navier Stokes term:

$$\nabla^{\langle\mu}u^{\nu\rangle} = \nabla^{\mu}u^{\nu} + \nabla^{\nu}u^{\mu} - \Delta^{\mu\nu}\nabla_{\alpha}u^{\alpha}$$

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