Viscous Evolution of a Quark Gluon Plasma

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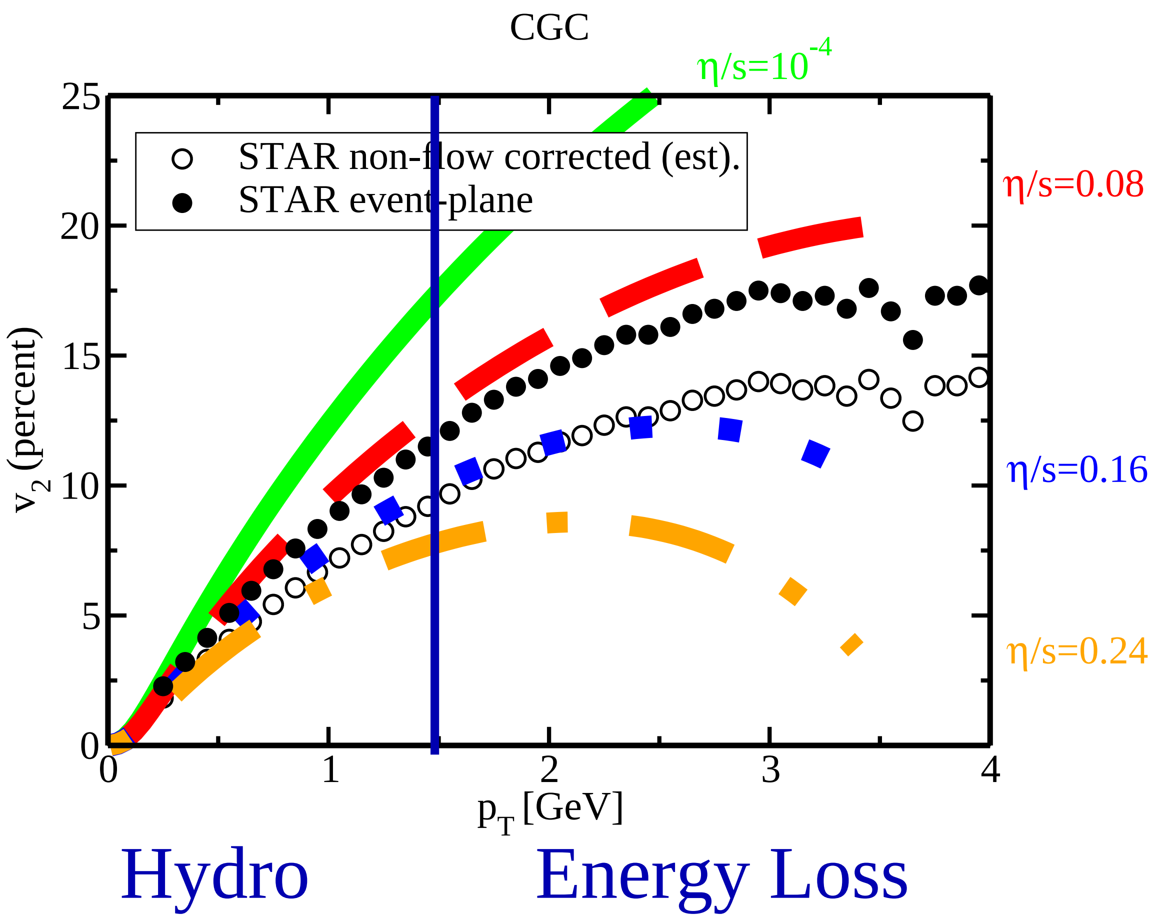
Kevin Dusling, Guy Moore, DT, arXiv:0907.4843

Talk and paper in two parts

1. Energy loss and $v_2(p_T)$
2. Coalescence hatred

Viscous hydro simulations (Romatschke Luzum)



What are the uncertainties?

How does this become energy loss ? $v_2(p_T)$ or $R_{AA}(\phi)$?

Viscous Corrections

1. Viscous corrections to the equation of motion

$$\partial_\mu T^{\mu\nu} = 0 \qquad \text{with} \qquad T^{\mu\nu} = \underbrace{(e+p)u^\mu u^\nu + pg^{\mu\nu}}_{\text{ideal}} \underbrace{-2\eta \left\langle \partial^\mu u^\nu \right\rangle}_{\text{viscous } \pi^{\mu\nu}}$$

2. Viscous corrections to the distribution function

$$f \to f_o + \delta f$$

   - Must be proportional to strains – must be a scalar
   - General form in rest frame and ansatz

$$\delta f = -\chi(p) \; \times f_o(p)\, \hat{p}^i \hat{p}^j \left\langle \partial_i u_j \right\rangle$$

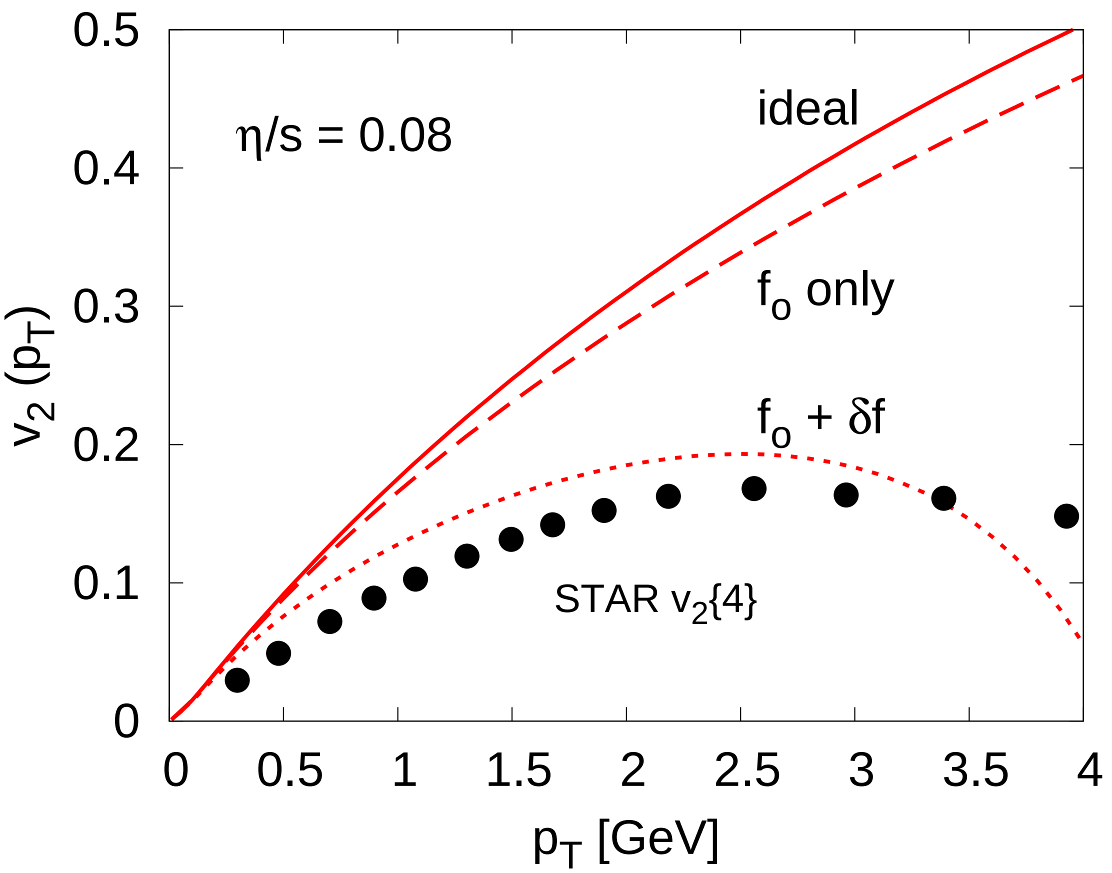
   - The Quadratic Ansatz $\chi(p) \propto p^2$

$$\delta f = -\frac{\eta}{sT^3} \times f_o(p)\, p^i p^j \left\langle \partial_i u_j \right\rangle$$

All simulations have used the quadratic ansatz!

The role of δf

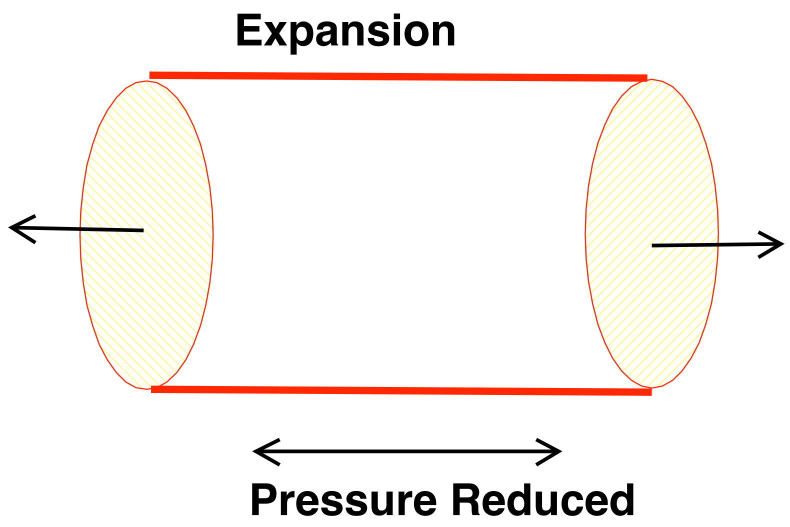
Pure glue, $e_{\mathrm{frz}} = 0.6\,\mathrm{GeV/fm^3}$



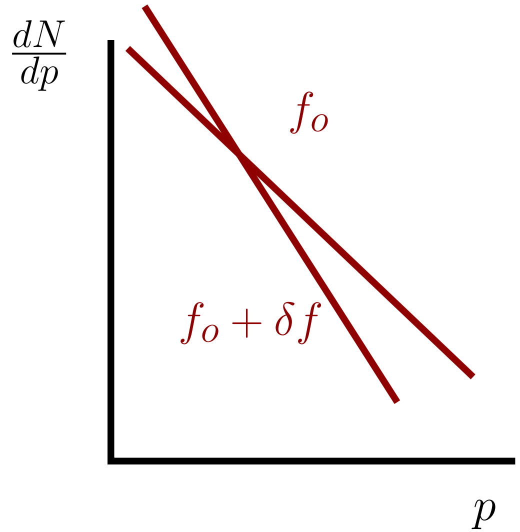
We should understand δf and the Quadratic Ansatz!

Basic Physics of δf

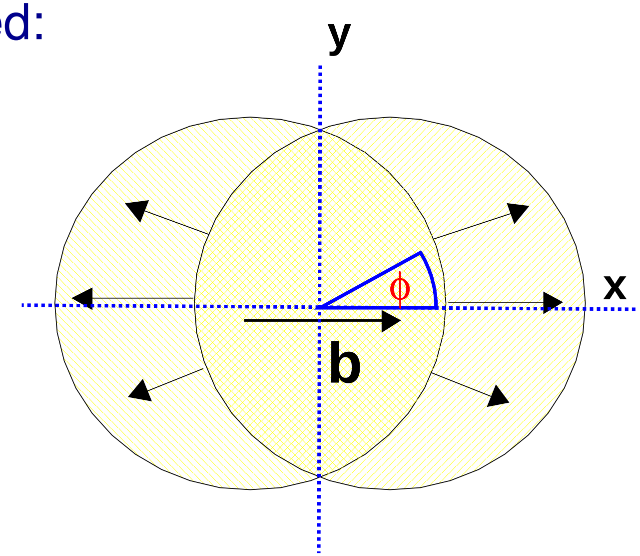
1. When the system is expanding the pressure is reduced:



So



2. Thus elliptic flow is reduced:



Calculating δf: Relaxation Time Approximation

$$\left[\partial_t + v_{\mathbf{p}}\frac{\partial}{\partial x}\right] f = -\frac{\delta f}{\tau_R(p)}$$

1. Substitute $f = n_p + \delta f$ with

$$\left[\partial_t + v_{\mathbf{p}}\frac{\partial}{\partial x}\right] n_p = -\frac{\delta f}{\tau_R(E_{\mathbf{p}})} \qquad \text{with} \qquad n_p = \frac{1}{e^{-P\cdot u(\mathbf{x},t)} \mp 1}$$

2. With a bit of algebra and classical statistics:

$$n_p\,\frac{p^i p^j}{T E_{\mathbf{p}}}\,\langle \partial_i u_j \rangle = -\frac{\delta f}{\tau_R(E_{\mathbf{p}})}$$

3. Find for massless gas

$$\delta f = -\frac{\tau_R(p)}{Tp}\, n_p\, p^i p^j \,\langle \partial_i u_j \rangle \qquad \text{or} \qquad \chi(p) = \tau_R(p)\frac{p}{T}$$

Quadratic ansatz corresponds to $\tau_R \propto p$.

What about $\tau_R \propto p^{\beta}$?

Two Extreme Limits: Quadratic and Linear Ansatz

$$\delta f = -\frac{\tau_R(p)}{Tp}\, n_p \, p^i p^j \, \langle \partial_i u_j \rangle$$

For the relaxation time take

$$\tau_R(p) \sim \frac{p}{\frac{dp}{dt}}$$

1. Relaxation time growing with parton energy – "collisional e-loss"

$$\tau_R \propto p \qquad \frac{dp}{dt} \propto \text{const} \qquad \chi(p) \propto p^2$$

2. Relaxation time independent of parton energy – "extreme rad. e-loss"

$$\tau_R \propto \text{Const} \qquad \frac{dp}{dt} \propto p \qquad \chi(p) \propto p$$

Reality is probably in-between

Relation between δf and shear viscosity

$$T^{ij} = p\delta^{ij} - \eta \left\langle \partial^i u^j \right\rangle = \int_{\mathbf{p}} \frac{p^i p^j}{E} (n_p + \delta f)$$

- First moment of δf determines the shear viscosity

$$\delta f = -\chi(p)\, n_p\, \hat{p}^i \hat{p}^j \, \langle \partial_i u_j \rangle \qquad \text{find} \qquad \eta = \frac{1}{15} \int \frac{d^3p}{(2\pi)^3} n_p \, \chi(p)\, p$$

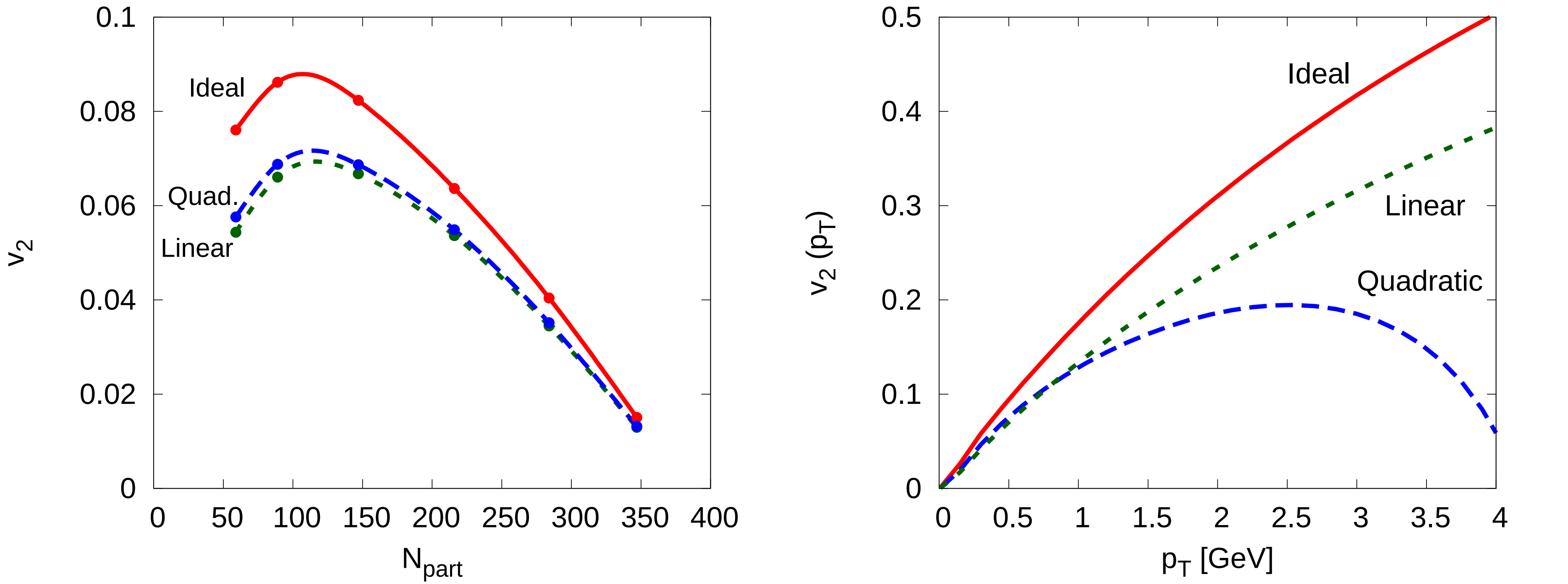
- More General Ansatz – massless gas

$$\chi(p) = Cp^{2-\alpha} \qquad \rightarrow \qquad C(\alpha) = \begin{cases} \frac{\eta}{sT} & \alpha = 0 \text{ (quadratic)}, \\ 5\,\frac{\eta}{sT} & \alpha = 1 \text{ (linear)}. \end{cases}$$

Ansätze partially constrained by shear viscosity

Two Limits: Quadratic and Linear Ansatz

pure glue, $e_{\rm frz} = 0.6\,{\rm GeV/fm^3}$, $\eta/s = 0.08$



- The $\overline{v}_2$ independent of δf – see arXiv:0905.2433
  - $\overline{v}_2$ largely determined by $T(e+\mathcal{P})$, u^μ, $\pi^{\alpha\beta}$

What is reality? Quadratic or Linear?

Solving for δf with the Boltzmann Equation:

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_x f = C \circ f$$

- Substitute $f = n_p + \delta f$

$$\partial_t n_p + v_{\mathbf{p}} \cdot \partial_x n_p = C \circ \delta f$$

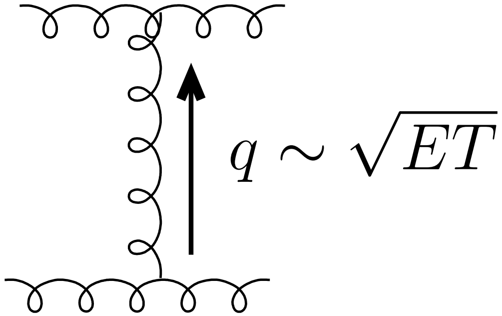
- with a bit of algebra

$$n_p \frac{p^i p^j}{T E_{\mathbf{p}}} \langle \partial_i u_j \rangle = C \circ \delta f$$

- The collisions and bremsstrahlung is all in C.

Can go invert this matrix and determine δf

Simple Scattering



- Transition Rate

$$\Gamma_{12\to 34} = \frac{|\mathcal{M}|^2}{(2E_1)(2E_2)(2E_3)(2E_4)}(2\pi)^4\delta^4(P_1 + P_2 - P_3 - P_4)$$

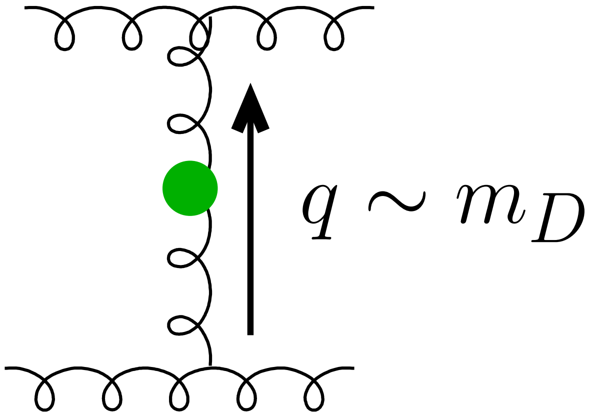
- Linearized equation

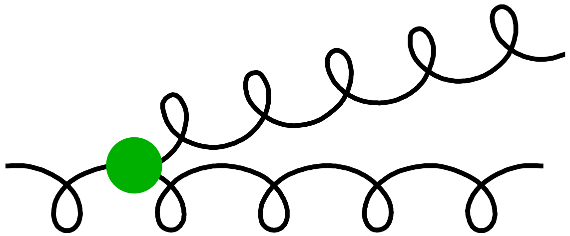
$$n^o_{\mathbf{p}}\,\frac{p^i p^j}{T E_{\mathbf{p}}}\,\langle\partial_i u_j\rangle \quad = \quad -\int_{234} \Gamma_{12\to 34}\, n^o_{\mathbf{p}} n^o_2 \left[\frac{\delta f(\mathbf{p})}{n^o_{\mathbf{p}}} + \frac{\delta f_2}{n^o_2} - \frac{\delta f_3}{n^o_3} - \frac{\delta f_4}{n^o_4}\right]$$

Matrix Equation for δf

$$b_p = [\Gamma]_{pp'}\ \delta f_{p'}$$

QCD Boltzmann equation is rich





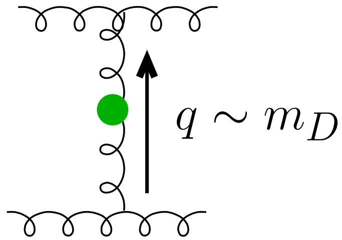
1. Scattering of soft classical field
2. Collinear Brem with interference

All these processes influence δf

Three Models of Energy Loss

1. Soft Scattering

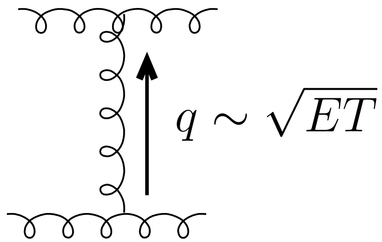
$$\frac{dp}{dt} \propto C_R\, \alpha_s^2 T^2 \log\left(\frac{T}{m_D}\right)$$



find $\chi(p) \propto p^2$

2. Collisional Energy Loss

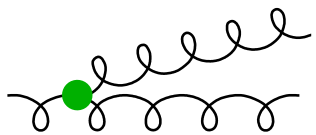
$$\frac{dp}{dt} \sim C_R\, \alpha_s^2 T^2 \log\left(\frac{p}{m_D}\right)$$



find $\chi(p) \propto \frac{p^2}{\log(p\mu)}$

3. Radiative + Collisional Energy Loss in Infinite medium

$$\frac{\Delta p}{\Delta t} \sim \alpha_s \sqrt{\hat{q} E_{\mathbf{p}}}$$

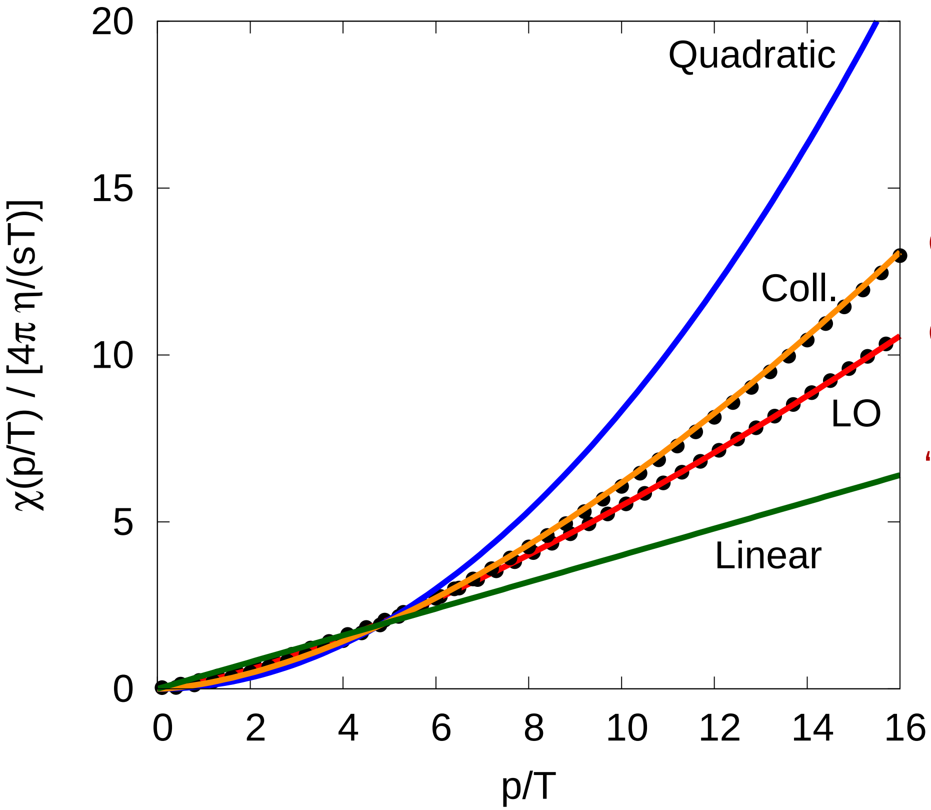


find $\chi \propto p^{3/2}$

These estimates are borne out by our numerical work

Summary – Energy Loss and δf

$$\delta f = -\chi(p)\, n_p\, \hat{p}^i \hat{p}^j \, \langle \partial_i u_j \rangle \qquad \text{fit with} \qquad C p^{2-\alpha}$$



Soft-Scatt $\propto p^2$

Coll $\propto p^{1.6}$

Coll + Rad $\propto p^{1.38}$

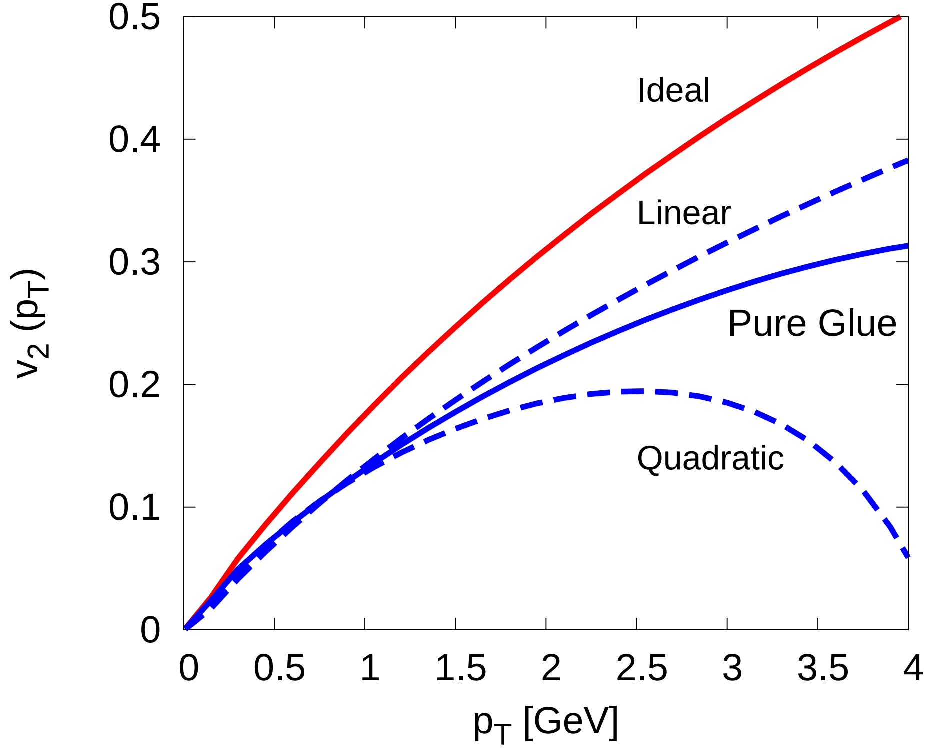
"Bethe-Heitler" $\propto p$

Energy loss determines $\chi(p)$

QCD kinetic theory expectation $\chi(p) \propto p^{1.38}$ in relevant range

Phenomenological Summary

pure glue, $\eta/s = 0.08$, $e_{\rm frz} = 0.6\,{\rm GeV/fm^3}$



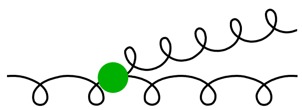
pQCD is closer to a linear ($\tau_R =$ const) rather than a quadratic ansatz

Connection to energy loss

1. At large momentum brem dominates the Boltzmann collision term

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_{\mathbf{x}} f = -\mathcal{C}^{1\leftrightarrow 2}[f].$$

2. The collision kernel is



$$\mathcal{C}^{1\rightarrow 2} \propto \underbrace{\int_0^1 dx}_{\text{Phase Space}} \times \underbrace{\gamma^g_{gg}(p; p', k)}_{|\mathcal{M}|^2} \times \underbrace{\left[f_p(1+f_{p'})(1+f_k)\right]}_{\text{Stimulation Factors}}$$

3. The QCD splitting function is medium modified see P.Arnold, C.Dogan, BDMPS

$$\gamma^g_{gg} \propto \alpha_s C_A d_A \sqrt{p\hat{q}}\,\frac{[1-x(1-x)]^{5/2}}{[x(1-x)]^{3/2}}$$

Linearizing the Boltzmann equation

$$\delta f = -\chi(p)\, n_p(1+n_p)\, \hat{p}^i \hat{p}^j \, \langle \partial_i u_j \rangle$$

1. The linearized Boltzmann equation becomes in high momentum limit

$$\frac{p}{T} = -\frac{(2\pi)^3}{32p} \int_0^\infty dx \, \underbrace{\gamma(p; xp, (1-x)p)}_{\propto\, \alpha_s \sqrt{\hat{q} p}} \, \left[\chi_p - \chi_{xp} - \chi_{(1-x)p}\right] \, .$$

2. Guess a solution $\chi = C p^{3/2}$

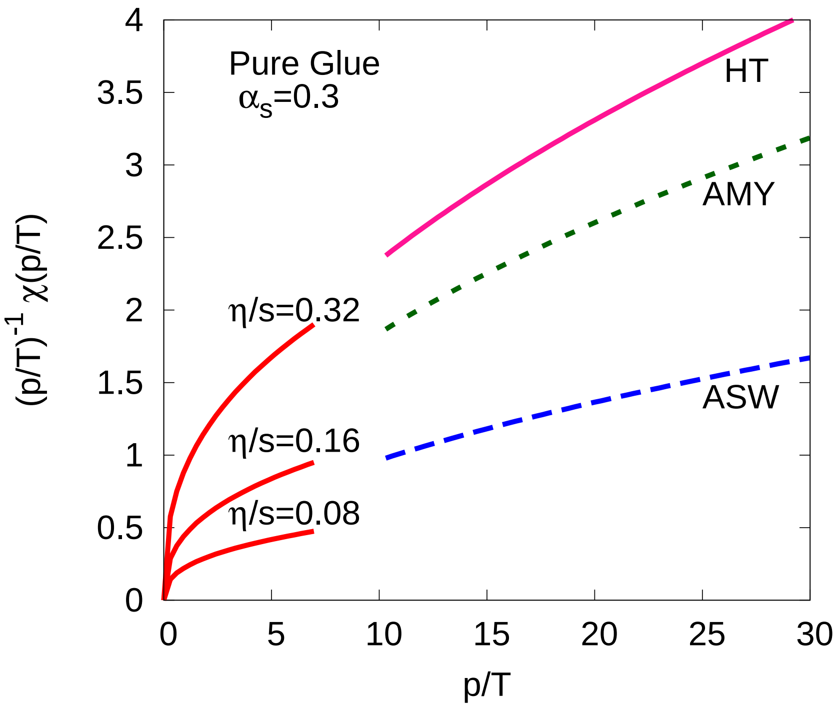
3. Find:

$$\chi(p) = \underbrace{0.7}_{\text{an } x \text{ integ. over split. fcn}} \times \frac{p^{3/2}}{\alpha_s T \sqrt{\hat{q}}}$$

$\hat{q}$ and viscous corrections at high momenta: A nifty formula

$$\underbrace{\chi(p)}_{\text{Viscous Correction}} \quad = \underbrace{0.7 \times \frac{p^{3/2}}{\alpha_s T \sqrt{\hat{q}}}}_{\text{radiative loss}}$$

1. At low momentum $\chi(p)$ is determined by the shear viscosity η/s
2. At high momentum $\chi(p)$ is determined by $\hat{q}$



$\hat{q}/T^3 = 10$

$\hat{q}/T^3 = 16$

$\hat{q}/T^3 = 60$

So far only single component (gluon) plasmas

Next: multi-component plasmas (coalesence hatred)

Quarks and Gluons (simple model)

- Quarks and Gluons have different relaxation times and δfj

$$\begin{aligned} \delta f^Q &= -C_q \, n_p p^i p^j \, \langle \partial_i u_j \rangle \\ \delta f^G &= -C_g \, n_p p^i p^j \, \langle \partial_i u_j \rangle \end{aligned}$$

- Casimir Scaling

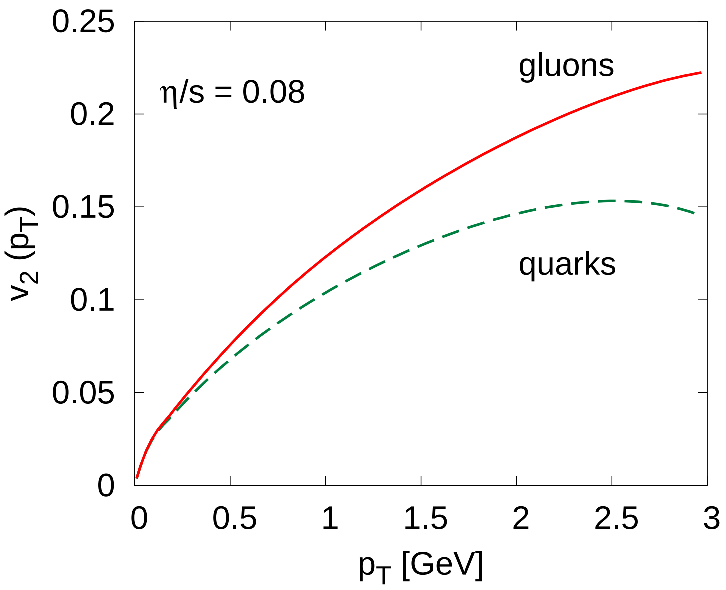
$$\frac{C_q}{C_g} = \frac{\tau_R^Q}{\tau_R^G} = \frac{\mathrm{C}_A}{\mathrm{C}_F} = \frac{9}{4}$$

- One constraint is provided by the shear viscosity

$$\eta = \frac{1}{15} \sum_{s=q,g} \nu_s C_s \int \frac{d^3p}{(2\pi)^3} p^3 n_p \, (1 \pm n_p) \, .$$

Can now solve for C_q and C_g

Simple Casimir Scaling – Quadratic Ansatz

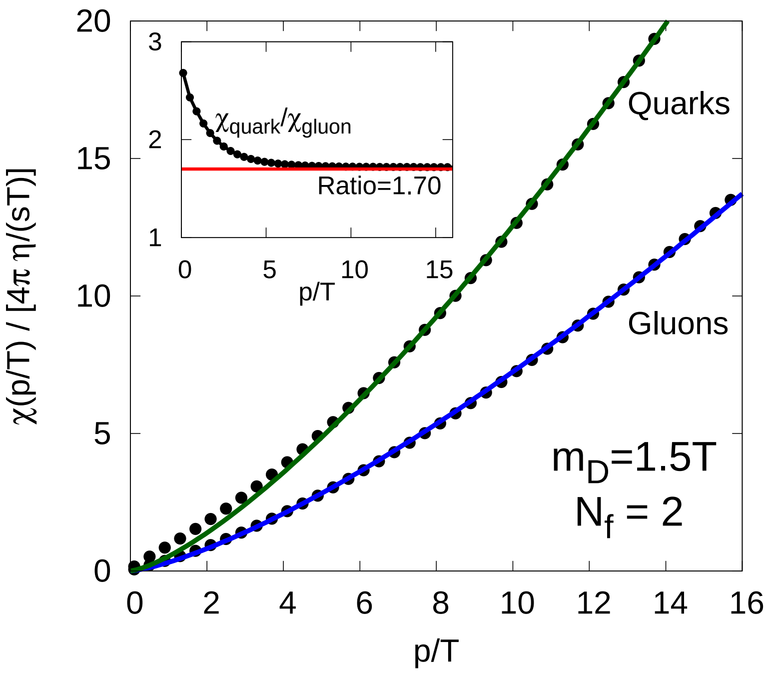


Now we can do a real calculation

- All kinds of processes: $g \longrightarrow q\bar{q}$, $gq \longrightarrow gq$
- As before we can linearize the Boltzmann equation and write a matrix equation

$$\begin{bmatrix} b_p^g \\ b_p^q \end{bmatrix} = \begin{bmatrix} \Gamma_{gg} & \Gamma_{gq} \\ \Gamma_{qg} & \Gamma_{qq} \end{bmatrix} \begin{bmatrix} \delta f_g \\ \delta f_q \end{bmatrix}$$

Quark and gluons:



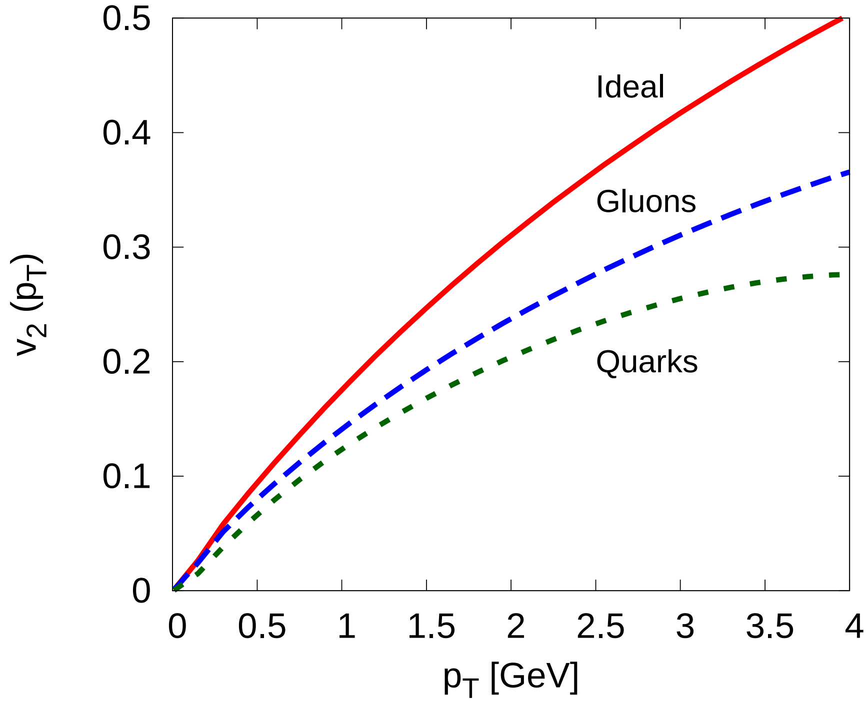
$$\frac{\chi_{\rm quark}}{\chi_{\rm glue}} \sim \frac{\tau_R^Q}{\tau_R^G} \sim 1.7$$

High momentum behavior – not just Casimirs

- Other splitting processes $g \longrightarrow q\bar{q}$ and spin dependence in splitting fcn.

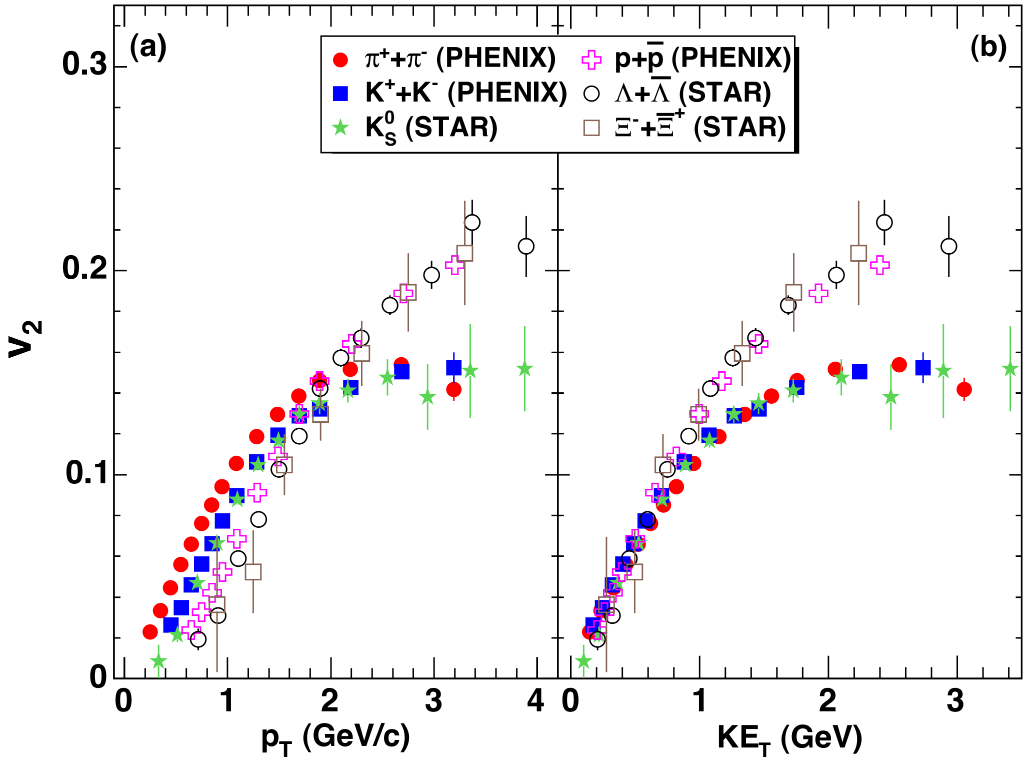
Derive the ratio $\frac{\chi_{\rm quark}}{\chi_{\rm glue}} = 1.7$ analytically by analyzing collinear emission.

Quarks and Gluons – Real Calculation



Different species with different relaxation times have different flows

Mesons and Baryons have different flows



Perhaps they have different relaxation times

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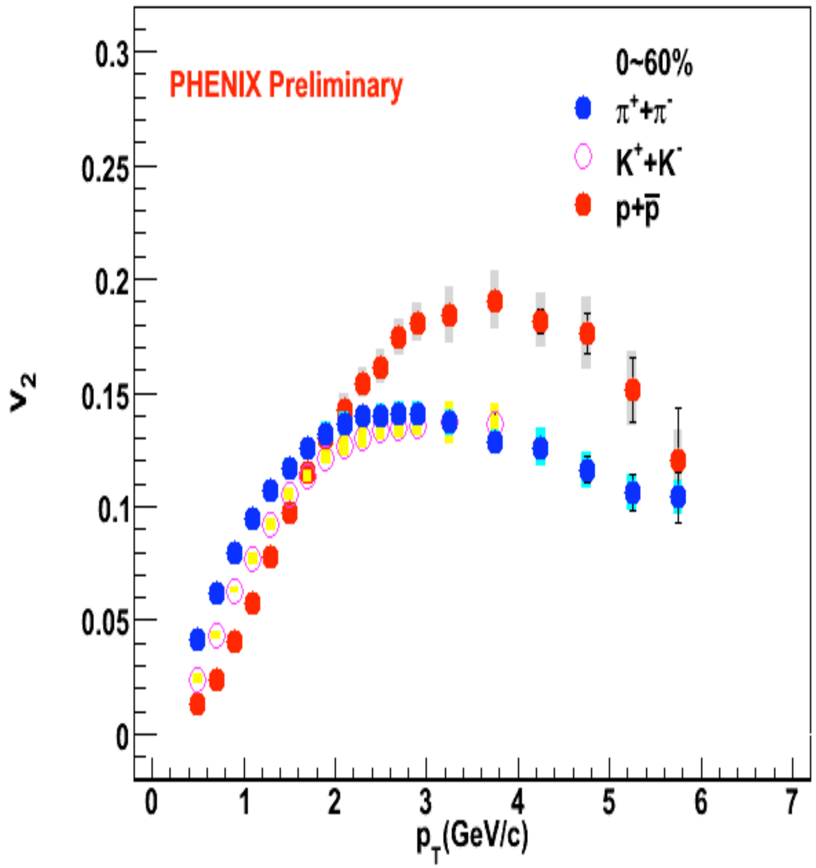
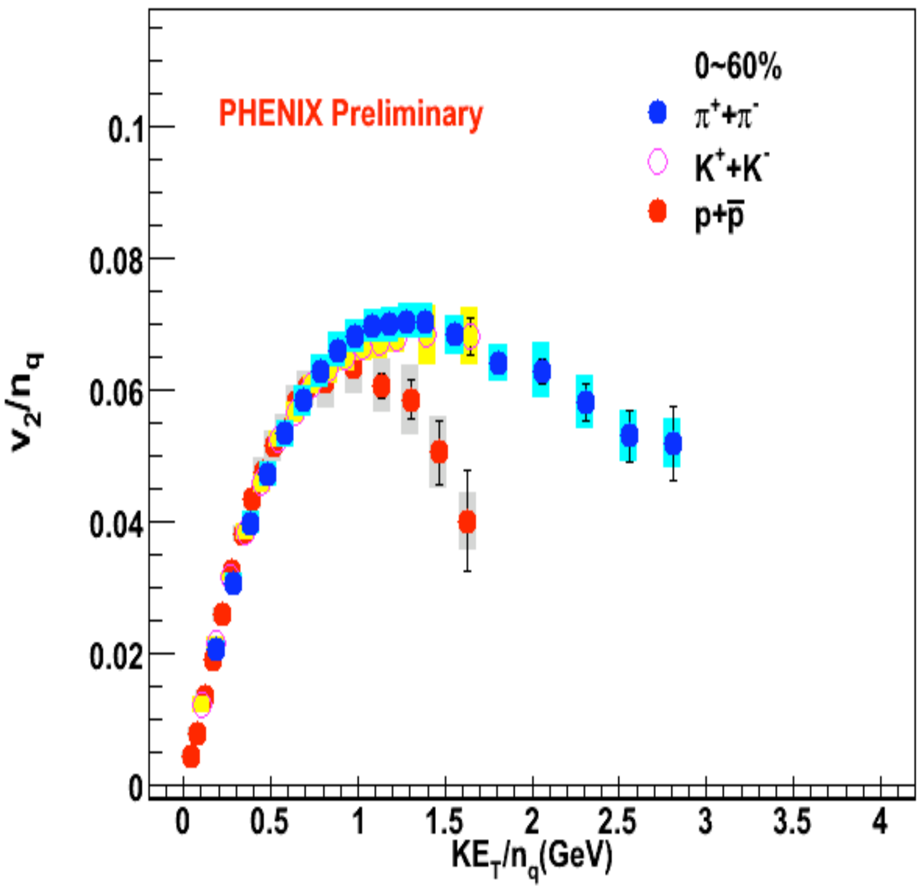
Two components interpreted with Coalescence





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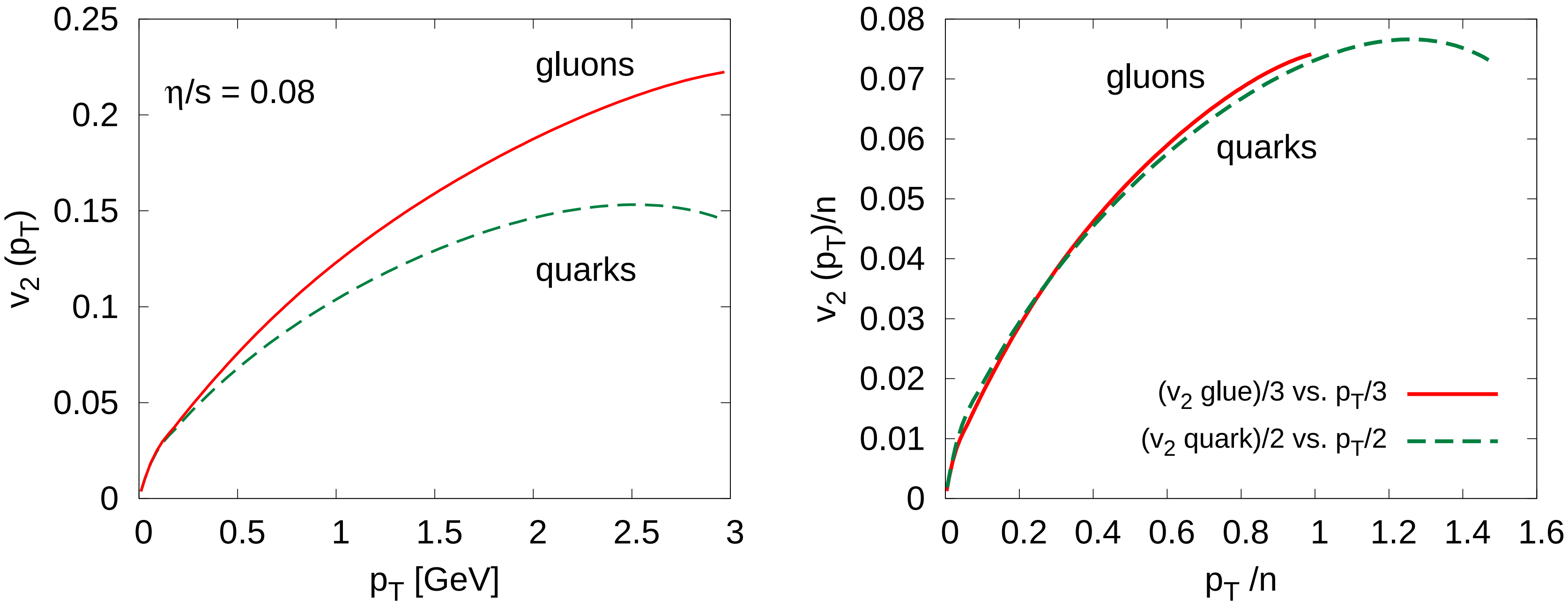
Simple quark and gluon model



"Scaling" can be an artifact of two different relaxation times

Try two different relaxation times for mesons and baryons

Two component meson/baryon gas – relaxation time

$$\begin{aligned} \delta f_m(p) &= -n_p(1+n_p)\chi_m(\tilde{p})\hat{p}^i\hat{p}^j \langle \partial_i u_j \rangle \\ \delta f_b(p) &= -n_p(1-n_p)\chi_b(\tilde{p})\hat{p}^i\hat{p}^j \langle \partial_i u_j \rangle \end{aligned}$$

- Parameterize the viscous corrections as

$$\begin{aligned} \chi_m(\tilde{p}) &= C_m p^2 \\ \chi_b(\tilde{p}) &= C_b p^2 \end{aligned}$$

- Fit

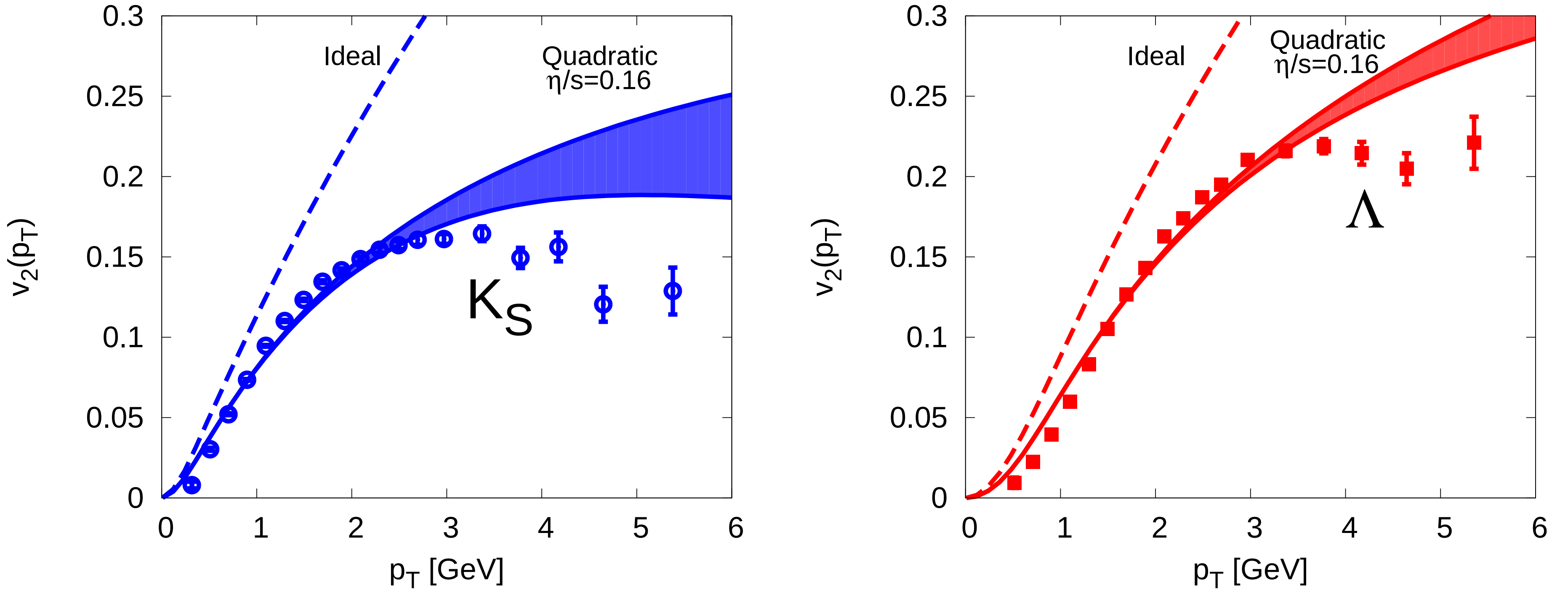
$$\frac{C_m}{C_b} = 1.6$$

- Constrained by shear viscosity

$$\eta = \frac{1}{15} \sum_{a=\pi,K,p,\ldots} \nu_a C_{m/b} \int \frac{d^3p}{(2\pi)^3 E_a} p^4 n(E_a) \left[1 \pm n(E_a)\right],$$

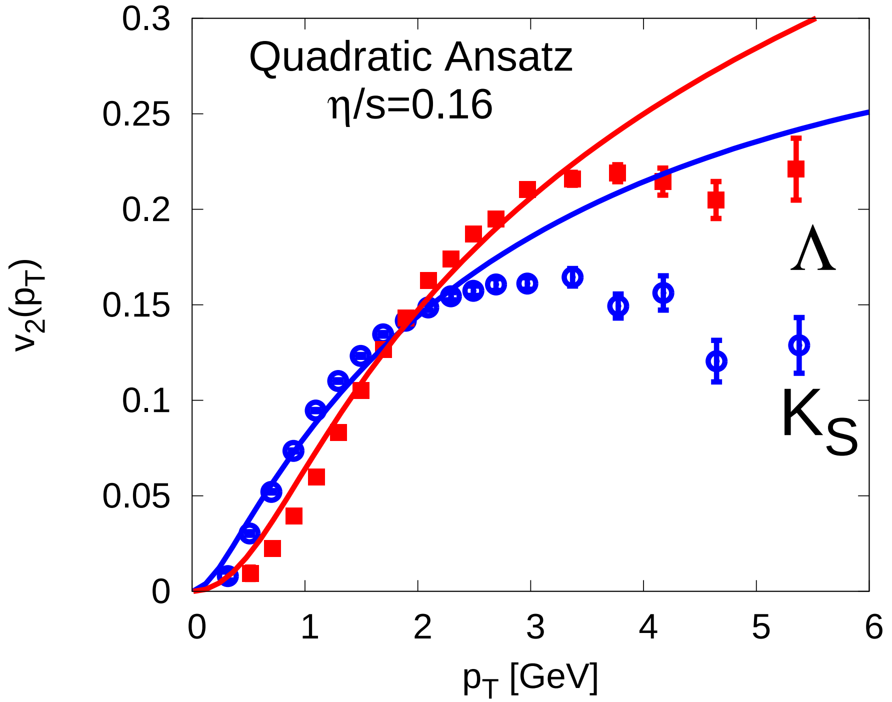
No reason to think the relaxation times of baryons are the same as mesons

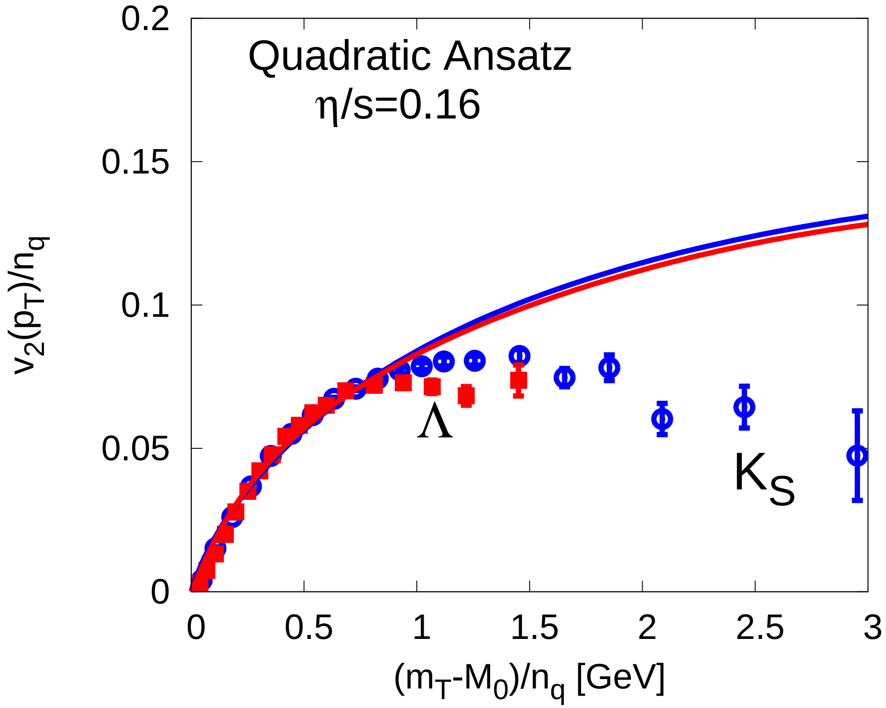
Results



0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
$v_2(p_T)$
p_T [GeV]
Ideal
Quadratic
η/s=0.16
K_S
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
$v_2(p_T)$
p_T [GeV]
Ideal
Quadratic
η/s=0.16
Λ

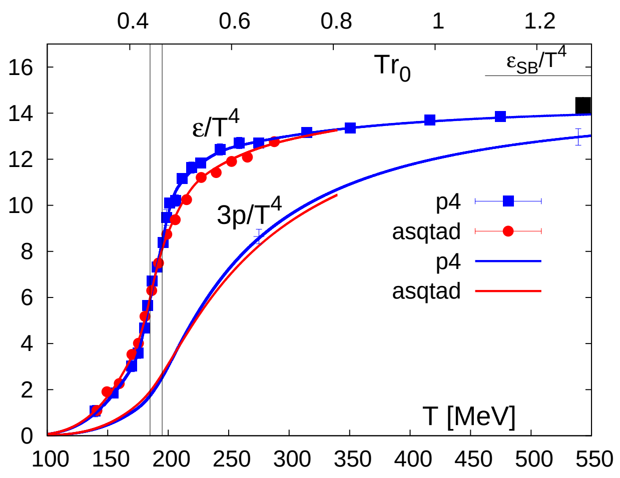
Scaling

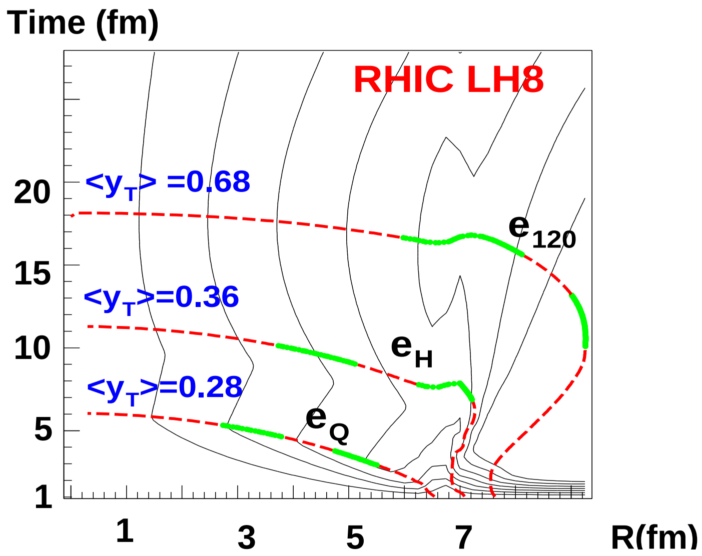




Perhaps quark number scaling is simply *Relaxation Time Scaling* (RTS)

Transition Region – long lived, not hadronic or partonic





1. The transition region is long lived $\sim 3\,\mathrm{fm}$

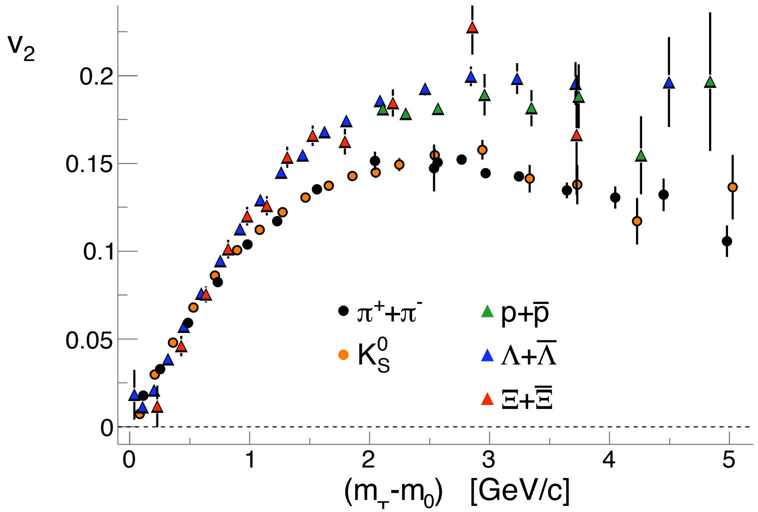
2. The interactions are very inelastic in this momentum range

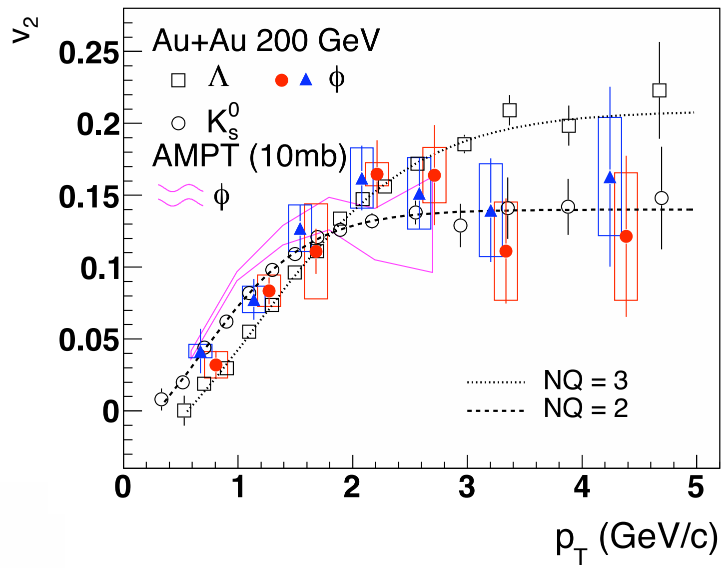
3. Results suggest the additive quark model (Bleicher et al)

$$\frac{C_m}{C_b} = \frac{\sigma_B}{\sigma_M} = 1.5$$

Transition Region – approximately $SU(3)$ symmetric

- In the high temperature range expect $SU(3)$ symmetric to be better
  - In $SU(3)$ symmetric world differences Baryon-Meson and spin diffs
  - $\underbrace{\pi, K}_{\tau_{R1}}$ and $\underbrace{p, \Lambda, \Sigma, \Xi}_{\tau_{R2}}$ and $\underbrace{\phi}_{\tau_{R3}}$ and $\underbrace{\Omega^-}_{\tau_{R4}}$





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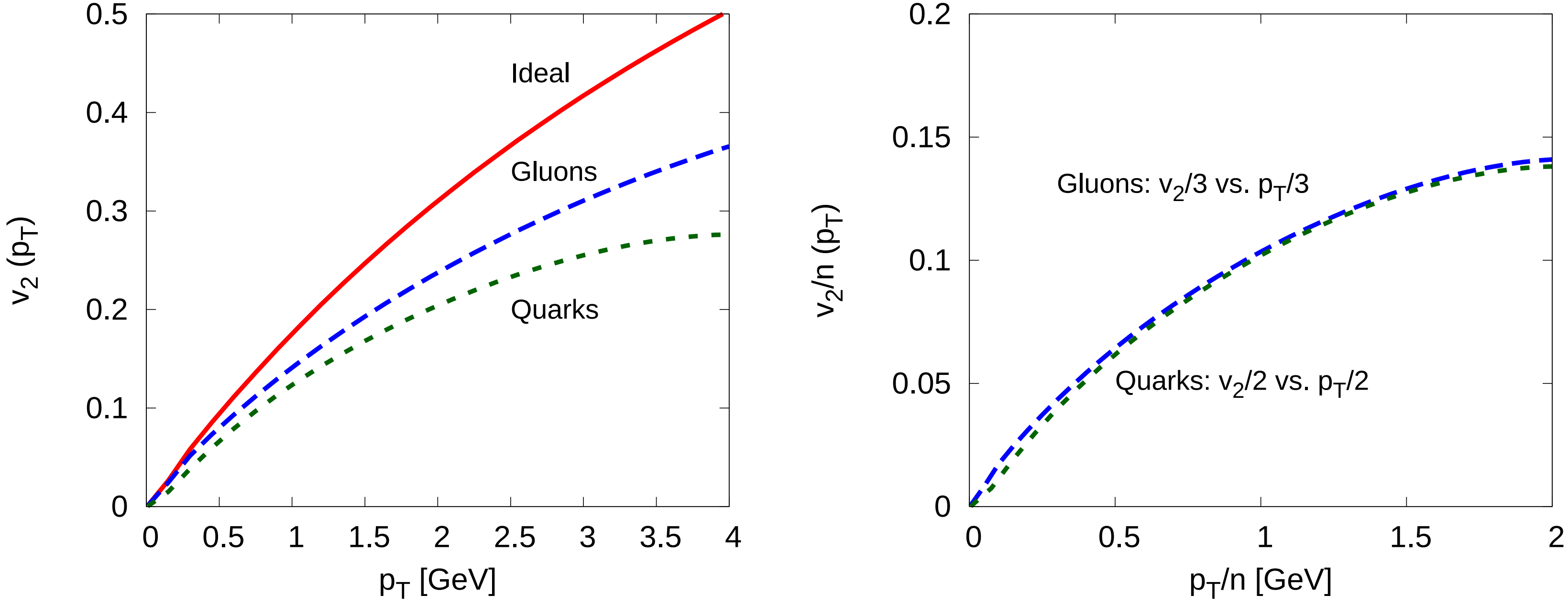
Conclusions

- Studied the kinetics of Quarks and Gluons and the imprints on elliptic flow.
- Radiative energy loss increases the elliptic flow in a certain range

$$p_T \simeq 1.5 \leftrightarrow 2.5\,\mathrm{GeV}$$

- Makes precise the connection between energy loss and viscosity.
- Observed *Relaxation Time Scaling (RTS)* in measured elliptic flow
  - I believe that such relaxation time fits will do as well as coalescence.

Backup I: perturbative quark and gluon model



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