Partial-Wave Analysis of Pion Scattering Reactions

R. Arndt, W. Briscoe, M. Paris,
I. Strakovsky, and R. Workman

The George Washington University

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Scattering Analysis Interactive Dial-in (SAID) – since 1976

[http://gwdac.phys.gwu.edu/]

[ssh -C -X said@gwdac.phys.gwu.edu [no passwd]

CNS Center for Nuclear Studies
Data Analysis Center

- CNS DAC Home
- CNS DAC [SAID]
- CNS Home

Partial-Wave Analyses at GW
[See Instructions]
- Pion-Nucleon
- Kaon-Nucleon
- Nucleon-Nucleon
- Pion Photoproduction
- Pion Electroproduction
- Kaon Photoproduction
- Eta Photoproduction
- Pion-Deuteron (elastic)
- Pion-Deuteron to Proton+Proton

Analyses From Other Sites
- Mainz (MAID – Analyses)
- Nijmegen (Nucleon-Nucleon OnLine)
- Hamburg (Pwa OnLine)

Contact
Richard A. Arndt
William J. Briscoe
Ron L. Workman
Igor I. Strakovsky
Mark W. Paris
Center for Nuclear Studies
Department of Physics
The George Washington University
-Virginia Campus-
20101 Academic Way
Ashburn, VA 20147, USA

CNS DAC Services [SAID Program]

- The Virginia Tech Partial-Wave Analysis Facility (SAID) has moved to GW!
- New features are being added and will first appear at this site. Suggestions for improvements are always welcomed.
- Once fully operational, this web page will become the main entry for the full range of services presently available through SAID.

Instructions for Using the Partial-Wave Analyses

The programs accessible with the left-hand side navigation bar allow the user to access a number of features available through the SAID program. Contact a member of our group if you are unfamiliar with the SSH version. If you enter choices which are unphysical, you may still get an answer (in accordance with the `garbage in, garbage out' rule). Please report unexpected garbage-out to the management.

Note: These programs use HTML forms to run the SAID code. If unfamiliar with the options, run the default setup first. The output is an (edited) echo of an interactive session which would have resulted had you used the SSH version. If the default example fails to clarify the specific task you have in mind, we can help (just send an e-mail message).

All programs expect energies in **MeV** units. All of the solutions and potentials have limited ranges of validity. Some are unstable beyond their upper energy limits. Extrapolated results may not make much sense.
Increments: The programs will not allow an arbitrary number of points to be generated. As a rule, stay below **100**.

ACKNOWLEDGMENTS

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The George Washington University WASHINGTON DC

The George Washington University VIRGINIA CAMPUS

N* and Δ* States Coupled to πN

- One of the most convincing ways to study Spectroscopy of N* & Δ* is πN PWA

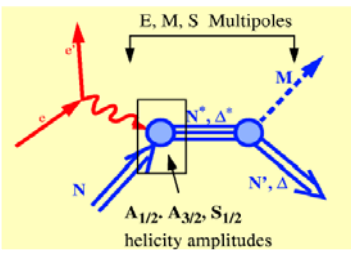
- Non-strange objects in the PDG Listings [http://pdg.lbl.gov/] come mainly from: Karlsruhe-Helsinki, Carnegie-Mellon-Berkeley, and GW/VPI
- The main source of EM couplings is the GW/VPI analysis

- GW DAC SAID program: $\pi N \to \pi N \Rightarrow \gamma N \to \pi N \Rightarrow \gamma^* N \to \pi N$

- πN elastic amplitudes from fits to the observables:
  σ^{tot}, $d\sigma/d\Omega$, and P plus a few Dpol R and A measurements [0.5 %]

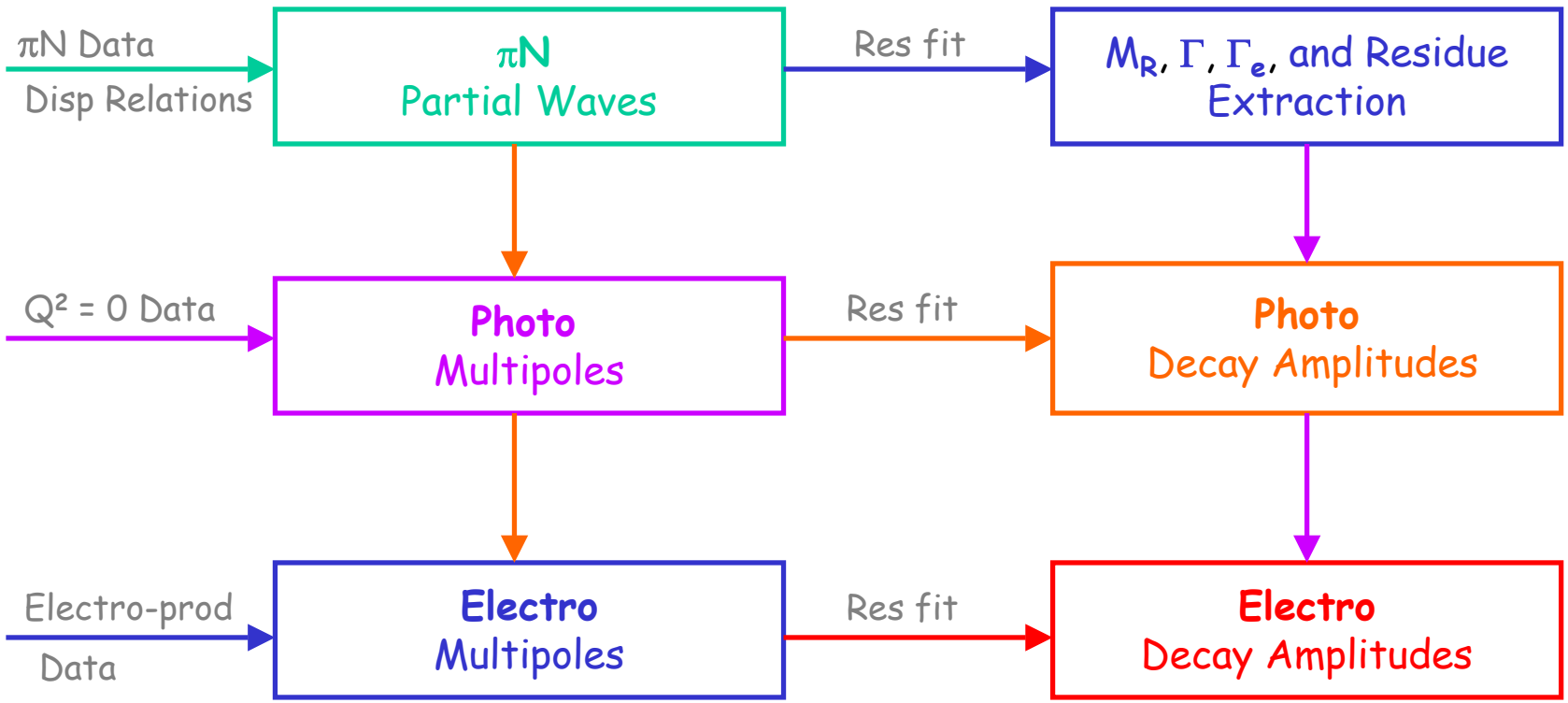
- Assuming dominance of 2-hadronic channels,
  can parameterize $\gamma^* N \to \pi N$ in terms of $\pi N \to \pi N$ amplitudes

- Resulting multipoles can be
  - Re-fitted in terms of Res/Bckgr contributions or
  - Used as input to multi-channel fits with more elaborate constraints



Road Map for GW SAID Analysis of Scattering Data

- πN PWA provides the base for Spectroscopy studies for *non-strange* baryons in all other processes



- Our PWAs have been as model-independent as possible, so as to avoid bias when used
  - in resonance extraction
  - or coupled-channel analysis

GW N* Program - Project Summary

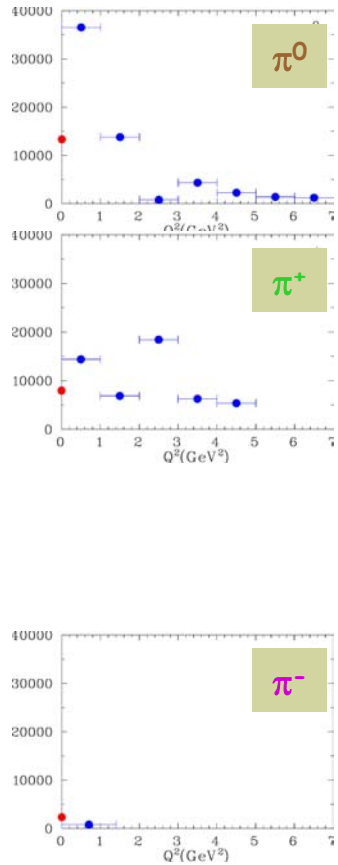
- Energy dependent SM08 and associated SQS
- W = 1080 – 2000 MeV Q^2 = 0 – 6 GeV²
- PWs = 60 [multipoles] [J < 6]
- Prms = 171
- Constraint: πN + pion Photo Prod PWAs (no theor input)

- PWA Problems:
  - Additional [S] Multipoles
  - Q^2 dependence

- Database Problems:
  - Most of data are unPol measurements
  - There are no $\pi^0 n$ data and very few $\pi^- p$ (no Pol measurements)
    That does not allow to determine n-couplings at $Q^2 > 0$

Reaction	Data	χ^2
$\gamma^* p \to \pi^0 p$	55,766	81,284
$\gamma^* p \to \pi^+ n$	51,312	80,004
Redundant	14,772	17,375
Total	124,453	178,663
$\gamma N \to \pi N$	24,888	50,684
All Photo	159,341	229,317
$\pi N \to \pi N$	31,876	57,255
All πN	191,217	286,572
$\gamma^* n \to \pi^- p$	801	
$\gamma^* n \to \pi^0 n$	No Data	

Q^2 Data



- There is no discrimination against any measurements

- World Electro Prod = 0.85 JLab CLAS + 0.15 Others

$\pi N \rightarrow \pi N$ Features

[R. Arndt, W. Briscoe, I. Strakovsky, R. Workman, Phys Rev C **74**, 045205 (2006)]

- Energy dependent WI08 and associated SES
- T_π = 0 – 2600 MeV [W = 1078 – 2460 MeV]
- 4-channel Chew-Mandelstam K-matrix parameterization [πN, $\pi\Delta$, ρN, ηN]
- 3 mapping variables: $g^2/4\pi$, a[$\pi^- p$], Eth
- PWs = 30 πN {15 [I=1/2] + 15 [I=3/2]} + 4 ηN [l < 9]
- Prms = 99 [I=1/2] + 89[I=3/2]

Reaction	Data	χ^2
$\pi^+ p \rightarrow \pi^+ p$	13,354	27,242
$\pi^- p \rightarrow \pi^- p$	11,978	22,705
$\pi^- p \rightarrow \pi^0 n$	2,975	6,091
$\pi^- p \rightarrow \eta n$	257	628
DR constraint	2,775	671
Total	28,564	57,241

[0 – 2600 MeV] 10 data/MeV

[550 – 800 MeV] 1 data/MeV

- 106 data above 800 MeV → 0.06 data/MeV [very few Pol measurements]

- There is no discrimination against any measurements
  Except
  - Some of data with huge χ^2 contribution
  - Redundant data
  - Pol measurements with Errs > 0.2, they have little influence
- However, all available data have been retained in the SAID database

- In the future, J-PARC, GSI, and CSR can contribute a lot of hadronic data

More PWA Features

$$\overline{T} = \overline{K}\,[I - C\,\overline{K}]^{-1}$$

$$K = K^{T} = K^{*}$$

$$C_{ij} = \delta_{ij}\, I_i \qquad [= R_i + i\,\rho_i]$$

$$I_i = (1/\pi)\int_0^{\infty} [\rho_i / (Z - E)]\, dZ$$

$$i = \pi[\pi N],\ \Delta[\pi\Delta],\ \rho[\rho N],\ \eta[\eta N]$$

$$T_{ij} = \overline{T}_{ij}[\rho_i\, \rho_j]^{1/2}$$

$$T_{\pi\pi} = \overline{T}_{\pi\pi}\, \rho_{\pi}$$

$$T_{\pi\eta} = \overline{T}_{\pi\eta}[\rho_{\pi}\, \rho_{\eta}]^{1/2}$$

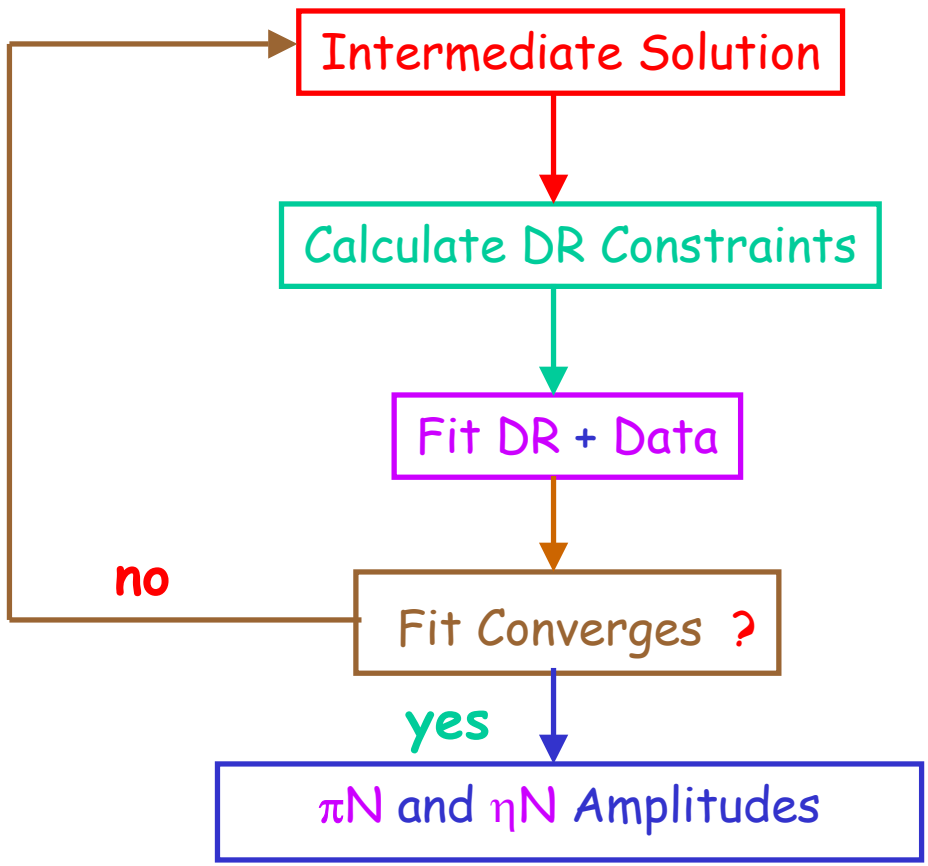
More Features...

- K_{ij} = Polynomial in Energy [P(E)]
- For P_{33}: K_{11} = [P(E) ± a] / [W_k – W ± b]
  a, b give a Hadronic charge splitting
- Coulomb Charge Splitting:
  - Nordita S, P waves to T_π = 500 MeV
    [B. Tromborg, S. Waldenstrom, and I. Overbo, Phys Rev D **15**, 725 (1977)]
  - 'Gibbs' to T_π = 985 MeV
    [W.R. Gibbs, private communication]
  - Else Coulomb Barrier: K → CK

Distributions of Search prms by Partial-Waves

L/I, J	$\frac{1}{2}, -\frac{1}{2}$	$\frac{1}{2}, +\frac{1}{2}$	$3/2, -\frac{1}{2}$	$3/2, +\frac{1}{2}$
0		14		10
1	10	8	7	9
2	10	9	8	7
3	9	4	6	6
4	4	4	3	2
5	5	3	3	3
6	2	2	2	1
7	1	2	2	1
Tot	41	46	31	39

πN Analysis Flow Chart



Cook until DONE !

Some Aternatives

$K_r = g_i g_j [W_k - W]^{-1}$ $\qquad \rho_{ij} = [\rho_i \rho_j]^{1/2}$

$T_{ij} = g_i g_j \rho_{ij} = [D_r - D_i]^{-1}$

$D_r = W_k - W - \Sigma(g_i^2 R_i)$

$D_i = \Sigma(g_i^2 \rho_i)$

$W_r = W_k \Sigma(g_i^2 R_i)$

$D_r \rightarrow \alpha[W_r - W]$ $\qquad \alpha = -1 - \Sigma[g_i^2 \, dR_i / dW]$

$T \rightarrow T[BW]$ $\qquad \Gamma_i = 2 g_i^2 \rho_i / \alpha$

Combine with Background

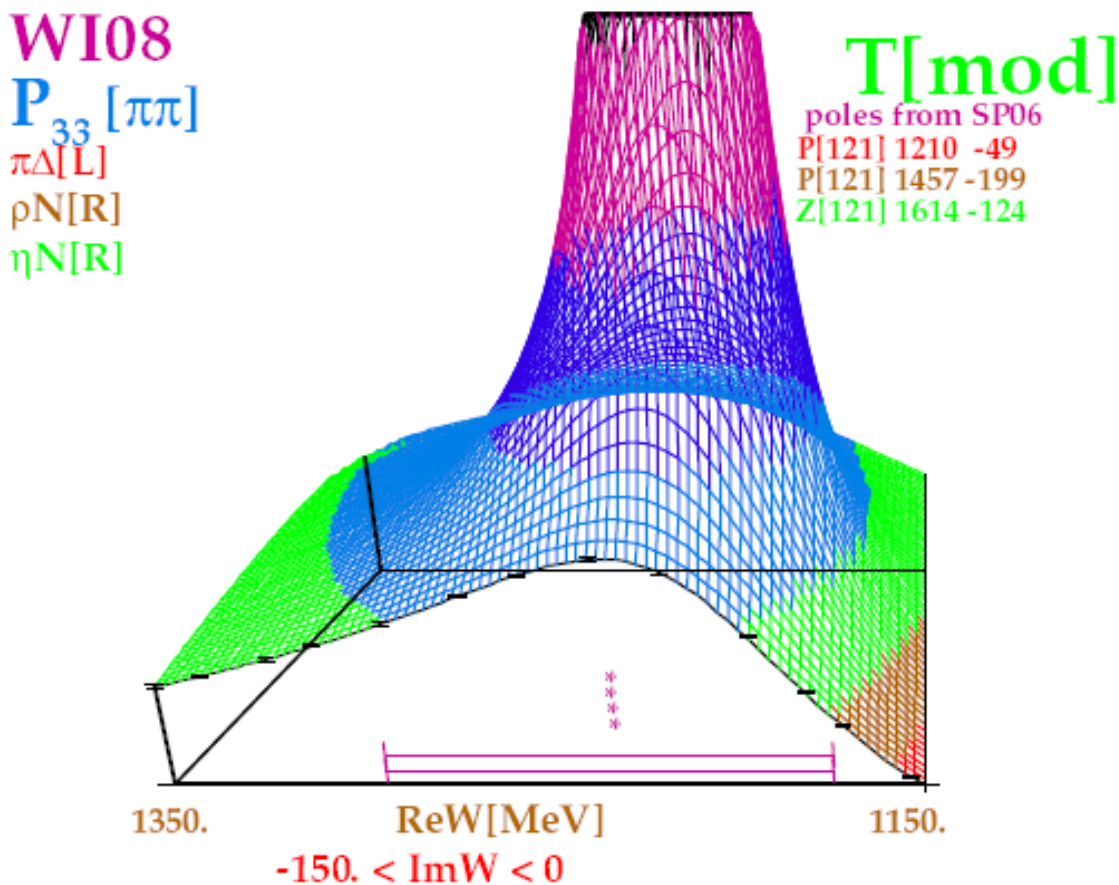
- Choice 1 [Sum] - $K = K_r + K_b[CM]$
- Choice 2 [Prod] - $S = S_b\, S_r\, S_b$

$T = T_r + S_r\, T_b + T_b\, S_r + 2i\, T_b\, S_r\, T$ @
$T_b = T$ [K = polynomial]

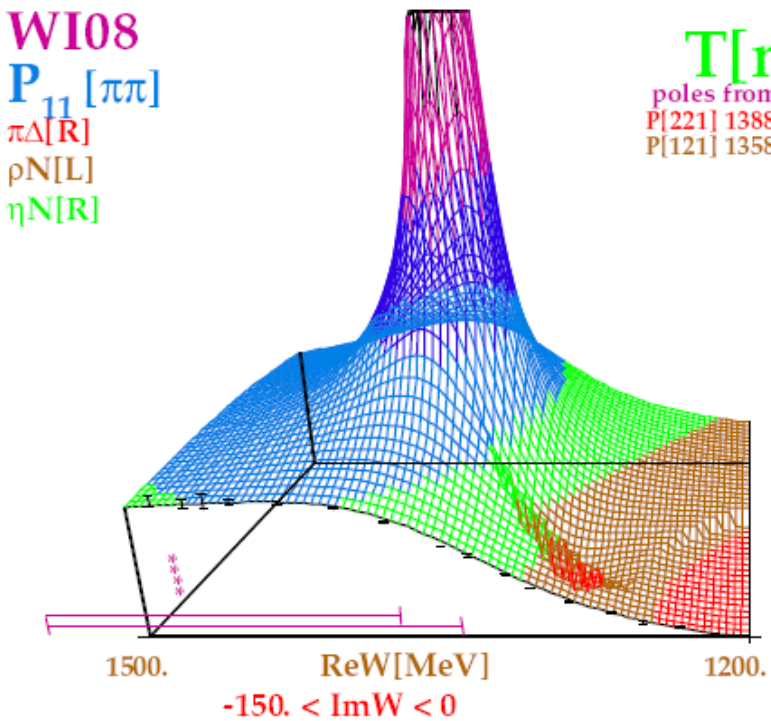
- *Point Resonance parameters:*

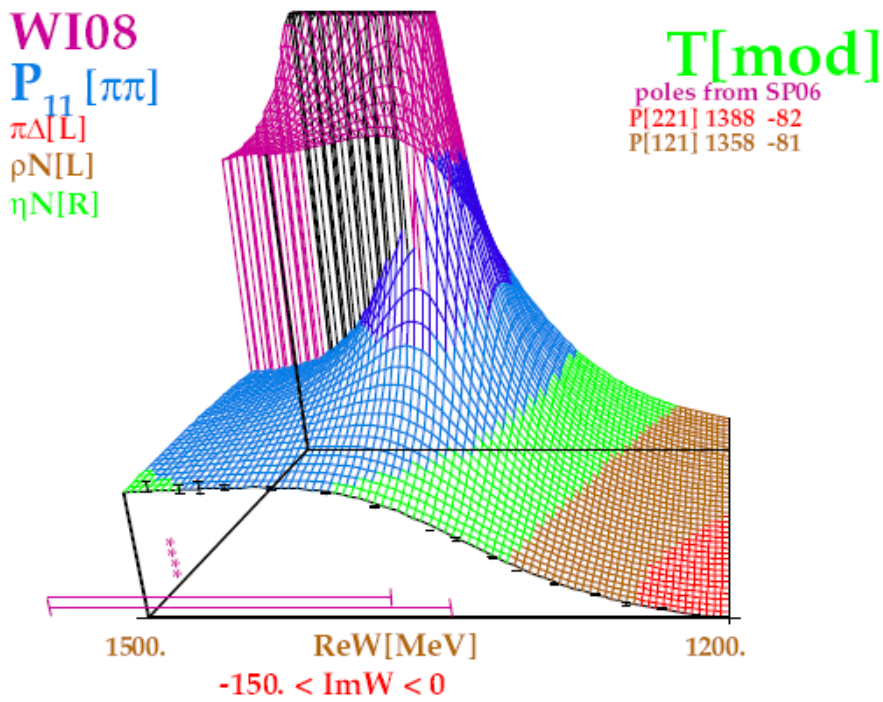
  If $T = \Gamma_e / 2 / [M - W - i\Gamma_t/2]$
  $T_r[M] = 0$ $\qquad R = G_e / G_t = T_i[M]$
  $\Gamma_t = -2\, R / [dT_i / dW]$

πN P_{33}

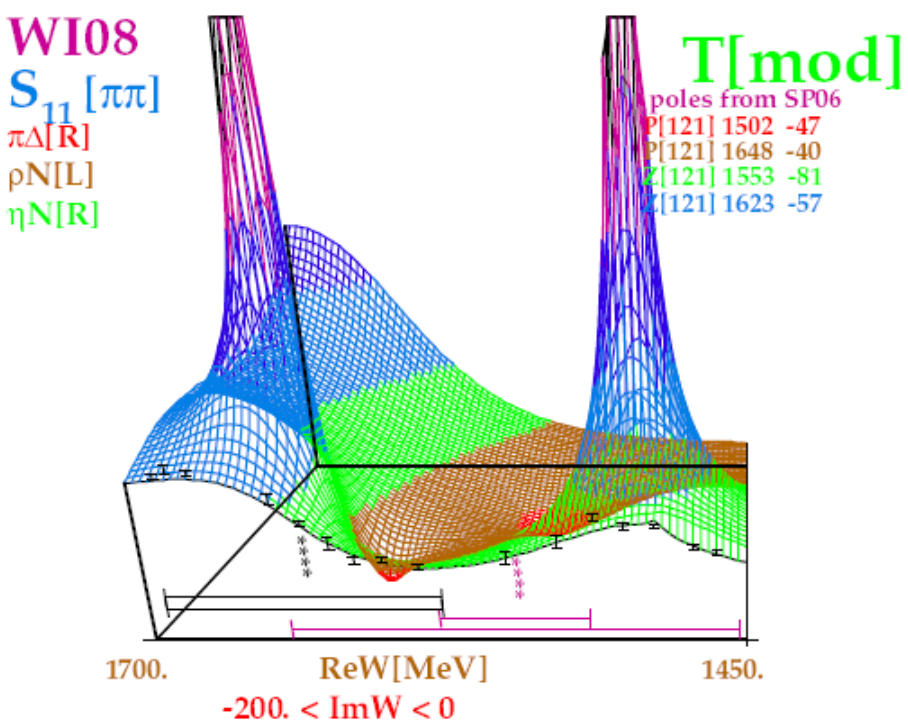

WI08
P33 [ππ]
πΔ[L]
ρN[R]
ηN[R]
T[mod]
poles from SP06
P[121] 1210 -49
P[121] 1457 -199
Z[121] 1614 -124
1350.
ReW[MeV]
1150.
-150. < ImW < 0

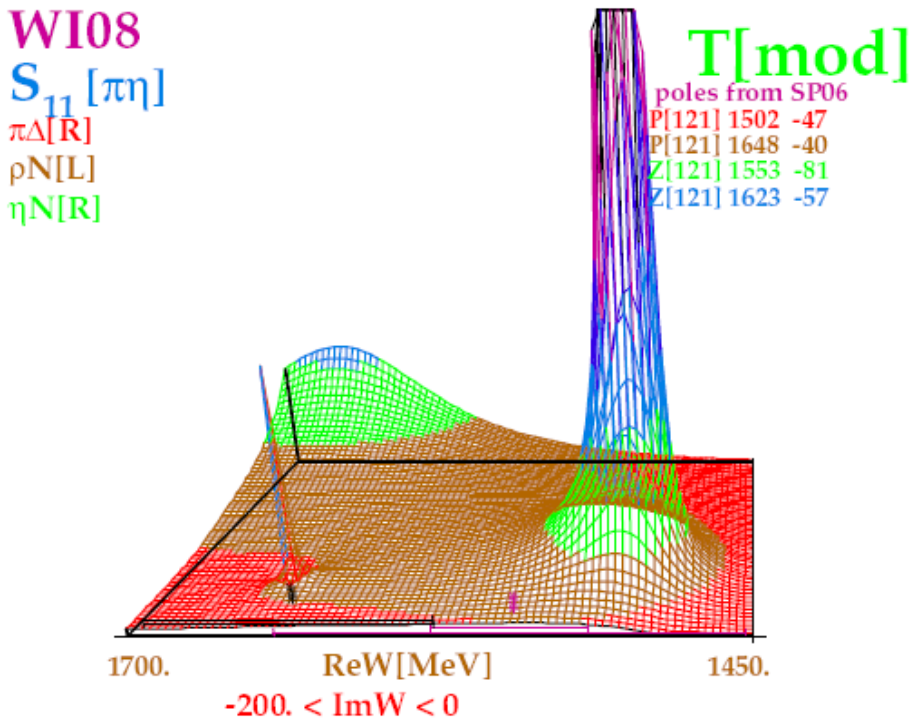
πN P_{11}


WI08
P11 [ππ]
πΔ[R]
ρN[L]
ηN[R]
T[mod]
poles from SP06
P[221] 1388 -82
P[121] 1358 -81
1500.
ReW[MeV]
1200.
-150. < ImW < 0

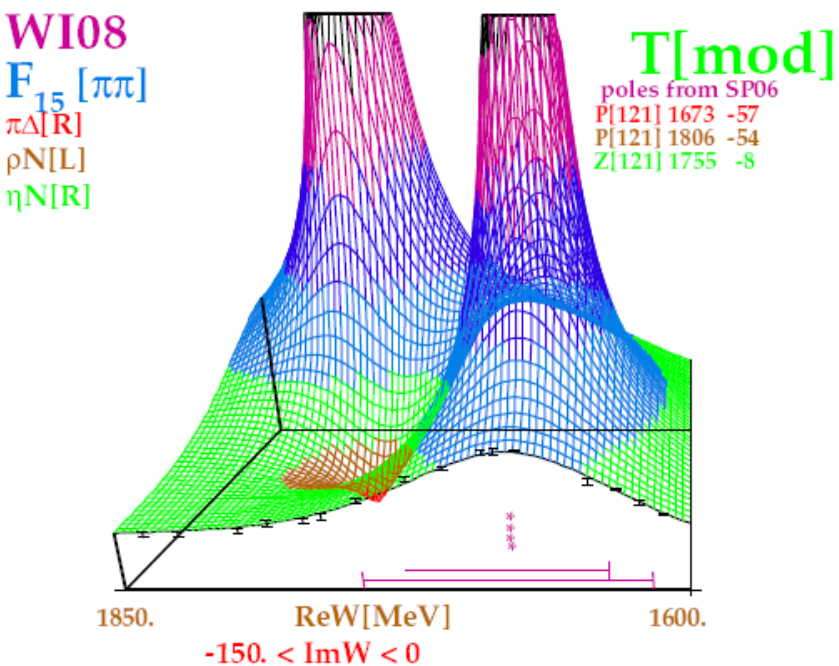

WI08
P11 [ππ]
πΔ[L]
ρN[L]
ηN[R]
T[mod]
poles from SP06
P[221] 1388 -82
P[121] 1358 -81
1500.
ReW[MeV]
1200.
-150. < ImW < 0

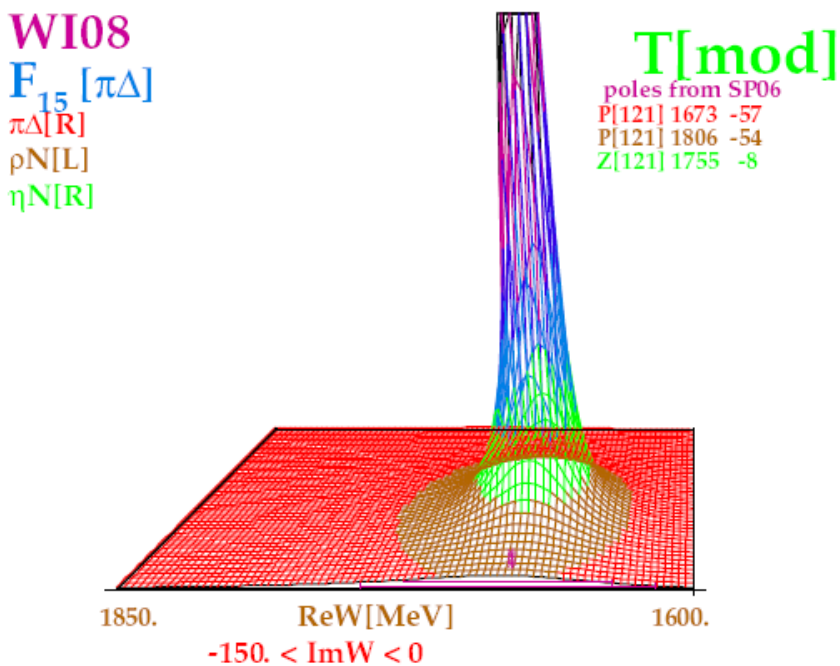
πN S_{11}


WI08
S11 [ππ]
πΔ[R]
ρN[L]
ηN[R]
T[mod]
poles from SP06
P[121] 1502 -47
P[121] 1648 -40
Z[121] 1553 -81
Z[121] 1623 -57
1700.
ReW[MeV]
1450.
-200. < ImW < 0


WI08
S11 [πη]
πΔ[R]
ρN[L]
ηN[R]
T[mod]
poles from SP06
P[121] 1502 -47
P[121] 1648 -40
Z[121] 1553 -81
Z[121] 1623 -57
1700.
ReW[MeV]
1450.
-200. < ImW < 0

πN F_{15}


WI08
F15 [ππ]
πΔ[R]
ρN[L]
ηN[R]
T[mod]
poles from SP06
P[121] 1673 -57
P[121] 1806 -54
Z[121] 1755 -8
1850.
ReW[MeV]
1600.
-150. < ImW < 0


WI08
F15 [πΔ]
πΔ[R]
ρN[L]
ηN[R]
T[mod]
poles from SP06
P[121] 1673 -57
P[121] 1806 -54
Z[121] 1755 -8
1850.
ReW[MeV]
1600.
-150. < ImW < 0

A Couple of Other Concepts

- Speed = abs(dT[$\pi\pi$] / dW)

- Time Delay τ#

  τ = dδ / dW δ = S[$\pi\pi$] phase

- Assertion

  The peak in these quantities marks the Real part of the Complex-plane Pole position

Resonance Resolution

- Ansatz 1 -

$T_r = \gamma_e / [M - W - i\gamma_t]$

$\gamma = (\Gamma/2)\, \rho[W] / \rho[M]$

$T_b = K / [1 - iK]$

$K = (\rho[W] / \rho[M])\, (K_r + D[W - M])$

$S[\pi\pi] = S_r\, S_b$

$\gamma_t = \gamma_e + \gamma_i$

$R = \Gamma_e / \Gamma_t$

$K = K_r + iK_i$

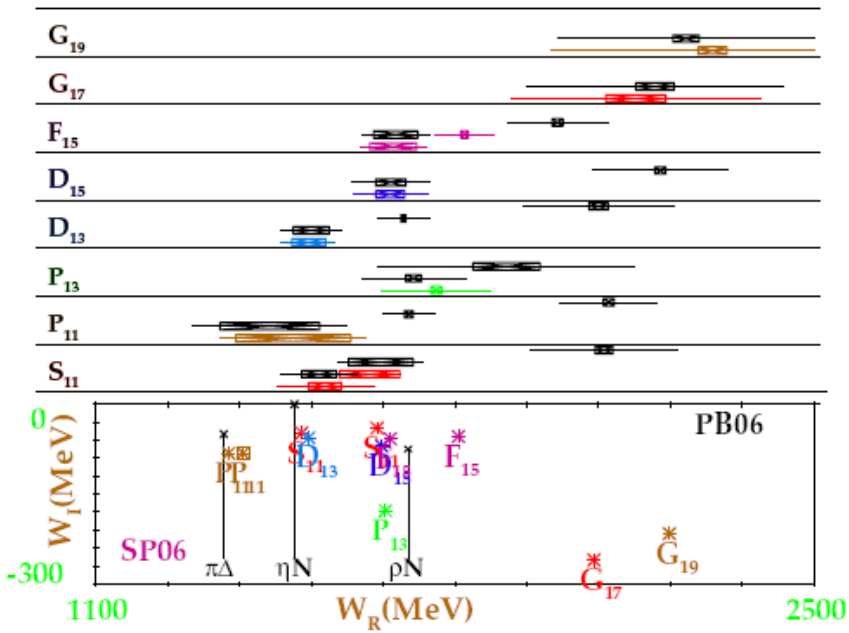
- Ansatz 2 -

Fit $T[\pi\pi]$ over narrow range of W

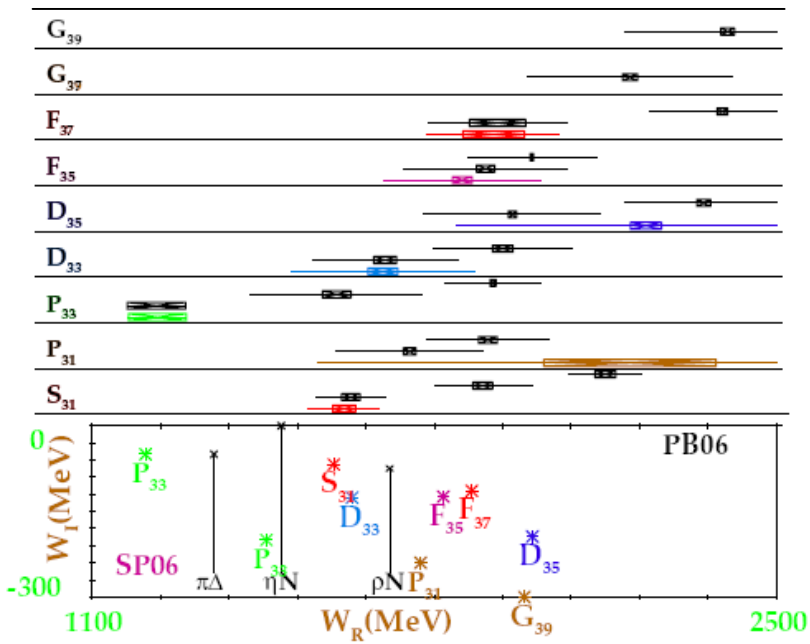
[Then fit DATA over same range]

Complex Energy Plane vs BW fits [GW vs PDG]

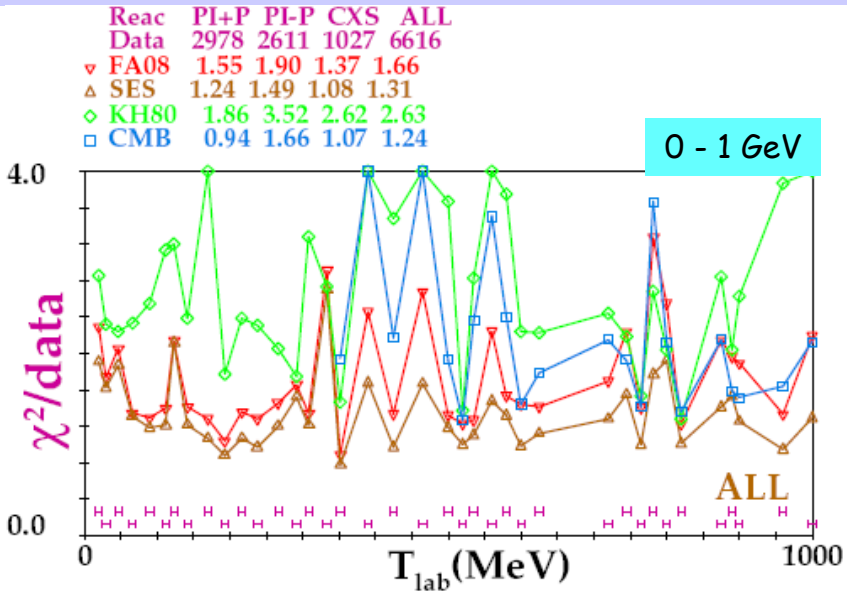
I = 1/2

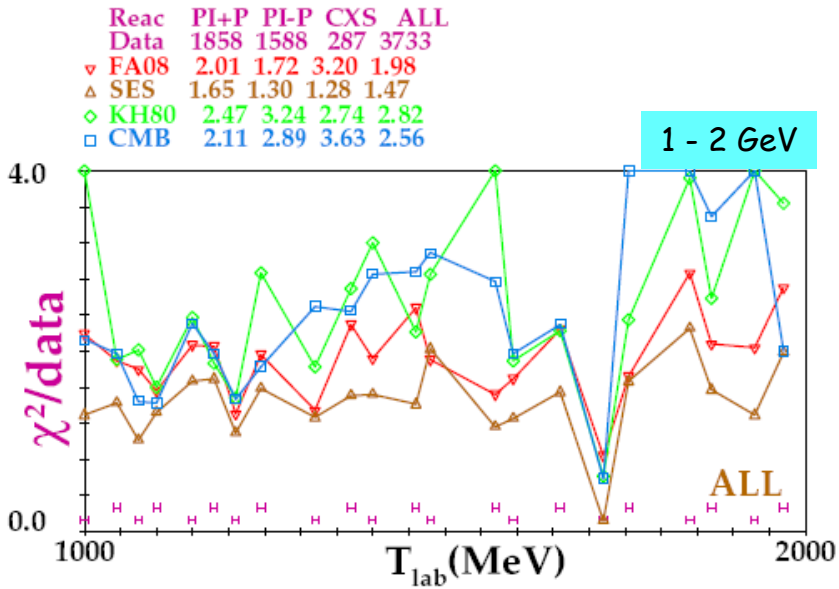


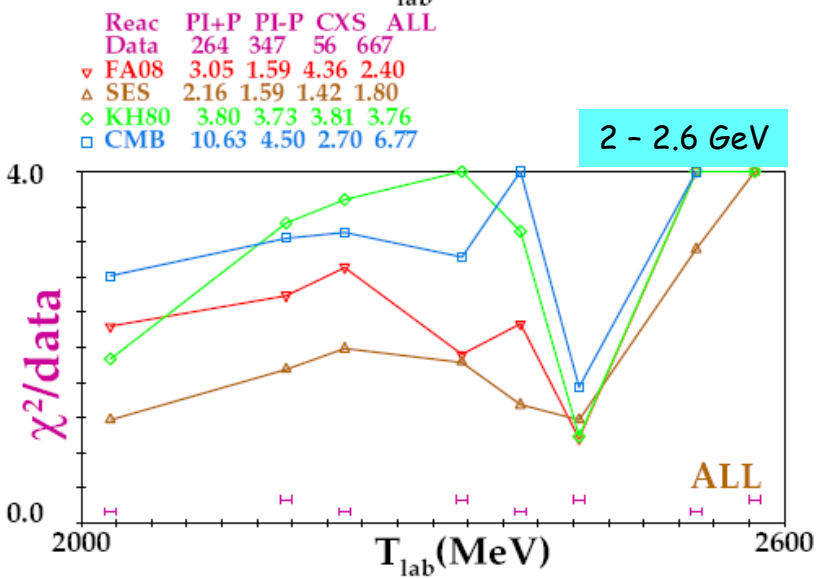
I = 3/2



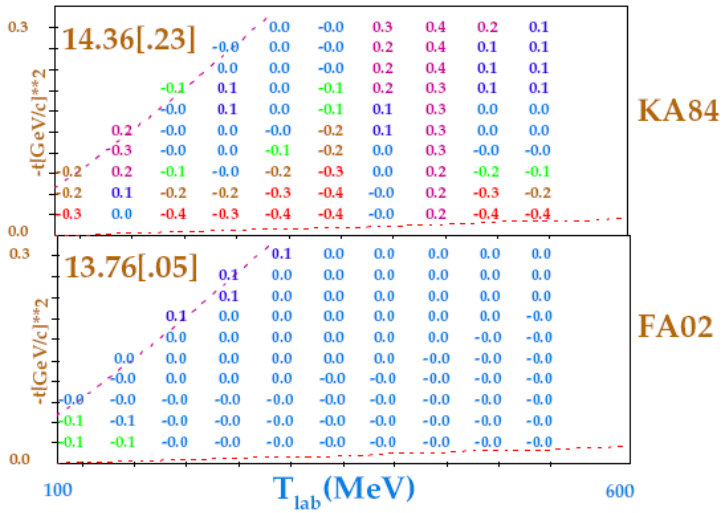
Fit to DATA

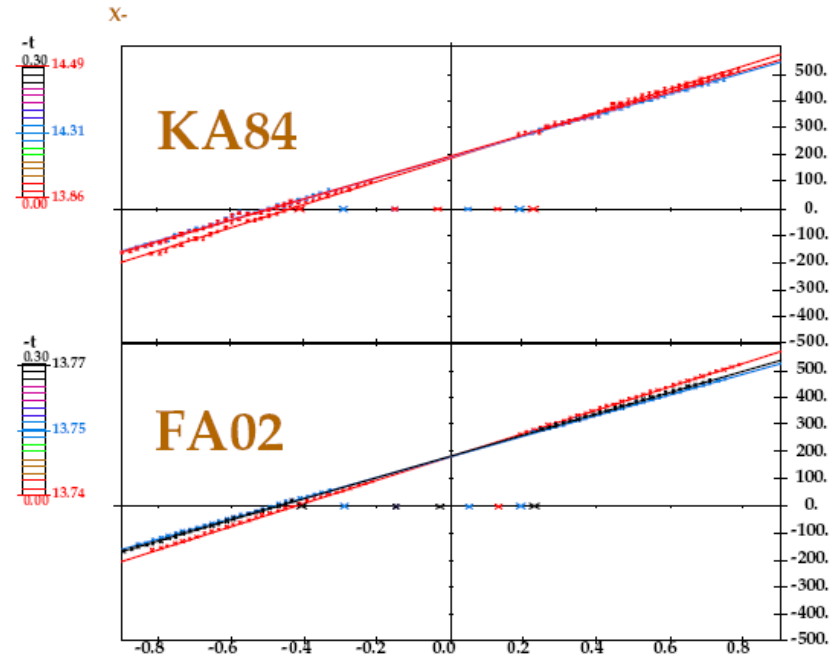

Reac PI+P PI-P CXS ALL
Data 2978 2611 1027 6616
FA08 1.55 1.90 1.37 1.66
SES 1.24 1.49 1.08 1.31
KH80 1.86 3.52 2.62 2.63
CMB 0.94 1.66 1.07 1.24
0 - 1 GeV
4.0
0.0
χ²/data
0
1000
T_lab(MeV)
ALL

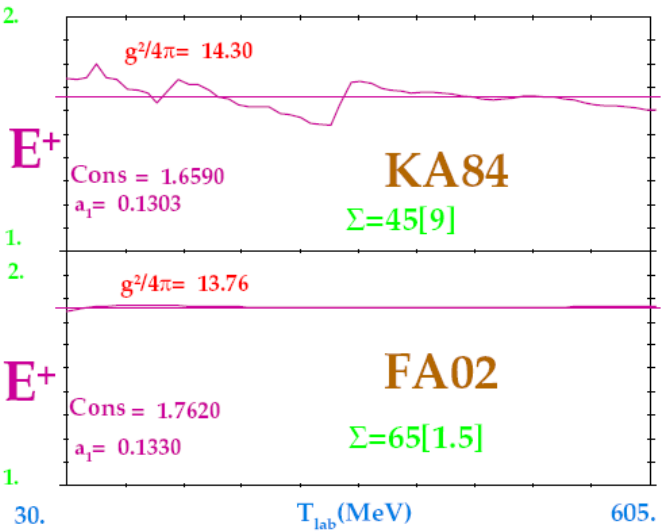

Reac PI+P PI-P CXS ALL
Data 1858 1588 287 3733
FA08 2.01 1.72 3.20 1.98
SES 1.65 1.30 1.28 1.47
KH80 2.47 3.24 2.74 2.82
CMB 2.11 2.89 3.63 2.56
1 - 2 GeV
4.0
0.0
χ²/data
1000
2000
T_lab(MeV)
ALL


Reac PI+P PI-P CXS ALL
Data 264 347 56 667
FA08 3.05 1.59 4.36 2.40
SES 2.16 1.59 1.42 1.80
KH80 3.80 3.73 3.81 3.76
CMB 10.63 4.50 2.70 6.77
2 - 2.6 GeV
4.0
0.0
χ²/data
2000
2600
T_lab(MeV)
ALL

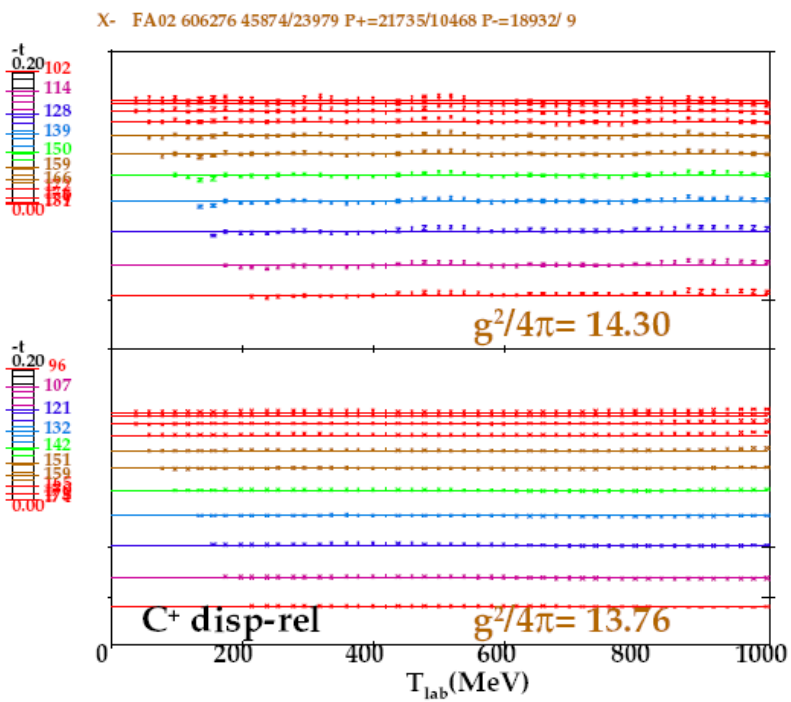
Dispersion Relations – πNN coupling

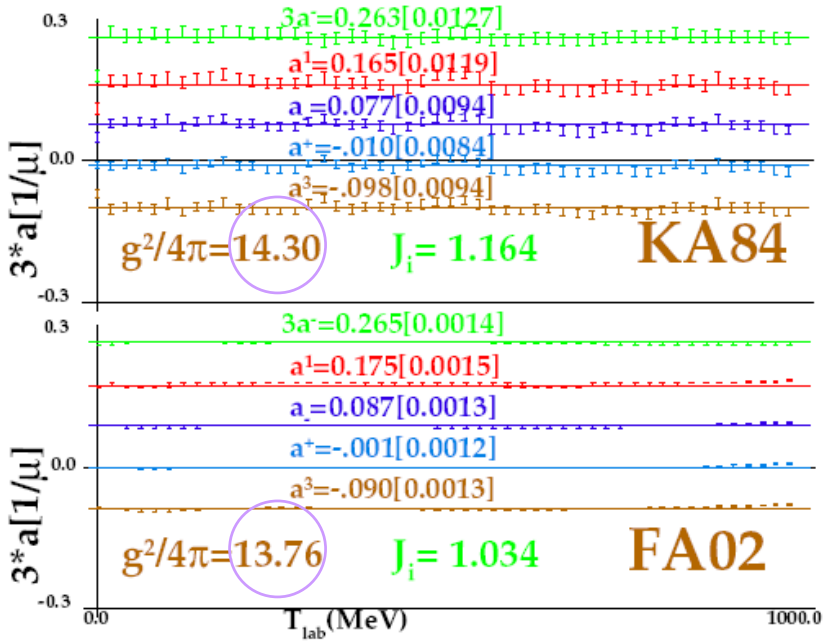




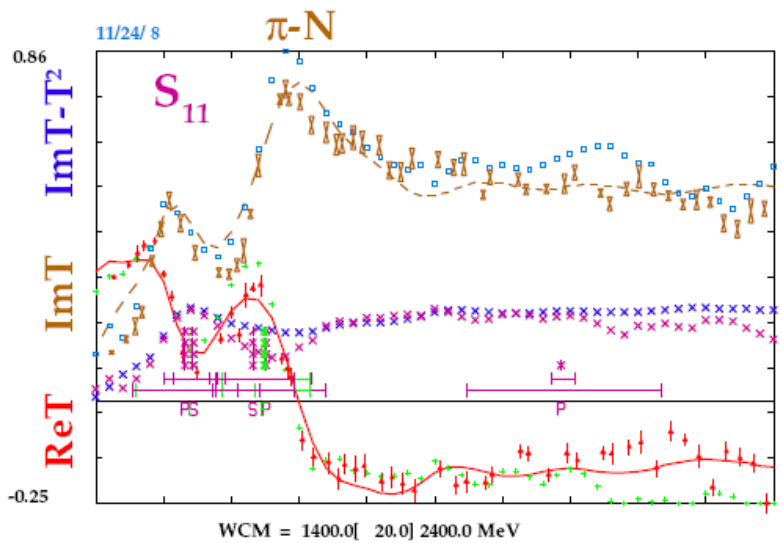


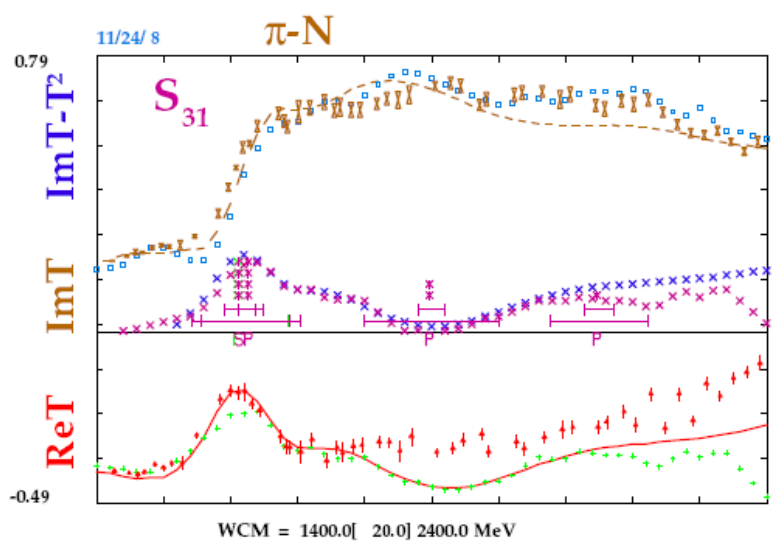
... More Dispersion Relations - πNN coupling



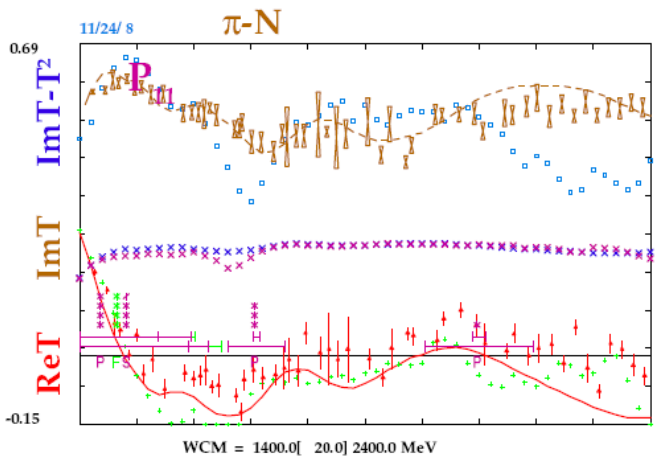


πN S - waves

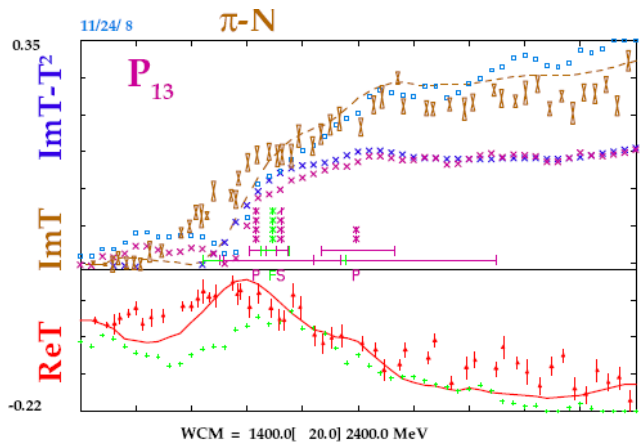

11/24/ 8
π-N
0.86
S11
ImT-T2
ImT
ReT
-0.25
PS
SP
P
WCM = 1400.0[20.0] 2400.0 MeV
KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19


11/24/ 8
π-N
0.79
S31
ImT-T2
ImT
ReT
-0.49
SP
P
P
WCM = 1400.0[20.0] 2400.0 MeV
KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19

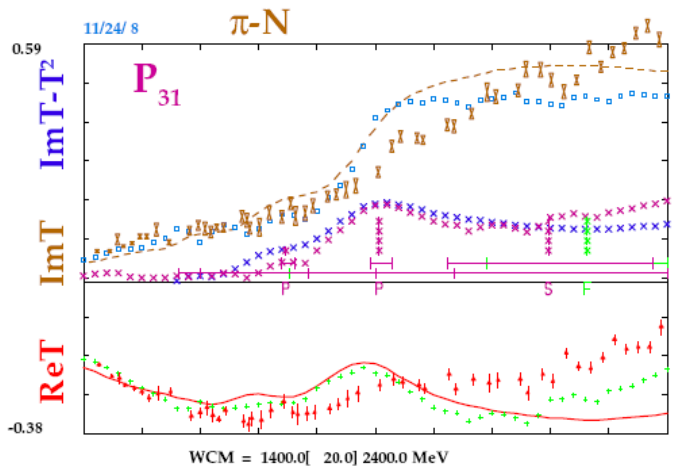
πN P - waves



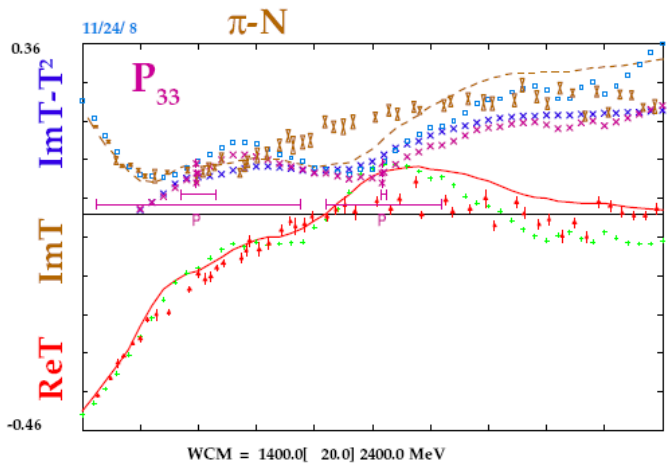
KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19



KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19

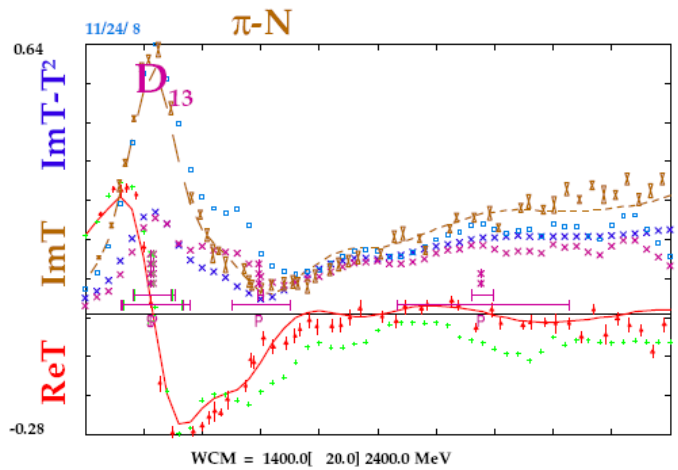


KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19

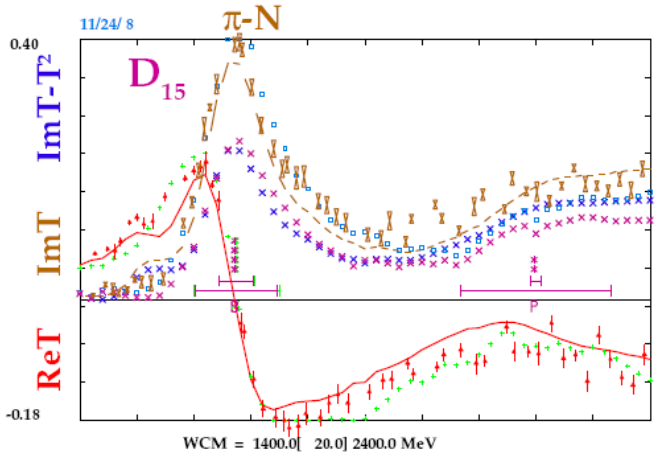


KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19

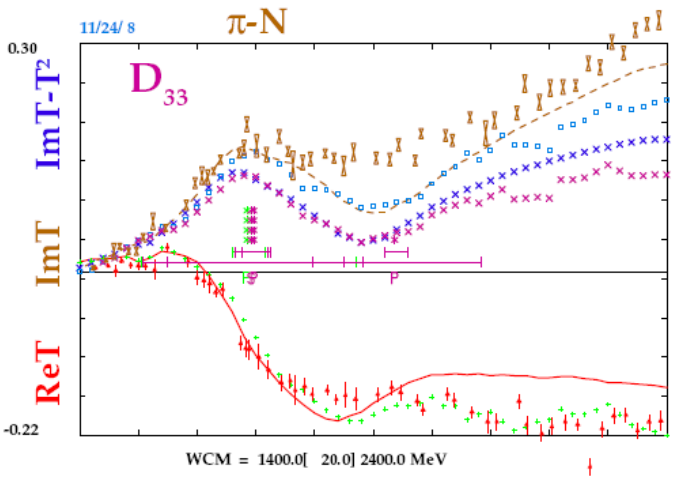
πN D - waves



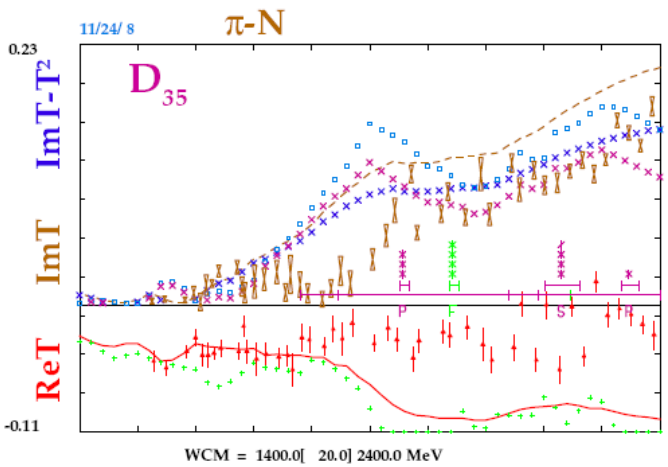
KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19



KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19

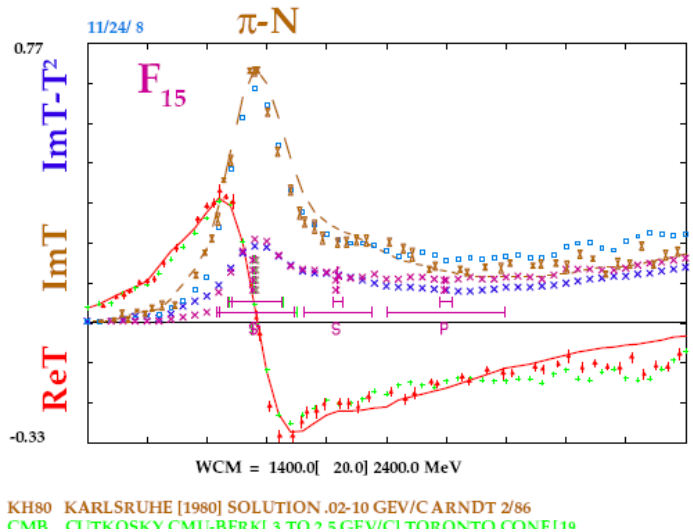


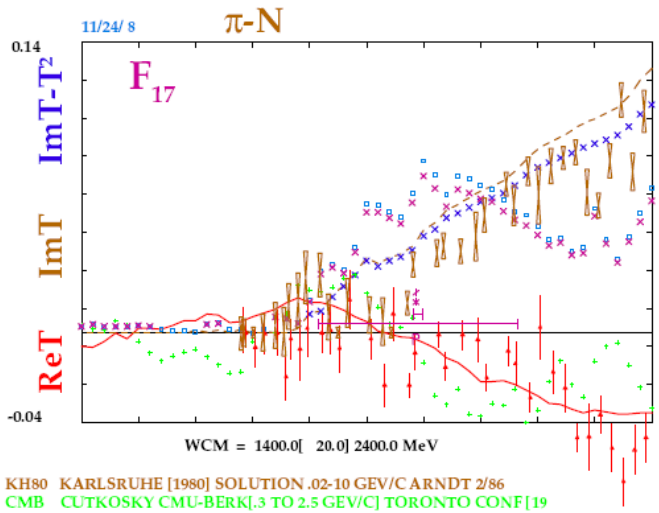
KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19

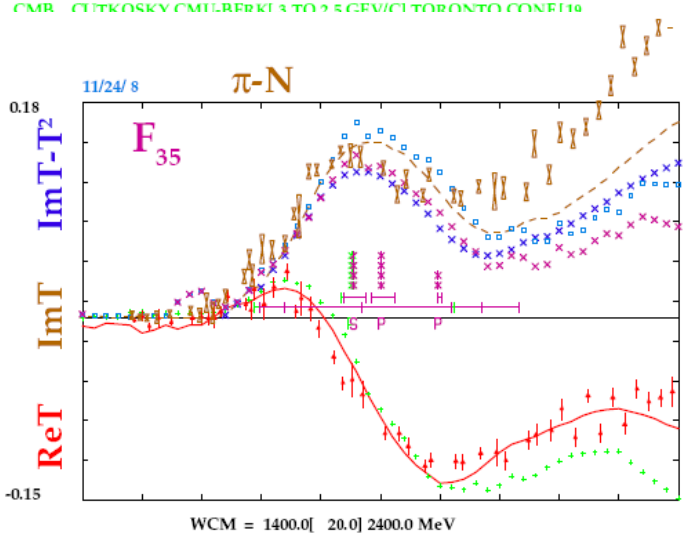


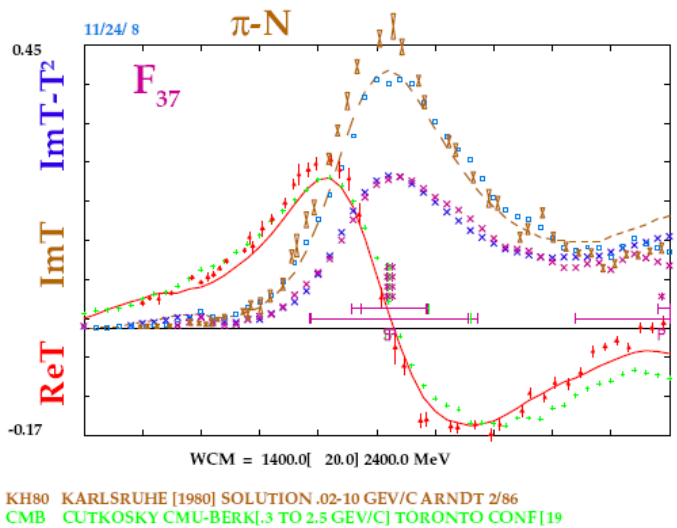
KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19

πN F - waves

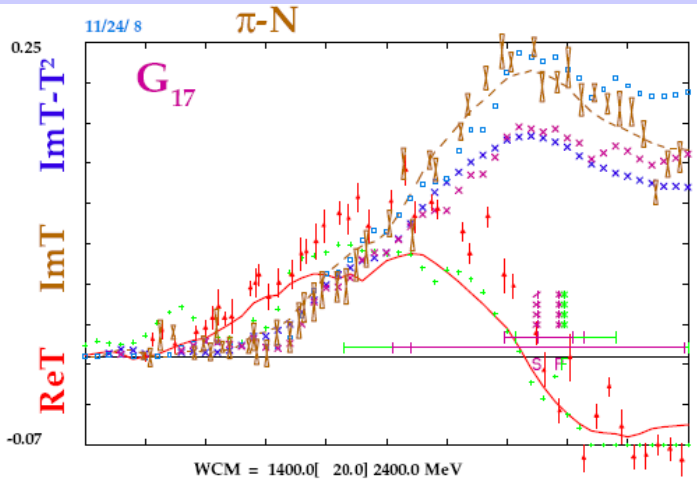

11/24/ 8
π-N
F15
ImT-T²
ImT
ReT
0.77
-0.33
WCM = 1400.0[20.0] 2400.0 MeV
KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19

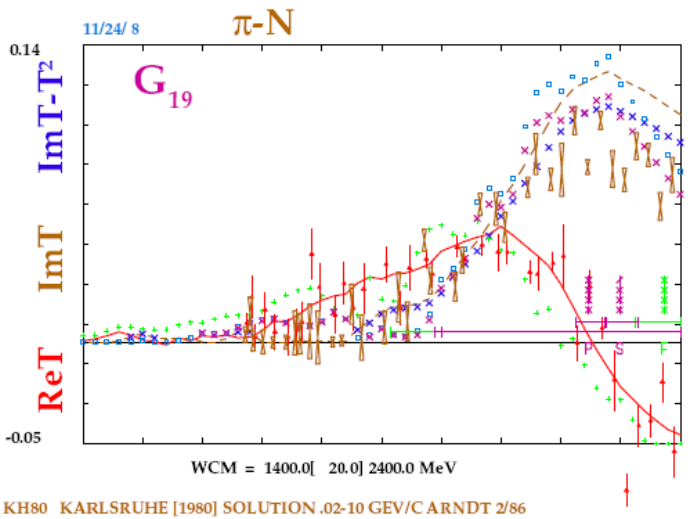

11/24/ 8
π-N
F17
ImT-T²
ImT
ReT
0.14
-0.04
WCM = 1400.0[20.0] 2400.0 MeV
KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19

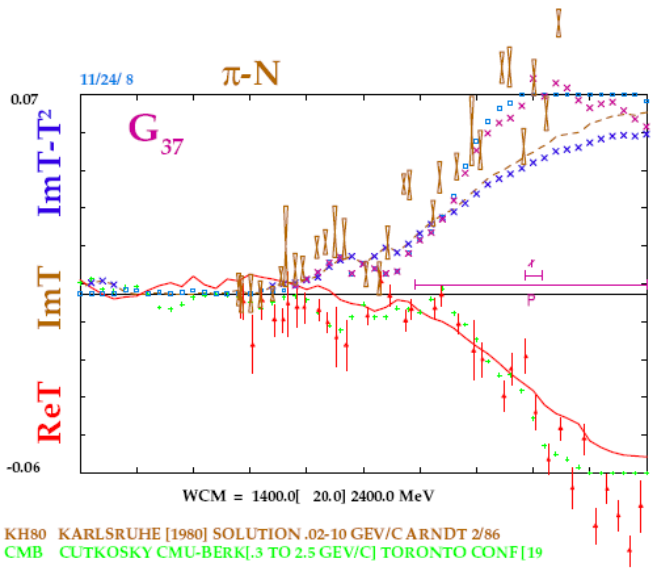

11/24/ 8
π-N
F35
ImT-T²
ImT
ReT
0.18
-0.15
WCM = 1400.0[20.0] 2400.0 MeV
KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19

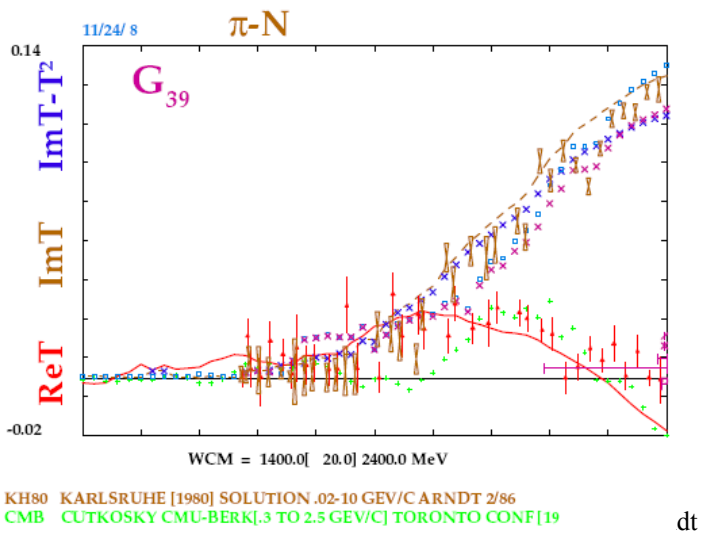

11/24/ 8
π-N
F37
ImT-T²
ImT
ReT
0.45
-0.17
WCM = 1400.0[20.0] 2400.0 MeV
KH80 KARLSRUHE [1980] SOLUTION .02-10 GEV/C ARNDT 2/86
CMB CUTKOSKY CMU-BERK[.3 TO 2.5 GEV/C] TORONTO CONF [19

πN G - waves









dt

Summary of N* and Δ* Finding from πN PWA

[R. Arndt, W. Briscoe, I. Strakovsky, R. Workman, Phys Rev C **74**, 045205 (2006)]

- Standard PWA
  - Allows to determine the N*s, Δ*s, and their quantum numbers using
    - The complex energy plane and
    - Breit-Wigner technique
  - Tends (by construction) to miss narrow Resonances with Γ < 30 MeV
  - Reveals only wide Resonances, but not too wide (Γ < 500 MeV) and possessing not too small BR (BR > 4%)

- PDG08 states

  The latest GWU analysis (Arndt06) finds no evidence for those resonances

PDG08 ***	$\Delta(1600)P_{33}$,	$N(1700)D_{13}$,	$N(1710)P_{11}$,	$\Delta(1920)P_{33}$
PDG08 **	$N(1900)P_{13}$,	$\Delta(1900)S_{31}$,	$N(1990)F_{17}$,	$\Delta(2000)F_{35}$,
	$N(2080)D_{13}$,	$N(2200)D_{15}$,	$\Delta(2300)H_{39}$,	$\Delta(2750)I_{313}$
PDG08 *	$\Delta(1750)P_{31}$,	$\Delta(1940)D_{33}$,	$N(2090)S_{11}$,	$N(2100)P_{11}$,
	$\Delta(2150)S_{31}$,	$\Delta(2200)G_{37}$,	$\Delta(2350)D_{35}$,	$\Delta(2390)F_{37}$

- Our study does suggest several 'new' N*s and Δ*s:

PDG08 ****	$\Delta(2420)H_{311}$	
PDG08 ***	$\Delta(1930)D_{35}$	
PDG08 **	$N(2000)F_{15}$,	$\Delta(2400)G_{39}$
PDG08 new	$N(2245)H_{111}$	[CLAS ?]