The $\pi^- p \rightarrow \pi^0 n$ reaction between 40 and 250 MeV and related topics

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(part of a Ph.D. thesis of Johannes Breitschopf)

Motivation and background

- Claims of isospin breaking at low energy $\pi^{\pm}p$ interaction can be tested with the $\pi^- p \to \pi^0 n$ charge exchange reaction.
- Sort out 2-3% differences between the TRIUMF CX total cross sections and the classical Bugg results.
- Transmission measurements of σ_T are totally different technically from measurements of annihilation gammas.

Amplitudes for pion-nucleon interaction, assuming isospin-symmetry:

$A(\pi^+p) = A_3, \quad A(\pi^-p) = \frac{1}{3}A_3 + \frac{2}{3}A_1$

$A(\pi^-p \to \pi^0 n) = A_{cx} = \frac{\sqrt{2}}{3}(A_3 - A_1)$

then $\sqrt{2}A_{cx} = A(\pi^+p) - A(\pi^-p)$

'the triangular relation' connects the three interactions.

A_3 and A_1 from e.g. 'SAID' analyses are dominated by elastic scattering data.

Departures from this relation could indicate isospin-symmetry breaking.

Experimental highlights

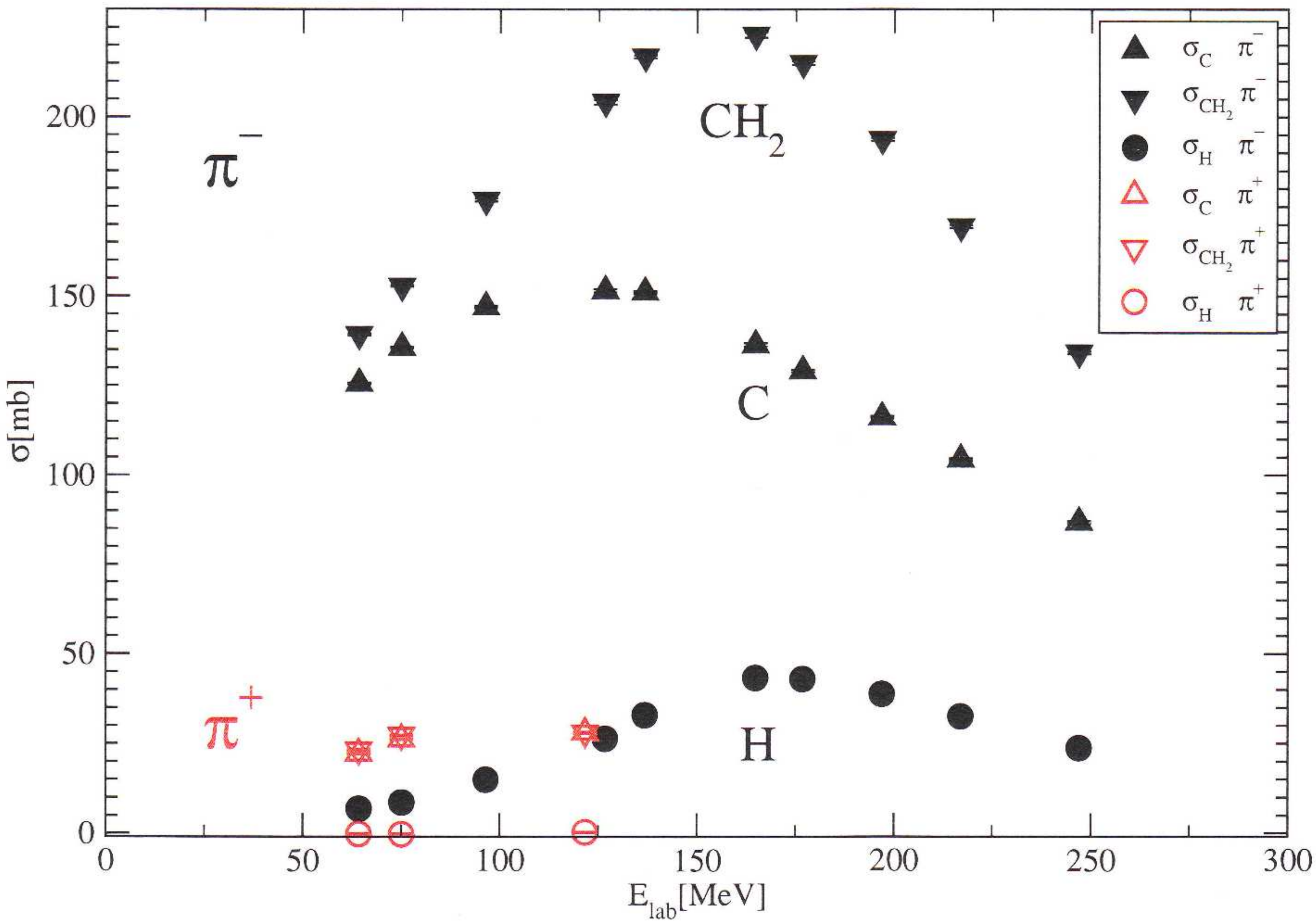
- piE3 and piM1 beamlines at PSI, own energy calibrations.
- Solid targets, CH_2 - C difference method.
  Same targets as in the TRIUMF experiment. Three independent chemical analyses. Energy-loss matched targets.
- For the first time event by event detection.

In a transmission measurement where $T = e^{-\alpha\sigma}$

the statistical error is

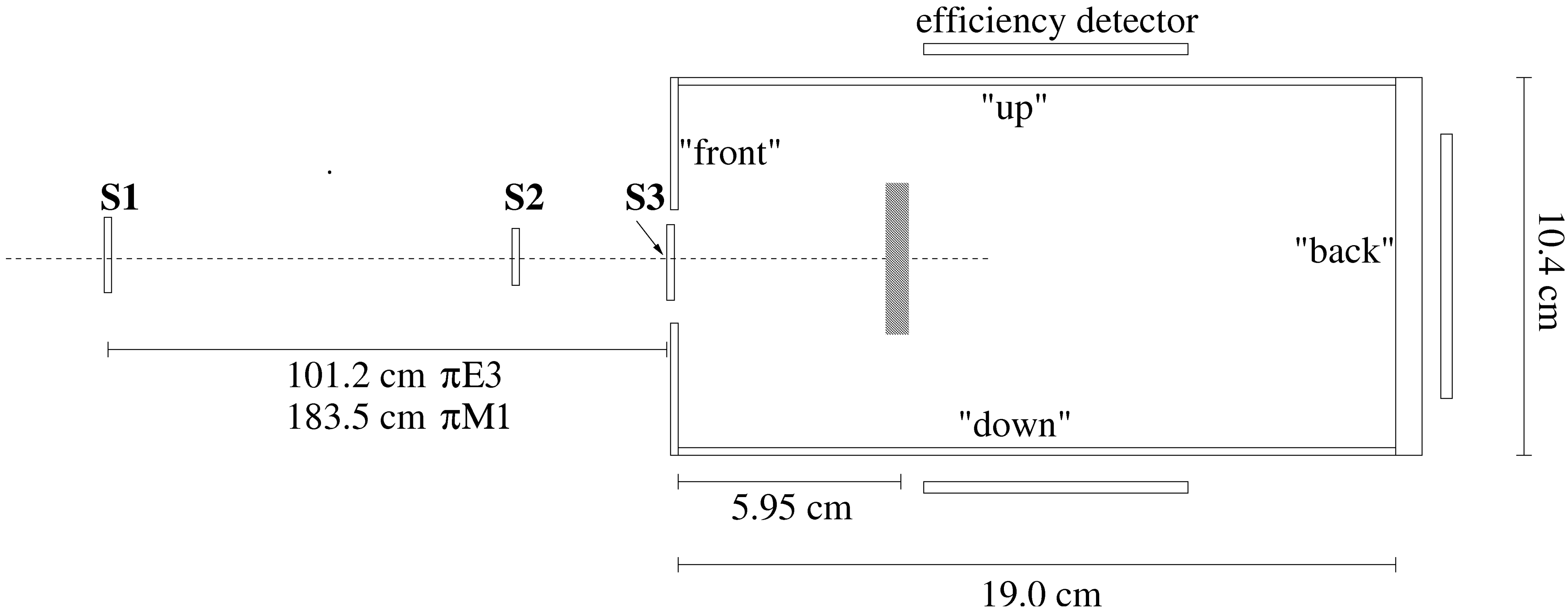
$$\frac{\Delta\sigma}{\sigma} = \frac{1}{\sqrt{N}} \left| \frac{\sqrt{1-T}}{T log T} \right|.$$

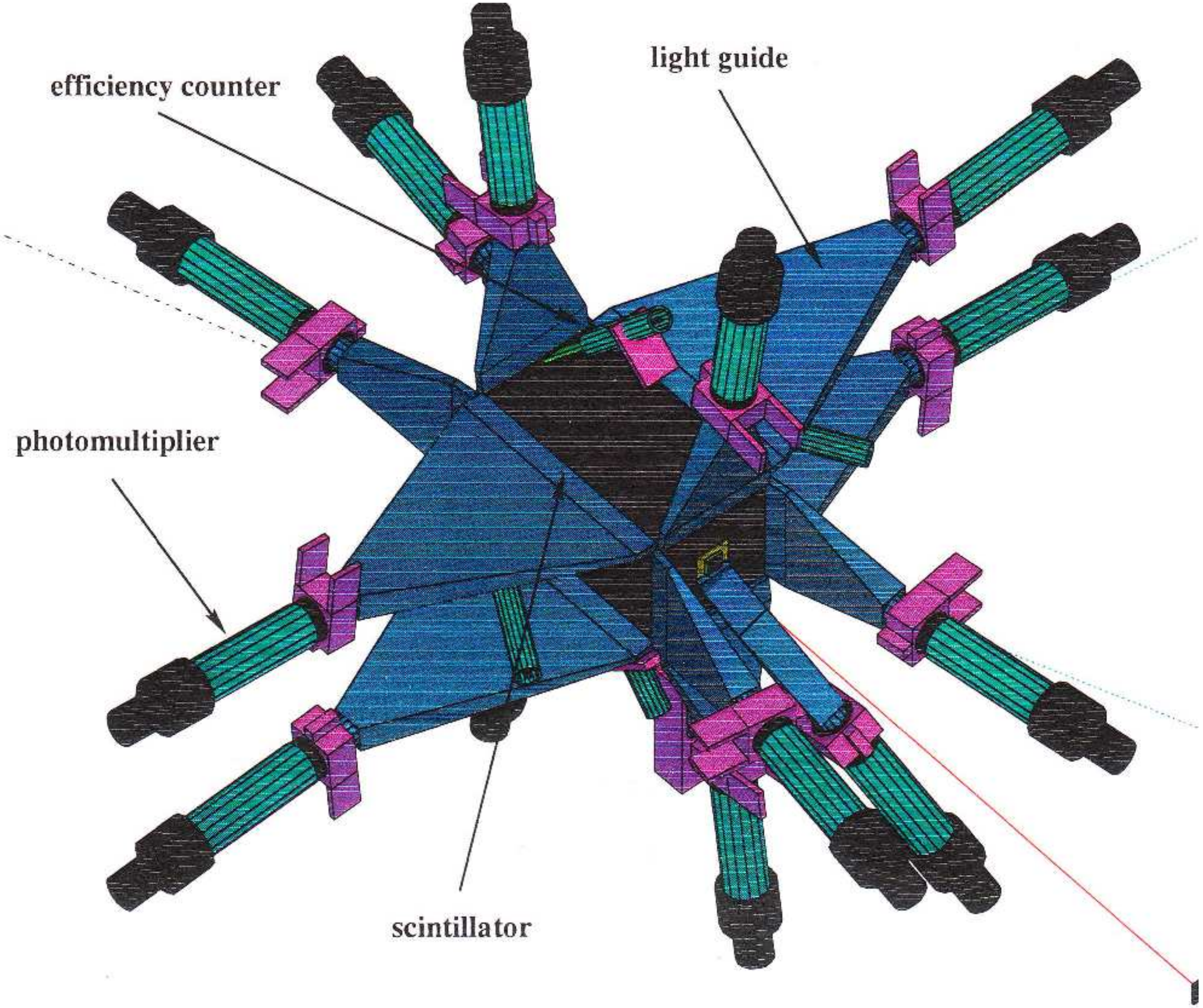
Then for 0.5% error when T=0.995 one needs $N \simeq 10^7$ counts per point, in addition to background measurements.



σC π−
σCH2 π−
σH π−
σC π+
σCH2 π+
σH π+
π−
π+
CH2
C
H
σ[mb]
Elab[MeV]
0
50
100
150
200
250
300

The 4π detector





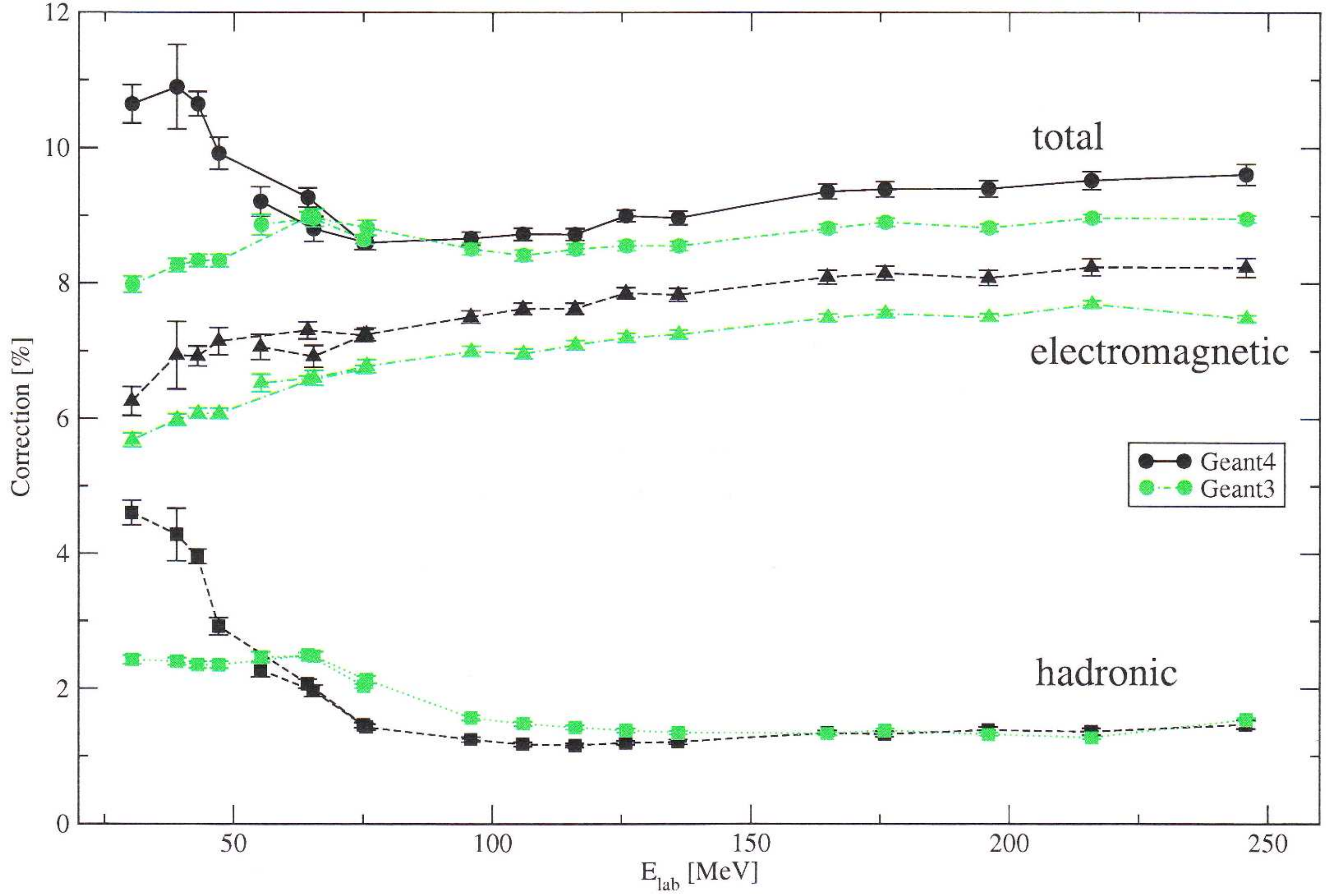
efficiency counter
light guide
photomultiplier
scintillator

Corrections to $\pi^- p \to \pi^0 n$ total cross section

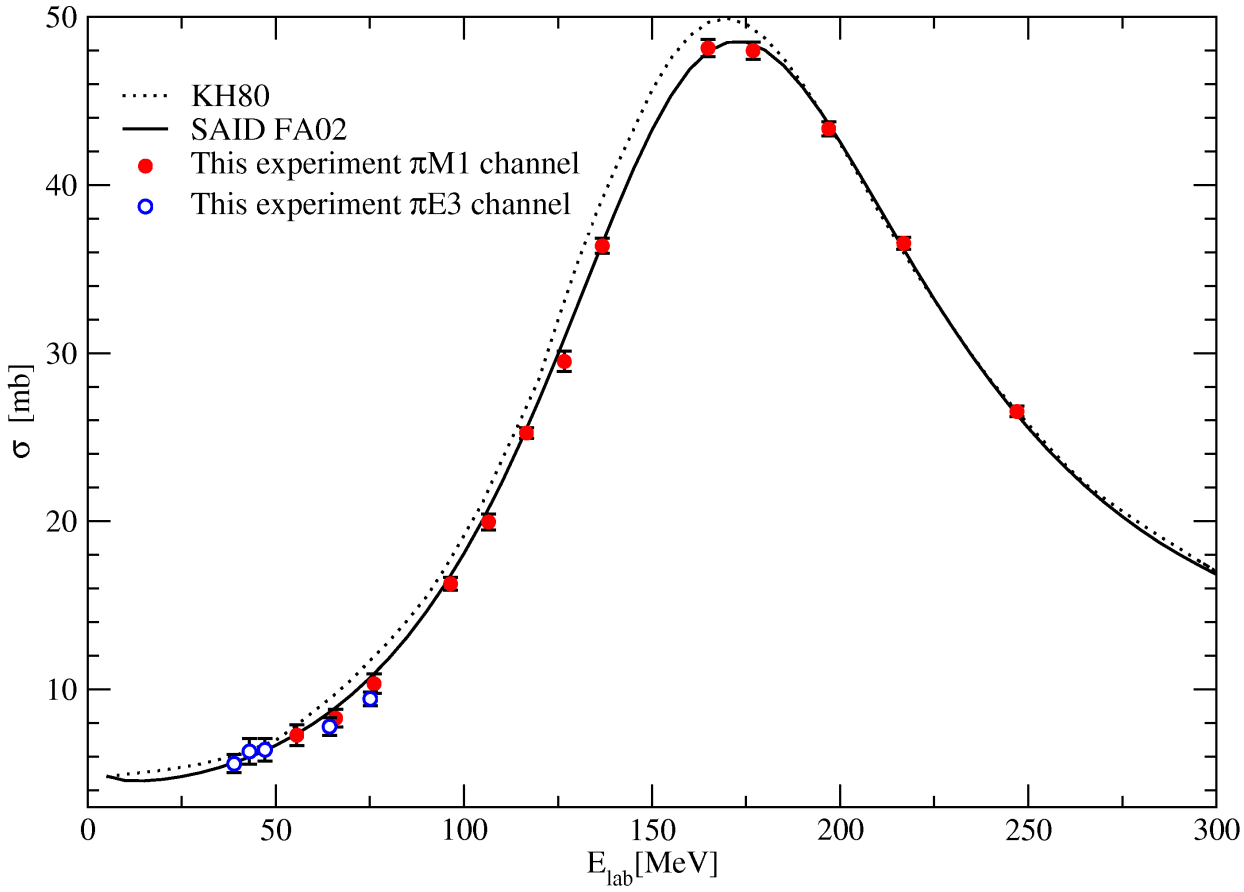
- Muon contamination of the pion beam
- detection of neutrals (gammas and neutrons)
- Dalitz pairs ($\pi^0 \to e^+ e^- \gamma$)
- π^- capture ($\pi^- p \to \gamma n$)

'Zero tests' with π^+, μ^- and e^-.

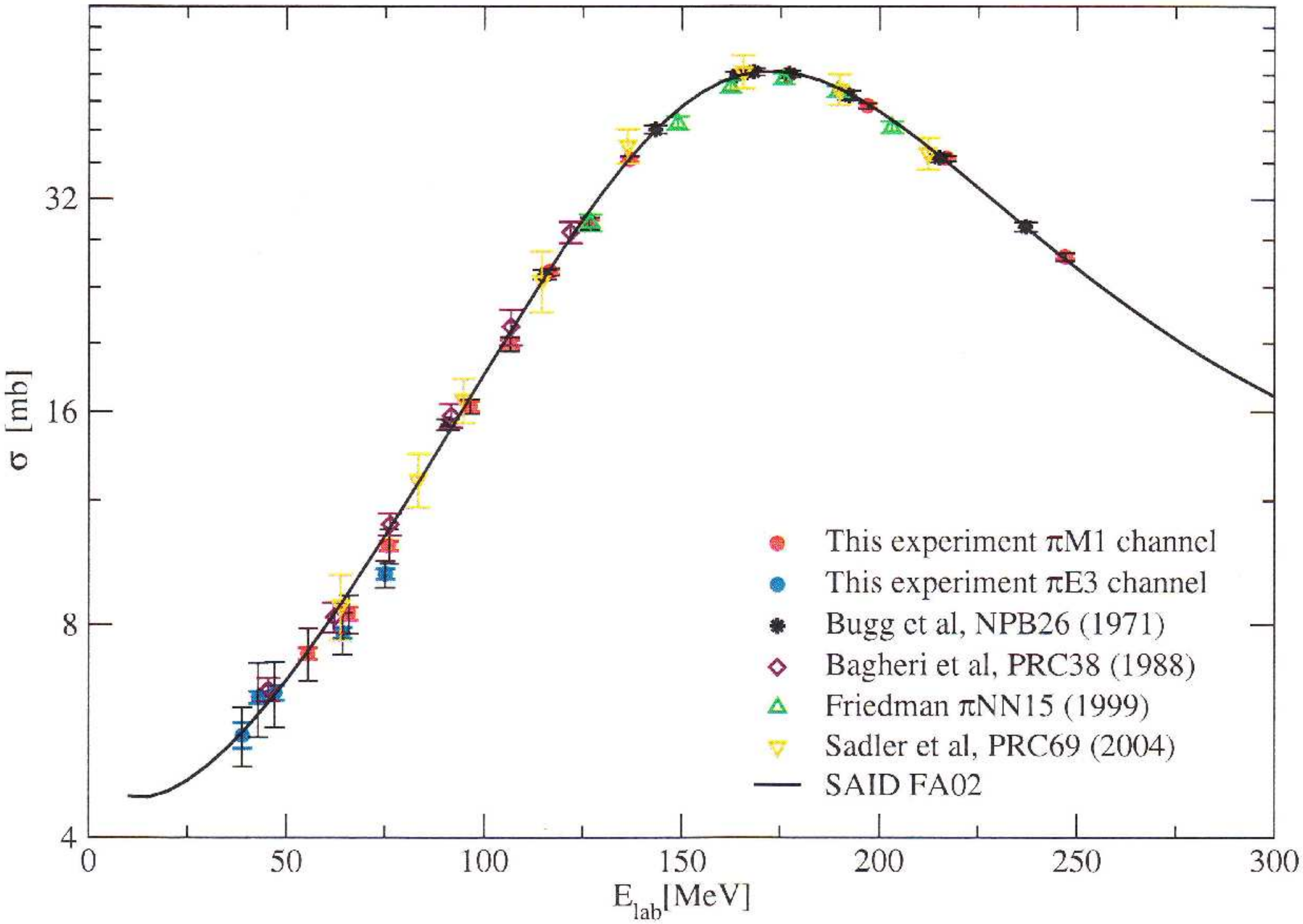
Detection of neutrals

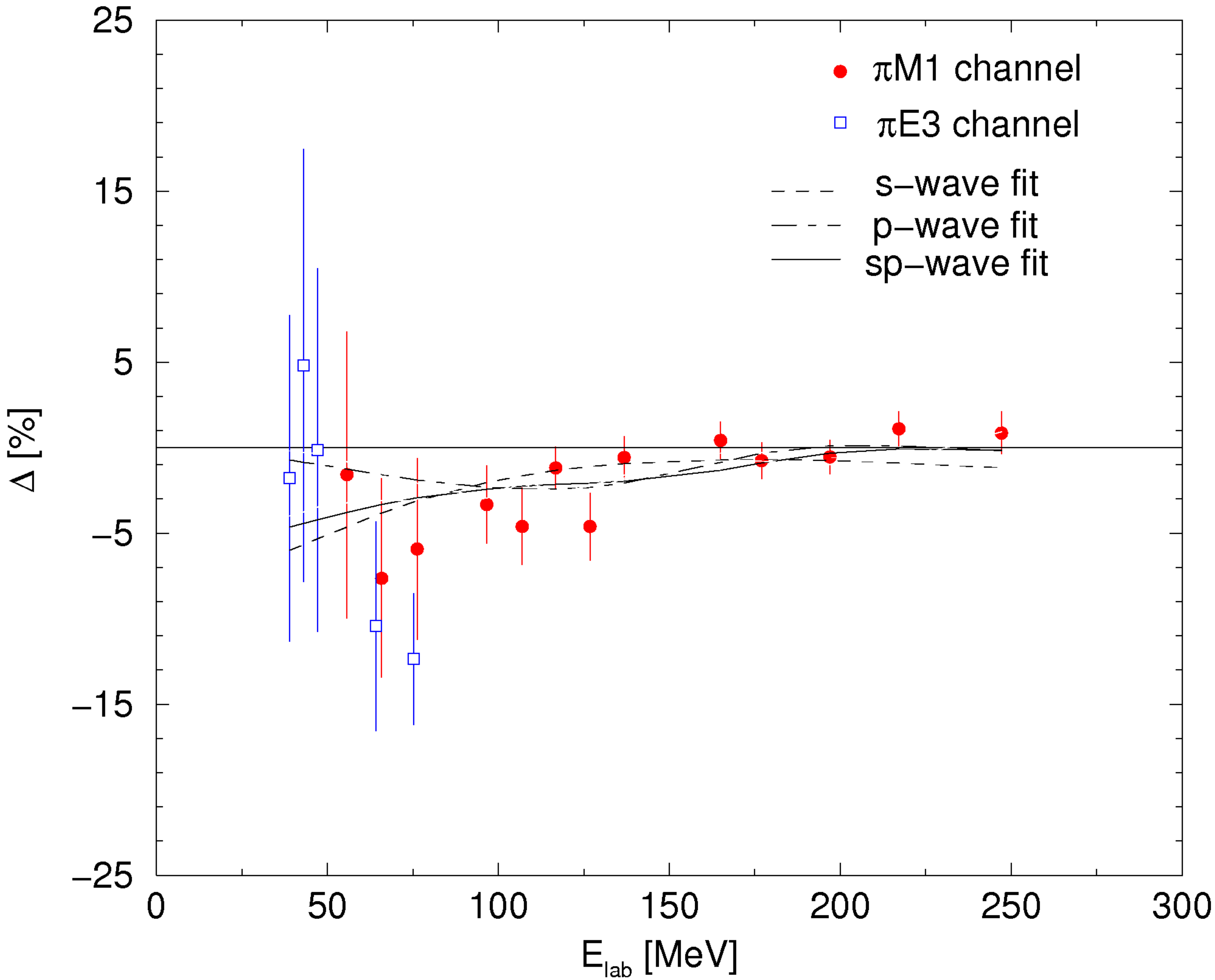


Total cross section for $\pi^- p \to \pi^0 n$



Comparisons with earlier results





πM1 channel
πE3 channel
s–wave fit
p–wave fit
sp–wave fit
25
15
5
–5
–15
–25
Δ [%]
0
50
100
150
200
250
300
E$_{lab}$ [MeV]

A ‘base-line’ expression for total CX cross sections, used in attempts to improve the agreement between experiment and the ‘SAID’-based calculations, uses s- and p- waves as follows:

$$BL = \frac{2}{9}\frac{4\pi}{k_{in}^2}\frac{k_{out}}{k_{in}}[|S_{11} - S_{31}|^2 + |P_{11} - P_{31}|^2 + 2|P_{13} - P_{33}|^2]$$

where S and P are the s- and p- wave amplitudes, respectively, from the SAID analysis. Wave numbers k are in the c.m. system. Indices are $2t$ $2j$.

Modifications to the s-wave term:

$|S_{11} - S_{31}| \rightarrow f \; |S_{11} - S_{31}|$.

Recall that $S_{kl} = \frac{1}{2i}(e^{2i\delta_{kl}} - 1)$.

Then $|S_{11} - S_{31}|^2 = sin^2(\delta_{11} - \delta_{31})$.

At low energies $|\delta_{11} - \delta_{31}| \leq 10^\circ$,

hence $f \simeq 1$ is acceptable.

In contrast, $|P_{13} - P_{33}| \simeq 1$ near 180 MeV. Consequently another approach is needed for modifying the p-wave terms.

(I) Establish that $|P_{33}|^2(W)$ can be described by a Breit-Wigner expression,

$BW(W_0, \Gamma_0, W)=$

$$= \frac{8\pi}{k^2} \frac{(\Gamma/2)^2}{(W - W_0)^2 + (\Gamma/2)^2} \simeq \frac{8\pi}{k^2} \frac{W_0^2 \Gamma^2}{(W_0^2 - W^2)^2 + W_0^2 \Gamma^2}.$$

W the total cm energy and with energy-dependent width:

$$\Gamma = \Gamma_0 (\frac{k}{k_0})^3 \frac{1 + k_0^2 R^2}{1 + k^2 R^2}.$$

R=1.15 fm, the 'interaction radius' (Pedroni *et al.* 1978).

For example, a fit between 105 and 245 MeV in 5 MeV steps and assuming 3% errors yields

$W_0 = 1231.1 \pm 0.5$ MeV; $\Gamma_0 = 113.1 \pm 1.3$ MeV,

in agreement with the 'mixed charges' $\Delta(1232)$ values due to the Particle Data Group.

As another test fit BW to $(2/9)|P_{33}|^2$ at the 18 energies of the present work and with the same errors.

$W_0 = 1230.8 \pm 0.4$ MeV; $\Gamma_0 = 113.3 \pm 1.1$ MeV.

Then modify $P_{33}(W) \to r(W)P_{33}(W)$

with $r(W) = [BW(W', \Gamma', W)/BW(W_0, \Gamma_0, W)]^{1/2}$.

This way the P_{33} phases are kept.

Get the parameters f, W' and Γ' (all or some) from χ^2 fits to the total CX cross section using the following:

$$\sigma_T(E) = \sigma_{SAID}(E) - (8\pi k_{out}/9k_{in}^3)[(1-f^2)|S_{11} - S_{31}|^2 + 2|P_{13} - P_{33}|^2 - 2|P_{13} - r(E)P_{33}|^2].$$

The small Coulomb and higher partial waves corrections which are included in SAID are therefore maintained.

(II) An alternative approach:

replace $|P_{13} - P_{33}|^2$ by a BW on account of the phase of P_{13} being very small ($\simeq -2.4°$).

As a check, fit to $|P_{13} - P_{33}|^2$ finds W''_0 too low by 3.3 MeV compared to expected 2.5 MeV from the above angle. Find $\Gamma'' = 113.2 \pm 1.3$ MeV.

Then fit f, W'' and Γ'' to our data

$$\sigma_T(E) = \sigma_{SAID}(E) - (8\pi k_{out}/9k^3_{in})[(1 - f^2)|S_{11} - S_{31}|^2$$
$$+ 2|P_{13} - P_{33}|^2 - 2BW(E, W'', \Gamma'')]$$

and correct W'' by the above 2.5 or 3.3 MeV.

(III)Replace P_{33} by

$$P'_{33} = \frac{\Gamma'/2}{W' - W - i\Gamma'/2}$$

and get the parameters f, W' and Γ' (all or some) from χ^2 fits to the total CX cross section using the following:

$$\sigma_T(E) = \sigma_{SAID}(E) - (8\pi k_{out}/9k_{in}^3)[(1 - f^2)|S_{11} - S_{31}|^2$$
$$+ 2|P_{13} - P_{33}|^2 - 2|P_{13} - P'_{33}(E)|^2].$$

Results of fits to the present data

model	χ^2 (18 points)	f	$W(\Delta^0)$(MeV)	$\Gamma(\Delta^0$(MeV)
SAID	32.6	–	–	–
s-wave	22.4	0.96±0.02	–	–
I	19.0	0.97±0.03	1231.7±0.6	112.9±2.0
I	20.6	–	1231.5±0.5	111.3±1.2
II	14.7	0.96±0.03	1231.4±0.5	113.2±1.7
II	16.9	–	1231.2±0.4	111.0±1.0
III	26.9	0.95±0.04	1231.6±0.6	114.7±2.4
III	31.4	–	1231.3±0.6	111.7±1.4

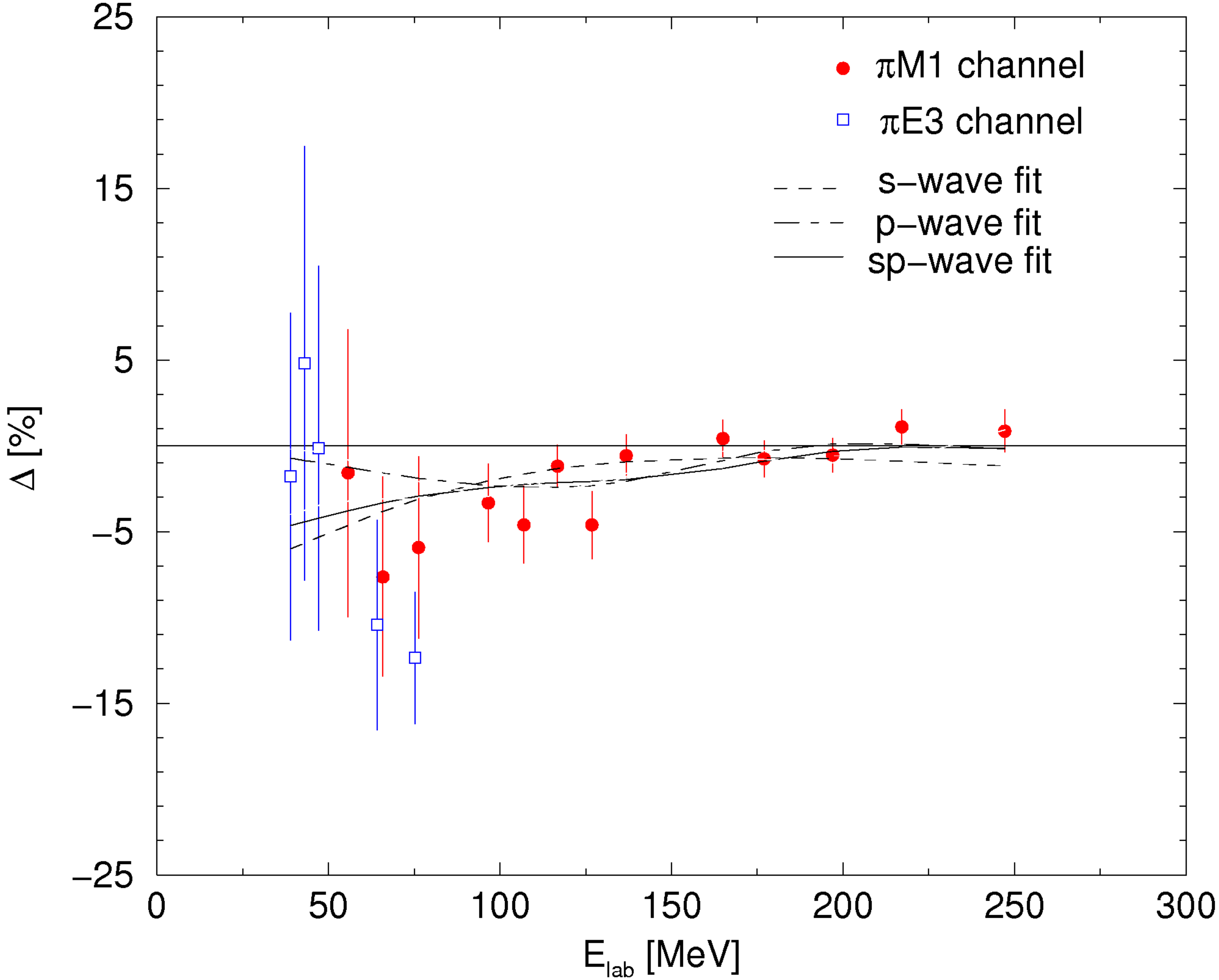
$\Delta f = -4\%$ means a $0.5\pm0.2^\circ$ smaller phase difference.

Note coupling between s- and p- wave terms.

Comparisons of derived BW parameters

model	χ^2 (18 points)	f	$W(\Delta^0)$(MeV)	$\Gamma(\Delta^0)$(MeV)
I	19.0	0.97±0.03	1231.7±0.6	112.9±2.0
II	14.7	0.96±0.03	1231.4±0.5	113.2±1.7
III	26.9	0.95±0.04	1231.6±0.6	114.7±2.4

$|P_{33}|^2$ fit: W_0 =1230.8±0.4 MeV, Γ_0=113.3±1.1 MeV.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300
E_lab [MeV]

Δ(1232) BREIT-WIGNER MASSES

MIXED CHARGES

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
1230 to 1234 (≈ 1232) OUR ESTIMATE				
1231±1	MANLEY	92	IPWA	$\pi N \to \pi N$ & $N\pi\pi$
1232±3	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
1233±2	HOEHLER	79	IPWA	$\pi N \to \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1228±1	PENNER	02C	DPWA	Multichannel
1234±5	VRANA	00	DPWA	Multichannel
1233	ARNDT	95	DPWA	$\pi N \to N\pi$

Δ(1232) BREIT-WIGNER WIDTHS

MIXED CHARGES

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
115 to 125 (≈ 120) OUR ESTIMATE				
118± 4	MANLEY	92	IPWA	$\pi N \to \pi N$ & $N\pi\pi$
120± 5	CUTKOSKY	80	IPWA	$\pi N \to \pi N$
116± 5	HOEHLER	79	IPWA	$\pi N \to \pi N$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
106± 1	PENNER	02C	DPWA	Multichannel
112±18	VRANA	00	DPWA	Multichannel
114	ARNDT	95	DPWA	$\pi N \to N\pi$

$\Delta(1232)^{++}$ WIDTH

VALUE (MeV)	DOCUMENT ID		TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •				
109.07±0.48	BERNICHA	96		Fit to PEDRONI 78
111.0 ±1.0	KOCH	80B	IPWA	$\pi N \to \pi N$
111.3 ±0.5	PEDRONI	78		$\pi N \to \pi N$ 70–370 MeV

Summary

- Precision measurements of total $\pi^- p \rightarrow \pi^0 n$ cross sections between 40-250 MeV in a *single* experiment.

- Phenomenological analysis suggest very similar $\Gamma(\Delta^0)$ and $\Gamma(\Delta(charged))$.

- Data available at Phys. Lett. B **639** (2006) 424-428 to include in proper PSA.