On the definition of physical properties of unstable particles

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Outline

1. Introduction

2. Toy model

3. Charge and magnetic moment of an unstable particle

4. Summary

Definition of the mass of an unstable particle as zero of the real part of the inverse propagator $\Longrightarrow$ field-redefinition and gauge-parameter dependence starting at two-loop order.

Defining the mass and width in terms of the complex pole of the propagator $\Longrightarrow$ field-redefinition and gauge-parameter independence.

S. Willenbrock and G. Valencia, Phys. Lett. B **247**, 341 (1990), Phys. Rev. D **42**, 853 (1990), Phys. Lett. B **259**, 373 (1991).

A. Sirlin, Phys. Rev. Lett. **67**, 2127 (1991), Phys. Lett. B **267**, 240 (1991).

J. Gegelia, G. Japaridze, A. Tkabladze, A. Khelashvili and K. Turashvili, in *Quarks '92, Proceedings of the International Seminar on Quarks*, Zvenigorod, Russia, 1992.

P. Gambino and P. A. Grassi, Phys. Rev. D **62**, 076002 (2000).

With the progress of lattice QCD and low-energy EFT of QCD definition of masses of baryon resonances becomes important.

As discussed in:

G. Höhler, *Against Breit-Wigner parameters — a pole-emic*, in C. Caso *et al.* [Particle Data Group], Eur. Phys. J. C **3**, 624 (1998).

conventional resonance mass and width determined from generalized Breit-Wigner formulas have problems regarding their relation to S-matrix theory and suffer from a strong model dependence.

In addition, these quantities depend on the field-redefinition parameter in a low-energy EFT of QCD.

D. Djukanovic, J. Gegelia and S. Scherer, Phys. Rev. D **76**, 037501 (2007).

S. Capstick *et al.*, Eur. Phys. J. A **35**, 253 (2008).

Toy model

Toy model Lagrangian respecting Lorentz invariance and P and C symmetries

$$
\begin{aligned}
\mathcal{L}_0 &= \bar\psi\,(i\not{D} - M_N)\psi + \bar\Psi(i\not{D} - M_R)\Psi \\
&\quad -\frac{1}{4}\,F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi - \frac{M^2}{2}\,\phi^2 \\
&\quad +g\,\phi\left(\bar\Psi\,\gamma^5\psi + \bar\psi\,\gamma^5\Psi\right) \\
&\quad +i\,\kappa\,F_{\mu\nu}\,\bar\Psi\,\sigma^{\mu\nu}\Psi + \cdots\,, \qquad (1) \\
F_{\alpha\beta} &= \partial_\alpha A_\beta - \partial_\beta A_\alpha\,, \\
D_\mu &= \partial_\mu - ieA_\mu\,,
\end{aligned}
$$

Ψ - heavy fermion, ψ - light fermion, ϕ - light pseudoscalar and A_μ - photon field. Dots stand for an infinite number of interaction terms.

Field transformation

$$\Psi \to \Psi + \xi\,\phi\,\gamma^5\psi\,, \quad \bar{\Psi} \to \bar{\Psi} + \xi\,\phi\,\bar{\psi}\,\gamma^5, \qquad (2)$$

ξ - an arbitrary parameter.

Physical quantities cannot depend on ξ.

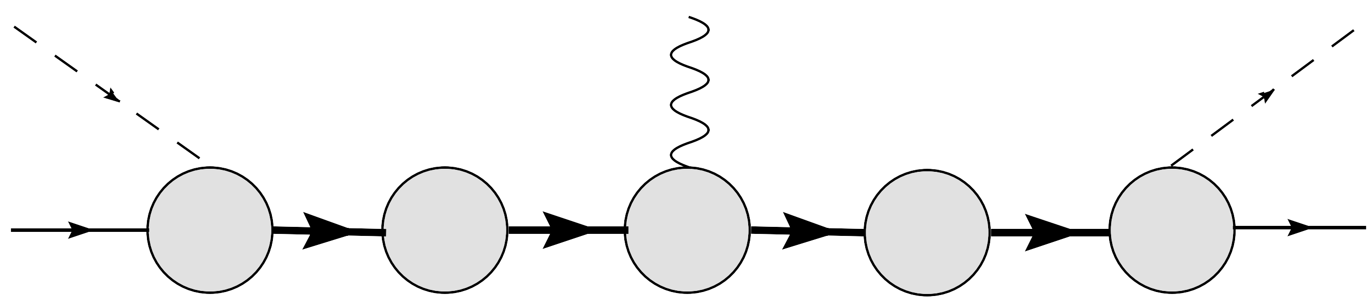
New interaction terms linear in ξ:

$$\begin{aligned}
\mathcal{L}_{\text{new}} \;=\; & -\xi\,M_R\,\phi\left(\bar{\psi}\,\gamma^5\Psi + \bar{\Psi}\,\gamma^5\psi\right) \\
& +\xi e\phi A_\mu\left(\bar{\psi}\,\gamma^5\gamma^\mu\Psi + \bar{\Psi}\gamma^\mu\,\gamma^5\psi\right) \\
& +i\,\kappa\,\xi\,F_{\mu\nu}\,\phi\left(\bar{\psi}\,\gamma^5\,\sigma^{\mu\nu}\Psi + \bar{\Psi}\,\sigma^{\mu\nu}\,\gamma^5\psi\right) \\
& +i\xi\phi\left(\bar{\psi}\,\gamma^5\gamma^\mu\partial_\mu\Psi + \bar{\Psi}\gamma^\mu\,\gamma^5\partial_\mu\psi\right) \\
& +i\xi\partial_\mu\phi\bar{\Psi}\gamma^\mu\,\gamma^5\psi + 2\,\xi\,g\,\phi^2\bar{\psi}\,\gamma^5\psi\,. \qquad (3)
\end{aligned}$$

Magnetic moment of the heavy fermion

No unstable "particle" in asymptotic states:

Consider the process $\phi\,\psi \to \gamma\,\phi\,\psi$ for $s \sim M_R^2$



Resonant part of the amplitude

$$A_{\mathbf{r}} \;=\; V_1(p_f,k')iS_R(p_f)\Gamma^\mu(p_f,p_i)iS_R(p_i)V_2(p_i,k)\,.$$

Dressed propagator of the heavy fermion

$$iS_R(p) = \frac{i}{\not{p} - M_R - \Sigma(\not{p})}\,, \qquad (4)$$

with $\Sigma(\not{p})$ self-energy.

S_R has a complex pole which is obtained from

$$z - M_R - \Sigma(z) = 0\,. \tag{5}$$

We define the pole mass as the real part of z.

In vicinity of the pole:

$$S_R(p) = \frac{Z}{\not{p} - z} + \text{n.p.} = \frac{Z\,(\not{p} + z)}{p^2 - z^2} + \text{n.p.}\,, \tag{6}$$

where the residue Z is given by

$$Z = 1 + \delta Z = 1/[1 - \Sigma'(z)]\,. \tag{7}$$

Introduce Dirac-spinors with complex masses

$$
\begin{aligned}
w^1(p) &= \begin{pmatrix} \sqrt{z+p_0} \\ 0 \\ \frac{p_3}{\sqrt{z+p_0}} \\ \frac{p_1+ip_2}{\sqrt{z+p_0}} \end{pmatrix}, \\
\bar{w}^1(p) &= \left(\sqrt{z+p_0}\,, \;\; 0\,, \;\; \frac{-p_3}{\sqrt{z+p_0}}\,, \;\; -\frac{p_1-ip_2}{\sqrt{z+p_0}} \right), \\
w^2(p) &= \begin{pmatrix} 0 \\ \sqrt{z+p_0} \\ \frac{p_1-ip_2}{\sqrt{z+p_0}} \\ \frac{-p_3}{\sqrt{z+p_0}} \end{pmatrix}, \\
\bar{w}^2(p) &= \left(0\,, \;\; \sqrt{z+p_0}\,, \;\; -\frac{p_1+ip_2}{\sqrt{z+p_0}}\,, \;\; \frac{p_3}{\sqrt{z+p_0}} \right),
\end{aligned}
\tag{8}
$$

which satisfy the following Dirac equations:

$$
\begin{aligned}
(\not{p} - z)\, w^i &= 0\,, \\
\bar{w}^i\, (\not{p} - z) &= 0\,.
\end{aligned}
\tag{9}
$$

Dirac-spinors satisfy the identity

$$w^i(p)\,\bar{w}^i(p) = \not{p} + z\,.$$

Substitute dressed propagator

$$S_R(p) = \frac{Z\,w^i(p)\,\bar{w}^i(p)}{p^2 - z^2} + \text{n.p.}$$

in the resonant amplitude and decompose in pole and non-pole parts

$$\begin{aligned} A_{\mathbf{r}} \;=\; & V_1(p_f,k')w^i(p_f)\sqrt{Z}\frac{i}{p_f^2 - z^2} \\ & \times\sqrt{Z}\,\bar{w}^i(p_f)\Gamma^\mu(p_f,p_i)w^j(p_i)\sqrt{Z} \\ & \times\frac{i}{p_i^2 - z^2}\sqrt{Z}\bar{w}^j(p_i)V_2(p_i,k) + \text{n.p.}\,. \end{aligned}$$

Parameterize the renormalized vertex function in terms of form factors

$$\begin{aligned} V^\mu(q^2) &= \sqrt{Z}\,\bar{w}^i(p_f)\Gamma^\mu(p_f,p_i)w^j(p_i)\sqrt{Z} = \\ &= i\,F_1(q^2)\,\bar{w}^i(p_f)\gamma_\mu w^j(p_i) \\ &+ i\,F_2(q^2)\,q^\nu\,\bar{w}^i(p_f)\,\sigma^{\mu\nu}\,w^j(p_i)\,, \end{aligned} \tag{10}$$

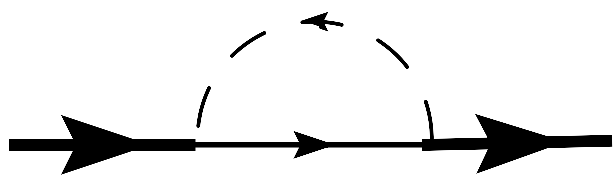
where

$$\sigma^{\mu\nu} = \frac{1}{2}\left(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu\right)\,.$$

Calculate $F_1(0)$ and $F_2(0)$ to one loop order:

Use dimensional regularization with $\overline{MS}$ scheme.

One-loop order self-energy of heavy fermion:



Residue of the propagator:

$$
\begin{aligned}
\delta Z \;=\; & -\frac{g^2 A_0\left(M^2\right)\left(3M_N^2+3M_R^2+2M_NM_R-3M^2\right)}{32\pi^2M_R^2\left(-M^2+M_N^2+M_R^2+2M_NM_R\right)} \\
& +\frac{g^2}{32\pi^2M_R^2\left(-M^2+M_N^2+M_R^2+2M_NM_R\right)} \\
& \times B_0\left(M_R^2,M^2,M_N^2\right)\{-3M^4+2M_R^2M^2-3M_N^4 \\
& +M_R^4-2M_N^3M_R+2M_NM_R\left(M^2+M_R^2\right) \\
& +2M_N^2\left(3M^2+M_R^2\right)\} \\
& +\frac{g^2\left(M^2+M_N^2-M_R^2\right)}{16\pi^2\left(-M^2+M_N^2+M_R^2+2M_NM_R\right)} \\
& -\frac{g^2\,A_0\left(M_N^2\right)\left(3M^2-3M_N^2+M_R^2-2M_NM_R\right)}{32\pi^2M_R^2\left(-M^2+M_N^2+M_R^2+2M_NM_R\right)}
\end{aligned}
$$

$$+\frac{\xi\, g}{16\pi^2 M_R}\{A_0\left(M^2\right) - A_0\left(M_N^2\right)$$

$$+B_0\left(M_R^2, M^2, M_N^2\right)[-M^2 + M_N^2$$

$$+M_R^2 - 2M_N M_R]\}\,. \qquad (11)$$

Loop functions:

$$A_0(m^2) = \frac{(2\pi)^{4-n}}{i\,\pi^2}\int \frac{d^n k}{k^2 - m^2 + i\,0^+}\,,$$

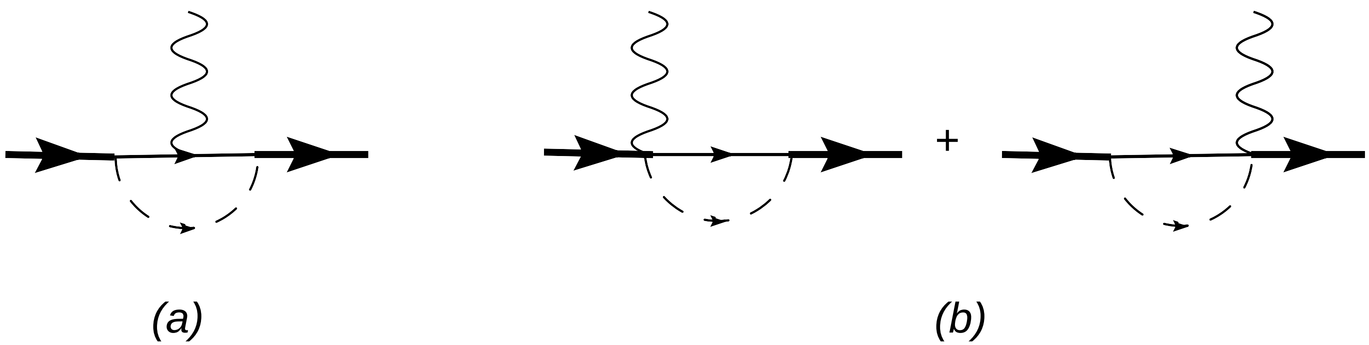
$$B_0(p^2, m_1^2, m_2^2) = \frac{(2\pi)^{4-n}}{i\,\pi^2}\times$$

$$\int \frac{d^n k}{\left[k^2 - m_1^2 + i\,0^+\right]\left[(p+k)^2 - m_2^2 + i\,0^+\right]}\,.$$

Tree order Vertex:

$$D^{\text{tree}} = i\left[\gamma^{\mu}\left(e + 4\kappa M_H\right) - 2\kappa\left(p_f^{\mu} + p_i^{\mu}\right)\right]. \quad (12)$$

One-loop order vertex diagrams:



The results of the diagrams (a) and (b) for $q^2 = 0$:

$$
\begin{aligned}
D^{(a)} \;=\;& -\frac{ie\left(p_i^{\mu}+p_f^{\mu}\right)g^2}{32\pi^2 M_N M_R^3\left(-M^2+M_N^2+M_R^2+2M_N M_R\right)} \\
& \times\{M_N\left(M^2+M_N^2+M_R^2+2M_N M_R\right)M_R^2 \\
& -2A_0\left(M^2\right)M_N\left(-M^2+M_N^2+M_N M_R\right) \\
& -2A_0\left(M_N^2\right)[-M_N^3-M_R M_N^2 \\
& +\left(M^2+M_R^2\right)M_N+M_R^3] \\
& -2B_0\left(M_R^2,M^2,M_N^2\right)M_N[M^4+M_N^4+M_N^3 M_R \\
& -M_N M_R\left(M^2+M_R^2\right)-M_N^2\left(2M^2+M_R^2\right)]\} \\
& -\frac{ie\,\gamma^{\mu}g^2}{32\pi^2 M_N M_R^3\left(-M^2+M_N^2+M_R^2+2M_N M_R\right)} \\
& \times\{-4M_N\left(M_N+M_R\right)M_R^4- \\
& A_0\left(M^2\right)M_N\left(M^2-M_N^2+3M_R^2-2M_N M_R\right)M_R \\
& +B_0\left(M_R^2,M^2,M_N^2\right)M_N[M_N^4+2M_R M_N^3 \\
& -2\left(M^2+M_R^2\right)M_N^2-2M_R\left(M^2+M_R^2\right)M_N
\end{aligned}
$$

$$
\begin{aligned}
&+B_0\left(M_R^2, M^2, M_N^2\right)[-M^2 + M_N^2 \\
&+M_R^2 - 2M_N M_R]\} \\
&-\frac{ig\xi\gamma^\mu(e + 4\kappa M_R)}{16\pi^2 M_R}\{A_0\left(M^2\right) - A_0\left(M_N^2\right) \\
&+B_0\left(M_R^2, M^2, M_N^2\right)[-M^2 + M_N^2 \\
&+M_R^2 - 2M_N M_R]\}.
\end{aligned} \tag{13}
$$

Form factors to one loop:

$$
\begin{aligned}
F_1(0) &= e\,, \\
F_2(0) &= -2\kappa + \frac{A_0\left(M^2\right)g^2}{16\pi^2 M_R^3\left(-M^2 + M_N^2 + M_R^2 + 2M_N M_R\right)} \\
&\quad \times [3\kappa M_R^3 - 3M^2\kappa M_R + M_N(e + 2\kappa M_R)M_R \\
&\quad -e\,M^2 + M_N^2(e + 3\kappa M_R)]
\end{aligned}
$$

$$
-\frac{g^2}{32\pi^2 M_R \left(-M^2+M_N^2+M_R^2+2M_N M_R\right)}
$$

$$
\times [-4\kappa M_R^3 + e\, M_R^2 + 4M^2\kappa M_R + 2e\, M_N M_R
$$

$$
+e\, M^2 + M_N^2\,(e\, + 4\kappa M_R)]
$$

$$
+\frac{A_0\left(M_N^2\right) g^2}{16\pi^2 M_N M_R^3 \left(-M^2+M_N^2+M_R^2+2M_N M_R\right)}
$$

$$
\times [-\,(e\, + 3\kappa M_R)\, M_N^3 - M_R\,(e\, + 2\kappa M_R)\, M_N^2
$$

$$
+\left(\kappa M_R^3 + eM_R^2 + 3M^2\kappa M_R + eM^2\right) M_N + eM_R^3]
$$

$$
-\frac{g^2 B_0\left(M_R^2, M^2, M_N^2\right)}{16\pi^2 M_R^3 \left(-M^2+M_N^2+M_R^2+2M_N M_R\right)}
$$

$$\times\,[\kappa M_R^5 + 2M^2\kappa M_R^3 - 3M^4\kappa M_R$$
$$-M_N^3\,(e + 2\kappa M_R)\,M_R + M_N\,(e + 2\kappa M_R)$$
$$\times\,\left(M^2 + M_R^2\right)M_R - e\,M^4 - M_N^4\,(e + 3\kappa M_R)$$
$$+M_N^2\left(2\kappa M_R^3 + e\,M_R^2 + 6M^2\kappa M_R + 2e\,M^2\right)] \tag{1.4}$$

1. Charge does not get renormalized and the magnetic moment does not depend on ξ.

2. In both quantities the ξ-dependent part of the residue of the propagator exactly cancels the ξ-dependent parts of the loop vertex diagrams.

3. The latter also contain imaginary parts.

4. Any definition which uses the real quantity as the wave function renormalization constant necessarily leads to ξ-dependence.

For example, the mass M_R^R defined from

$$M_R^R - M_R - \mathrm{Re}\,\Sigma(M_R^R) = 0 \qquad (15)$$

depends on field redefinition parameter at two-loop order.

Below we encounter similar problems already at one-loop order.

Close to $\not p \sim M_R^R$ the dressed propagator can be written as

$$
\begin{aligned}
iS_R(p) &= \frac{i}{\left(\not p - M_R^R\right)\left[1 - \Sigma'(M_R^R)\right] - i\,\mathrm{Im}\Sigma(M_R^R)} \\
&= \frac{iZ_R}{\left(\not p - M_R^R\right)\left[1 - iZ_R\mathrm{Im}\Sigma'(M_R^R)\right] - i\,Z_R\mathrm{Im}\Sigma(M_R^R)} \\
Z_R &= 1/[1 - \mathrm{Re}\Sigma'(M_R^R)]\,. \qquad (16)
\end{aligned}
$$

Up to one-loop accuracy:

$$
\begin{aligned}
iS_R(p) &= \frac{iZ_R}{\left(\not p - M_R^R\right)\left[1 - i\,\mathrm{Im}\Sigma'(M_R^R)\right] - i\,\mathrm{Im}\Sigma(M_R^R)} \\
Z_R &= 1 + \mathrm{Re}\Sigma'(M_R^R)\,, \qquad (17)
\end{aligned}
$$

which has the characteristic Breit-Wigner form, with energy dependent width and real wave function renormalization constant.

Using the Dirac spinors with the mass M_R^R, putting the external legs of vertex functions "on-mass-shell", i.e. $\not{p} = M_R^R$ and taking Z_R as the wave function renormalization constant:

$$
\begin{aligned}
F_1(0) \;=\; & e + \frac{e\, g^2\, i\, \mathrm{Im}\left[B_0\left(M_R^2, M^2, M_N^2\right)\right]}{32\pi^2 M_R^2\left(-M^2 + M_N^2 + M_R^2 + 2M_N M_R\right)} \\
& \times\{3M_N^4 + 2M_R M_N^3 - 6M^2 M_N^2 \\
& -2M_R^2 M_N^2 - 2M_R\left(M^2 + M_R^2\right) M_N \\
& +3M^2\left(M^2 - M_R^2\right) + M_R^2\left(M^2 - M_R^2\right)\}\,, \\
& + \frac{i\,\xi\, e\, g\, \mathrm{Im}\left[B_0\left(M_R^2, M^2, M_N^2\right)\right]}{32\pi^2 M_R^2\left(-M^2 + M_N^2 + M_R^2 + 2M_N M_R\right)} \\
& \times\{-2M_R M_N^4 + 4M_R^3 M_N^2 + 4M^2 M_R M_N^2 \\
& +2M_R^3\left(M^2 - M_R^2\right) - 2M^2 M_R\left(M^2 - M_R^2\right)\}
\end{aligned}
$$

$$
\begin{aligned}
F_2(0) \;=\; & -2\kappa + \frac{g^2 A_0\left(M^2\right)}{16\pi^2 M_R^3\left(-M^2+M_N^2+M_R^2+2M_N M_R\right)} \\
& \times [3\kappa M_R^3 - 3M^2\kappa M_R + M_N\left(e+2\kappa M_R\right) M_R \\
& - e\,M^2 + M_N^2\left(e+3\kappa M_R\right)] \\
& - \frac{g^2}{32\pi^2 M_R\left(-M^2+M_N^2+M_R^2+2M_N M_R\right)} \\
& \times [-4\kappa M_R^3 + e\,M_R^2 + 4M^2\kappa M_R \\
& + 2e\,M_N M_R + e\,M^2 + M_N^2\left(e+4\kappa M_R\right)] \\
& + \frac{e\,B_0\left(M_R^2, M^2, M_N^2\right) g^2}{16\pi^2 M_R^3\left(-M^2+M_N^2+M_R^2+2M_N M_R\right)} \\
& \times [M^4 + M_N^4 + M_N^3 M_R \\
& - M_N M_R\left(M^2+M_R^2\right) - M_N^2\left(2M^2+M_R^2\right)]
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{\kappa\,\mathrm{Re}\left[B_0\left(M_R^2, M^2, M_N^2\right)\right] g^2}{16\pi^2 M_R^2\left(-M^2+M_N^2+M_R^2+2M_N M_R\right)} \\
&\times[-3M^4+2M_R^2M^2-3M_N^4+M_R^4-2M_N^3M_R \\
&+2M_NM_R\left(M^2+M_R^2\right)+2M_N^2\left(3M^2+M_R^2\right)] \\
&+\frac{g^2A_0\left(M_N^2\right)}{16\pi^2M_NM_R^3\left(-M^2+M_N^2+M_R^2+2M_NM_R\right)} \\
&\times[\left(\kappa M_R^3+eM_R^2+3M^2\kappa M_R+eM^2\right)M_N+e\,M_R^3 \\
&-(e+3\kappa M_R)\,M_N^3-M_R\,(e+2\kappa M_R)\,M_N^2] \\
&+\frac{g\,\kappa\,\xi}{8\pi^2M_R}\,i\,\mathrm{Im}\left[B_0\left(M_R^2, M^2, M_N^2\right)\right] \\
&\times\left(-M^2+M_N^2+M_R^2-2M_NM_R\right)\,. \qquad (18)
\end{aligned}
$$

Within ”on-mass-shell” scheme the charge of an unstable heavy fermion gets ”strong” corrections.

Charge and the magnetic moment get imaginary parts which depend on ξ.

Summary

1. We performed field transformation with arbitrary parameter ξ in EFT Lagrangian .

2. Physical quantities characterizing unstable resonances defined through the residues of the S-matrix in complex plane are ξ-independent.

3. In "on-mass-shell" scheme the charge of an unstable particle gets strong contributions and both the charge and the magnetic moment depend on field redefinition parameter ξ.