What is a Resonance?

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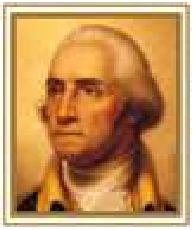

What is a Resonance? — Some Thoughts...

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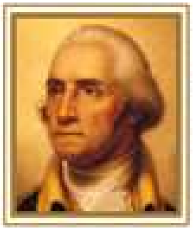

Thanks to: S. Krewald, K. Nakayama, R. Workman

Overview

- Resonance Basics
- Time Delay
- Elementary vs. Dynamical Resonances
- Can one define an interface between quark and hadronic models at the level of bare masses and bare vertices? [A: Not without a lot of work.]
- Summary

Resonance Basics

Experiment

- 'Bump' in the cross section
- Phase shifts show rapid change through $\pi/2$
- Time delay
- Exponential decay law

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Theory

Resonant structures arise from three types of mechanisms:

- Poles of the S-matrix corresponding to elementary resonances
- Poles of the S-matrix corresponding to dynamic resonances
- Structures that produce the usual signals of resonances (see above) *without* accompanying poles of the S-matrix [Calucci/Ghirardi, Phys. Rev. **169**, 1339 (1968)]

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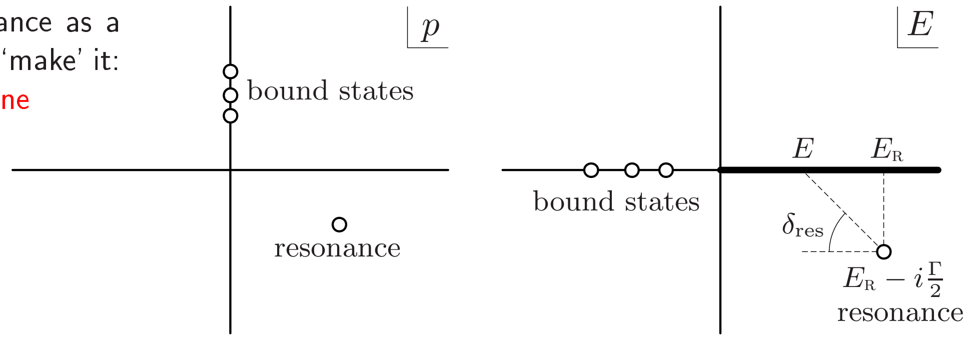
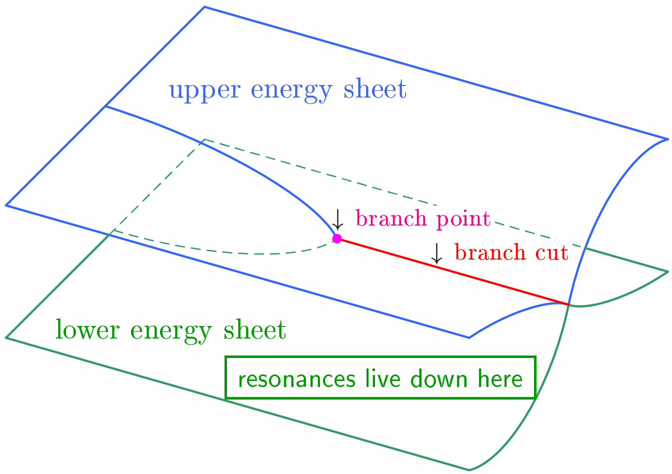
■ **Will ignore last item because it cannot be treated generically. Caveat: Its experimental manifestation may lead to erroneous phenomenological pole-type description.**

Resonance Basics

- Usually, we think of a resonance as a bound state that didn't quite 'make' it:
  $\Longrightarrow$ Pole in the complex plane





Scattering phase shifts:

$$\delta(E) = \delta_{\mathsf{res}}(E) + \delta_{\mathsf{bg}}(E)$$

Resonance phase shift:

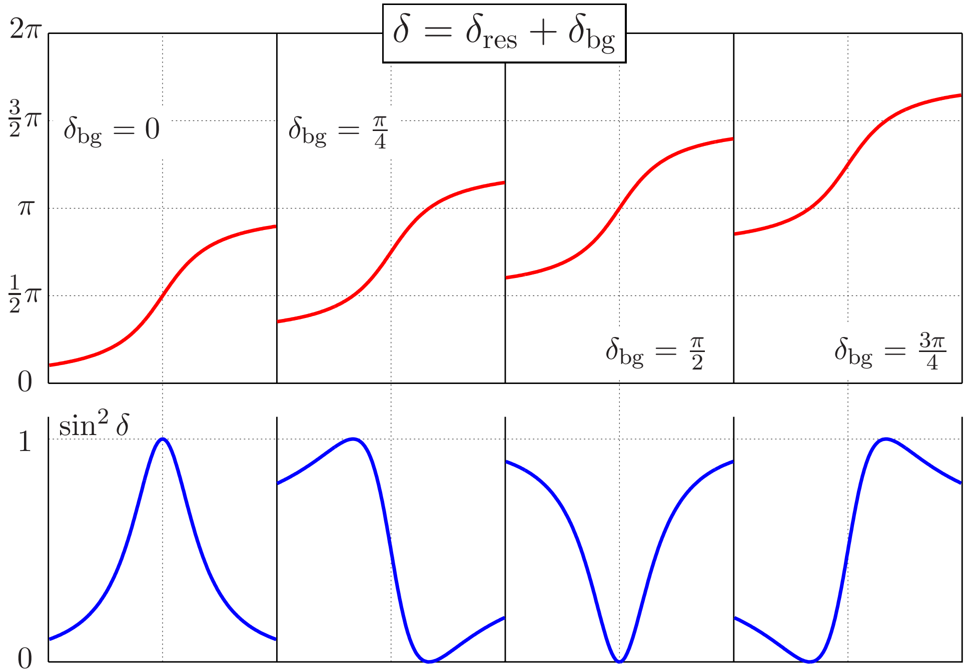
$$\tan \delta_{\mathsf{res}} = -\frac{\Gamma}{2(E - E_R)}$$

Breit–Wigner peak:

$$\sin^2 \delta_{\mathsf{res}} = \frac{\Gamma^2}{4(E - E_R)^2 + \Gamma^2}$$

Resonance Basics

Resonant phase shifts with various constant background contribution



Each tile 300 MeV wide; width $\Gamma = 100$ MeV

Time Delay — Single-Channel Case [energy resolution $\Delta E \ll \Gamma$]

- Scattered particles in resonance experience a time delay within the interaction region:

$$\Delta t = \hbar \frac{d\delta}{dE}$$

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$$\Delta t = \hbar \frac{d\delta}{dE} \gtrsim -\frac{R_0}{v_0}$$

R_0: size of interaction region
v_0: velocity through interaction region

In a wave-packet picture, this limit arises from the fact that the scattered wave must not arrive at the target before the incident wave.

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- **True resonance:**

$$\Delta t = \hbar \frac{d\delta}{dE} > 0$$

Time Delay — Multi-Channel Case

- Eisenbud: [PhD Thesis, Princeton, 1948 (unpubl.)]

$$\Delta t = \hbar \frac{d}{dE} \arg(S - 1)$$

Matrix!

Same as previous result for single channel where $S = e^{2i\delta}$.

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- **True resonance in channel i:**

$$\Delta t_{ii} > 0$$

$S_{ii} = \eta_i \mathrm{e}^{2i\delta_i}$, with $\eta_1 = \eta_2$ for $N = 2$ and $\sum_{i=1}^{N} \eta_i = N - 2$.

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- Smith claims: [PR **118**, 349 (1960)]

$$\Delta t_{ij} = \mathrm{Re}\left[-i\hbar (S_{ij})^{-1} \frac{dS_{ij}}{dE} \right]$$

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Wrong! — See HH&RW, PRC **76**, 058201 (2007)

Problem due to the fact that Eisenbud's notation does not distinguish between phases and eigenphases.

Smith's Lifetime Matrix

[PR 118, 349 (1960)]

- Lifetime matrix:

$$Q = -i\hbar \frac{dS}{dE} S^\dagger$$

Same as previous result for single channel where $S = \mathrm{e}^{2i\delta}$.

- Diagonal elements, Q_{ii}, are a measure for the time delay in the resonant channel i.

$$Q_{ii} = i\hbar \left[\sum_j S_{ij} \frac{dS^*_{ij}}{dE} \right]_{E=E_R} = 2\hbar \left[\frac{dT^*_{ii}}{dE} + \sum_j (2i\hbar T_{ij}) \frac{dT_{ij}}{dE} \right]_{E=E_R}$$

Direct relationship to Eisenbud's time delay can only be established if there is one single resonant channel. Otherwise one needs to consider trace of Q:

$$\mathrm{tr}\, Q = -i\hbar \mathrm{tr} \left(\frac{dS_D}{dE} S^*_D \right) = 2\hbar \sum_j \frac{d\phi_j}{dE} = 2 \sum_j \Delta t_{jj} \qquad \phi_j\text{: eigenphase}$$

- Relationship to speed plot:

$$Q_{ii} = 2\hbar\, \mathrm{Sp}(E_R)\ , \qquad \text{with} \qquad \mathrm{Sp}(E) = \left| \frac{dT_{ii}(E)}{dE} \right|$$

Elementary vs. Dynamic Resonances

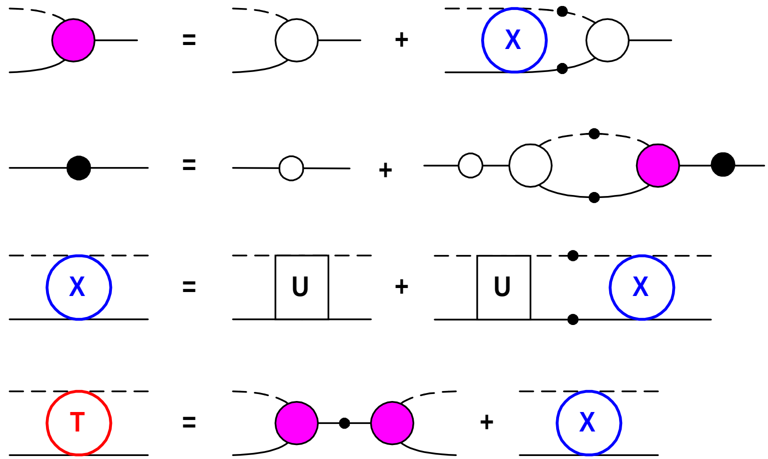
$\pi N \to \pi N$

dressed vertex

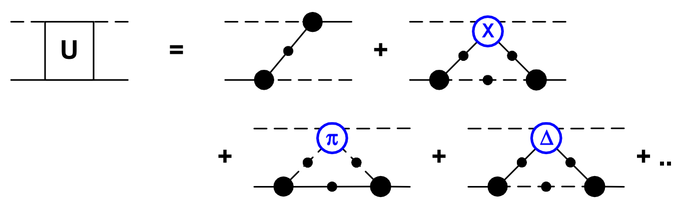
dressed propagator

nonpolar T matrix

full T matrix



two-particle irreducible kernel



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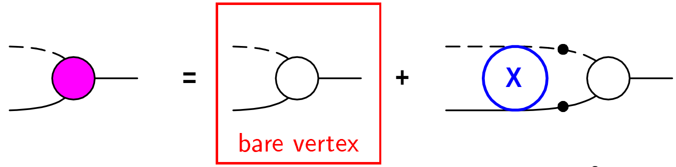

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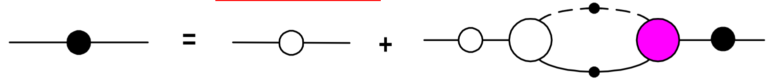
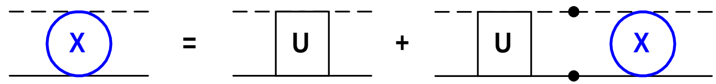
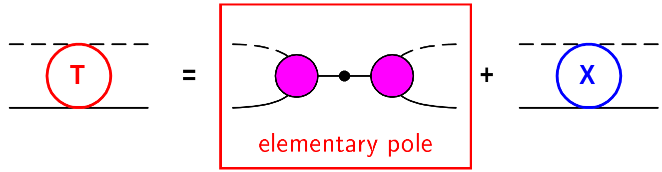


nonpolar T matrix



full T matrix



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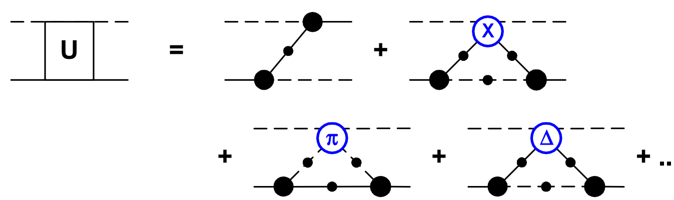

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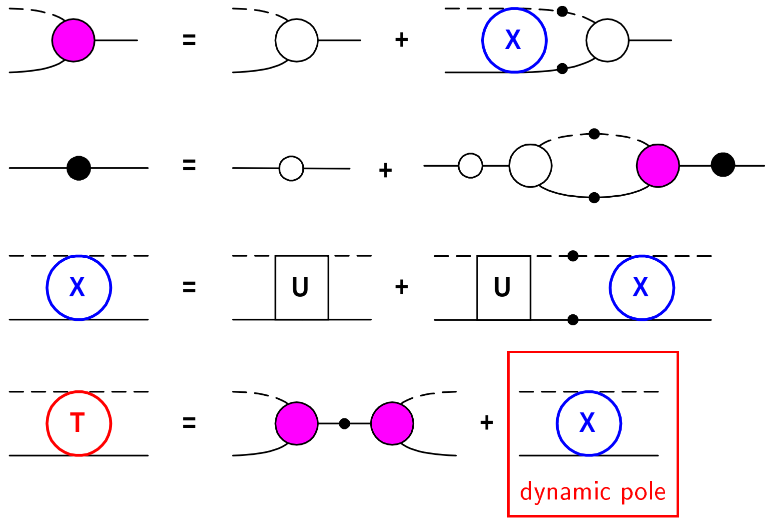
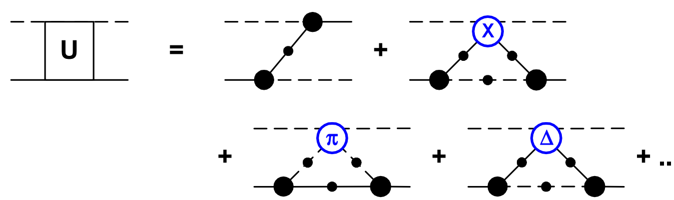


dynamic pole arises from
two-particle irreducible kernel



Elementary vs. Dynamic Resonances

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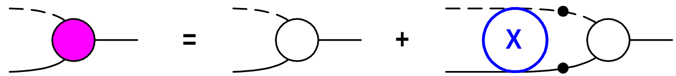
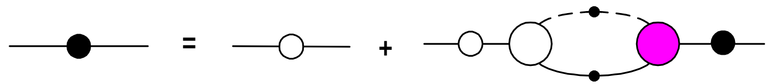


dressed propagator



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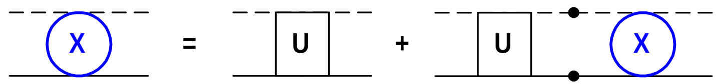
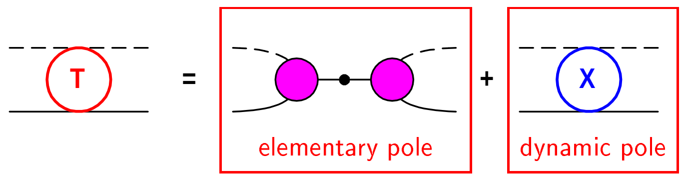


full T matrix



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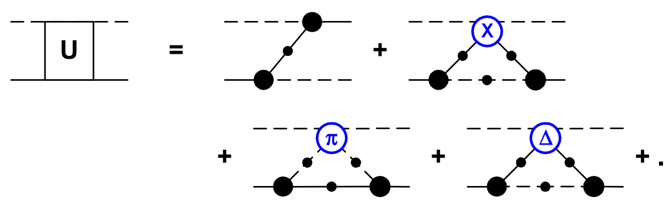
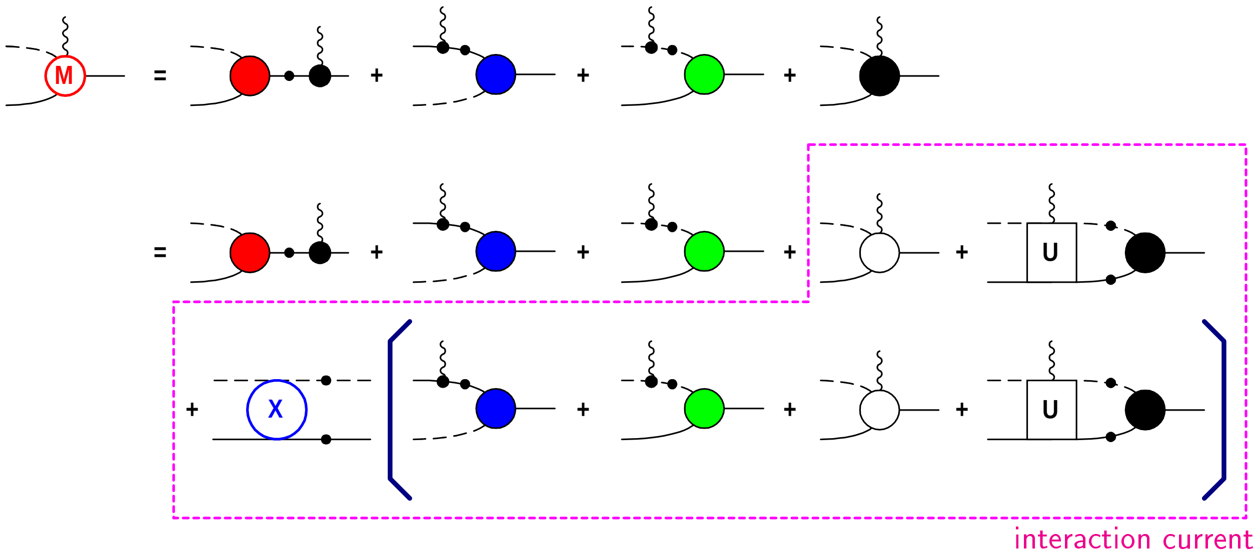


Elementary vs. Dynamic Resonances

$\gamma N \to \pi N$



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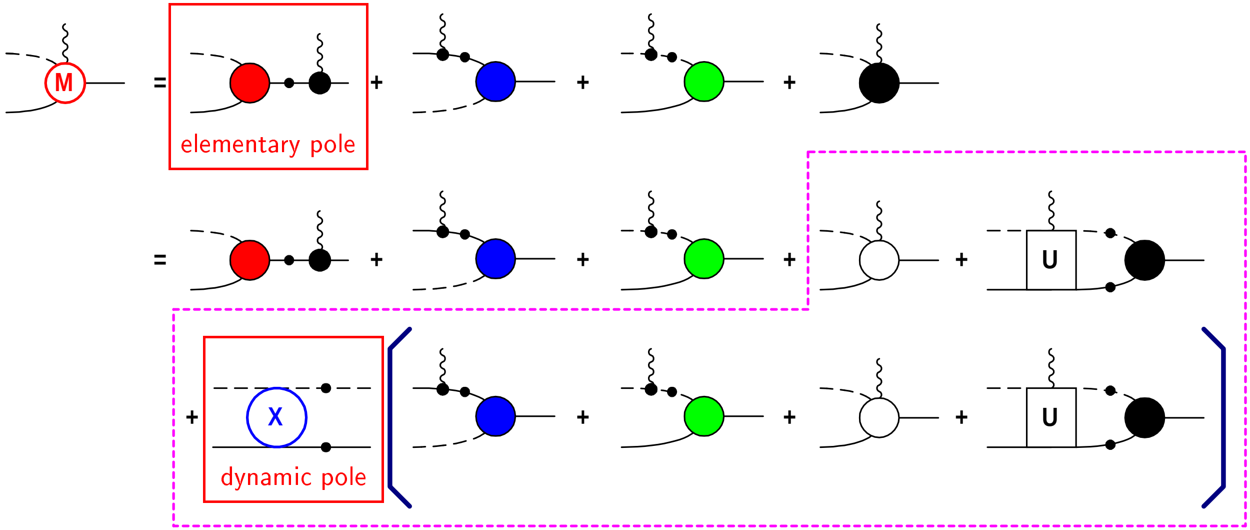

- Same mechanisms as in hadronic reaction.

Elementary vs. Dynamic Resonances

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dressed propagator

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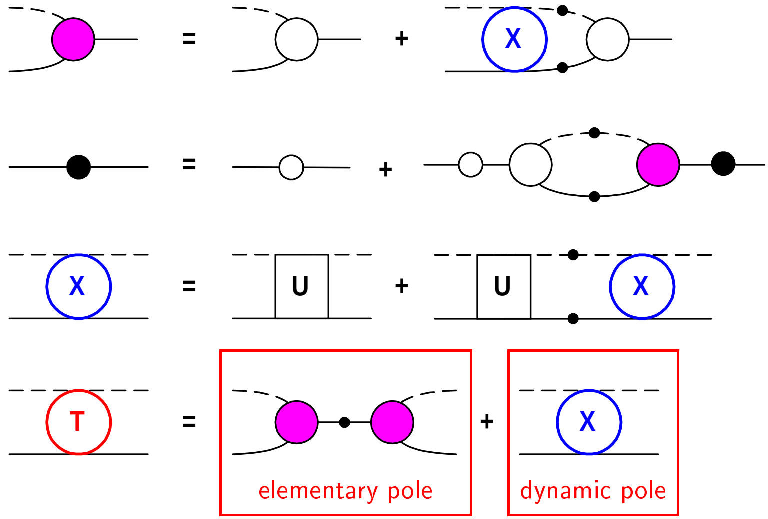
full T matrix



- **Phenomenologically, one cannot distinguish between elementary and dynamic resonances.**

Elementary vs. Dynamic Resonances

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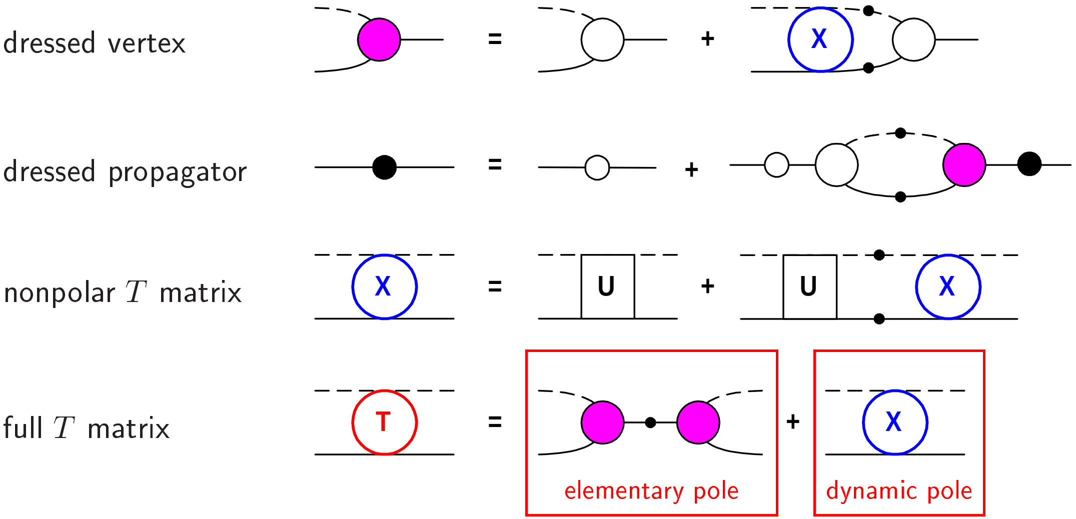

- **Phenomenologically, one cannot distinguish between elementary and dynamic resonances.**
- **Theoretically, it makes a huge difference whether a resonance is elementary or dynamic.**

Elementary vs. Dynamic Resonances

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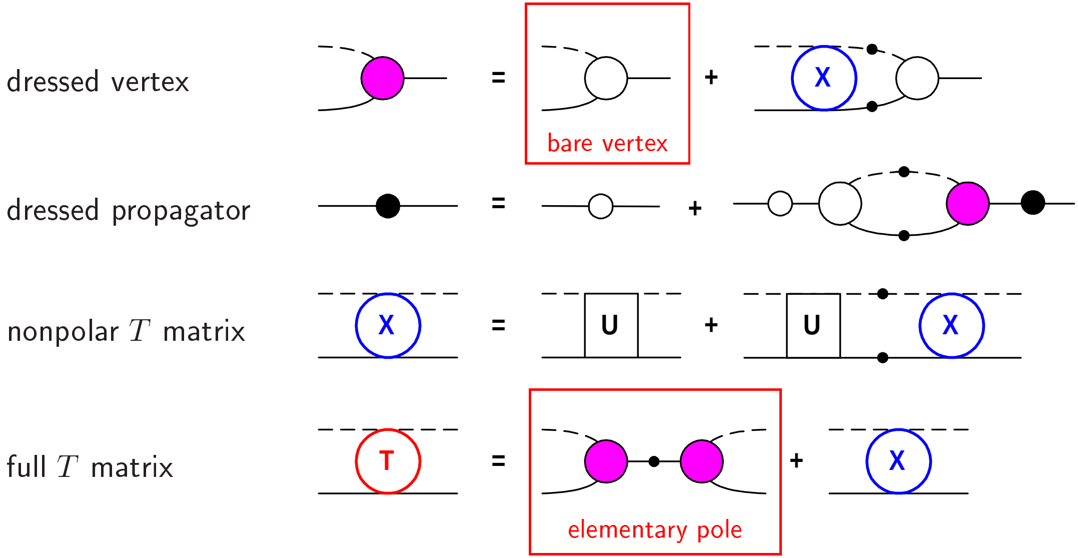

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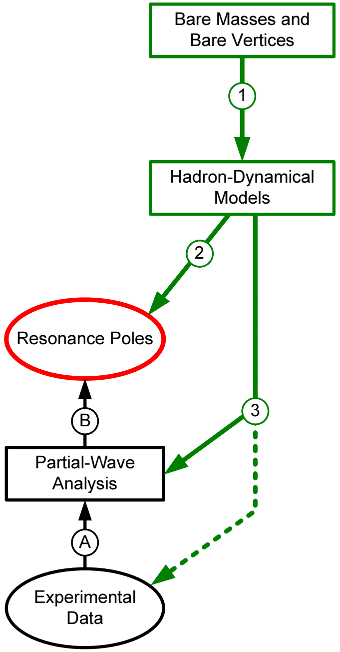
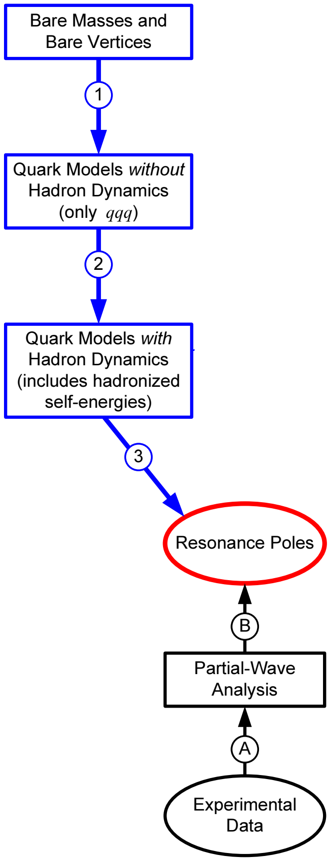
Quark dynamics $\Longrightarrow$ Bare hadronic vertex $\Longrightarrow$ Elementary resonance

Elementary Resonances and Hadron-Dynamical Models



Hadron-Dynamical Model

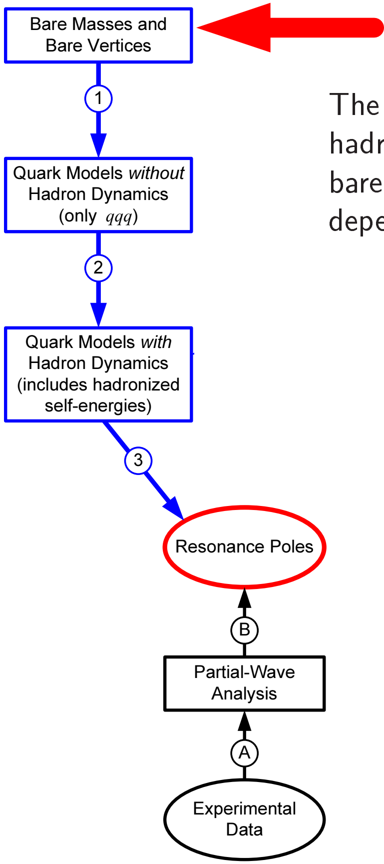
Elementary Resonances and Constituent Quark Models



Constituent Quark Model

Elementary Resonances

The interface between quark and hadron dynamics — bare masses and bare vertices — is not just model-dependent, it is unphysical.



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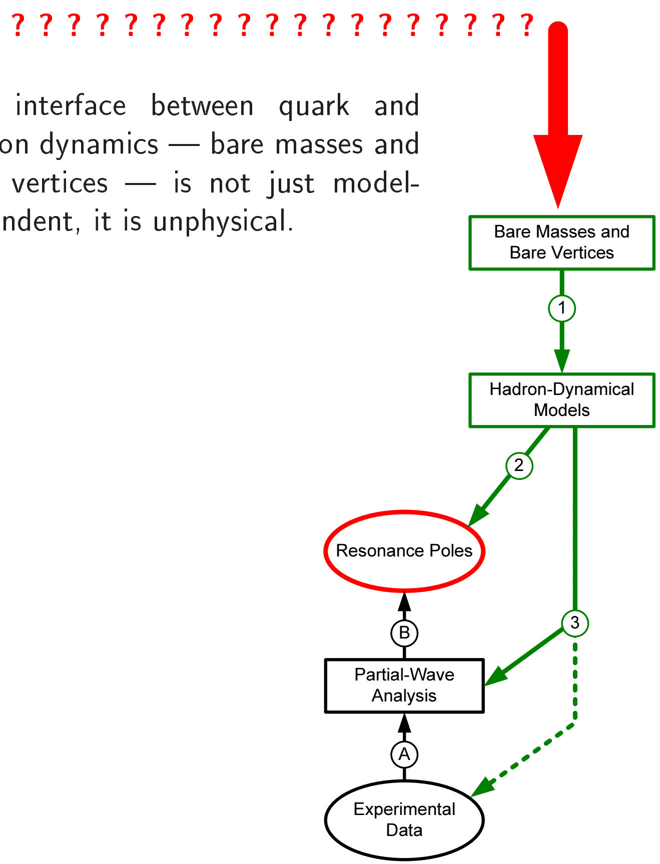
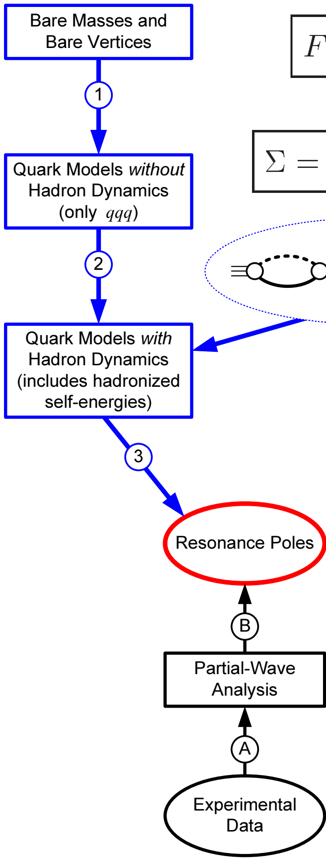


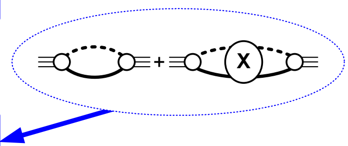
Hadron-Dynamical Model

Elementary Resonances

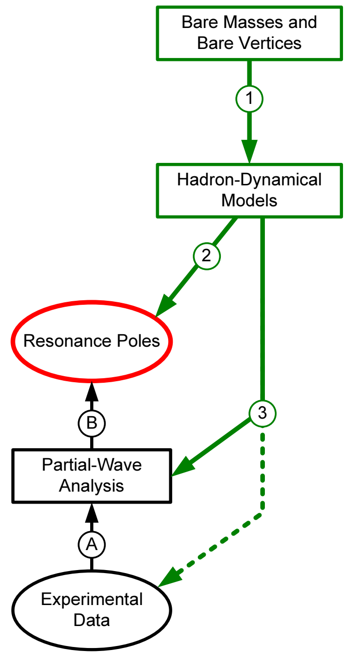


$$F^{(0)} = V_h \psi_{qqq}$$ bare CQM vertex

$$\Sigma = \langle F^{(0)} | G_0 | F^{(0)} \rangle + \langle F^{(0)} | G_0 \, X \, G_0 | F^{(0)} \rangle$$

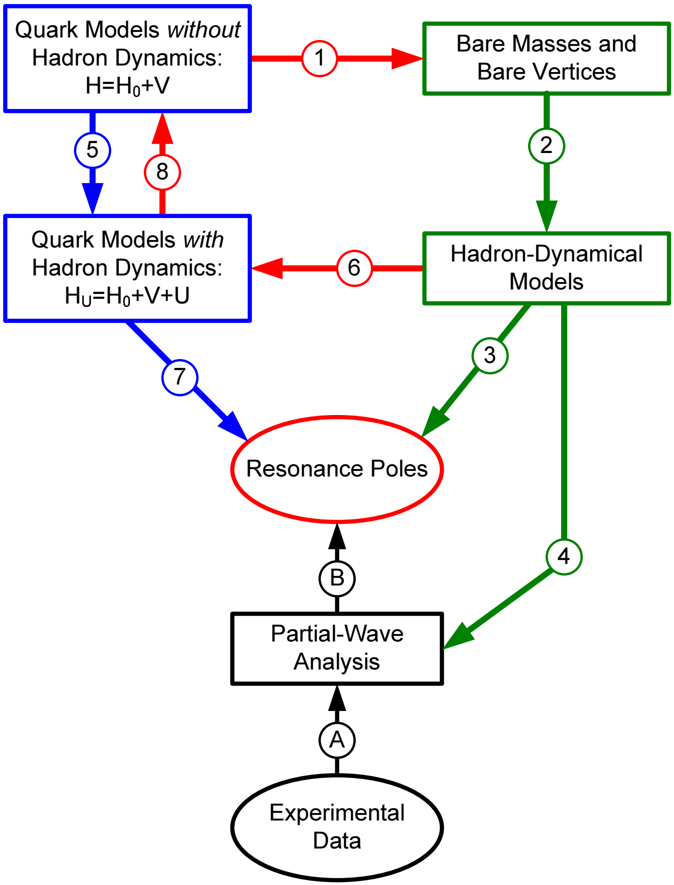


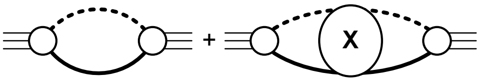
hadronized self-energy



Constituent Quark Model **Hadron-Dynamical Model**

Elementary Resonances — Self-Consistency

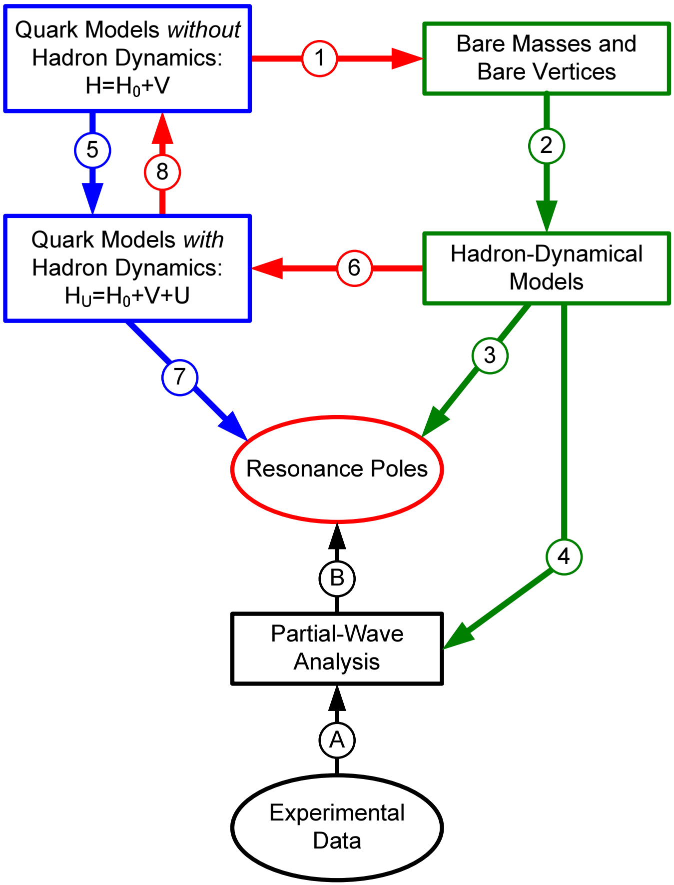


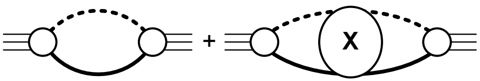


Optical potential

Schematic connection between quark-model calculations (top two boxes on the left) and field-theory-inspired hadron-dynamical models (top two boxes on the right). Both quark-models *with* explicit hadron degrees of freedom and hadron-dynamical models can be used to directly extract resonance-pole parameters (masses and widths). The experimental data are linked to these parameters via partial-wave analyses. The lines labeled 1–4, taken by themselves, describe an approach where there is no feedback between the hadronic dynamics that link to the data and the quark model. The feedback mechanism enters via lines 6 and 8: Line 6 supplies the optical potential into the quark model which may then be used to calculate the physical resonance-pole parameters directly. Comparison of the corresponding values obtained via the hadronic or quark routes 3 or 7, respectively, provide a feedback that, via line 8, can be used to improve, along line 1, the bare input for the hadronic approach.

Elementary Resonances — Self-Consistency





Optical potential

- Self-consistency scheme very elaborate
- Presumably not very practical
- Serves only to show that idea of *defining* an interface between CQM and hadronic models without any correction mechanism is ill-advised

Summary

- There are many structures — dynamical or otherwise — that produce signatures usually attributed to resonant behavior.

- True resonances are characterized by an enhanced dwell time of reacting particles in the interaction region $\longrightarrow$ positive delay time.

- Whether poles of T- or S-matrices are elementary or dynamic in origin cannot be unambiguously decided.

- Bare input for hadron-dynamical models cannot be directly related to quark constituent models. (At least not without a *lot* of work.)

Thank You!