

Pole Extraction from a World Collection of PWA

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Questions

- How many partial wave analyses do we have up to now, and how many we will have from now on?
- How many extraction methods for resonance parameters, poles ... do we have up to now?
- What can we do with all of them?
- Can we compare them?

Let see what happens if we do so.

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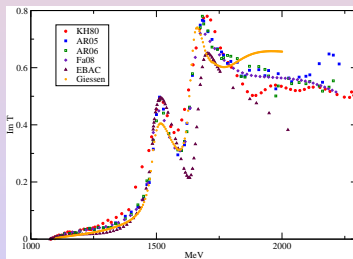
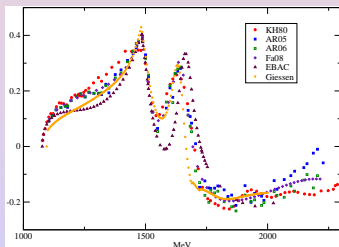
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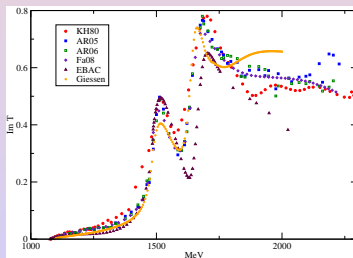
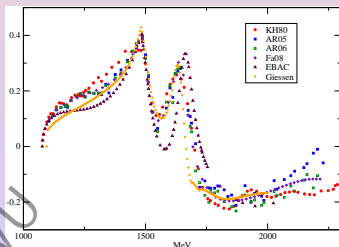
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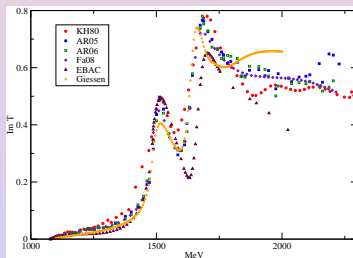
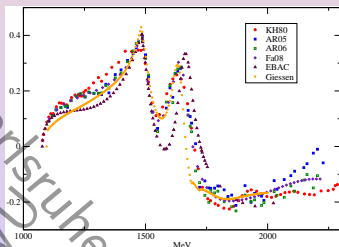
Comparison ... of apples and pears



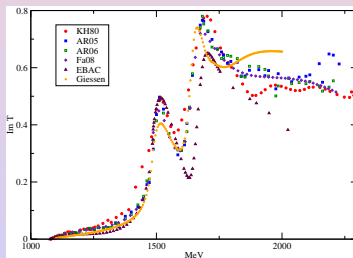
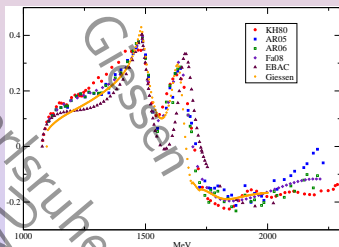
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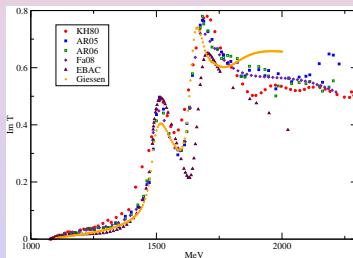
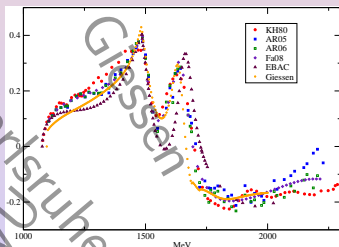
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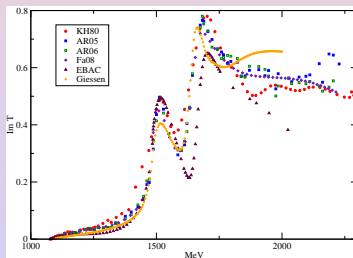
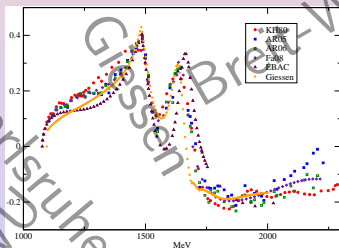
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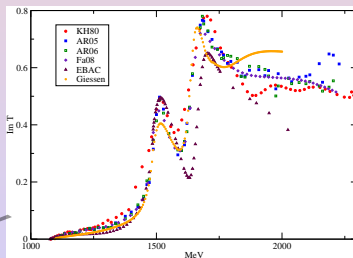
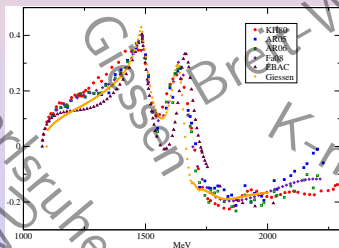


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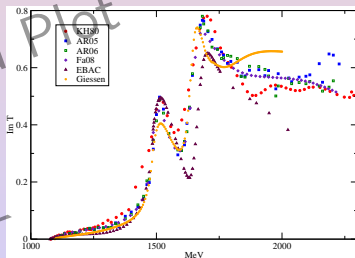
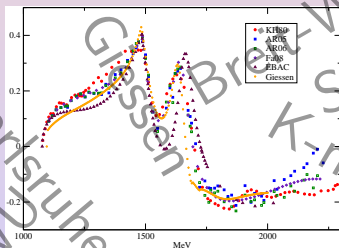
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Parametrization
After parametrization
Fitting procedure (Dressed poles)
Results
Conclusions

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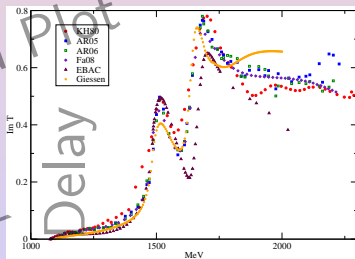
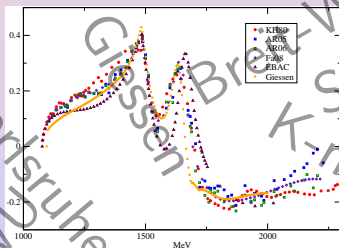
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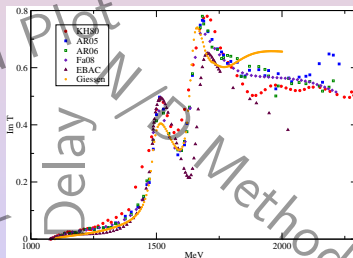
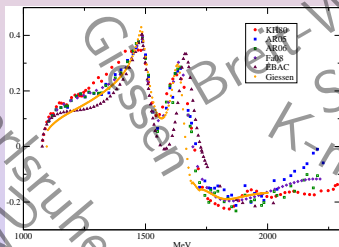
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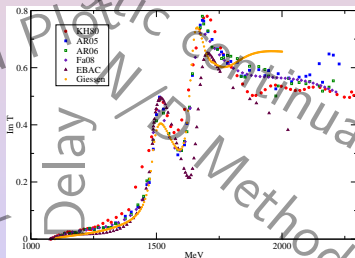
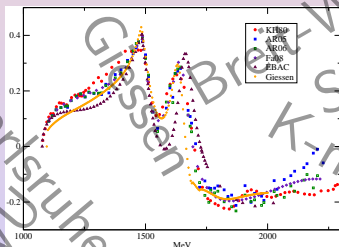
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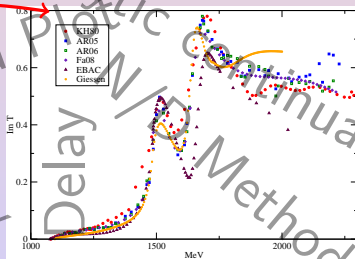
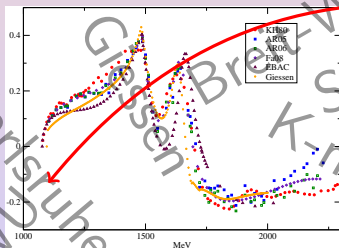
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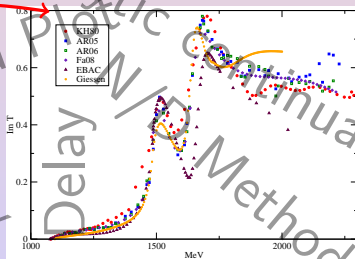
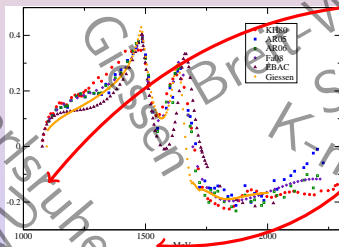
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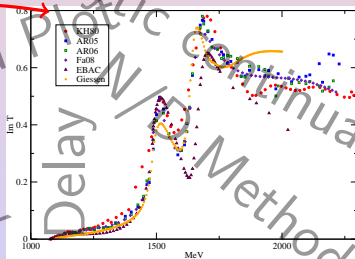
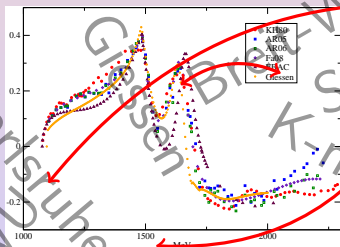
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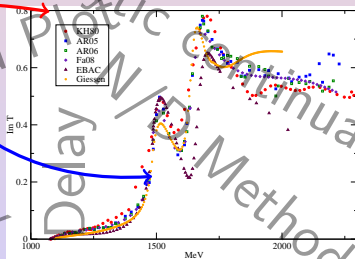
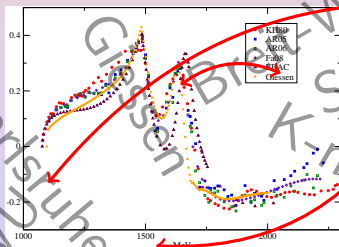
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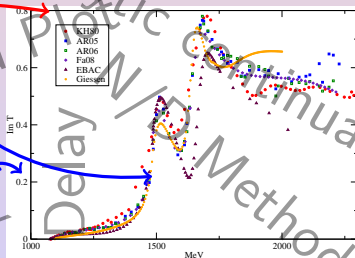
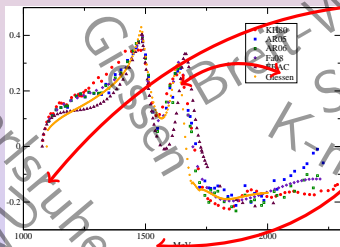
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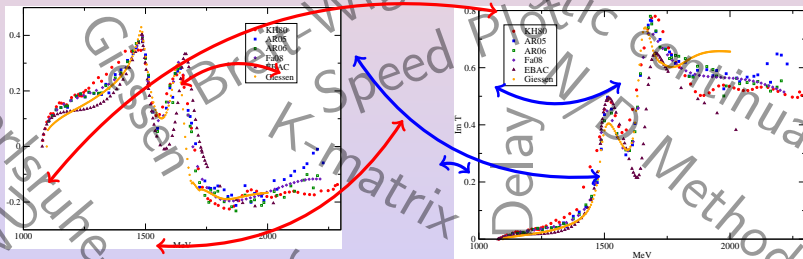
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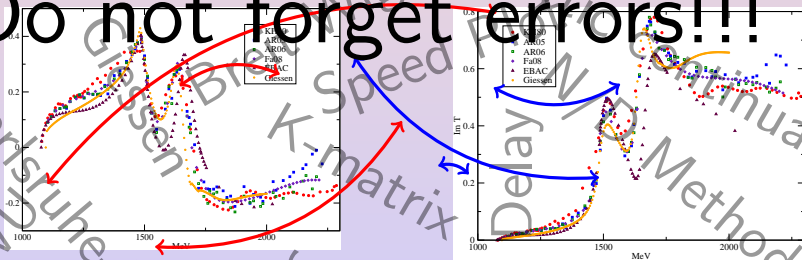
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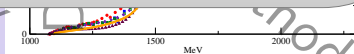
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Do not forget errors!!!



Comparison ... of apples and pears

Messy, isn't it?



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T - matrix

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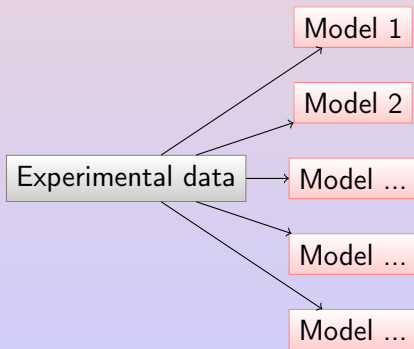
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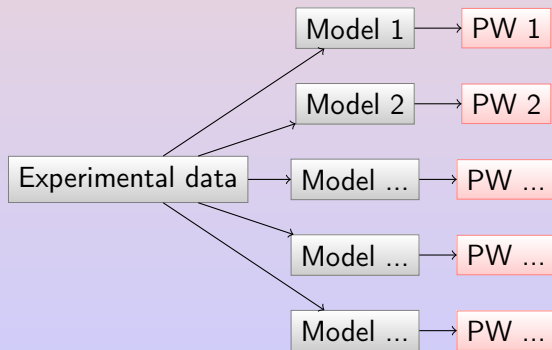
Resonance parameters

Experimental data

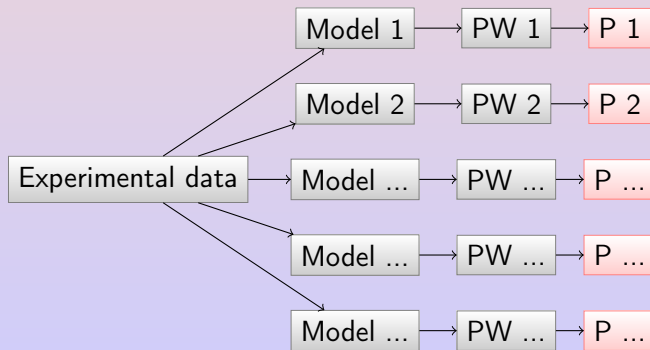
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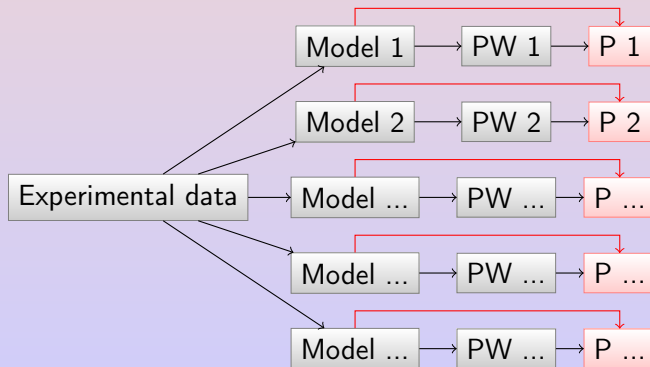
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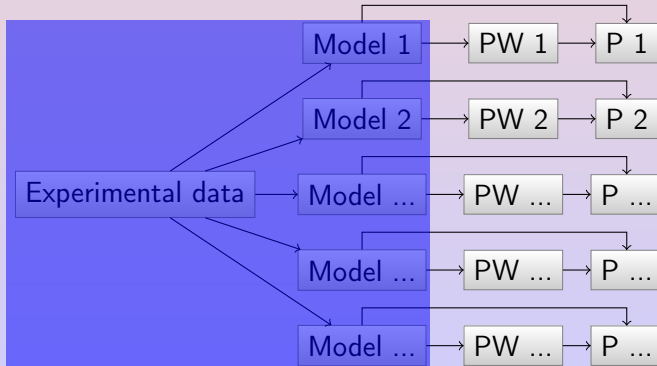
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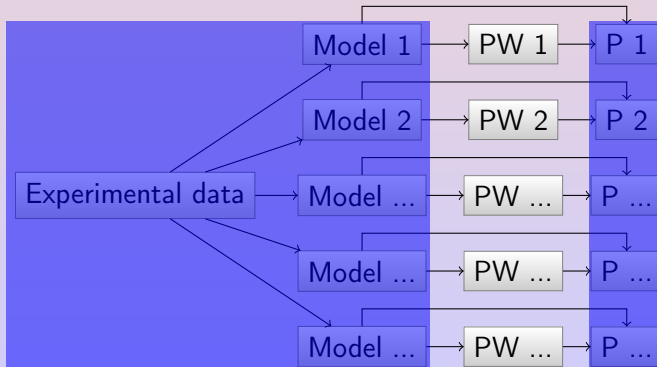
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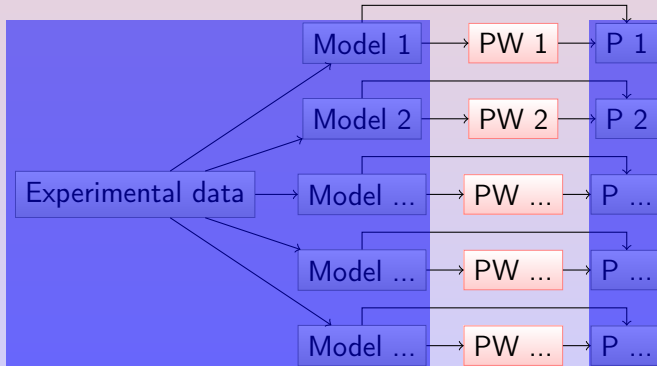
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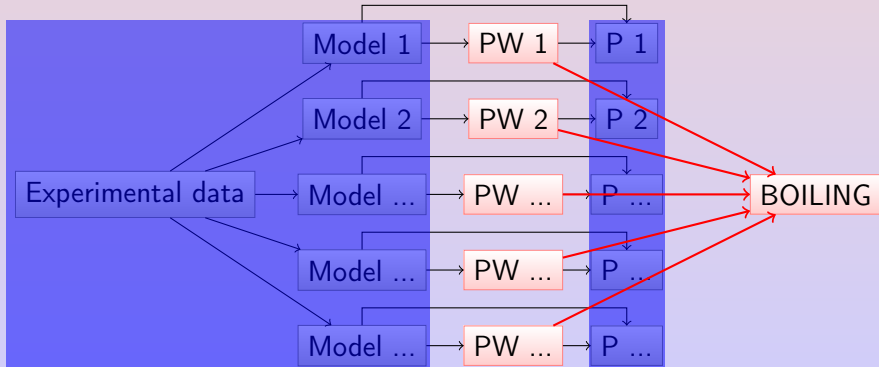
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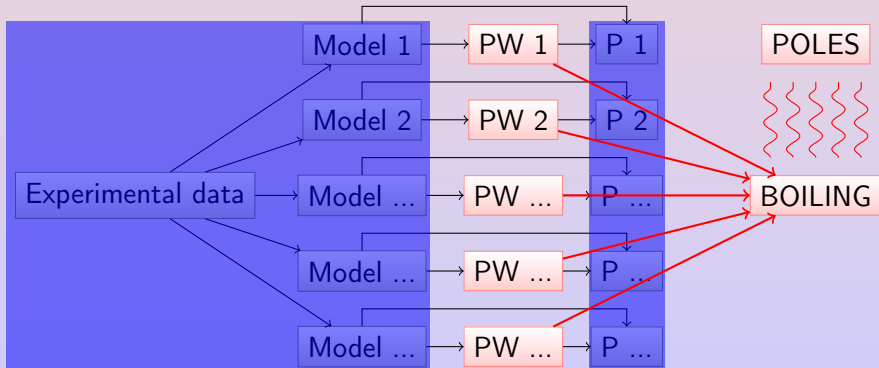
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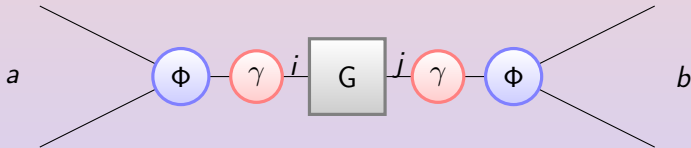
Resonance parameters



Resonance parameters



T - matrix



$$T_{ab} = \sum_{i,j=1}^N \sqrt{\mathfrak{S}m \phi_a} \gamma_{ai} G_{ij}(s) \gamma_{jb} \sqrt{\mathfrak{S}m \phi_b}$$

T - matrix

Something about all components of T - matrix in the CMB approach

$$T_{ab} = \sum_{i,j=1}^N \mathfrak{I}m\Phi_a(s)^{1/2} \gamma_{ai} G_{ij}(s) \gamma_{jb} \mathfrak{I}m\Phi_b(s)^{1/2}$$

- s - total center-of-mass energy squared
- a, b denote channels which can be either
 - stable two particle system (πN or ηN)
 - quasi two particle state ($\pi\pi N$)
- i, j denote intermediate states (resonances) between states denoted with indices a and b

T - matrix

Something about all components of T - matrix in the CMB approach

$$T_{ab} = \sum_{i,j=1}^N \boxed{f_a(s) \sqrt{\rho_a(s)}} \gamma_{ai} G_{ij}(s) \gamma_{jb} \boxed{\sqrt{\rho_b(s)} f_b(s)}$$

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- $f_a(s)$ - form factor
- γ_{ai} - strenght constant
 - The strenght constant and form factor define the decay of the i th resonance into channel a
- $\rho_a(s)$ - phase space factor
- G_{ij} - dressed propagator

Parametrization

Form factor and phase space factor

form factor

$$f_a(s) = \left(\frac{q_a}{Q_1 + \sqrt{q_a^2 + Q_2^2}} \right)^{l_a}$$

- Q_1 and Q_2 have to be defined for each channel
 - for $\pi N \rightarrow \pi N$,
 $Q_1 = Q_2 = m_\pi$,
 - for $\pi N \rightarrow \eta N$,
 $Q_1 = Q_2 = m_\eta$

phase space factor

$$\rho_a = \frac{q_a}{\sqrt{s}}$$

- q_a is center-of-mass-momentum for channel a , defined as:

$$q_a = \sqrt{\frac{(s - (m_{\pi, \eta} + m_N)^2)(s - (m_{\pi, \eta} - m_N)^2)}{4s}}$$

Parametrization

Channel propagator

Imaginary part of the channel propagator

$$\Im m \phi_a(s) = f_a^2 \rho_a(s)$$

Real part of the channel propagator

Once subtracted dispersion relations

$$\phi_a = \frac{s - s_0}{\pi} \int_{s_{th}}^{\infty} \frac{\Im m \phi_a(s')}{(s' - s)(s' - s_0)} ds'$$

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Extrapolation

Pietarinen's expansion

We also use Pietarinen's expansion

$$\varphi(s) = \sum_{n=0}^N c_n Z^n(s)$$

to extrapolate imaginary and real part of channel propagator.
Function $Z(s)$ can be either

$$Z(s) = \frac{\alpha - \sqrt{s - s_0}}{\alpha + \sqrt{s - s_0}}$$

or

$$Z(s) = \frac{\alpha + \sqrt{s - s_0}}{\alpha - \sqrt{s - s_0}}$$

T - matrix again

3 channels and 3 poles

T-matrix element

$$T_{ab} = \sum_{1,j=1}^N \sqrt{\mathfrak{S}m \phi_a} \gamma_{ai} G_{ij} \gamma_{jb} \sqrt{\mathfrak{S}m \phi_b}$$

T-matrix

$$T = \sqrt{\mathfrak{S}m \Phi} \gamma^T G \gamma \sqrt{\mathfrak{S}m \Phi}$$

$$\begin{array}{c}
 \left[\begin{array}{ccc} \pi\pi & \pi\eta & \pi\pi^2 \\ \eta\pi & \eta\eta & \eta\pi^2 \\ \pi^2\pi & \pi^2\eta & \pi^2\pi^2 \end{array} \right] \\
 \underbrace{\hspace{10em}} \\
 \left[\begin{array}{ccc} \pi\pi & 0 & 0 \\ 0 & \pi\eta & 0 \\ 0 & 0 & \pi\pi^2 \end{array} \right] \left[\begin{array}{ccc} \pi\pi 1 & \pi\pi 2 & \pi\pi 3 \\ \pi\eta 1 & \pi\eta 2 & \pi\eta 3 \\ \pi\pi^2 1 & \pi\pi^2 2 & \pi\pi^2 3 \end{array} \right] \underbrace{\hspace{1em}}_G \left[\begin{array}{ccc} 1\pi\pi & 1\pi\eta & 1\pi\pi^2 \\ 2\pi\pi & 2\pi\eta & 2\pi\pi^2 \\ 3\pi\pi & 3\pi\eta & 3\pi\pi^2 \end{array} \right] \left[\begin{array}{ccc} \pi\pi & 0 & 0 \\ 0 & \pi\eta & 0 \\ 0 & 0 & \pi\pi^2 \end{array} \right] \\
 \underbrace{\hspace{2em}}_{\sqrt{\mathfrak{S}m \Phi}} \quad \underbrace{\hspace{2em}}_{\gamma^T} \quad \underbrace{\hspace{2em}}_{\gamma} \quad \underbrace{\hspace{2em}}_{\sqrt{\mathfrak{S}m \Phi}}
 \end{array}$$

T - matrix again

3 channels and 3 poles

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 \underbrace{\hspace{10em}} \\
 \left[\begin{array}{ccc|ccc|ccc} \pi\pi & 0 & 0 & \pi\pi 1 & \pi\pi 2 & \pi\pi 3 & 1\pi\pi & 1\pi\eta & 1\pi\pi^2 & \pi\pi & 0 & 0 \\ 0 & \pi\eta & 0 & \pi\eta 1 & \pi\eta 2 & \pi\eta 3 & 2\pi\pi & 2\pi\eta & 2\pi\pi^2 & 0 & \pi\eta & 0 \\ 0 & 0 & \pi\pi^2 & \pi\pi^2 1 & \pi\pi^2 2 & \pi\pi^2 3 & 3\pi\pi & 3\pi\eta & 3\pi\pi^2 & 0 & 0 & \pi\pi^2 \end{array} \right] \underbrace{\hspace{2em}}_G \underbrace{\hspace{2em}}_{????????} \\
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Dressed propagator

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2 channels and 2 poles & 3 channels and 2 poles

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 \left[\begin{array}{cc} \pi\pi & \pi\pi^2 \\ \pi^2\pi & \pi^2\pi^2 \end{array} \right] \\
 \hline
 \left[\begin{array}{cc} \pi\pi & 0 \\ 0 & \pi\pi^2 \end{array} \right] \left[\begin{array}{cc} \pi\pi 1 & \pi\pi 2 \\ \pi\pi^2 1 & \pi\pi^2 2 \end{array} \right] \underbrace{G}_{????????} \left[\begin{array}{cc} 1\pi\pi & 1\pi\pi^2 \\ 2\pi\pi & 2\pi\pi^2 \end{array} \right] \left[\begin{array}{cc} \pi\pi & 0 \\ 0 & \pi\pi^2 \end{array} \right] \\
 \sqrt{\Im m \Phi} \quad \gamma^T \quad \quad \quad \gamma \quad \quad \sqrt{\Im m \Phi}
 \end{array}$$

$$\begin{array}{c}
 \left[\begin{array}{ccc} \pi\pi & \pi\eta & \pi\pi^2 \\ \eta\pi & \eta\eta & \eta\pi^2 \\ \pi^2\pi & \pi^2\eta & \pi^2\pi^2 \end{array} \right] \\
 \hline
 \left[\begin{array}{ccc} \pi\pi & 0 & 0 \\ 0 & \pi\eta & 0 \\ 0 & 0 & \pi\pi^2 \end{array} \right] \left[\begin{array}{cc} \pi\pi 1 & \pi\pi 2 \\ \pi\eta 1 & \pi\eta 2 \\ \pi\pi^2 1 & \pi\pi^2 2 \end{array} \right] \underbrace{G}_{????????} \left[\begin{array}{ccc} 1\pi\pi & 1\pi\eta & 1\pi\pi^2 \\ 2\pi\pi & 2\pi\eta & 2\pi\pi^2 \end{array} \right] \left[\begin{array}{ccc} \pi\pi & 0 & 0 \\ 0 & \pi\eta & 0 \\ 0 & 0 & \pi\pi^2 \end{array} \right] \\
 \sqrt{\Im m \Phi} \quad \gamma^T \quad \quad \quad \gamma \quad \quad \sqrt{\Im m \Phi}
 \end{array}$$

Number of parameters

So far

$$\underbrace{\begin{bmatrix} \pi\pi & \pi\eta & \pi\pi^2 \\ \eta\pi & \eta\eta & \eta\pi^2 \\ \pi^2\pi & \pi^2\eta & \pi^2\pi^2 \end{bmatrix}}_T$$

$$\parallel$$

$$\underbrace{\begin{bmatrix} \pi\pi & 0 & 0 \\ 0 & \pi\eta & 0 \\ 0 & 0 & \pi\pi^2 \end{bmatrix}}_{\sqrt{3m\Phi}} \underbrace{\begin{bmatrix} \pi\pi^1 & \pi\pi^2 & \pi\pi^3 \\ \pi\eta^1 & \pi\eta^2 & \pi\eta^3 \\ \pi\pi^2^1 & \pi\pi^2^2 & \pi\pi^2^3 \end{bmatrix}}_{\gamma^T} \underbrace{\begin{matrix} G \\ \text{?????????} \end{matrix}}_G \underbrace{\begin{bmatrix} 1\pi\pi & 1\pi\eta & 1\pi\pi^2 \\ 2\pi\pi & 2\pi\eta & 2\pi\pi^2 \\ 3\pi\pi & 3\pi\eta & 3\pi\pi^2 \end{bmatrix}}_{\gamma} \underbrace{\begin{bmatrix} \pi\pi & 0 & 0 \\ 0 & \pi\eta & 0 \\ 0 & 0 & \pi\pi^2 \end{bmatrix}}_{\sqrt{3m\Phi}}$$

As we can see in this example, we have nine parameters. Generally the number of the paramaters are given by

$$N_C \times N_P$$

which is the dimension of γ matrices, where N_C is number of channels, and N_P is the number of poles included in fitting procedure.

Dressed propagator

Contains... & gives more parameters

Bare propagator

$$G_{ij}^0 = \frac{\delta_{ij} e_i}{s - s_{0,i}}$$

Self energy term

$$\Sigma_{ij} = \sum_{c=1}^M \gamma_{ci} \Phi_c(s) \gamma_{cj}$$

Dressed propagator G is obtained by solving Dyson-Schwinger equation

$$G = G^0 + G^0 \Sigma G.$$

This equation can be solved by inverting a $N \times N$ matrix,

$$G = \begin{bmatrix} s - s_{0,1} - \Sigma_{11} & -\Sigma_{12} & -\Sigma_{13} \\ -\Sigma_{21} & s - s_{0,2} - \Sigma_{22} & -\Sigma_{23} \\ -\Sigma_{31} & -\Sigma_{32} & s - s_{0,3} - \Sigma_{33} \end{bmatrix}^{-1}$$

- where $s_{0,i}$ are bare poles
- So, finally we have $(N_C \times N_P) + N_P$ parameters.

T - matrix

After parametrization

$$\underbrace{\begin{bmatrix} \pi\pi & \pi\eta & \pi\pi^2 \\ \eta\pi & \eta\eta & \eta\pi^2 \\ \pi^2\pi & \pi^2\eta & \pi^2\pi^2 \end{bmatrix}}_T$$

||

$$\underbrace{\begin{bmatrix} \pi\pi & 0 & 0 \\ 0 & \pi\eta & 0 \\ 0 & 0 & \pi\pi^2 \end{bmatrix}}_{\sqrt{\Im m \Phi}} \underbrace{\begin{bmatrix} \pi\pi^1 & \pi\pi^2 & \pi\pi^3 \\ \pi\eta^1 & \pi\eta^2 & \pi\eta^3 \\ \pi\pi^2{}^1 & \pi\pi^2{}^2 & \pi\pi^2{}^3 \end{bmatrix}}_{\gamma T} \underbrace{G}_{\Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow} \underbrace{\begin{bmatrix} 1\pi\pi & 1\pi\eta & 1\pi\pi^2 \\ 2\pi\pi & 2\pi\eta & 2\pi\pi^2 \\ 3\pi\pi & 3\pi\eta & 3\pi\pi^2 \end{bmatrix}}_{\gamma} \underbrace{\begin{bmatrix} \pi\pi & 0 & 0 \\ 0 & \pi\eta & 0 \\ 0 & 0 & \pi\pi^2 \end{bmatrix}}_{\sqrt{\Im m \Phi}}$$

$$\Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow \Downarrow$$

$$\begin{bmatrix} s - s_{0,1} - \Sigma_{11} & -\Sigma_{12} & -\Sigma_{13} \\ -\Sigma_{21} & s - s_{0,2} - \Sigma_{22} & -\Sigma_{23} \\ -\Sigma_{31} & -\Sigma_{32} & s - s_{0,3} - \Sigma_{33} \end{bmatrix}^{-1}$$

Fitting procedure

$$\underbrace{\begin{bmatrix} \pi\pi & \pi\eta & \pi\pi^2 \\ \eta\pi & \eta\eta & \eta\pi^2 \\ \pi^2\pi & \pi^2\eta & \pi^2\pi^2 \end{bmatrix}}_T$$

||

$$\underbrace{\begin{bmatrix} \pi\pi & 0 & 0 \\ 0 & \pi\eta & 0 \\ 0 & 0 & \pi\pi^2 \end{bmatrix}}_{\sqrt{3m\phi}} \underbrace{\begin{bmatrix} \pi\pi^1 & \pi\pi^2 & \pi\pi^3 \\ \pi\eta^1 & \pi\eta^2 & \pi\eta^3 \\ \pi\pi^2^1 & \pi\pi^2^2 & \pi\pi^2^3 \end{bmatrix}}_{\gamma^T} \underbrace{G}_{\text{}} \underbrace{\begin{bmatrix} 1\pi\pi & 1\pi\eta & 1\pi\pi^2 \\ 2\pi\pi & 2\pi\eta & 2\pi\pi^2 \\ 3\pi\pi & 3\pi\eta & 3\pi\pi^2 \end{bmatrix}}_{\gamma} \underbrace{\begin{bmatrix} \pi\pi & 0 & 0 \\ 0 & \pi\eta & 0 \\ 0 & 0 & \pi\pi^2 \end{bmatrix}}_{\sqrt{3m\phi}}$$

⇓

$$\begin{bmatrix} s - s_{0,1} - \Sigma_{11} & -\Sigma_{12} & -\Sigma_{13} \\ -\Sigma_{21} & s - s_{0,2} - \Sigma_{22} & -\Sigma_{23} \\ -\Sigma_{31} & -\Sigma_{32} & s - s_{0,3} - \Sigma_{33} \end{bmatrix}^{-1}$$

- Cyan → PWD or PWA
- Magenta → Theoretically obtained function
- Red and blue → parameters to fit data

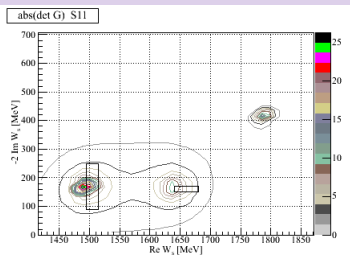
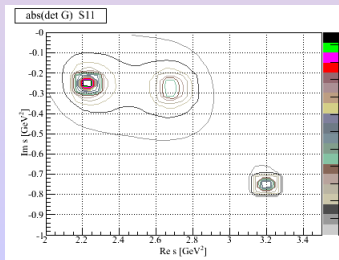
Dressed poles

Dressed poles can be obtained by solving an equation

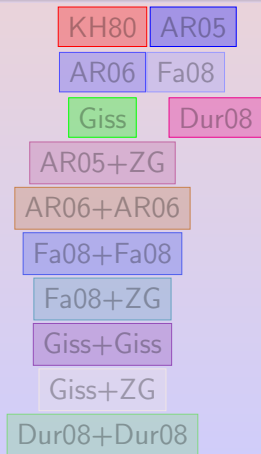
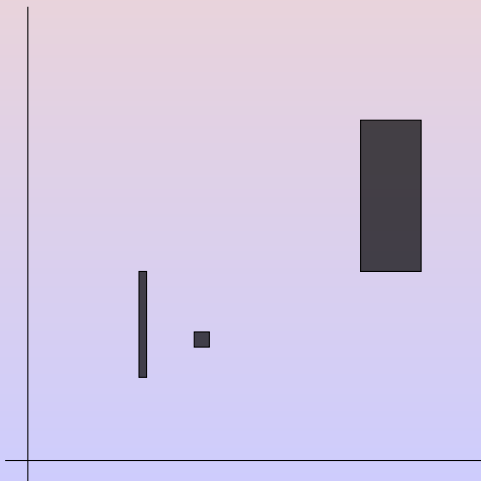
$$\det(G^{-1}) = \begin{vmatrix} s - s_{0,1} - \Sigma_{11} & -\Sigma_{12} & -\Sigma_{13} \\ -\Sigma_{21} & s - s_{0,2} - \Sigma_{22} & -\Sigma_{23} \\ -\Sigma_{31} & -\Sigma_{32} & s - s_{0,3} - \Sigma_{33} \end{vmatrix} = 0$$

Example: S11 partial wave

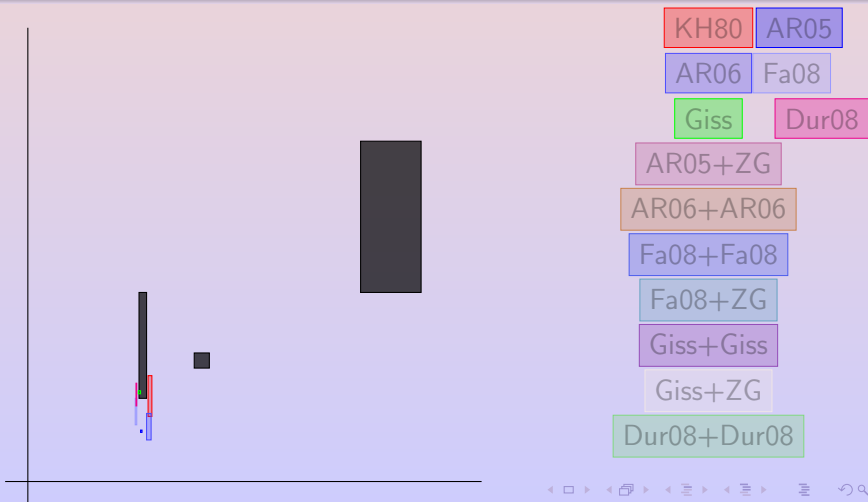
- 3 channels
- 3 poles
- Fa08 $\pi N \rightarrow \pi N$
- ZG98 $\pi N \rightarrow \eta N$



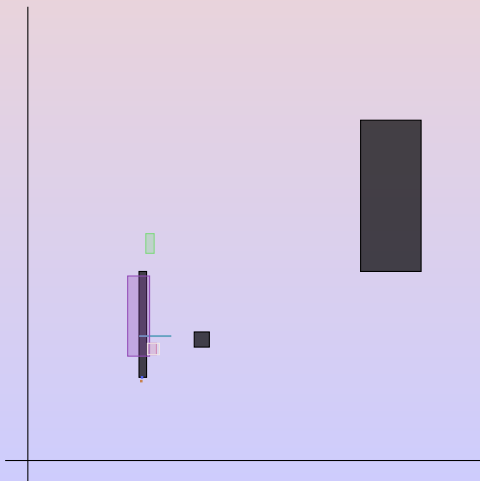
Results



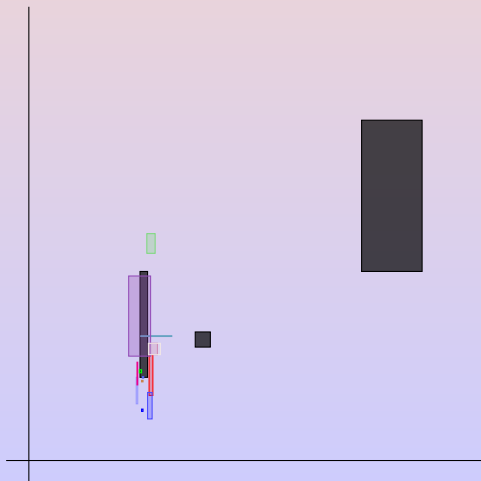
Results



Results

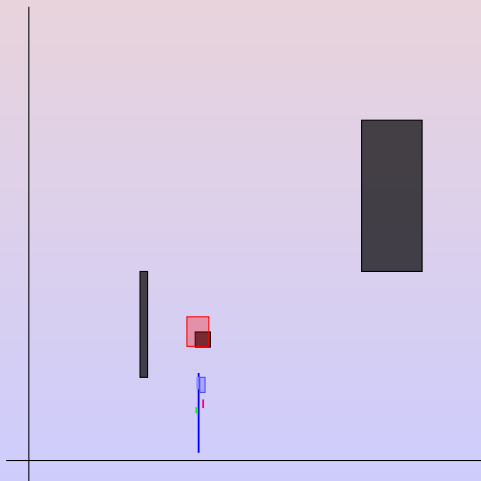


Results

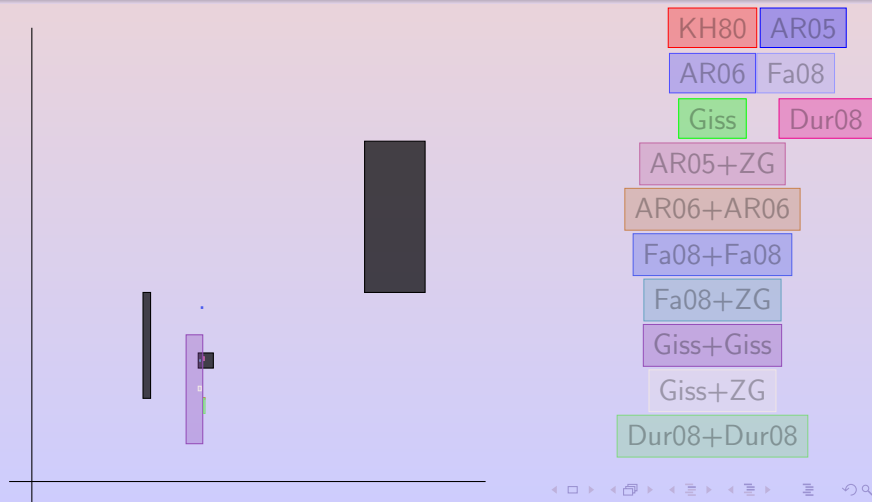


- KH80
- AR05
- AR06
- Fa08
- Giss
- Dur08
- AR05+ZG
- AR06+AR06
- Fa08+Fa08
- Fa08+ZG
- Giss+Giss
- Giss+ZG
- Dur08+Dur08

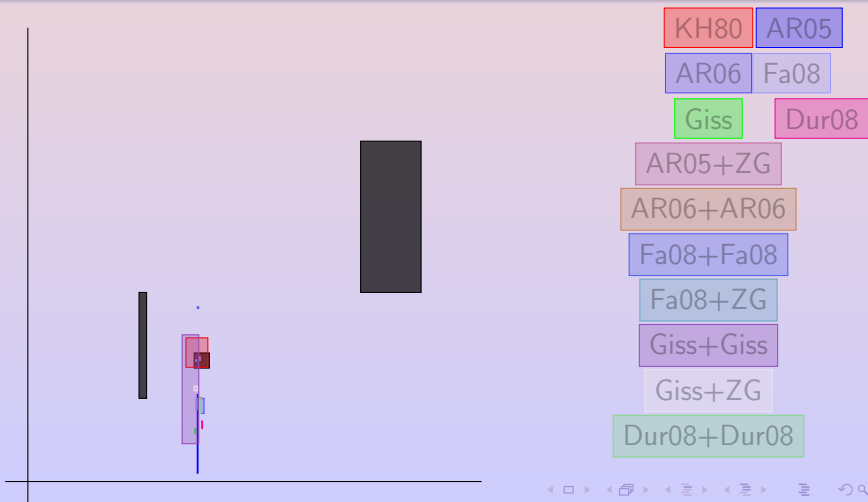
Results



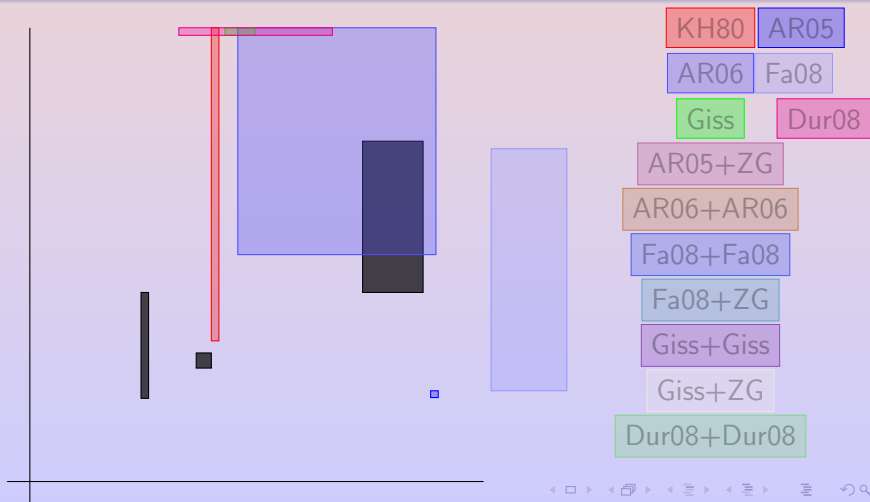
Results



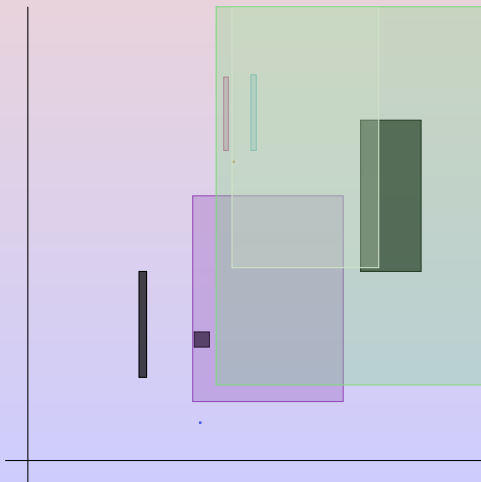
Results



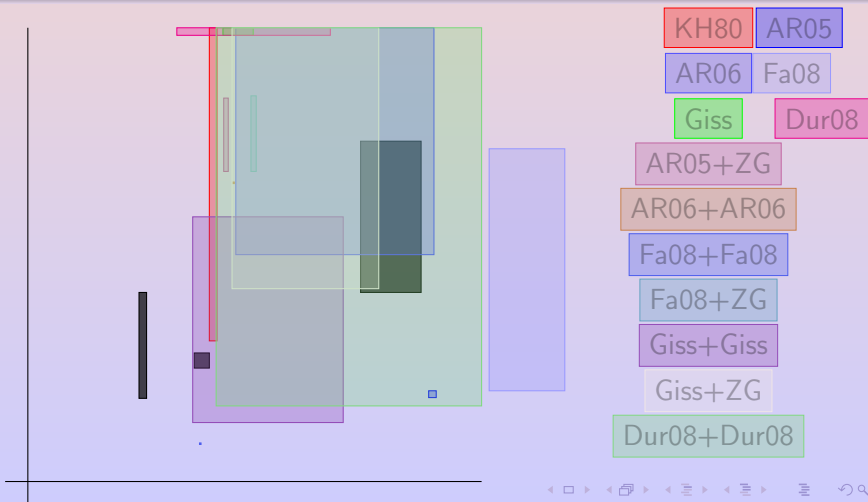
Results



Results



Results



Conclusions

- The first and the second pole position are stabile,
- Positions of the third pole (most of them are in beetwen 1700 MeV and 1900 MeV) indicate existance of the third pole somewhere about 1800 MeV.
- If we want the third pole to be stabile it is nescesary to include another channel in the fitting procedure.

Thank you !!!