Nucleon Resonance Analysis from Dubna-Mainz-Taipei Dynamical Model

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ECT, Trento, June 1-5, 2009

June 2, 2009

Outline

Relativistic meson exchange model

$$\pi(q) + N(p) \to \pi(q') + N'(p')$$

The starting point is the Bethe-Salpeter (BS) equation for πN

$$T_{\pi N} = B_{\pi N} + B_{\pi N} G_0 T_{\pi N},$$

where $B_{\pi N}$ is the sum of all irreducible two-particle Feynman amplitudes and G_0 is the free relativistic pion-nucleon propagator. In c.m. BS contains 4-dimensional integrals (very hard to solve!) However, with appropriate propagator G_0 we can present BS as a Lippman-Schwinger equation with relativistic kinematics, modified potential $v(\vec{q}', \vec{q}; E)$ and πN propagator $g(\vec{q}; E)$

$$t(\vec{q}', \vec{q}; E) = v(\vec{q}', \vec{q}; E) + \int v(\vec{q}', \vec{q}''; E) g(\vec{q}''; E) t(\vec{q}'', \vec{q}; E) d\vec{q}''$$

DMT relativistic meson exchange model

$$t_{ij}(E) = v_{ij}(E) + \sum_k v_{ik}(E)\, g_k(E)\, t_{kj}(E)\,,$$

where $v_{ij(E)}$ $(i = \pi, \eta)$ constructed using model Lagrangian with π, N, R, ρ, σ, η fields (background) and resonance contributions.

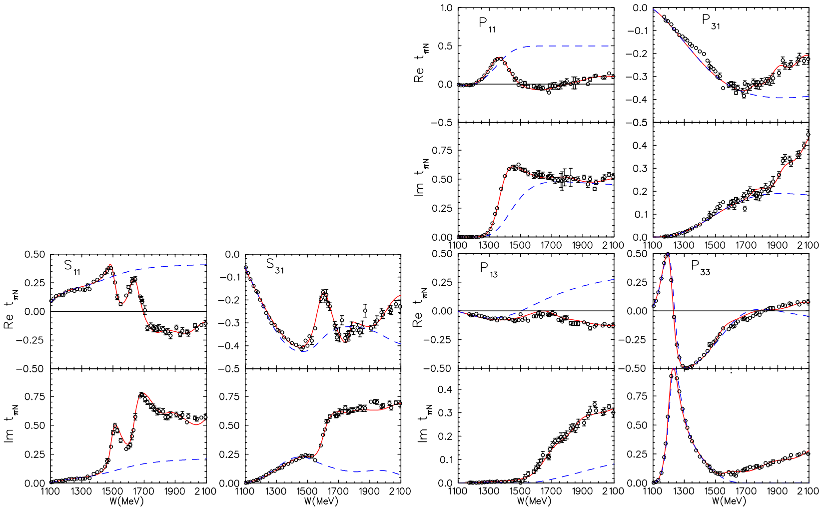
$$v_{ij}(E) = v_{ij}^{B}(E) + \sum_{n=1}^{N} v^{R_{ij}^{n}}(E)$$

$$v^{R_{ij}^{n}}(q, q'; E) = \frac{f_i^n(\tilde{\Lambda}_i^n, q; E)\, g_i^{(0)n}\, g_j^{(0)n}\, f_j^n(\tilde{\Lambda}_j^n, q'; E)}{E - M_R^{(0)n} + i\frac{1}{2}\Gamma_{2\pi}^{n}(E)}\,.$$

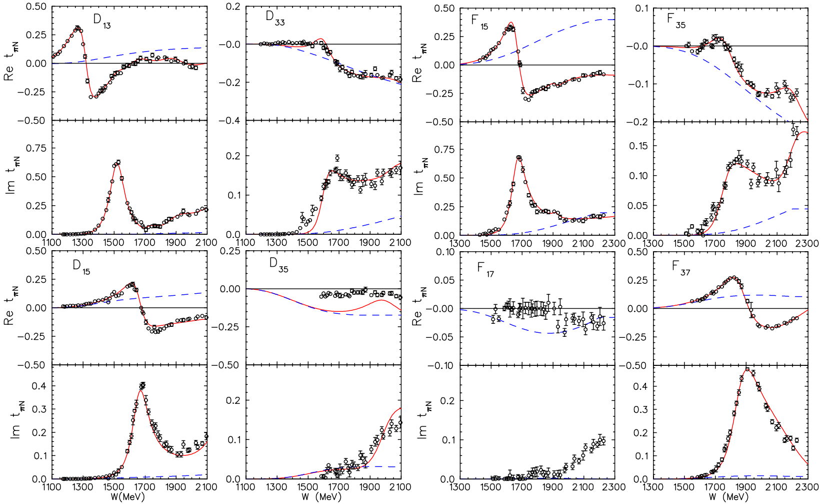
For each resonance we take $\Gamma_{2\pi}(E)$ as

$$\Gamma_{2\pi}(E) = \Gamma_{2\pi}^{0}\left(\frac{q_{2\pi}}{q_0}\right)^{2l+4}\left(\frac{X^2 + q_0^2}{X^2 + q_{2\pi}^2}\right)^{l+2}\,.$$

Results of the fit



Results of the fit



Extraction of the resonance parameters

Two- potential formulation (Afnan and collaborators)

$$t_{\pi N}(E) = \tilde{t}^{B}_{\pi N}(E) + \tilde{t}^{R}_{\pi N}(E) = v_{\pi N}(E) + v_{\pi N}(E) g_0(E) t_{\pi N}(E),$$

where $v_{\pi N}(E) = v^{B}_{\pi N}(E) + v^{R}_{\pi N}(E)$ and

$$\tilde{t}^{B}_{\pi N}(E) = v^{B}_{\pi N} + v^{B}_{\pi N}\, g_0(E)\, \tilde{t}^{B}_{\pi N}(E).$$

$\tilde{t}^{B}_{\pi N}(E)$ is the "non-resonant" background. The resonance term $\tilde{t}^{R}_{\pi N}(E)$ takes the form

$$\tilde{t}^{R}_{\pi N}(E) = \bar{h}_{\pi R}(E) \frac{1}{E - M^{(0)}_{R} - \Sigma_R(E)} h_{\pi R}(E)$$

with

$$h_{\pi R}(E) = h^{(0)}_{\pi R} + h^{(0)}_{\pi R}\, g_0(E)\, \tilde{t}^{B}_{\pi N}(E)\,,$$
$$\bar{h}_{\pi R}(E) = h^{(0)\dagger}_{\pi R} + \tilde{t}^{B}_{\pi N}(E)\, g_0(E)\, h^{(0)\dagger}_{\pi R}\,.$$

Self-energy

$$\Sigma_R = h^{(0)}_{\pi R}\, g_0\, h^{\dagger}_{\pi R} = h^{(0)}_{\pi R}\, g_0\, h^{(0)\dagger}_{\pi R} + h^{(0)}_{\pi R}\, g_0\, \tilde{t}^{B}_{\pi N}\, g_0\, h^{(0)\dagger}_{\pi R}\,.$$

The information about the physical mass and the total width of the resonance R are contained in the dressed resonance propagator

$$E - M^{(0)}_R - Re\,\Sigma_R(E) = 0\,.$$

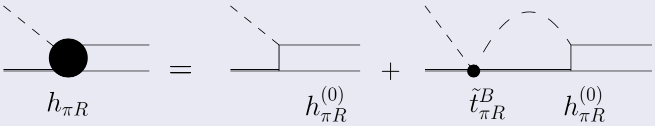
The solution of this equation, $E = M_R$, corresponds to the energy at which the dressed propagator is purely imaginary.
Next we define the "physical" or "dressed" mass,

$$M_R = M^{(0)}_R + Re\,\Sigma_R(M_R)$$

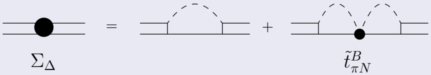
and the width of the resonance

$$\Gamma_R(E) = -2\, Im\,\Sigma_R(E)\,.$$

dressed vertex



self-energy



Overlapping resonances

$$t_{\pi N}(E) \neq \tilde{t}^{B}_{\pi N}(E) + \sum_{i=1}^{N} \tilde{t}^{R_i}_{\pi N}(E)\,.$$

We therefore prefer to separate the resonance and background contributions in the framework of DMT model.

$$t_{\pi N}(E) = t^{B}_{\pi N}(E) + \sum_{i=1}^{N} t^{R_i}_{\pi N}(E)\,.\,,$$

where

$$t^{B}_{\pi N}(E) \quad = \quad v^{B}_{\pi N} + v^{B}_{\pi N}\, g_0(E)\, t_{\pi N}(E)\,,$$

$$t^{R_i}_{\pi N}(E) = \frac{\bar{h}_{\pi R_i}(E) h^{(0)}_{\pi R_i}}{E - M^{(0)}_{R_i} - \Sigma^{1\pi}_{R_i}(E) - \Sigma^{2\pi}_{R_i}(E)}\,.$$

Results for the resonance parameters (I=1/2)

N^*	$M_R^{(0)}$	M_R	Γ_R	$\beta_R^{1\pi}(\%)$	ϕ_R(deg)
$P_{11}(1440)$	1612	1418	436	44	32
$\ast\ast\ast\ast$		1445 ± 25	325 ± 125	65 ± 10	
$D_{13}(1520)$	1590	1520	94	62	1.2
$\ast\ast\ast\ast$		1520 ± 5	115 ± 15	60 ± 5	
$S_{11}(1535)$	1559	1520	130	43	20
$\ast\ast\ast\ast$		1535 ± 10	150 ± 25	45 ± 10	
$S_{11}(1650)$	1727	1678	200	73	24
$\ast\ast\ast\ast$		1655 ± 10	165 ± 20	77 ± 17	
$D_{15}(1675)$	1710	1670	154	18	49
$\ast\ast\ast\ast$		1675 ± 5	147 ± 17	40 ± 5	
$F_{15}(1680)$	1748	1687	156	67	7.9
$\ast\ast\ast\ast$		1685 ± 5	130 ± 10	67 ± 2	
$D_{13}(1700)$	1753	1747	156	5	-1
$\ast\ast\ast$		1700 ± 50	100 ± 50	10 ± 5	
$P_{11}(1710)$	1798	1803	508	32	40
$\ast\ast\ast$		1710 ± 30	180 ± 100	15 ± 5	

Results for the resonance parameters (I=3/2)

N^*	$M_R^{(0)}$	M_R	Γ_R	$\beta_R^{1\pi}(\%)$	$\phi_R(\text{deg})$
$P_{33}(1232)$	1425	1233	132	100	12
$****$		1232 ± 1	118 ± 2	100	
$P_{33}(1600)$	1575	1562	216	6	-9
$***$		1600 ± 100	350 ± 100	17 ± 7	
$S_{31}(1620)$	1654	1616	160	32	-41
$****$		1630 ± 30	142 ± 18	25 ± 5	
$D_{33}(1700)$	1690	1650	260	15	-5
$****$		1710 ± 40	300 ± 100	15 ± 5	
$P_{31}(1750)$	1765	1746	554	4	-24
$*$		1744 ± 36	300 ± 120	8 ± 3	
$S_{31}(1900)$	1796	1770	430	8	-44
$**$		1900 ± 50	190 ± 50	2 ± 1	
$F_{35}(1905)$	1891	1854	534	11	-12
$****$		1890 ± 25	335 ± 65	12 ± 3	

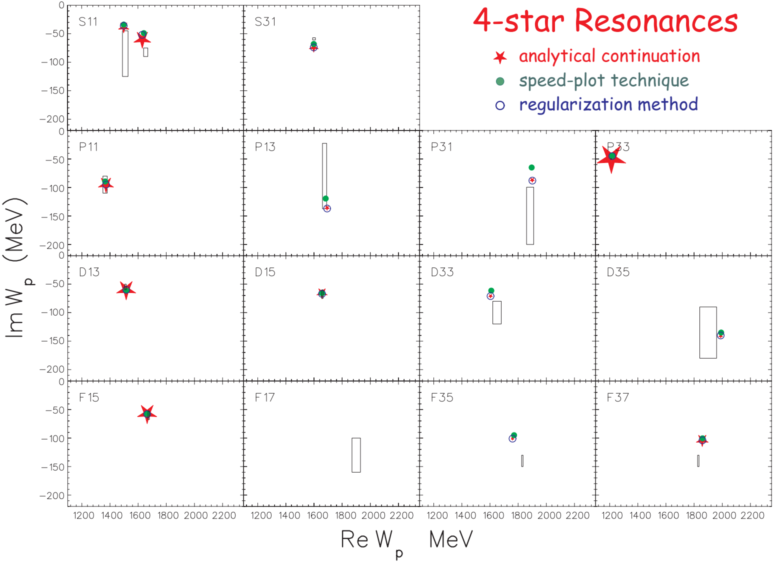
Pole Extraction Methods

- speed-plot (favoured by G. Höhler)
- regularization technique (S. Ceci, A. Svarc)
- analitic continuation
  - Gauss-Plots (S. Krewald, Julich group)
  - Pearce-Gibson method with diff. Reimann sheets (T. Sato, H. Lee, EBAC group)
  - $\mid T \mid$ contour plots and direct zero search of $1/T$ (DMT group)

In DMT method from $1/T(z) = 0 \Rightarrow W_p = ReW_p + ImW_p$
Rezidues:

$$\lim_{z \to W_p} T(z) = r = \mid r \mid \exp(i\theta)$$

Pole positions for the 4star resonances



contour plots for the D_{15} and F_{17} (simple cases)

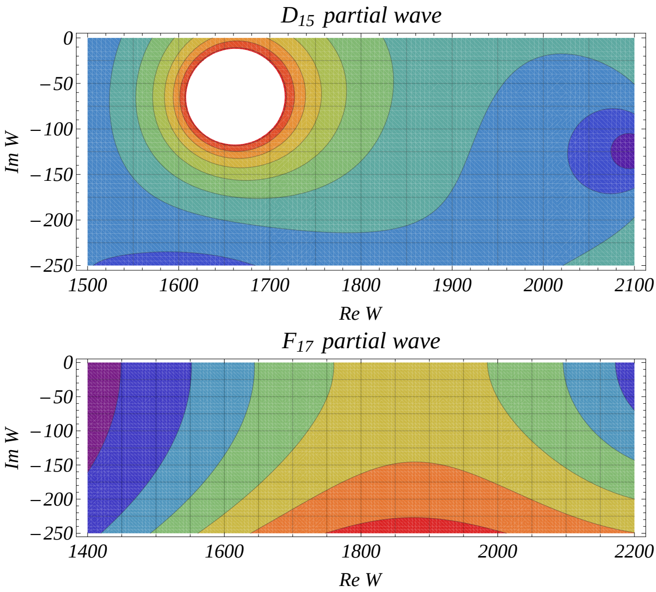


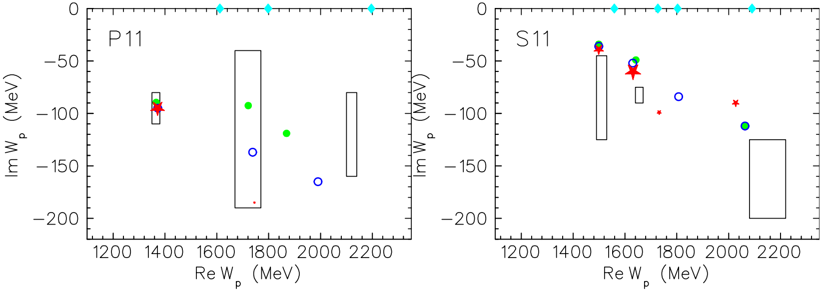
Table: Pole positions $W_p = M_p - \frac{1}{2} i\Gamma_p$ and absolute values of the residues $\mid r \mid$ at the pole, all in MeV, as well as the phases θ of the residues for S-wave resonances.

N^*	ReW_p	$-ImW_p$	$\mid r \mid$	θ(deg)
$S_{11}(1535)$	1499	39	14	-45
SP	1499	34	11	-46
RM	1499	36	12	
****	1510 ± 20	85 ± 40	96 ± 63	15 ± 45
$S_{11}(1650)$	1631	60	35	-83
SP	1642	49	21	-73
RM	1630	52	28	
****	1655 ± 15	83 ± 8	55 ± 15	-75 ± 25
$S_{31}(1620)$	1598	74	23	-98
SP	1598	68	22	-99
RM	1598	74	24	
****	1600 ± 10	59 ± 2	16 ± 3	-110 ± 20

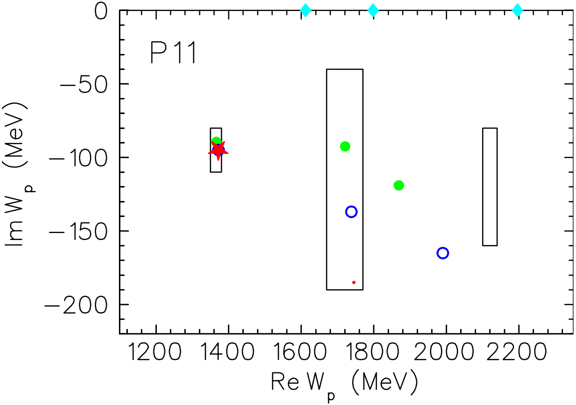
Table: Pole positions and residues for P-wave resonances.

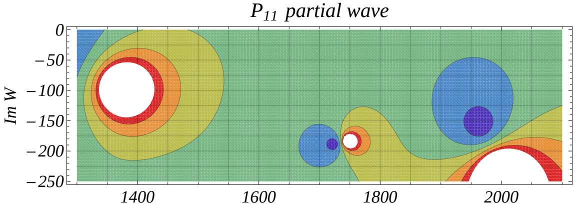
N^*	ReW_p	$-ImW_p$	$\mid r \mid$	θ(deg)
$P_{33}(1232)$	1212	49	49	-42
SP	1218	45	42	-35
RM	1218	45	40	
****	1210 ± 1	50 ± 1	53 ± 2	-47 ± 1
$P_{11}(1440)$	1371	95	50	-79
SP	1366	90	47	-87
RM	1372	95	49	
****	1365 ± 15	95 ± 15	46 ± 10	-100 ± 35
$P_{13}(1720)$	1693	136	20	-43
SP	1683	120	15	-64
RM	1693	137	22	
****	1675 ± 15	98 ± 40	13 ± 7	-139 ± 51

poles in P_{11} and S_{11} channels



contour plot for P_{11}





poles and residues in P_{11} and S_{11} channels

N^*	ReW_p	$-ImW_p$	$\|r\|$	θ(deg)
$P_{11}(1440)$	1371	95	50	-79
SP	1366	90	47	-87
RM	1372	95	49	
****	1365 ± 15	95 ± 15	46 ± 10	-100 ± 35
$P_{11}(1710)$	1745	184	11	-54
SP	1721	93	5	-163
RM	1738	137	5	
***	1720 ± 50	115 ± 75	10 ± 4	-175 ± 35
$P_{11}(2100)$	1996	229	56	-144
SP	1869	119	10	-216
RM	1990	165	4	
*	2120 ± 240	120 ± 40	14 ± 7	35 ± 35

N^*	ReW_p	$-ImW_p$	$\|r\|$	θ(deg)
$S_{11}(1535)$	1499	39	14	-45
SP	1499	34	11	-46
RM	1499	36	12	
****	1510 ± 20	85 ± 40	96 ± 63	15 ± 45
$S_{11}(1650)$	1631	60	35	-83
SP	1642	49	21	-73
RM	1630	52	28	
****	1655 ± 15	83 ± 8	55 ± 15	-75 ± 25
$S_{11}(1880)$	1732	99	15	-36
SP				
RM	1807	84		
new				
$S_{11}(2090)$	2027	90	23	
SP	2063	112		
RM	2063	112		
*	2150 ± 70	175 ± 50	40 ± 20	0 ± 90

Summary

Summary

- a meson-exchange model has been presented for pion-nucleon scattering that fits the S, P, D and F waves very well up to $W = 2.3$ GeV
- for all PDG resonances below 2.2 GeV we can determine conventional resonance parameters and pole positions
- for all 4-star resonances SP, RM and our analytic continuation methods give very close results for the pole position and residues and consistent with PDG values
- this model is very well suited for an application of pion and eta photo- and electroproduction to calculate pion loop contributions in the e.m. form factors

Table: Pole positions and residues for *D*-wave resonances.

N^*	ReW_p	$-ImW_p$	$\mid r \mid$	θ(deg)
$D_{13}(1520)$	1515	60	40	-7
SP	1516	62	40	-6
RM	1516	60	42	
****	1510 ± 5	55 ± 5	35 ± 3	-10 ± 4
$D_{15}(1675)$	1657	66	24	-22
SP	1657	66	24	-22
RM	1657	67	26	
****	1660 ± 5	68 ± 6	29 ± 6	-30 ± 10
$D_{33}(1700)$	1604	71	9.4	-63
SP	1609	67	9.5	-52
RM	1604	71	9.9	
****	1650 ± 30	100 ± 20	13 ± 3	-20 ± 25

Table: Pole positions and residues for F-wave resonances.

N^*	ReW_p	$-ImW_p$	$\mid r \mid$	θ(deg)
$F_{15}(1680)$	1664	57	38	-26
SP	1663	58	38	-28
RM	1664	58	42	
****	1672 ± 8	61 ± 6	38 ± 3	-23 ± 7
$F_{35}(1905)$	1760	100	10	-66
SP	1771	95	11	-47
RM	1760	101	10	
****	1830 ± 5	140 ± 10	25 ± 8	-50 ± 20
$F_{37}(1950)$	1858	104	43	-48
SP	1860	101	43	-45
RM	1859	104	44	
****	1880 ± 10	140 ± 10	50 ± 7	-33 ± 8

Results for the resonance parameters

N^*	$M_R^{(0)}$	M_R	Γ_R	$\beta_R^{1\pi}(\%)$	$\phi_R(\text{deg})$
$P_{13}(1720)$	1725	1711	278	13	0
$****$		1725 ± 25	225 ± 75	15 ± 5	
$P_{13}(1900)$	1922	1861	1000	18	-3.5
$**$		1879 ± 17	498 ± 78	26 ± 6	
$F_{15}(2000)$	1928	1926	58	4	18
$**$		1903 ± 87	490 ± 310	8 ± 5	
$D_{13}(2080)$	1972	1946	494	15	5
$**$		1804 ± 55	450 ± 185	~ 4	
$S_{11}(xxx)$	1803	1878	508	41	-5
$S_{11}(2090)$	2090	2124	388	37	-18
$*$		2180 ± 80	350 ± 100	18 ± 8	
$P_{11}(2100)$	2196	2247	1020	42	32
$*$		2125 ± 75	260 ± 100	12 ± 2	
$D_{13}(xxx)$	2162	2152	292	14	7
$P_{13}(xxx)$	2220	2204	406	15	-4

N^*	$M_R^{(0)}$	M_R	Γ_R	$\beta_R^{1\pi}(\%)$	ϕ_R(deg)
$P_{31}(1910)$	1953	1937	226	14	-21
$\ast\ast\ast\ast$		1895 ± 25	230 ± 40	22 ± 7	
$P_{33}(1920)$	1856	1827	834	12	3
$\ast\ast\ast$		1935 ± 35	220 ± 70	12 ± 7	
$D_{35}(1930)$	2100	2068	426	15	-20
$\ast\ast\ast$		1960 ± 60	360 ± 140	10 ± 5	
$D_{33}(1940)$	2100	2092	310	6	-10
$\ast$		2057 ± 110	460 ± 320	18 ± 12	
$F_{37}(1950)$	1974	1916	338	47	13
$\ast\ast\ast\ast$		1932 ± 17	285 ± 50	40 ± 5	
$F_{35}(2000)$	2277	2260	356	11	-26
$\ast\ast$		2200 ± 125	400 ± 125	16 ± 5	
$P_{31}(new)$	2160	2100	492	35	-25
$S_{31}(2150)$	2118	1942	416	70	-44
$\ast$		2150 ± 100	200 ± 100	8 ± 2	