

---

# Nucleon Excited States on the Lattice

David Richards (Jefferson Laboratory)

*Hadron Spectrum Collaboration*

PWA Workshop, ECT\* Trento, 1-5 June, 2009

# Hadron Spectrum Collaboration

---

- **University of Pacific**
  - J Juge
- **JLAB**
  - S Cohen
  - J Dudek
  - R Edwards
  - B Joo
  - H-W Lin
  - D Richards
  - C Thomas
- **CMU**
  - J Bulava
  - J Foley
  - C Morningstar
- **UMD**
  - E Engelson
  - S Wallace
- **Tata Institute (Mumbai)**
  - N Mathur
- **Trinity College Dublin**
  - Mike Peardon
  - Sinead Ryan

# USQCD: Computing Resources

[www.usqcd.org](http://www.usqcd.org)

Chroma, MILC, QDP



Leadership-class (ORNL, ANL) – **petaflop at ORNL**

Clusters at  
FNAL and  
JLAB



LQCD2: Proposal to DOE for computational facility 2010-2014  
ARRA Funding for LQCD

# Outline

---

- Baryons as seen from the lattice – challenges of the hypercubic lattice
- Recent results
  - Spectrum with two flavors of quarks
  - First transition form factor to  $P_{11}$
- Future Challenges
  - Unstable Resonances and multi-particle states
  - Lattices for spectroscopy
  - Identifying the Spin
- Conclusions

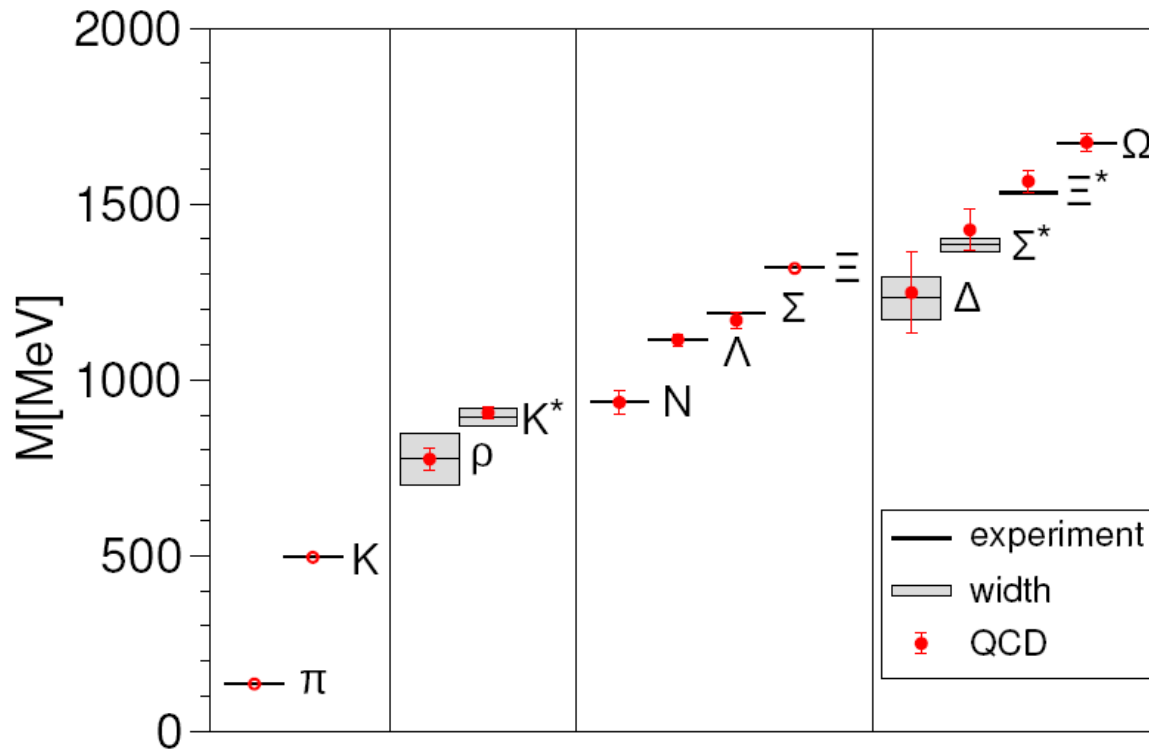
# Spectrum from correlation functions

- **Euclidean space:** stationary state energies can be extracted from asymptotic decay rate of temporal correlations of the fields
- Spectral representation of a simple correlation function
  - assume transfer matrix, ignore temporal boundary conditions

$$\begin{aligned} C(t) = \langle 0 | N(t) \bar{N}(0) | 0 \rangle &= \sum_n \langle 0 | e^{Ht} N(0) e^{-Ht} | n \rangle \langle n | \bar{N}(0) | 0 \rangle \\ &= |\langle n | N(0) | 0 \rangle|^2 e^{-E_n t} = \sum_n A_n e^{-E_n t} \end{aligned}$$

- **Extract lowest energy and amplitude as  $t \rightarrow \infty$**

# Low-lying Hadron Spectrum



Ch. Hoelbling et al.  
(BMW  
Collaboration),  
Science 2008

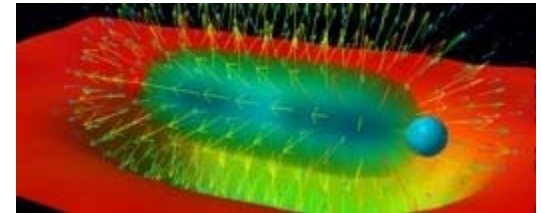
Control over:

- *Quark-mass dependence*
- *Continuum extrapolation*
- *finite-volume effects*  
(pions, resonances)

# Resonance Spectrum of QCD

- *Why is it important?*

- *What are the key degrees of freedom describing the bound states?*
- *What is the role of the gluon in the spectrum – **search for exotics?***
- *What is the origin of confinement, describing 99% of observed matter?*
- ***If QCD is correct and we understand it, expt. data must confront ab initio calculations***

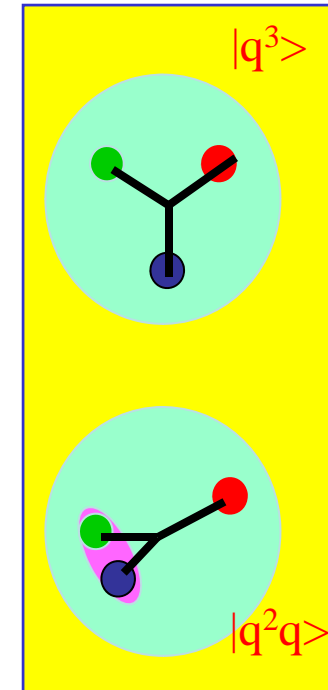
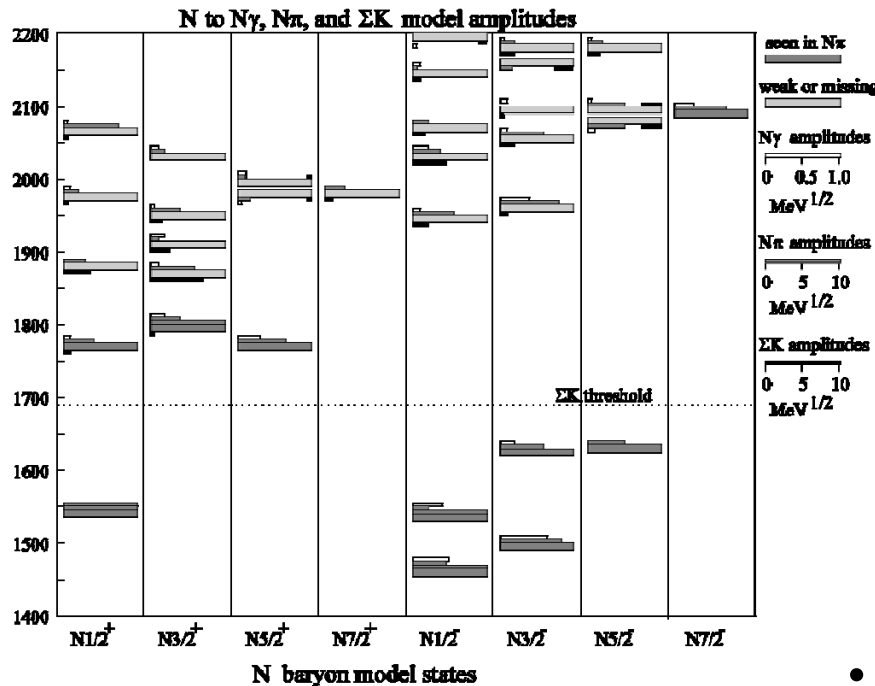


- **NSAC Performance Measures**

- *“Complete the combined analysis of available data on single  $\pi$ ,  $\eta$ , and  $K$  photo-production of nucleon resonances...” (HP3:2009)*
- *“Measure the electromagnetic excitations of low-lying baryon states (<2 GeV) and their transition form factors...” (HP12)*
- *“First results on the search for exotic mesons using photon beams will be completed” (HP15)*

# Spectroscopy - I

- Nucleon Spectroscopy: Quark model masses and amplitudes – states classified by isospin, parity and **spin**.

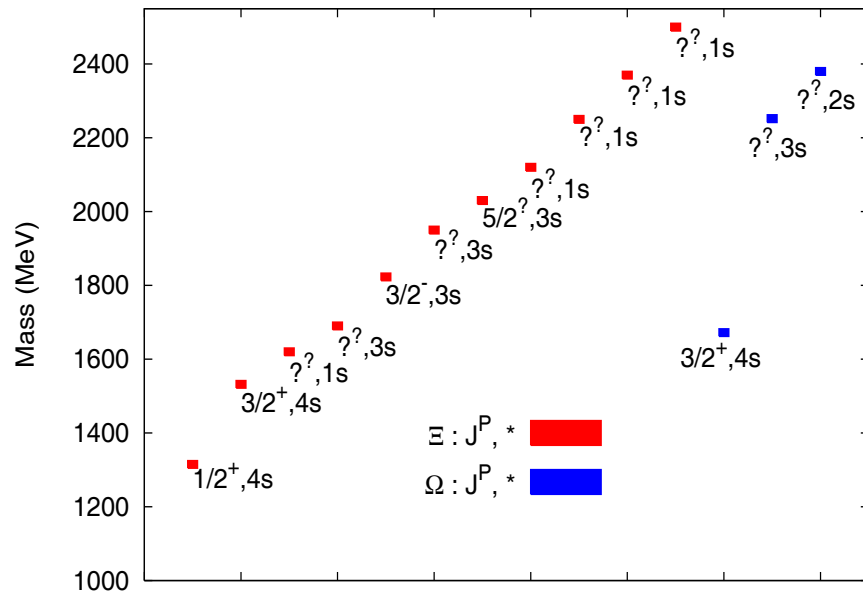


*Capstick and Roberts,  
PRD58 (1998) 074011*

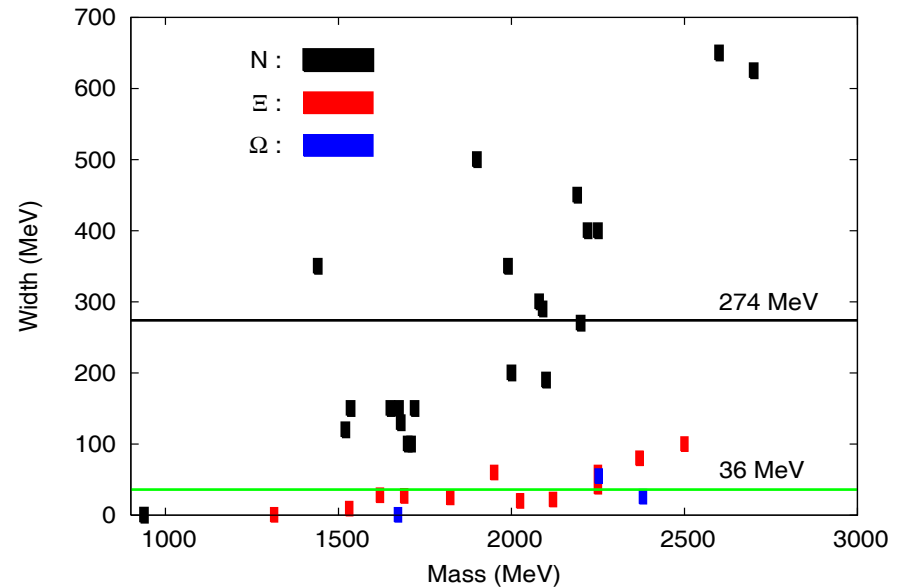
- Are states **Missing**, because our pictures are not expressed in correct degrees of freedom?
- Do they just not couple to **probes**?

# Discovery: cascade physics

Cascades (uss) are largely *terra incognita*



Thanks to N. Mathur



# Variational Method

- Extracting excited-state energies described in C. Michael, NPB 259, 58 (1985) and Luscher and Wolff, NPB 339, 222 (1990)
- Can be viewed as exploiting the *variational method*
- Given  $N \times N$  correlator matrix  $C_{\alpha\beta}(t) = \langle 0 | O_{\alpha}(t) O_{\beta}(0) | 0 \rangle$ , one defines the N *principal correlators*  $\lambda_i(t, t_0)$  as the eigenvalues of

$$C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)$$

- Principal effective masses defined from correlators plateau to lowest-lying energies

$$\lambda_i(t, t_0) \rightarrow e^{-E_i(t-t_0)} \left( 1 + O(e^{-\Delta E(t-t_0)}) \right)$$

**Eigenvectors**, with metric  $C(t_0)$ , are orthonormal and project onto the respective states

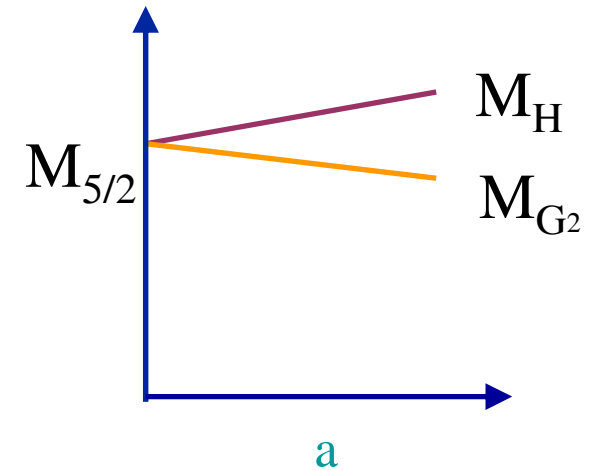
# Variational Method - II

- Spectrum on lattice looks different – states at rest classified by isospin, parity and **representation under cubic group**

Illustration	Name	Explicit form ( $ i  \neq  j  \neq  k $ )
	single-site	$\phi_{ABC}^F \varepsilon_{abc} \tilde{\psi}_{Aa\alpha} \tilde{\psi}_{Bb\beta} \tilde{\psi}_{Cc\gamma}$
	singly-displaced	$\phi_{ABC}^F \varepsilon_{abc} \tilde{\psi}_{Aa\alpha} \tilde{\psi}_{Bb\beta} (\tilde{D}_j^{(p)} \tilde{\psi})_{Cc\gamma}$
	doubly-displaced-I	$\phi_{ABC}^F \varepsilon_{abc} \tilde{\psi}_{Aa\alpha} (\tilde{D}_{-j}^{(p)} \tilde{\psi})_{Bb\beta} (\tilde{D}_j^{(p)} \tilde{\psi})_{Cc\gamma}$
	doubly-displaced-L	$\phi_{ABC}^F \varepsilon_{abc} \tilde{\psi}_{Aa\alpha} (\tilde{D}_j^{(p)} \tilde{\psi})_{Bb\beta} (\tilde{D}_k^{(p)} \tilde{\psi})_{Cc\gamma}$
	triply-displaced-T	$\phi_{ABC}^F \varepsilon_{abc} (\tilde{D}_{-j}^{(p)} \tilde{\psi})_{Aa\alpha} (\tilde{D}_j^{(p)} \tilde{\psi})_{Bb\beta} (\tilde{D}_k^{(p)} \tilde{\psi})_{Cc\gamma}$
	triply-displaced-O	$\phi_{ABC}^F \varepsilon_{abc} (\tilde{D}_i^{(p)} \tilde{\psi})_{Aa\alpha} (\tilde{D}_j^{(p)} \tilde{\psi})_{Bb\beta} (\tilde{D}_k^{(p)} \tilde{\psi})_{Cc\gamma}$

Lattice PWA

$J$	$n_{G_1}^J$	$n_{G_2}^J$	$n_H^J$
$\frac{1}{2}$	1	0	0
$\frac{3}{2}$	0	0	1
$\frac{5}{2}$	0	1	1
$\frac{7}{2}$	1	1	1
$\frac{9}{2}$	1	0	2
$\frac{11}{2}$	1	1	2
$\frac{13}{2}$	1	2	2
$\frac{15}{2}$	1	1	3
$\frac{17}{2}$	2	1	3



Extension to  $qqq \bar{q}q$

# A lattice theorist's view

DOE NP2012 milestone:

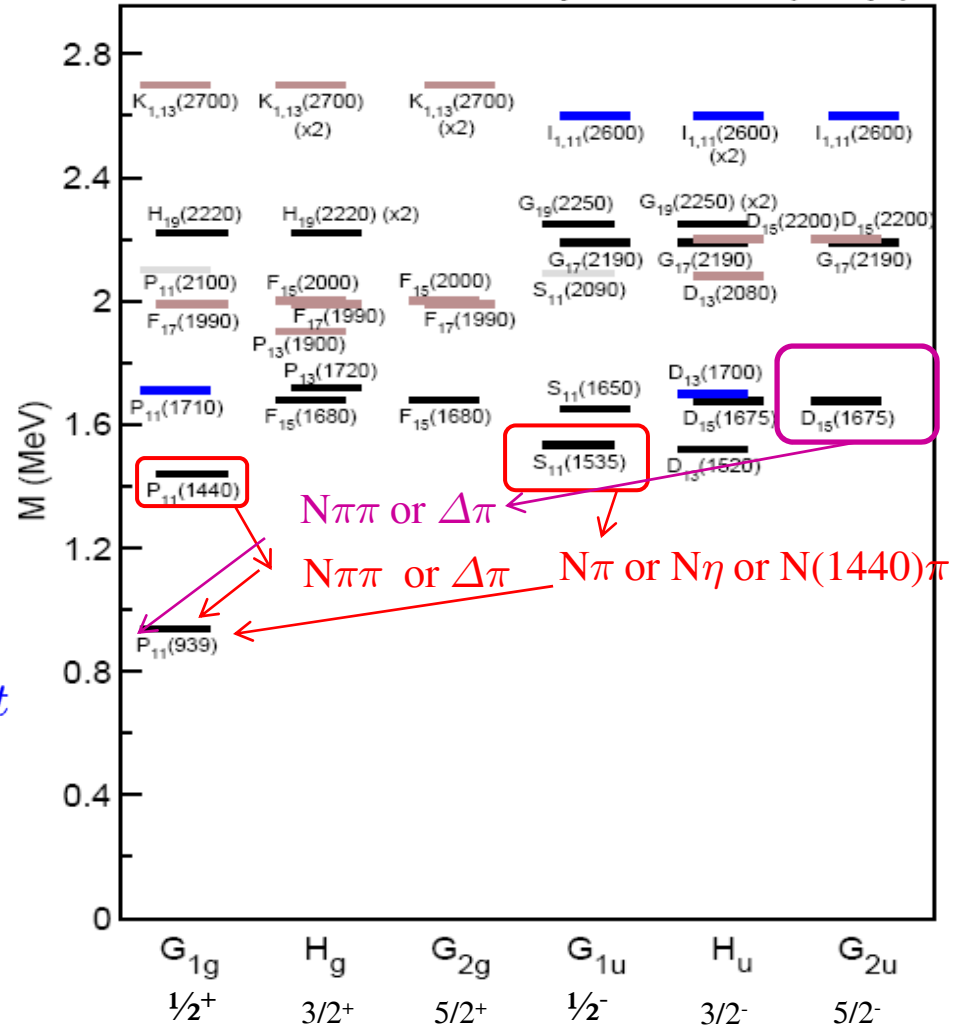
Spectrum & E&M transitions up to  $Q^2 = 7 \text{ GeV}^2$

- Challenges/opportunities:

- Compute excited energies
- Compute decays

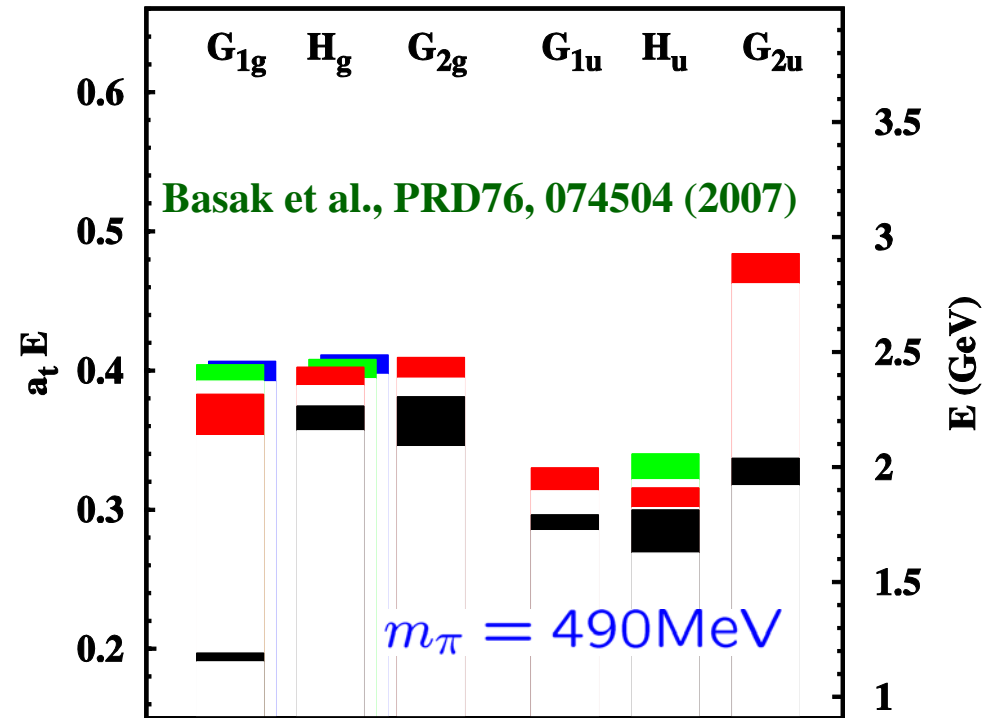
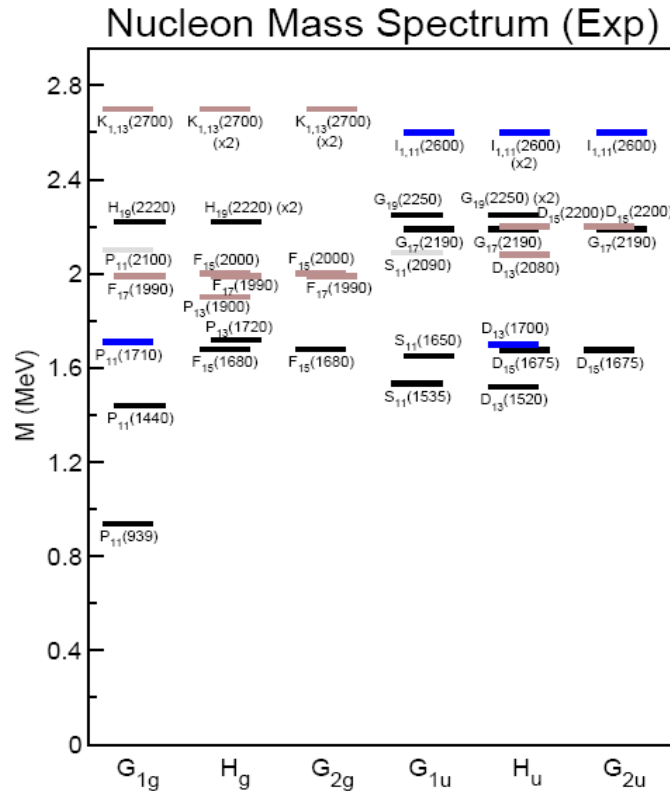
signal-to-noise  $\simeq e^{-(m_H - \frac{3}{2}m_\pi)t}$

Nucleon Mass Spectrum (Exp)



Thanks: R. Edwards

# Resonance Spectrum - Quenched

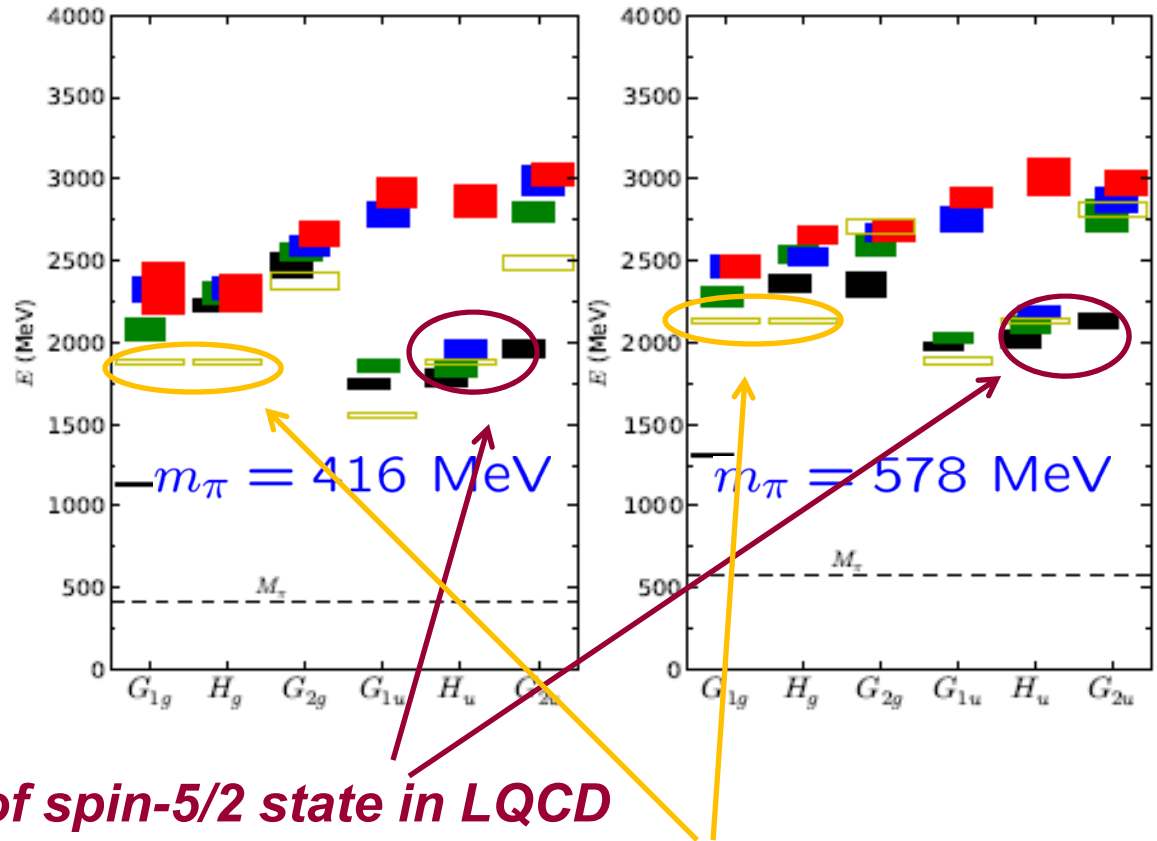
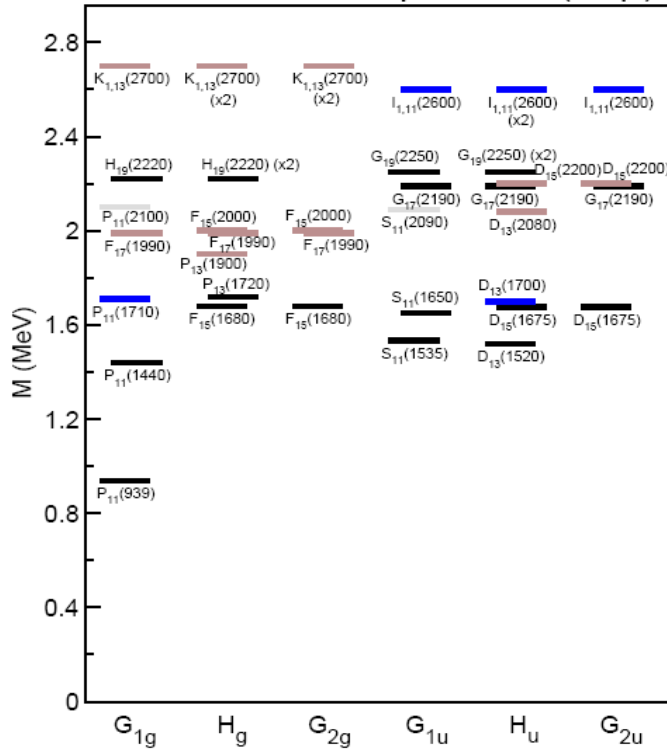


- Demonstration of our ability to extract nucleon resonance spectrum
- Hints of patterns seen in experimental spectrum
- Methodology central to remainder of project
- **Do not recover ordering of  $P_{11}$  and  $S_{11}$**

# Resonance Spectrum – Nf=2

N<sub>f</sub>=2: Hadron Spectrum Collab., Phys.Rev.D79:034505 (2009)

Nucleon Mass Spectrum (Exp)

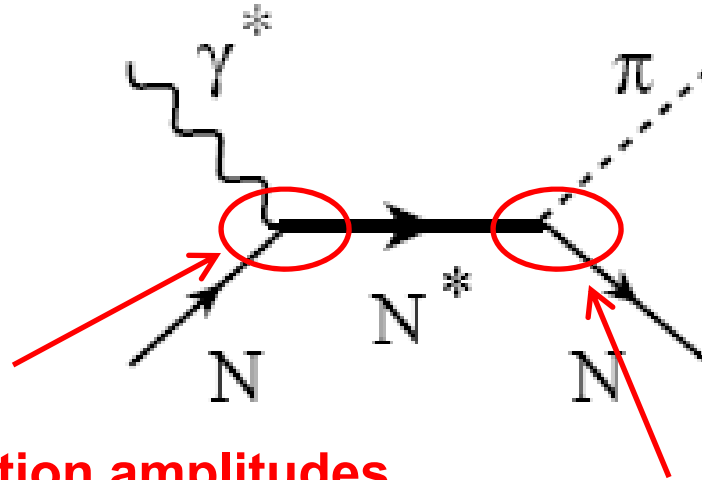


• First identification of spin-5/2 state in LQCD

Little evidence for multi-particle states

# EM Transitions and Lattice QCD

Example: Single-pion photoproduction

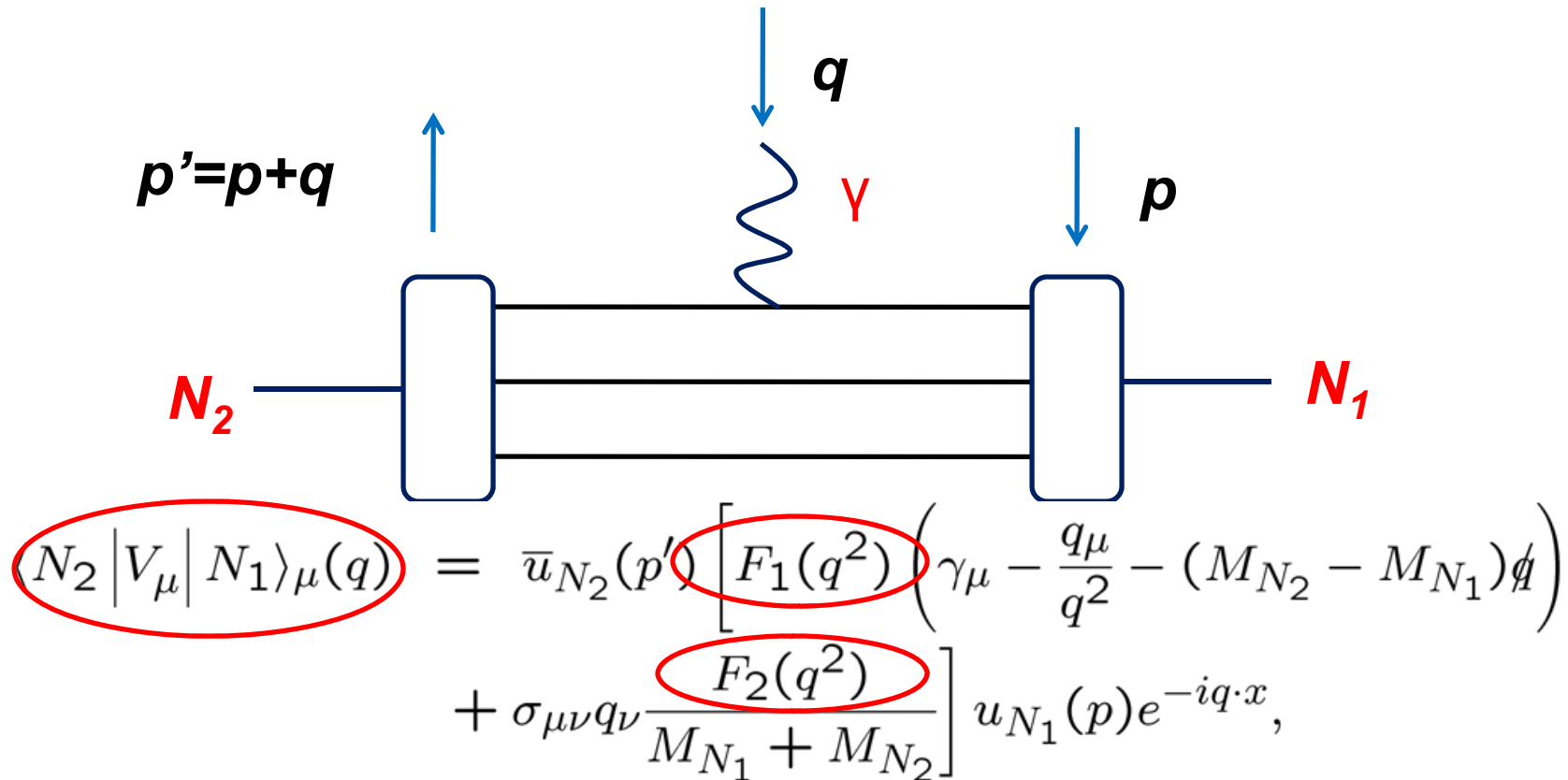


Radiative transition amplitudes

Axial-vector Couplings?

# Anatomy of a Calculation - I

- Lattice QCD computes the transition between isolated states



# Anatomy of a Calculation - II

$$\Gamma_{3\text{pt}}(\vec{p}, \vec{q}; t_f, t) = \sum_{\vec{x}, \vec{y}} \langle N_1(\vec{x}, t_f) V_\mu(\vec{y}, t) N_2(0) \rangle e^{-i\vec{p}\cdot\vec{x}} e^{-i\vec{q}\cdot\vec{y}}$$

Complete set of states

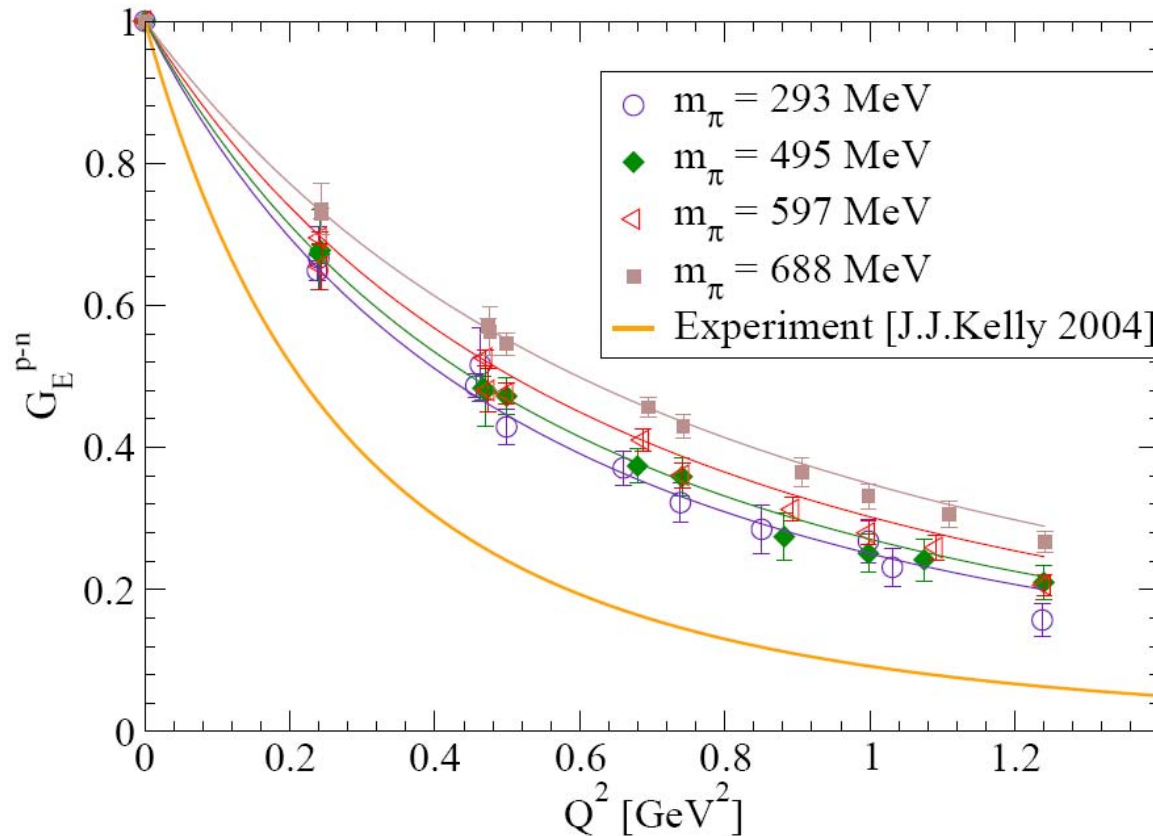
At large  $t_f - t$ , and  $t$ , correlator is dominated by *lowest lying state*

$$\langle 0 | N_1(0) | N_1, \vec{p} \rangle \langle N_1, \vec{p} | V_\mu(0) | N_2, \vec{p} + \vec{q} \rangle \langle N_2, \vec{p} + \vec{q} | \phi^\dagger | 0 \rangle e^{-E(\vec{p} + \vec{q})(t - t_i)} e^{-E(\vec{p})(t_f - t)}$$

Lattice calculations of electromagnetic properties of some lowest-lying states well established, eg:

- EM form factors of nucleon and pion
- Moments of GPDs in DVCS for nucleon
- N-Delta transition Form Factor

# Isovector Form Factor



J.D.Bratt et al (LHPC),  
arXiv:0810.1933

Euclidean lattice: form  
factors in space-like  
region

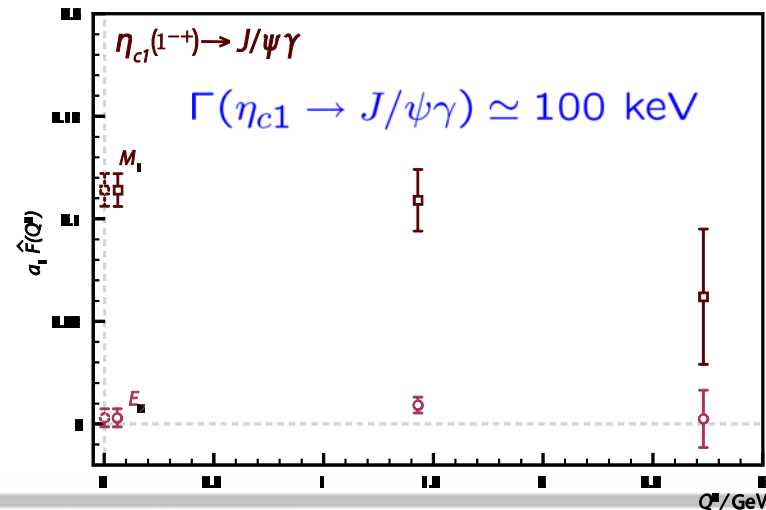
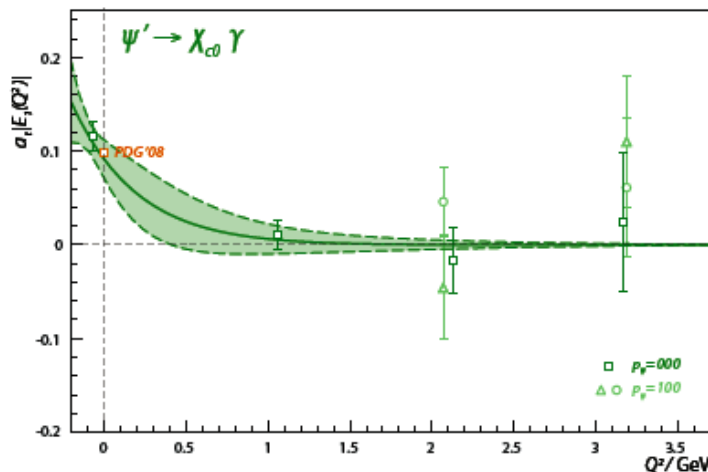
Extension to higher  $Q^2$

# Transitions to “Radial” excitations

- At currently accessible masses, the delta is **Stable and lowest energy level** in its respective channel
- Transitions to resonances: use variational method to obtain interpolating operator to  $n^{\text{th}}$  excited state

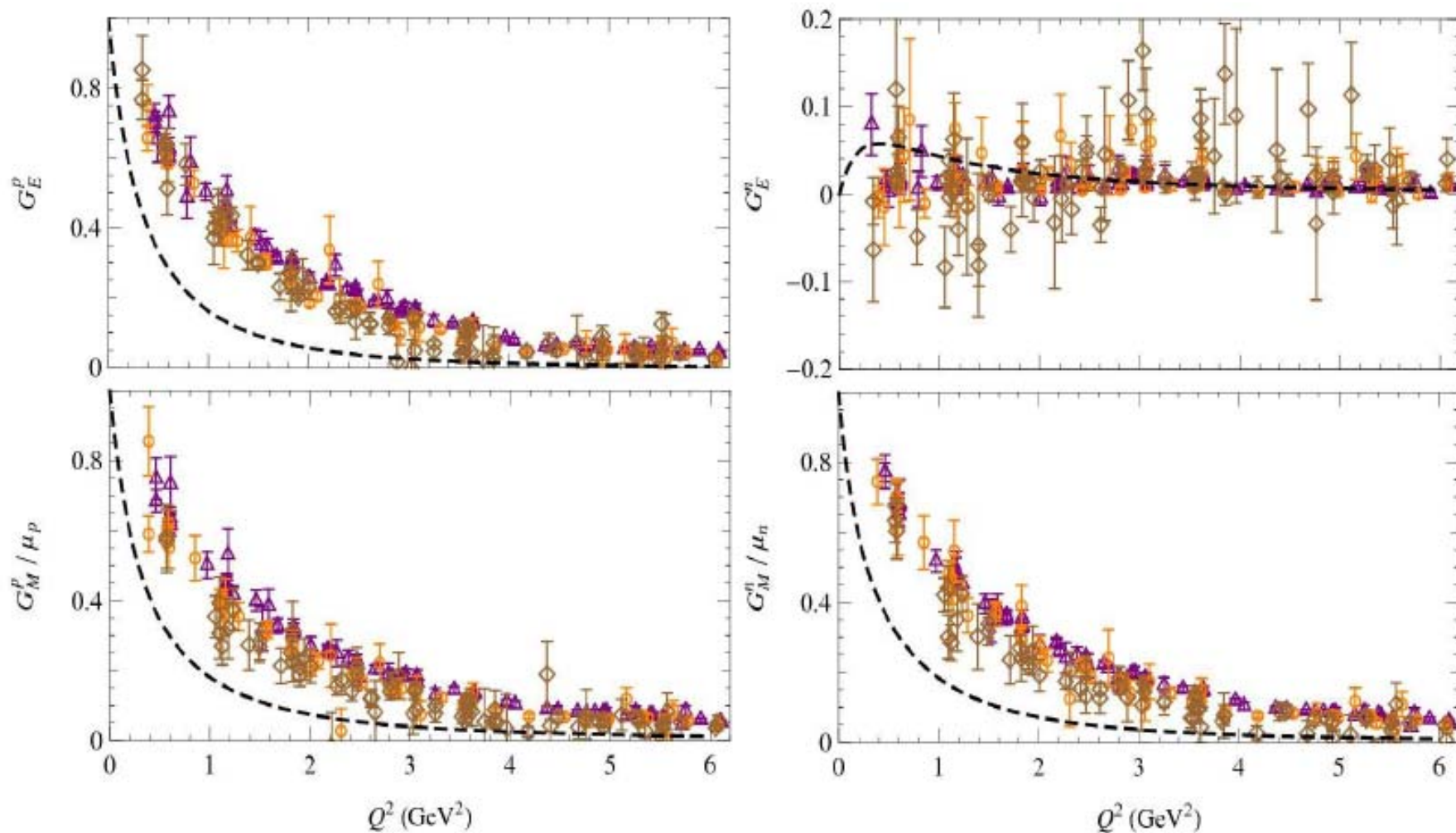
$$\tilde{N}^n = \sum_j A_j^n N_j$$

J Dudek, R Edwards, C Thomas,  
arXiv:0902.2241, PRD



# Nucleon Form-Factors

- $N_f=0$  anisotropic lattices,  $M_\pi = 480, 720, 1080$  MeV



# Nucleon Radiative Transition - I

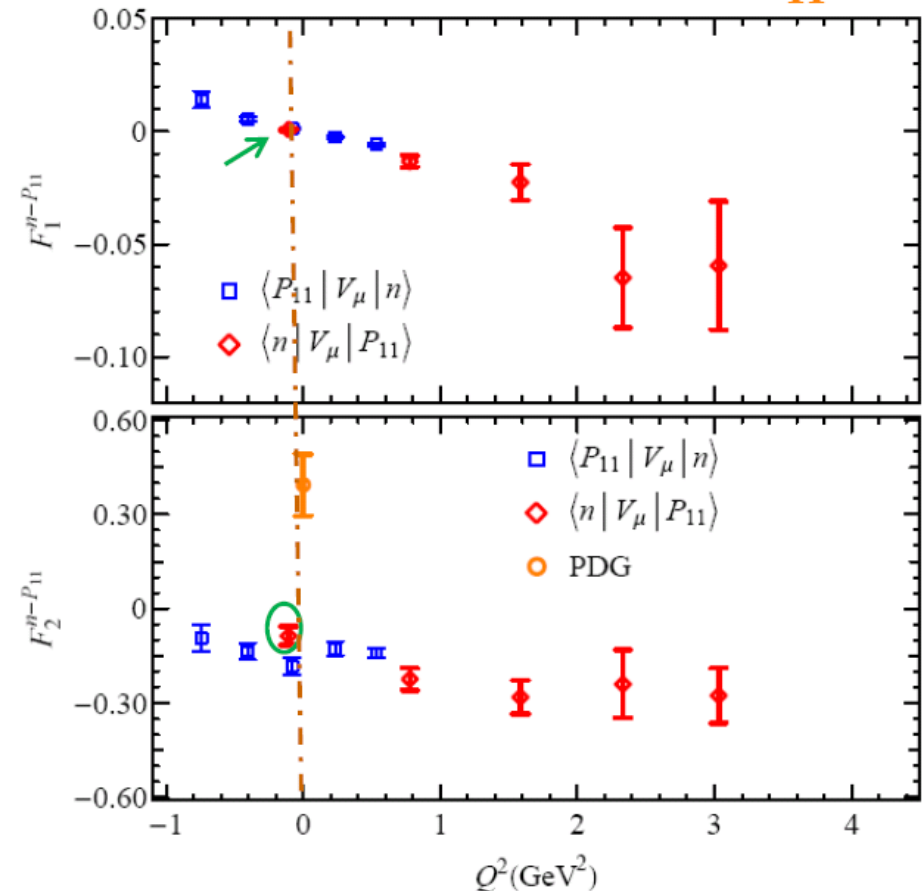
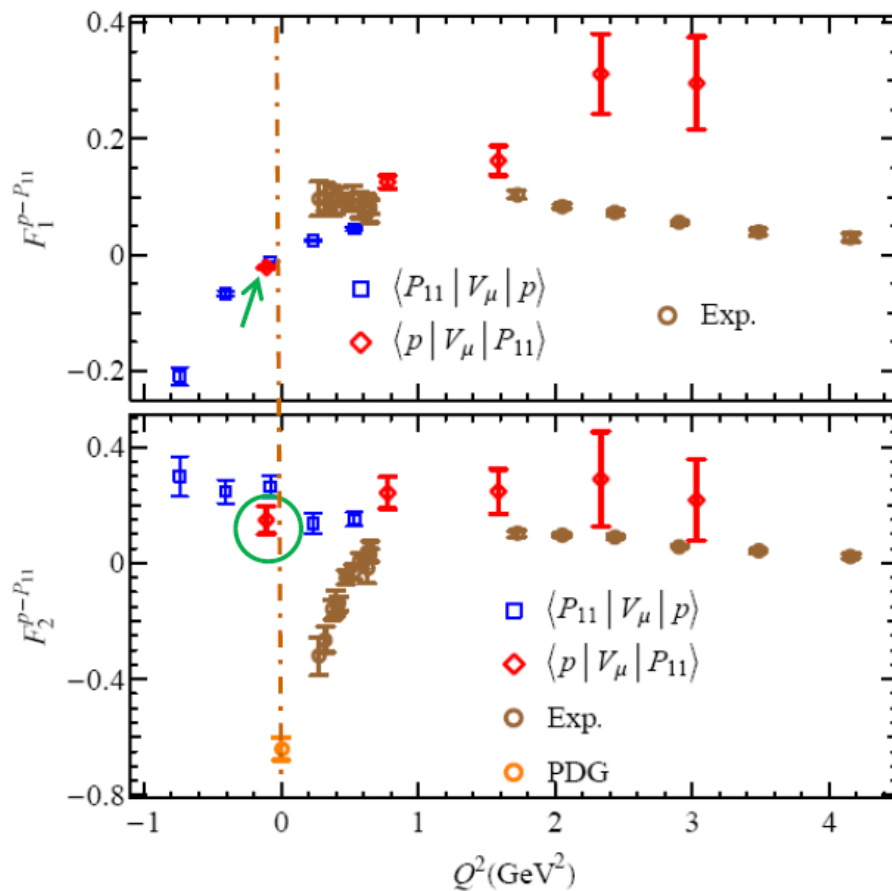
$N_f=0$  exploratory:  $P_{11} \rightarrow$  Nucleon transition

H-W Lin et al.,  
Phys.Rev.D78:114508 (2008).

Proton- $P_{11}$

720 MeV Pion

Neutron- $P_{11}$



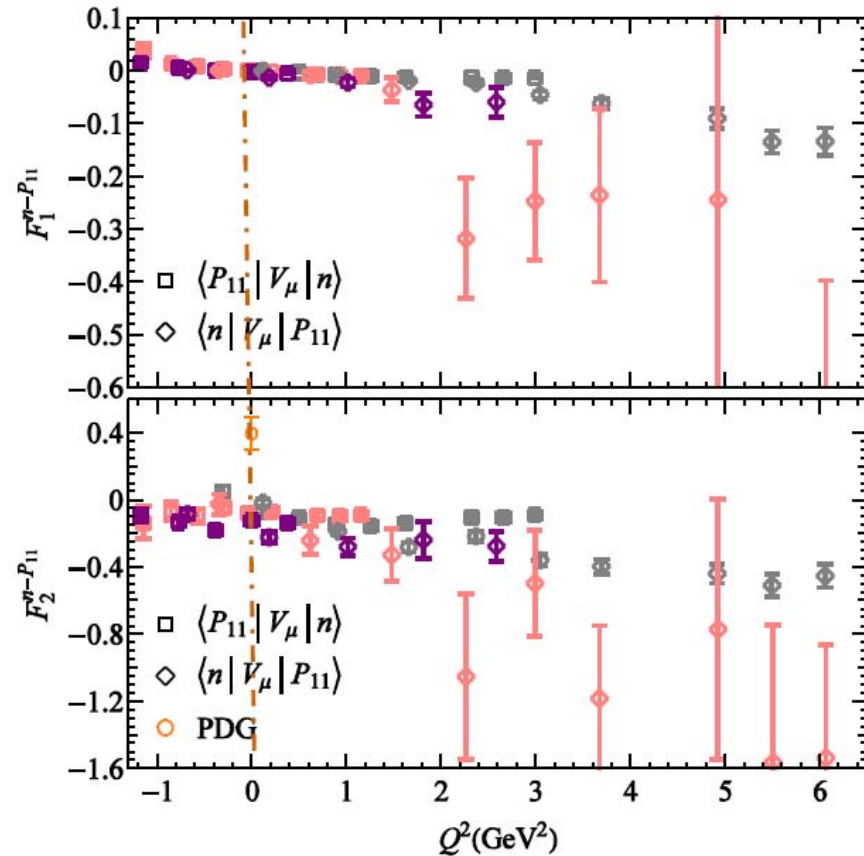
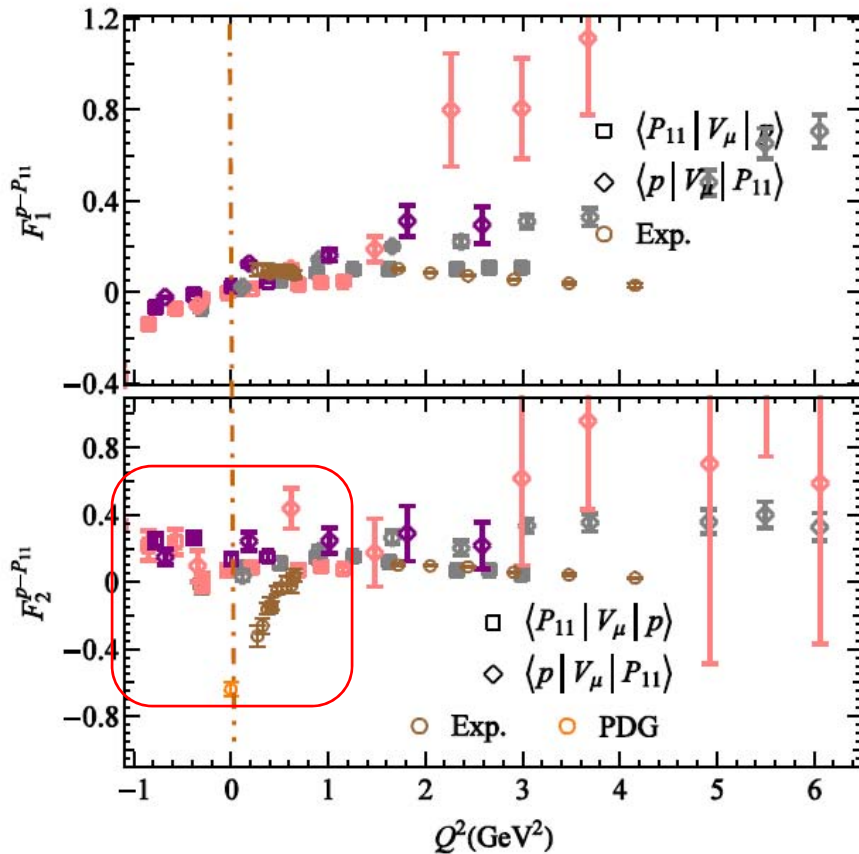
# Nucleon Radiative Transition - II

Excited transition: large “pion cloud” effects → small mass

Proton- $P_{11}$

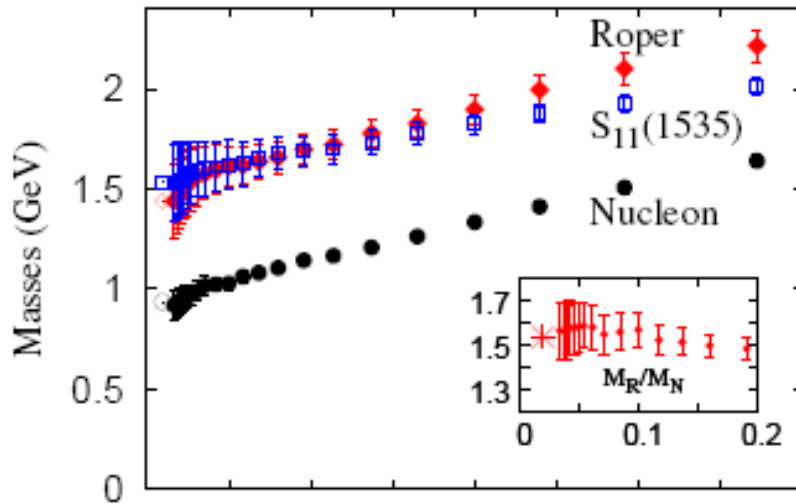
$m_\pi = 480, 720, 1100$  MeV

Neutron- $P_{11}$



H-W Lin et al., Lattice 2008

# Roper Resonance

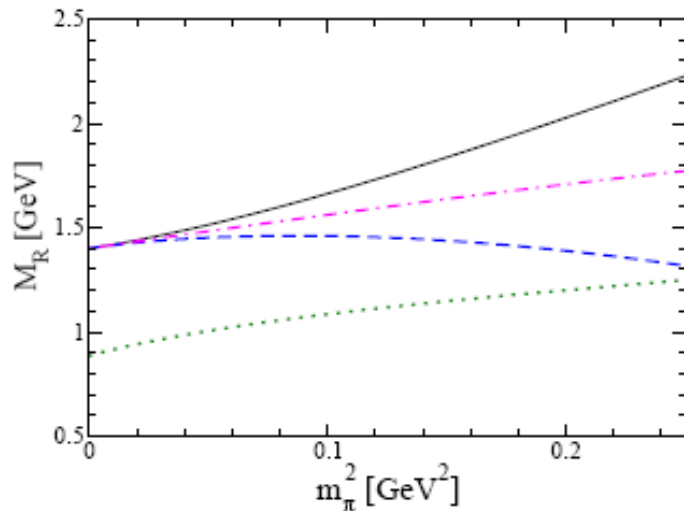


- Bayesian statistics and constrained curve fitting
- Used simple three-quark operator

Dong et al., PLB605, 137 (2005)

Borasoy et al., Phys.Lett. B641 (2006) 294-300

We have not recovered expt ordering



# Future Challenges

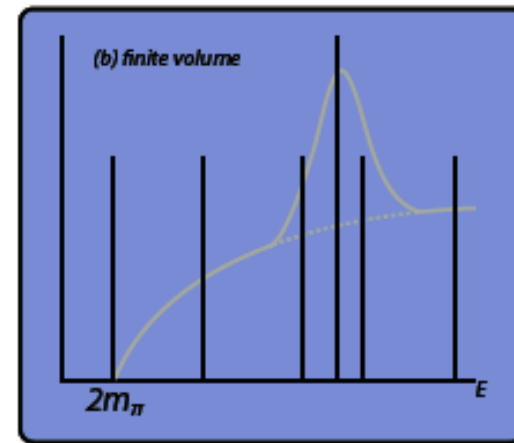
---

- Unstable Resonances
- Lattices for spectroscopy
- Identification of spin

# Unstable resonances

*Thanks to Jo Dudek*

- In QCD, even  $\rho$  is unstable under strong interactions – *resonance in  $\pi$ - $\pi$  scattering (quenched QCD not a theory – won't discuss).*
- Spectral function continuous; finite volume yields discrete set of energy eigenvalues



Momenta quantised: known set of free-energy eigenvalues

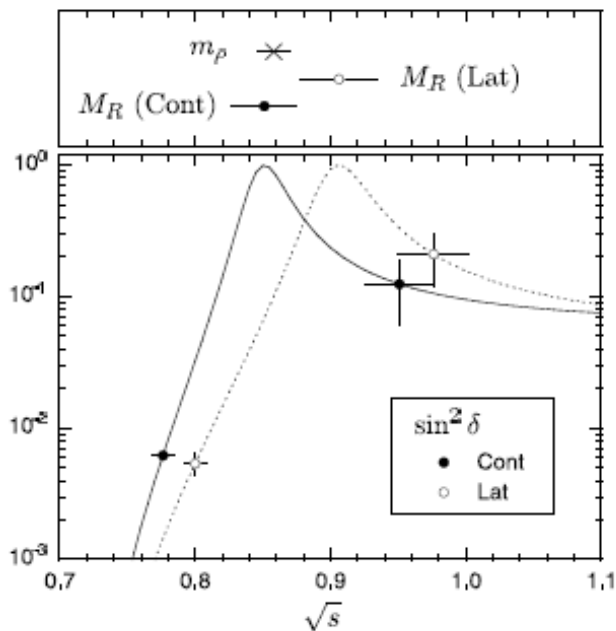
$$E_n = 2\sqrt{m_\pi^2 + \left(\frac{2n\pi}{L}\right)^2}$$

# Unstable Resonance - II

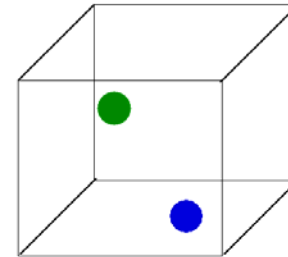
- For interacting particles, energies are shifted from their free-particle values, by an amount that depends on the energy.
- Luscher: relates shift in the free-particle energy levels to the phase shift at the corresponding E.

Breit-Wigner fit

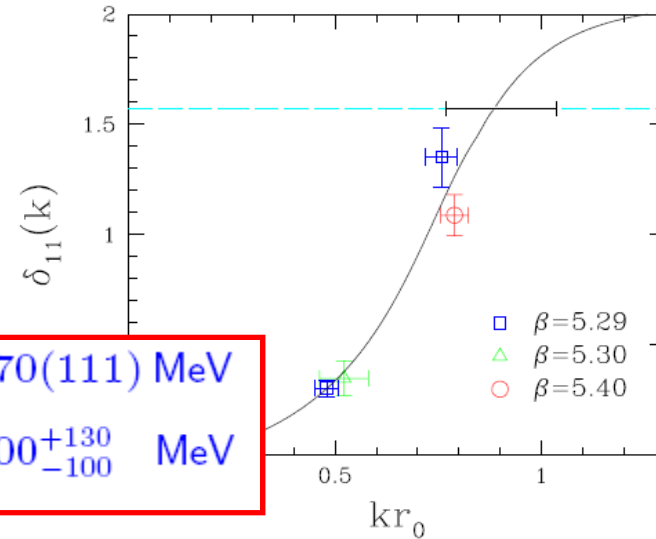
CP-PACS, arXiv:0708.3705



$$\delta E(L) \leftrightarrow \delta(E)$$



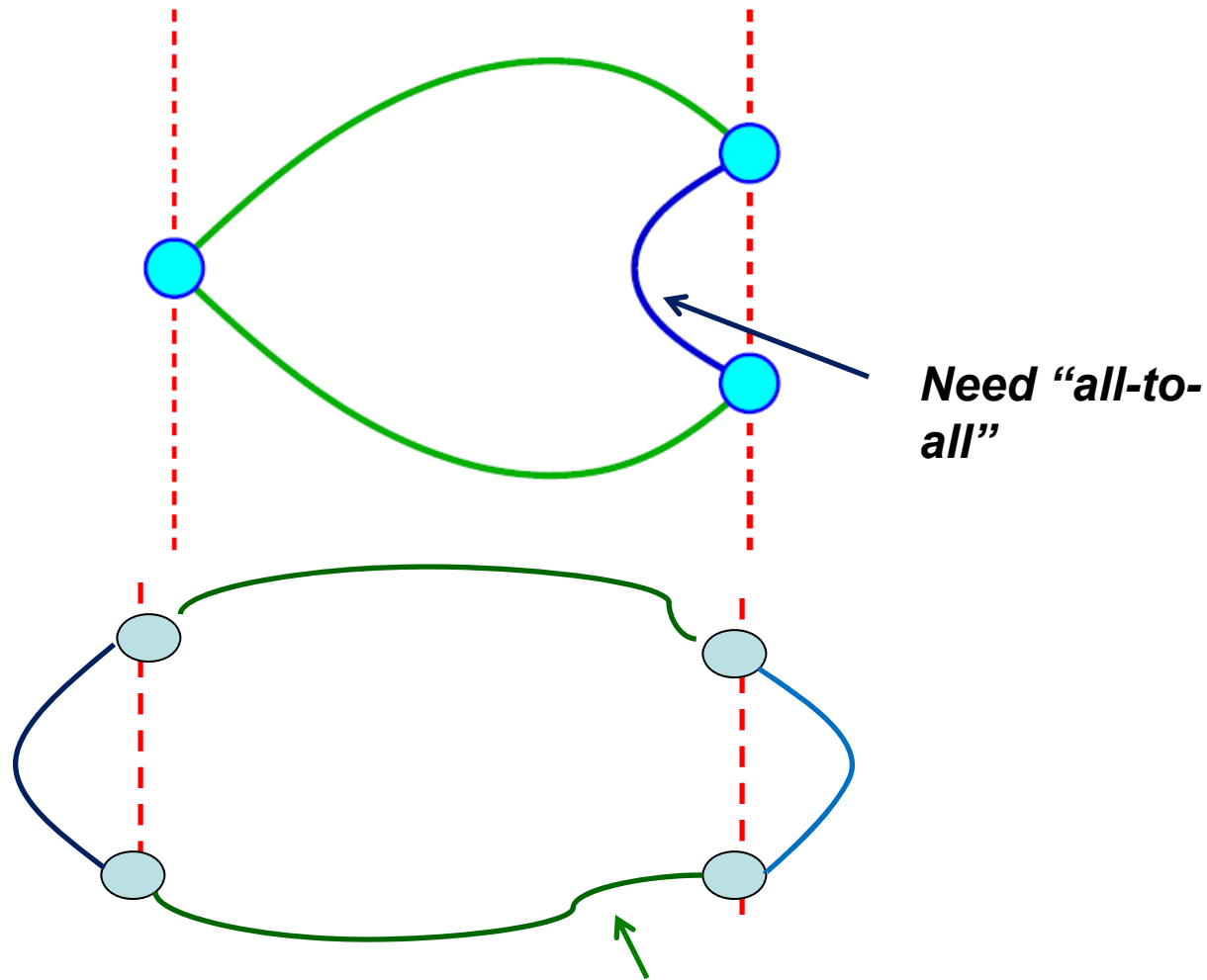
QCDSF, 2008



$$m_\rho = 770(111) \text{ MeV}$$

$$\Gamma_\rho = 200^{+130}_{-100} \text{ MeV}$$

# Multi-hadron Operators



Usual methods give "point-to-all"

# Correlation functions: Distillation

- Use the new “distillation” method.
- Observe

*Eigenvectors of  
Laplacian*

$$L^{(J)} \equiv \left(1 - \frac{\kappa}{n} \Delta\right)^n = \sum_{i=1} f(\lambda_i) v^{(i)} \otimes v^{*(i)}$$

- Truncate sum at sufficient  $i$  to capture relevant physics modes – we use 64: set “weights”  $f$  to be unity
- Meson correlation function

*Includes displacements*

$$C_M(t, t') = \langle 0 | \bar{d}(t') \Gamma^B(t') u(t') \bar{u}(t) \Gamma^A(t) d(t) | 0 \rangle$$

↑   ↑   ↑   ↑

- Decompose using “distillation” operator as

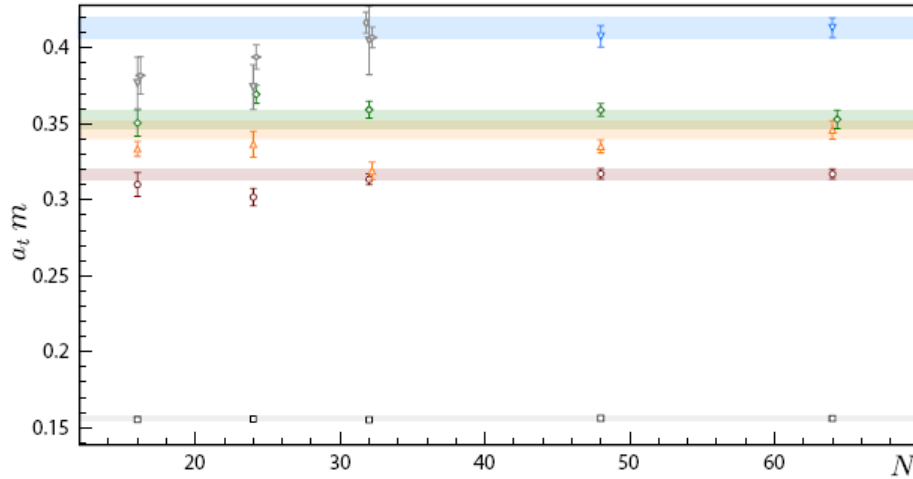
M. Peardon *et al.*,  
arXiv:0905.2160  
where

$$C_M(t, t') = \text{Tr} \langle \phi^A(t') \tau(t', t) \Phi^B(t) \tau^\dagger(t', t), \rangle$$

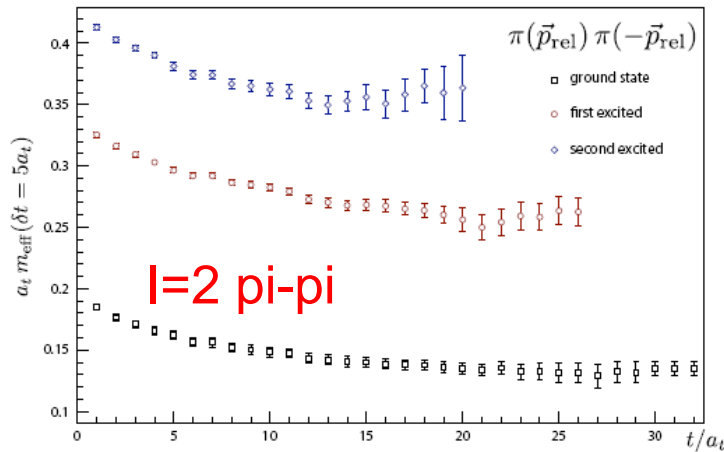
**Perambulators**  $\longrightarrow$

$$\begin{aligned} \Phi_{\alpha\beta}^{A,ij} &= v^{*(i)}(t) [\Gamma^A(t) \gamma_5]_{\alpha\beta} v^{(j)}(t') \\ \tau_{\alpha\beta}^{ij}(t, t') &= v^{*(i)}(t') M_{\alpha\beta}^{-1}(t', t) v^{(j)}(t). \end{aligned}$$

# Distillation Results



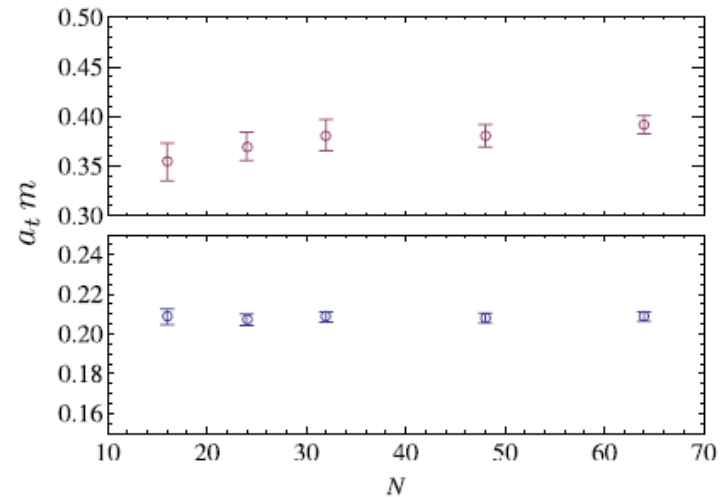
Nucleon Variational Analysis



I=2 pi-pi

## $\rho$ Variational Analysis

Errors < 3%



Overall momentum 0  
Basis: pairs of back-to-back operators at momentum  $p$

# Anisotropic Clover Generation - I

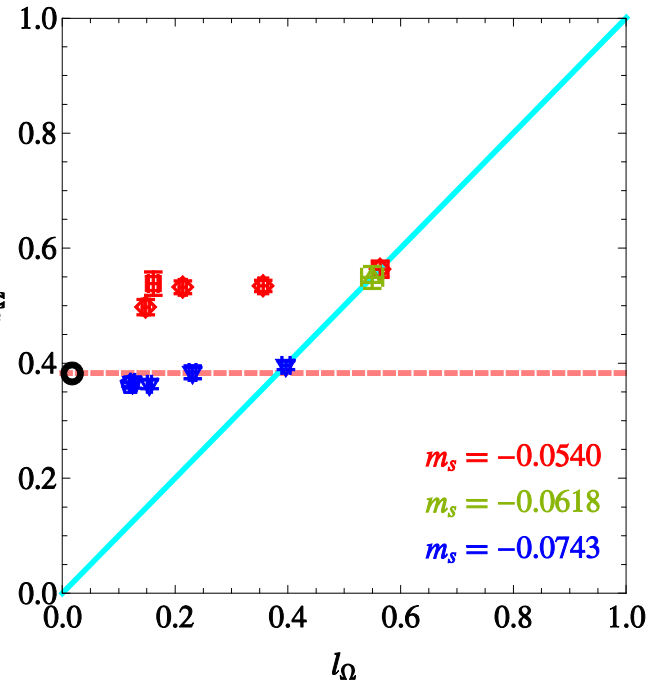
- “Clover” Anisotropic lattices  $a_t < a_s$ : major gauge generation program under INCITE and discretionary time at ORNL designed for spectroscopy

Challenge: setting scale and strange-quark mass

$$s_X = (9/4)[2m_K^2 - m_\pi^2]/m_X^2$$

Omega

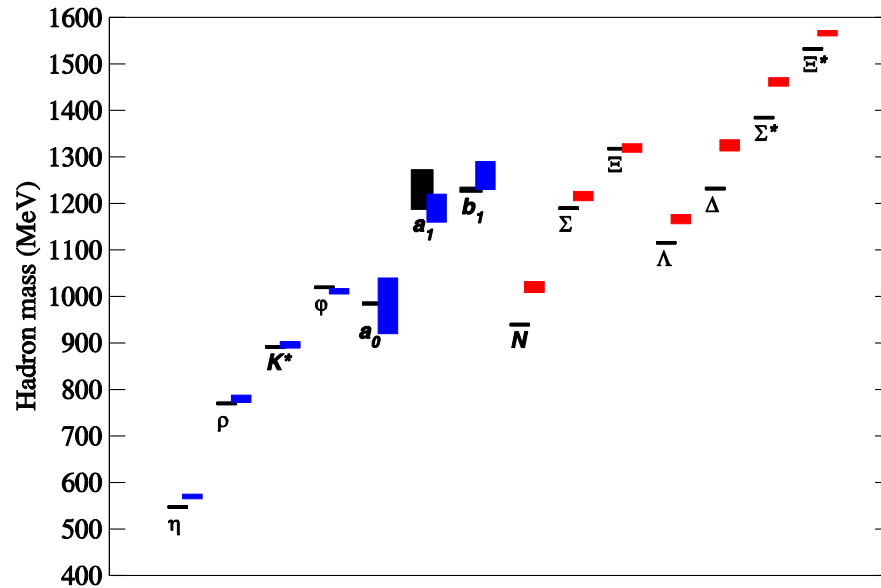
Express physics in (dimensionless)  
(l,s) coordinates



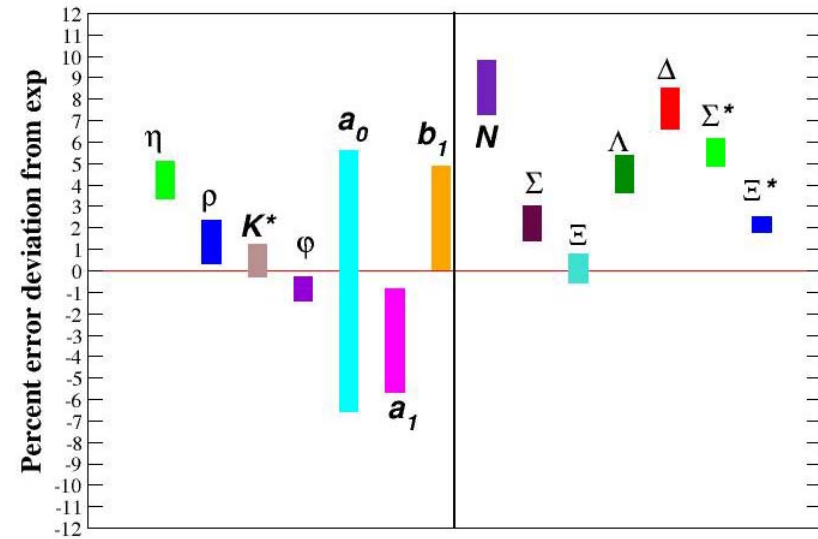
$$l_X = (9/4)m_\pi^2/m_X^2$$

H-W Lin et al (Hadron Spectrum Collaboration),  
PRD79, 034502 (2009 )

# Anisotropic Clover – II



$N_f=2+1$  Hadron Spectrum: NN Leading Order Extrapolation



$L_s$ (fm)	2.45fm	2.95fm	3.93fm	4.91fm
$m_\pi$ (MeV)	$20^3 \times 128$	$24^3 \times 128$	$32^3 \times 256$	$40^3 \times 256$
833	6k, TACC[1.0M](10)			
560	7k, TACC[1.5M](6.7)			
448	8k, TACC[2.1M](5.4)			
383	13k, done	13k, done	11k, Tenn[22M](7.4)	
230		6k, PSC[6M](3.2)	11k, ORNL[70M](4.2)	
140				11k, INCITE[390M](3.4)

Two volumes

# Identification of Spin

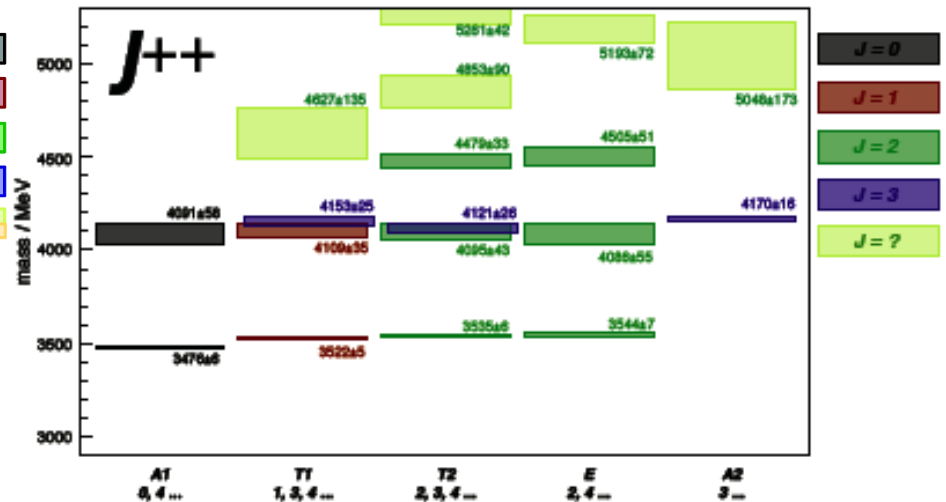
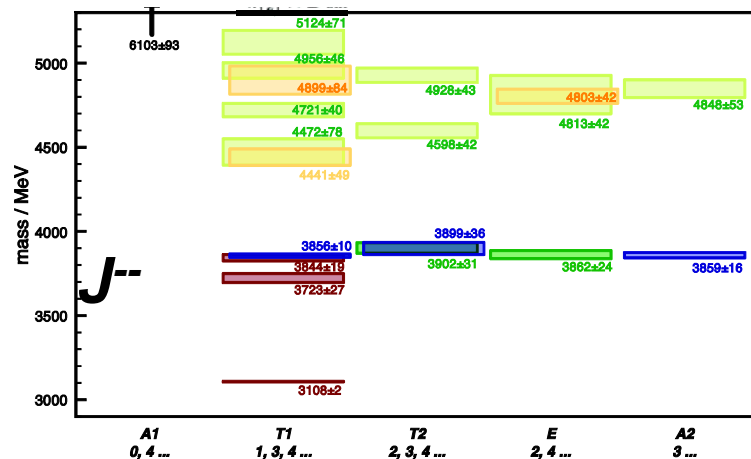
Many overlap resonances in each irrep – **degeneracies in continuum challenging**

→ *Operators with known continuum behavior*

$$\langle 0 | \psi \gamma^i D^j \psi | | 2^{++}(\vec{0}, r) \rangle = Z \epsilon^i(\vec{0}, r).$$

$$\langle 0 | (\rho \times \nabla)_{T_2}^i | 2^{++}(\vec{0}, r) \rangle = |\epsilon_{ijk}| \langle 0 | \psi \gamma^j D^k \psi | 2^{++}(\vec{0}, r) \rangle = Z |\epsilon_{ijk}| \epsilon^{jk}(\vec{0}, r)$$

$$\langle 0 | (\rho \times \nabla)_E^i | 2^{++}(\vec{0}, r) \rangle = Q_{ijk} \langle 0 | \psi \gamma^j D^k \psi | 2^{++}(\vec{0}, r) \rangle = Z Q_{ijk} \epsilon^{jk}(\vec{0}, r).$$



Extension to baryons in progress

*"Charmonium excited state spectrum in lattice QCD"*  
(JJD, RGE, DGR, Nilmani Mathur) *Phys.Rev.D77:034501,2008*

# Conclusions

---

- Lattice calculations evolving from studies of properties of ground-state hadrons to those of resonances
- Variational method enabling us to isolate not only ground state, but first few excited states
  - Good interpolating operators → electromagnetic properties
  - Major progress: **P11 transition**
- Major effort supported by USQCD:
  - Generation of Lattices
  - Development of new methods for computing correlators (“distillation”, “dilution”)
- Challenges:
  - **Identification of spins**
  - **Delineating the single- and multiparticle states**
  - **Transition Form Factors at higher  $Q^2$**
  - **Mapping to Chiral Perturbation Theory**

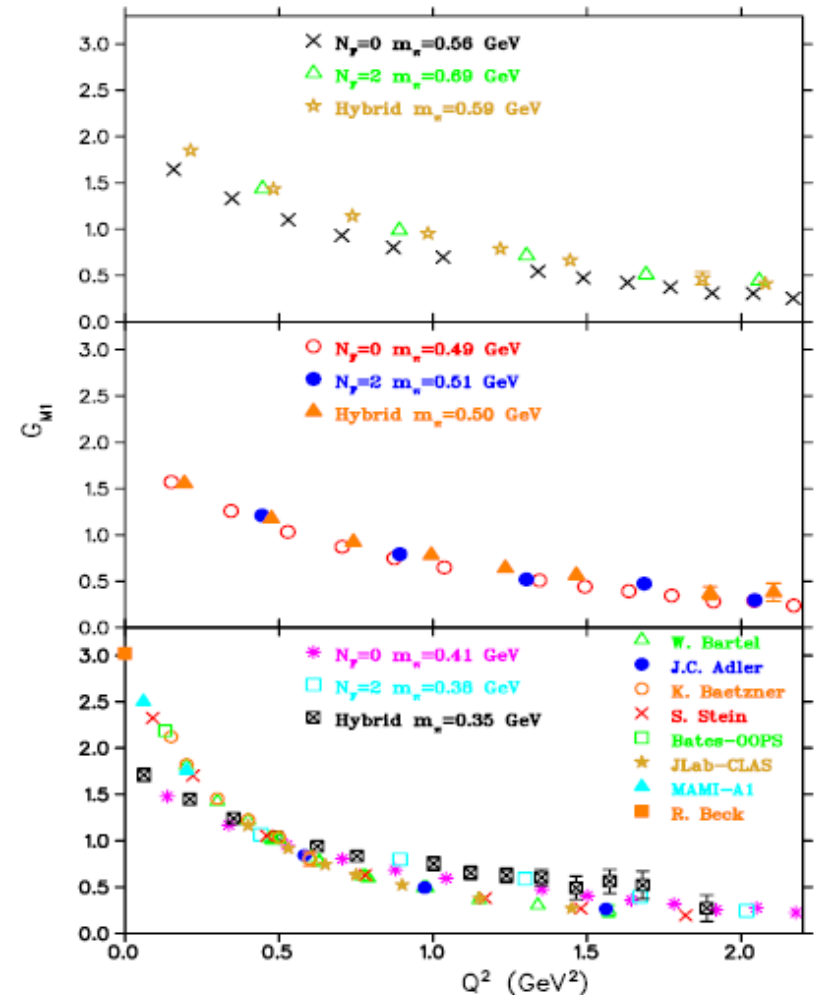
# BACKUP

---

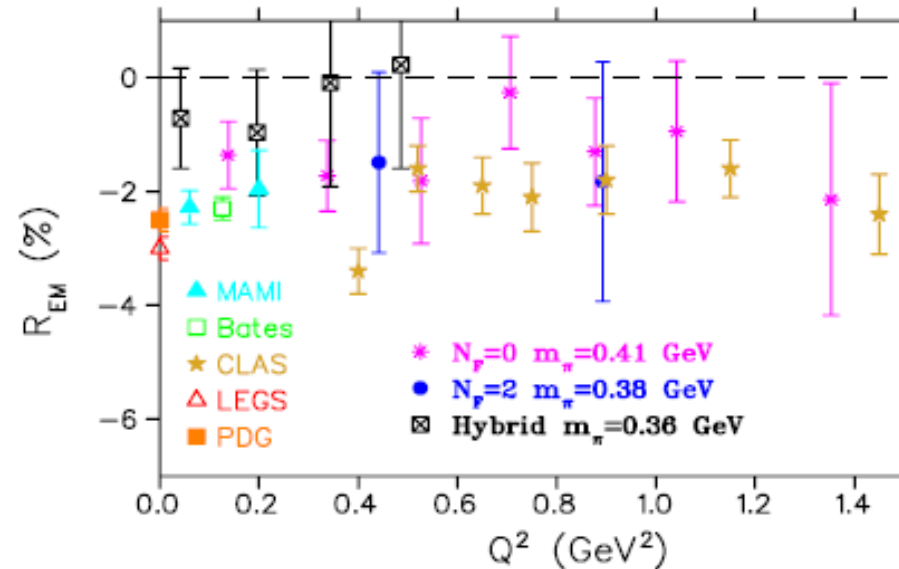
# N- $\Delta$ Transition Form Factor - I

- Transition between lowest lying  $I=3/2, J=3/2$  ( $\Delta$ ), and  $I=1/2, J=1/2$  (N)
- Comparison between different lattice calculations and expt.
  - Milder  $Q^2$  dependence than experiment but
  - *Quark masses corresponding to pion masses around 350 MeV*
  - *$Q^2$  range up to around 2 GeV<sup>2</sup>*

*Alexandrou et al, arXiv:0710.4621*

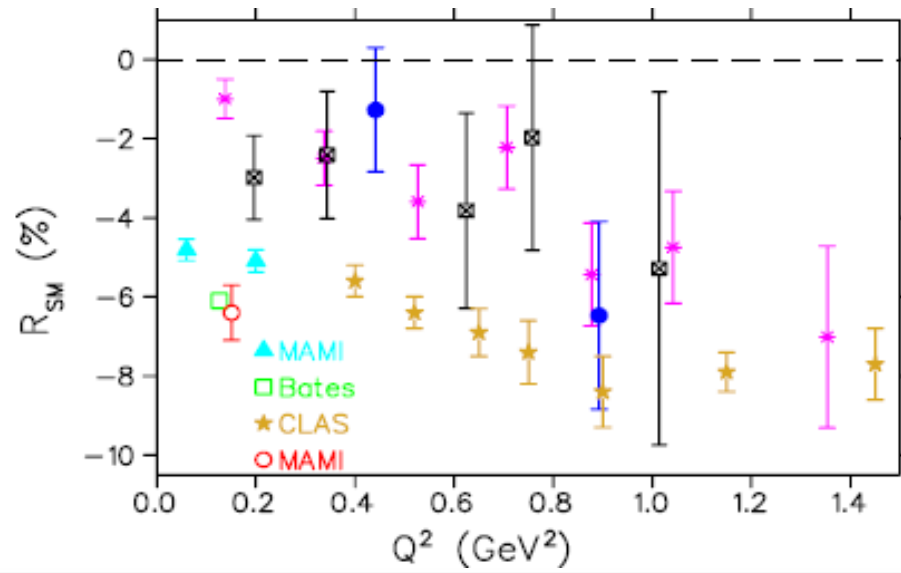


# N- $\Delta$ Transition Form Factor - II



$R_{EM} \rightarrow +1$

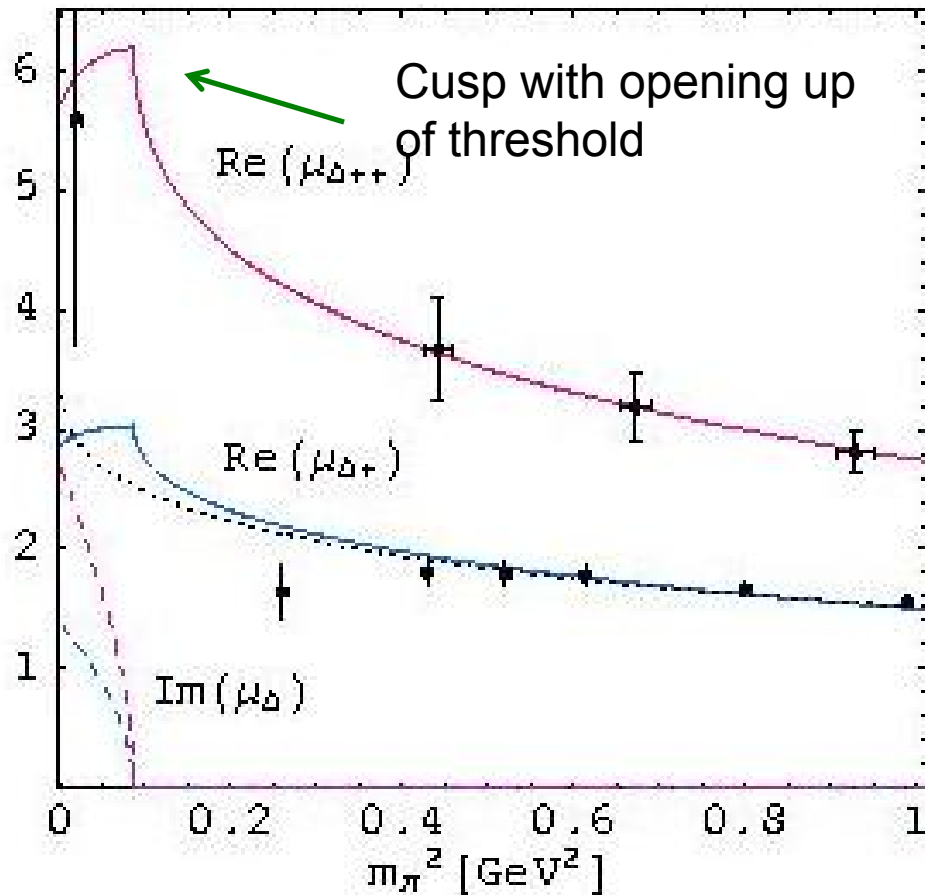
*Alexandrou et al, arXiv:0710.4621*



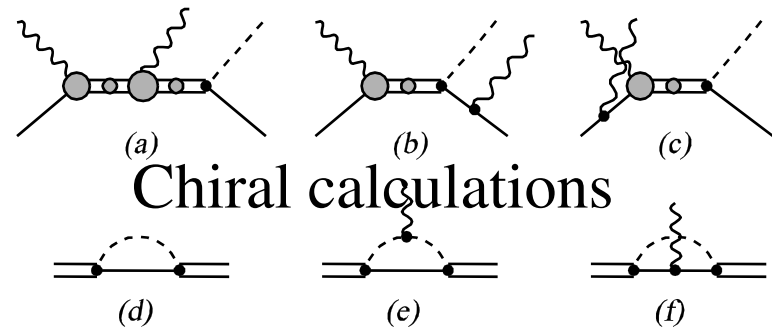
Deformation in nucleon or delta

# Entering resonance region

Current calculations:  $\Delta$  *stable* – P-wave decay suppressed at  $p = 0$



Lattice points from  
**Leinweber (1992)**  
**Cloet, Leinweber, Thomas (2003)**  
**Lee *et al.* (2004)**

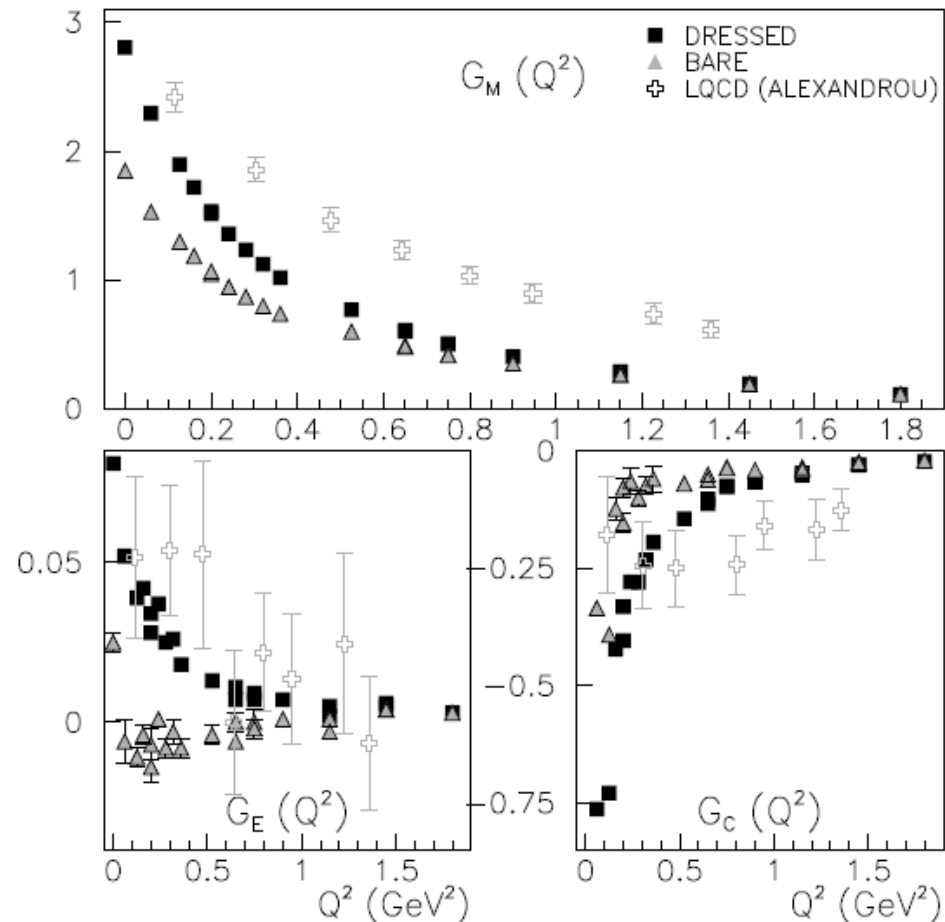


**Pascalutsa, Vanderhaeghen (2004)**  
**Thomas, Young (...)**

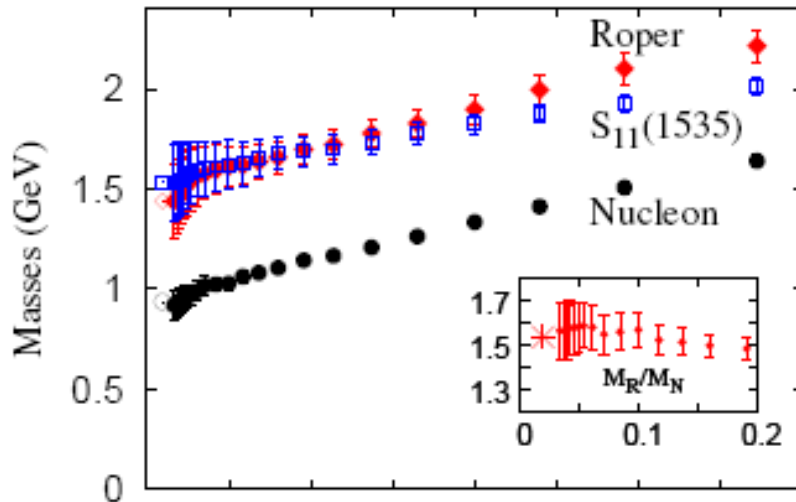
# Interpretation of Parameters

Julia-Diaz et al., Phys.Rev.  
C75 (2007) 015205

Comparison of LQCD, EFT +  
expt: **lattice QCD can vary**  
**quark masses**

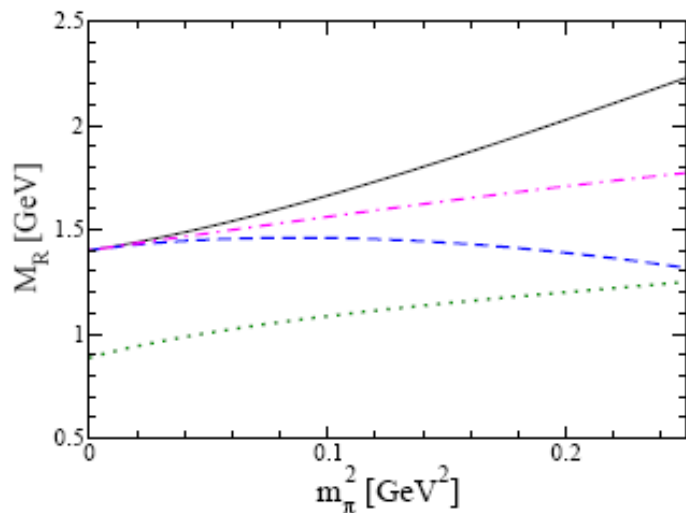


# Roper Resonance



- Bayesian statistics and constrained curve fitting
- Used simple three-quark operator

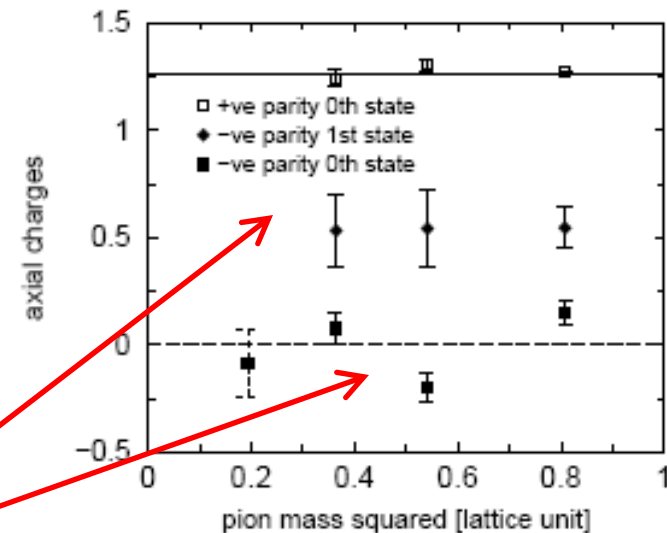
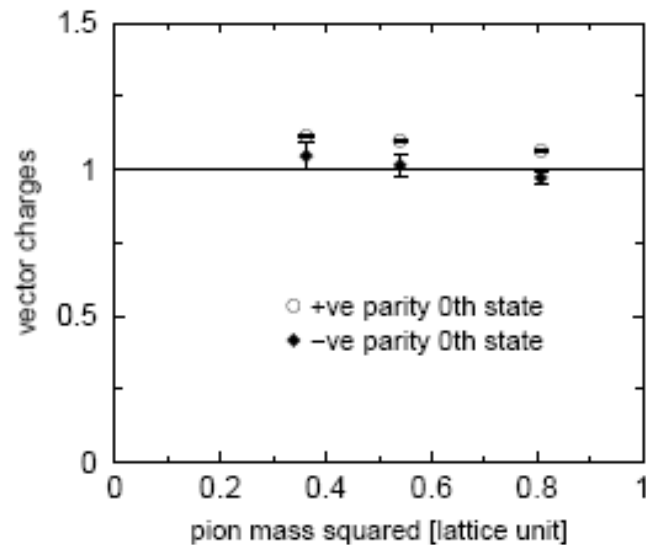
Dong et al., PLB605, 137 (2005)



Borasoy et al., Phys.Lett. B641 (2006) 294-300

# Axial-vector Charges

- The axial-vector charges  $g_A^{N1 N2}$  can provide additional insight into hadron structure
- Recent calculation of axial-vector charges of two lowest-lying  $\frac{1}{2}$ -states, associated with N(1535) and N(1650).



[Takahashi, Kunihiro,](#)  
[arXiv:0801.4707](#)

Consistent with NR quark model