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# Analytic Properties of Karlsruhe-Helsinki Type PWA

Mikko Sainio

Helsinki Institute of Physics

P.O. Box 64, FI-00014 University of Helsinki, Finland

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# Contents

- Generalities of $\pi$N interaction
- The Karlsruhe analysis
- A note of the CMU-LBL analysis
- Recent activity with the expansion techniques
- Conclusions

# Generalities of $\pi$N interaction

The pion-nucleon amplitude can be presented as

$$T_{\pi N} = \bar{u}'[A(\nu,t) + \frac{1}{2}\gamma^{\mu}(q+q')_{\mu}B(\nu,t)]u$$

where $$\nu = \frac{s-u}{4m} = \omega + \frac{t}{4m}.$$

$$C(\nu,t) = A(\nu,t) + \frac{\nu}{(1-t/4m^2)}B(\nu,t)$$

Optical theorem: $\mathrm{Im}\, C(\omega, t=0) = k_{\,\mathrm{lab}}\,\sigma$

Isospin: $C^{\pm} = \frac{1}{2}(C_{\pi^- p} \pm C_{\pi^+ p})$

**Basic principles:**

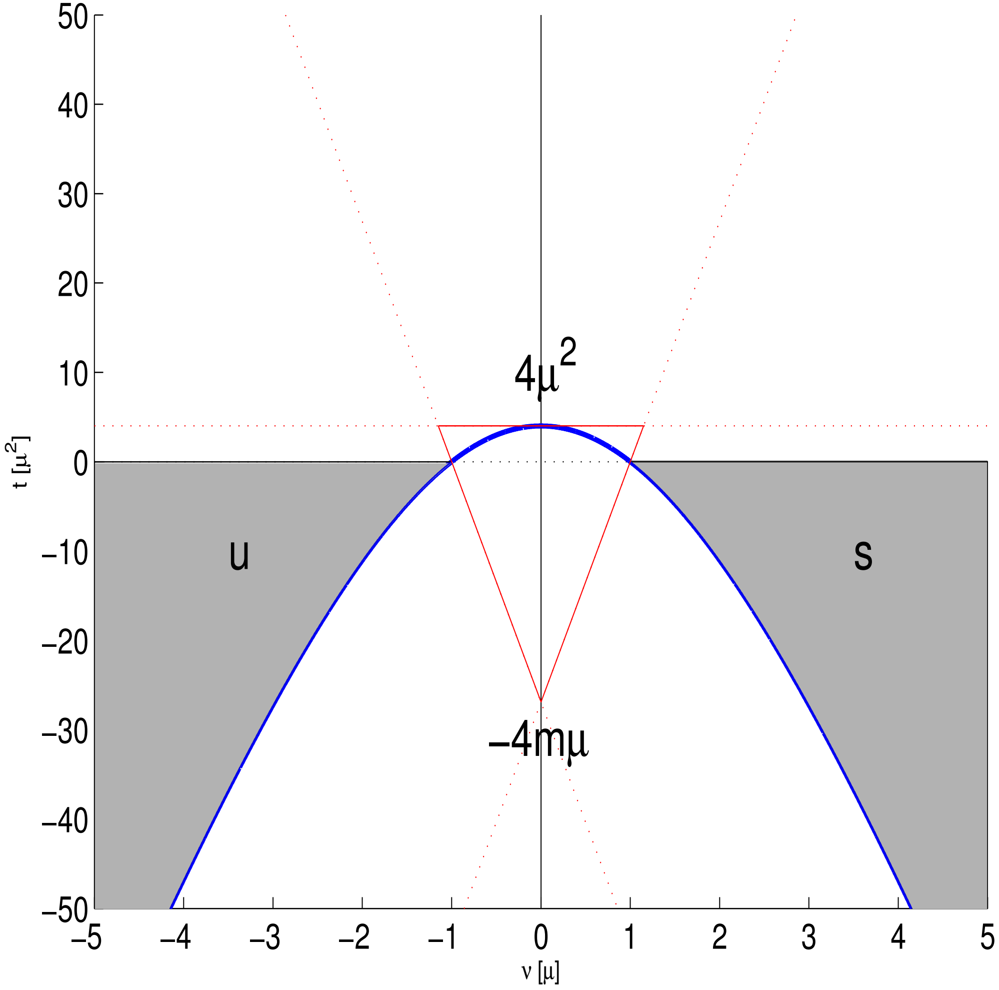
Analyticity, unitarity and crossing.

Fixed-$t$ dispersion relations can be proven from first principles for $4\,\mu^2 > t > -18\,\mu^2 \simeq -0.35\,\mathrm{GeV}^2$.

Isospin is an approximate symmetry: violated by the electromagnetic interaction and $m_u \neq m_d$.

Constraints from fixed-$t$ analyticity and isospin invariance are strong enough to resolve the ambiguities of phase-shift analysis.

# Mandelstam diagram:



# The Karlsruhe analysis

G. Höhler, Landolt-Börnstein, Vol. 9 b2, ed. H. Schopper (Springer, Berlin, 1983).

Analysis in 3 stages

- fixed-$t$ analysis
- fixed centre-of-mass angle analysis
- phase-shift analysis

which are performed iteratively until the amplitudes agree to about 3 %; solutions KH78 and KH80.

For the fixed-$t$ and the fixed-$\theta_{CM}$ analyses the Karlsruhe group uses the expansion techniques.

**Pietarinen's expansion** for the amplitudes at fixed-$t$:

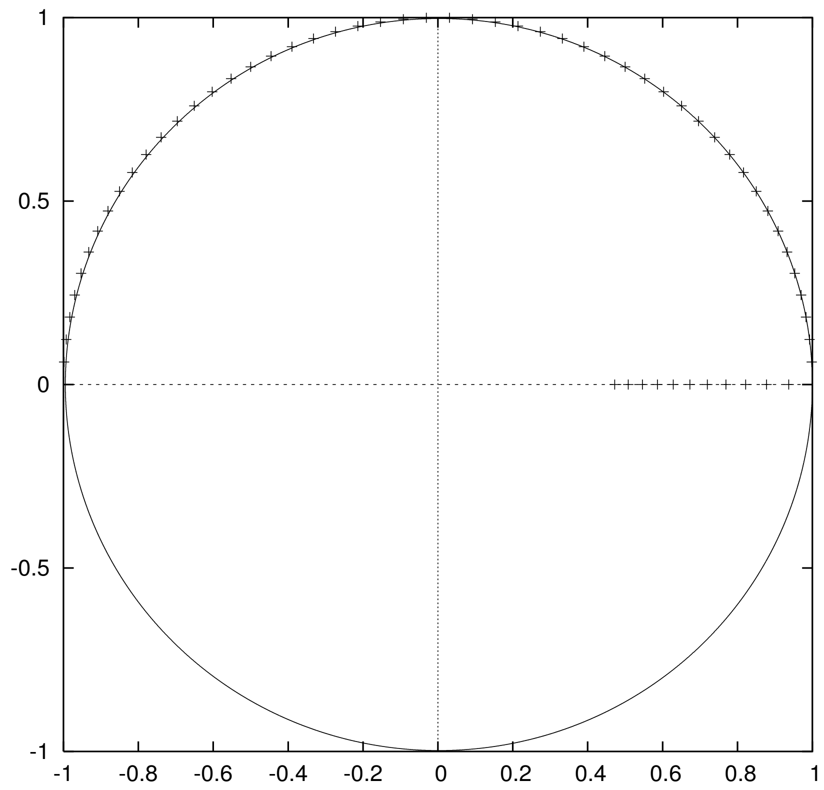
$$C^+(\nu,t) = C_N^+(\nu,t) + H(Z,t)\sum_{n=0}^{N} c_n^+ Z^n,$$

where H is adjusted to the asymptotic behaviour of the amplitude and

$$Z(\nu^2,t) = \frac{\alpha - \sqrt{\nu_{th}^2 - \nu^2}}{\alpha + \sqrt{\nu_{th}^2 - \nu^2}},$$

where $\alpha = 0.72$ GeV and $\nu_{th} = \mu + \frac{t}{4m}$.

This maps the physical region on the upper semicircle of the unit circle



The coefficients $c_n^+$ in the expansion can be determined by minimizing

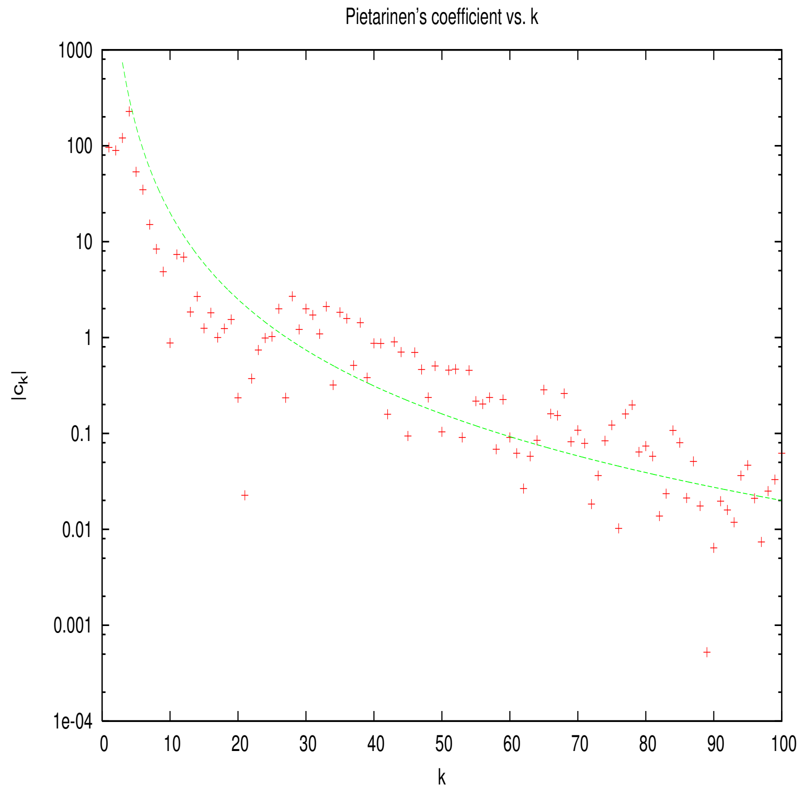
$$\chi^2 = \chi^2_{DATA} + \chi^2_{PW} + \chi^2_T(4\,\text{terms}),$$

where $\chi^2_{DATA}$ and $\chi^2_{PW}$ refer to the contributions from data and the existing partial wave solution respectively. The convergence and smoothing is taken care by a convergence test function

$$\chi^2_T = \lambda \sum_{n=0}^{N} (c_n^+)^2 (n+1)^3,$$

which is added to the $\chi^2$ expression with similar terms for the $C^-$ and $B^\pm$ amplitudes.

The coefficients $c_n$ are expected to go as $n^{-3}$ for large $n$ and for the $C^+$-amplitude we obtain:



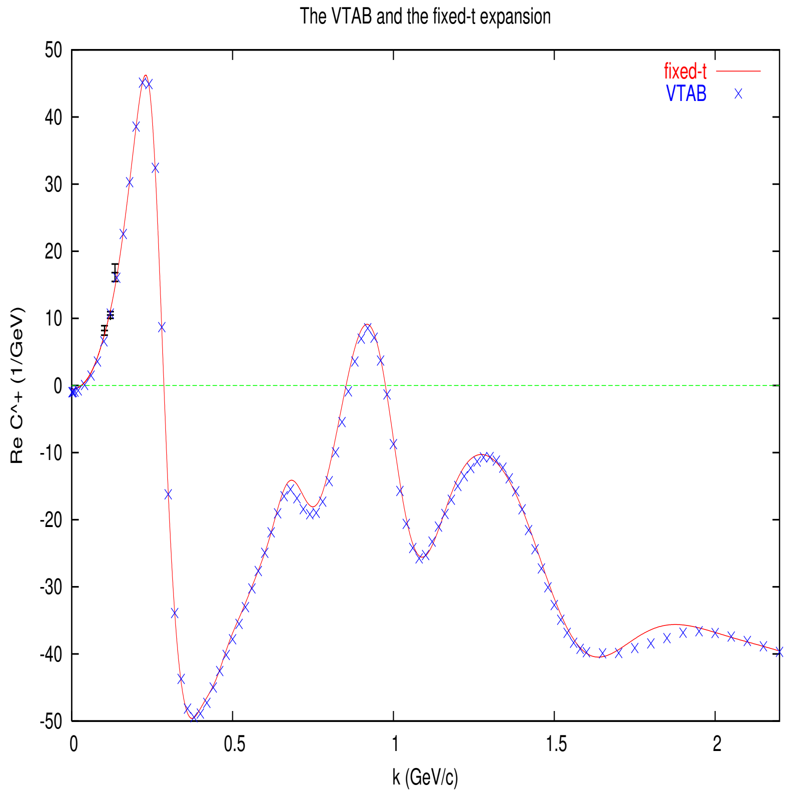
For the expansion techniques for the fixed centre-of-mass angle analysis, see e.g. G. Höhler et al., Handbook of Pion-Nucleon Scattering (1979).

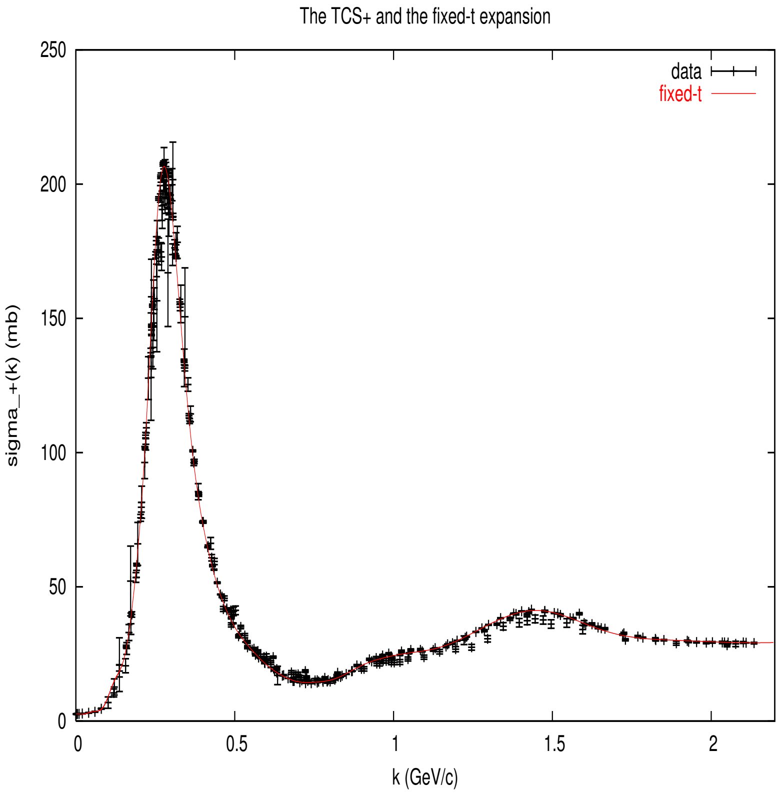
The KH78 analysis covers the range $k_{lab} = 0.$ - 10. GeV/c and the KH80 analysis the range $k_{lab} = 0.$ - 0.5 GeV/c.

Partial wave dispersion relations (PWDR) have been checked separately (Hutt and Koch) as well as partial wave relations (Koch). Agreement has been found satisfactory.

For the details of the KA84 solution, see R. Koch, Z. Phys. **C29** (1985) 597.

For the forward direction, $t = 0$, with N=40 we have:



# A note of the CMU-LBL analysis

The CMU-LBL analysis,

R. Cutkosky et al., Phys. Rev. **D20** (1979) 2782, 2804, 2839 and the Baryon'80 Proceedings,

focused on the determination of the parameters of the nucleon resonances and considered the momentum range $k_{lab}$ = 0.429 - 2.5 GeV/c at 43 different momenta (partial wave amplitudes can be found e.g. in Höhler's book).

The analysis was performed in 3 main steps:

- data amalgamation,
- energy-independent partial wave analysis incorporating iteratively constructed dispersion relation constraints,
- parametrization of the partial wave energy dependence to identify resonances and extract their properties.

The **data amalgamation** involves combining and interpolating experimental data to fixed angular bins at predetermined momenta. Particular attention was paid to the error analysis and the full covariance matrix was calculated.

For the **partial wave analysis** the real and imaginary parts of each invariant amplitude was separately written in the form

$$F(x) = B(x) + R(x)\,P(z(x)), \quad \text{where} \;\; x = \cos\theta.$$

The "Born term" $B(x)$ involves the $\rho$, $f$, $N$ and $\Delta$ exchanges in the Regge model. The variable $z(x)$ is constructed in a way which accelerates the convergence of the polynomial $P(z)$. In the physical region it converges more rapidly than the usual partial wave expansion.

The fixed modulating factor $R(x)$ takes care of the appropriate $s$-channel Regge behaviour.

The coefficients of the polynomial are the adjustable parameters in the $\chi^2$ fit, which involves, in addition to the normal $\chi^2$ contribution of the amalgamated data, a truncation function and terms related to the unitarity and hyperbolic dispersion relation constraints.

In addition to the forward dispersion relations, 5 hyperbolic dispersion relations are used to constrain the fit corresponding to $t = \mu^2,\ 2\,\mu^2,\ 2.5\,\mu^2,\ 3\,\mu^2$ and $4\,\mu^2$ at $\nu = 0$ and $t = 0$ at $\nu = \pm\mu$.

It turns out that 5 hyperbolas are sufficient for a resolution of all discrete ambiguities.

# Recent activity with the expansion techniques

In Helsinki we have been working on a $\pi$N PWA for quite some time (together with Pekko Metsä).

The idea has been to build in the fixed-$t$ constraints with the Pietarinen's expansion, i.e. in this respect to repeat the Karlsruhe analysis with an improved data base and more computing power. The latter has made it possible for us to pay more attention to the error analysis.

Our focus is more on the low-energy scattering to make contact to the ChPT domain.

The analysis of the forward data has been completed and published: V. Abaev et al., EPJ **A32** (2007) 321; P. Metsä, EPJ **A33** (2007) 349.

Goldberger-Miyazawa-Oehme sum rule:

$$C^-(\mu) = \frac{8\pi f^2}{\mu(1-(\frac{\mu}{2m})^2)} + 4\pi\mu J^- = 4\pi(1+\frac{\mu}{m})a^-_{0+}$$

where

$$J^- = \frac{1}{4\pi^2}\int_0^\infty \frac{\sigma_{\pi^-p}(k) - \sigma_{\pi^+p}(k)}{\omega}\,dk, = -1.060 \pm 0.030 \text{ mb}.$$

This agrees exactly with the Höhler-Kaiser value of 1980, they did not, however, perform an error analysis.

If pionic hydrogen information is used, giving

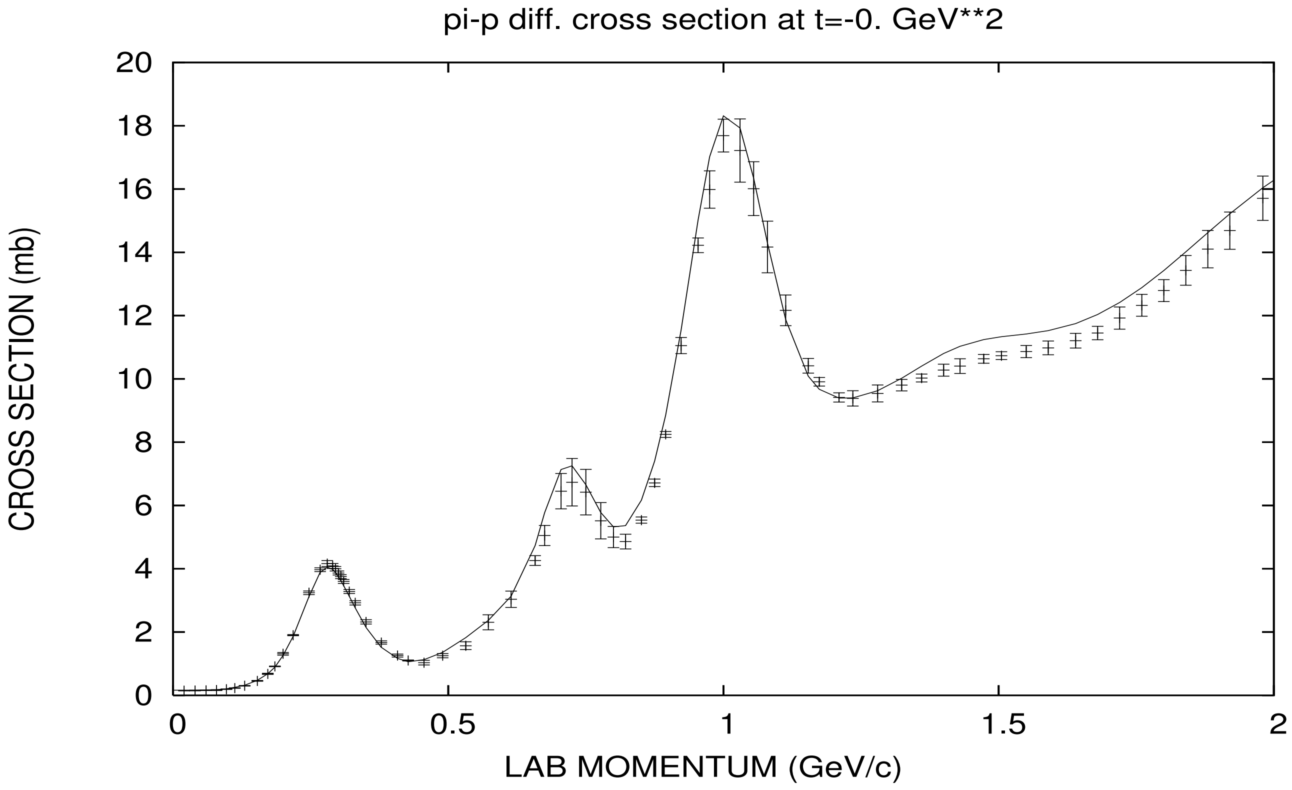
$a_{\pi^- p} = 0.0933 \pm 0.0029 \ 1/\mu$,

together with a value for the $s$-wave $\pi^+ p$ scattering length,

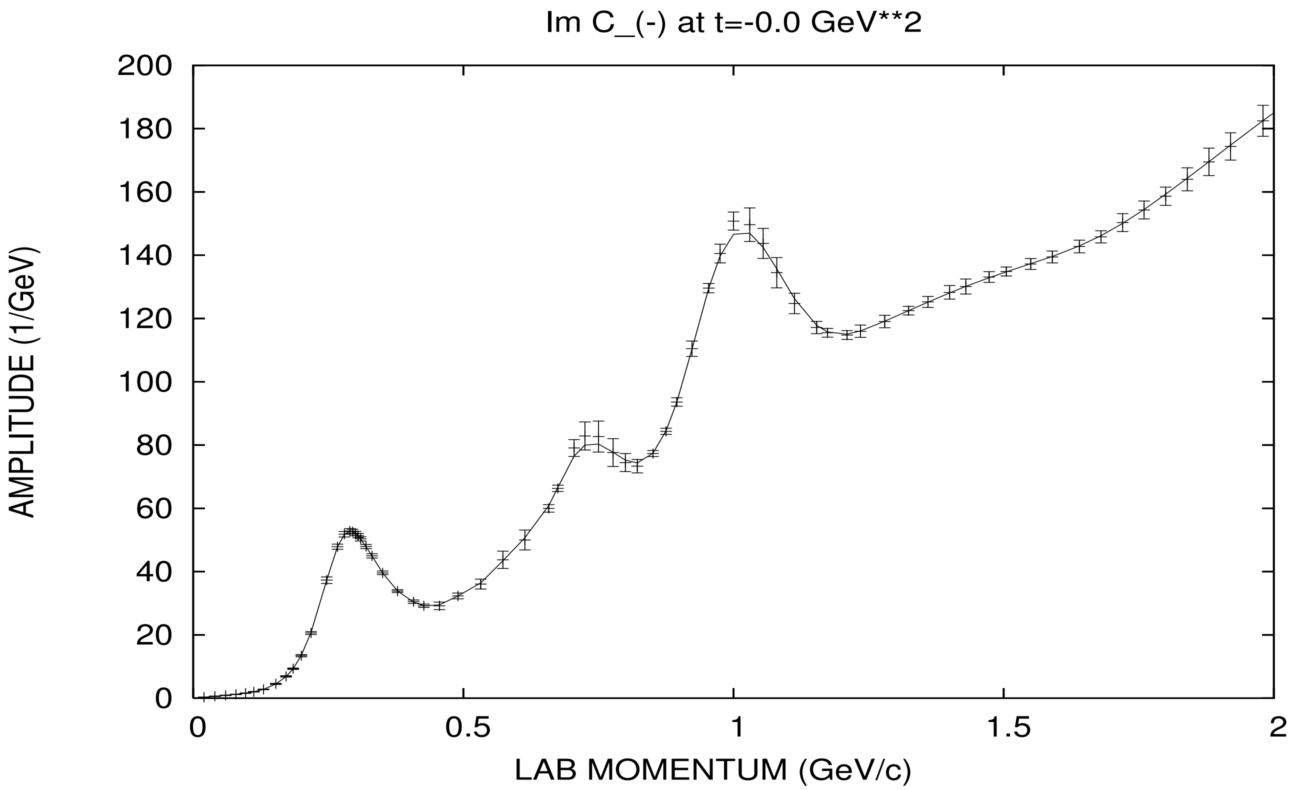
$a_{\pi^+ p} = -0.0764 \pm 0.0014 \ 1/\mu$,

we get $f^2 = 0.075 \pm 0.002$ for the pion-nucleon coupling constant.

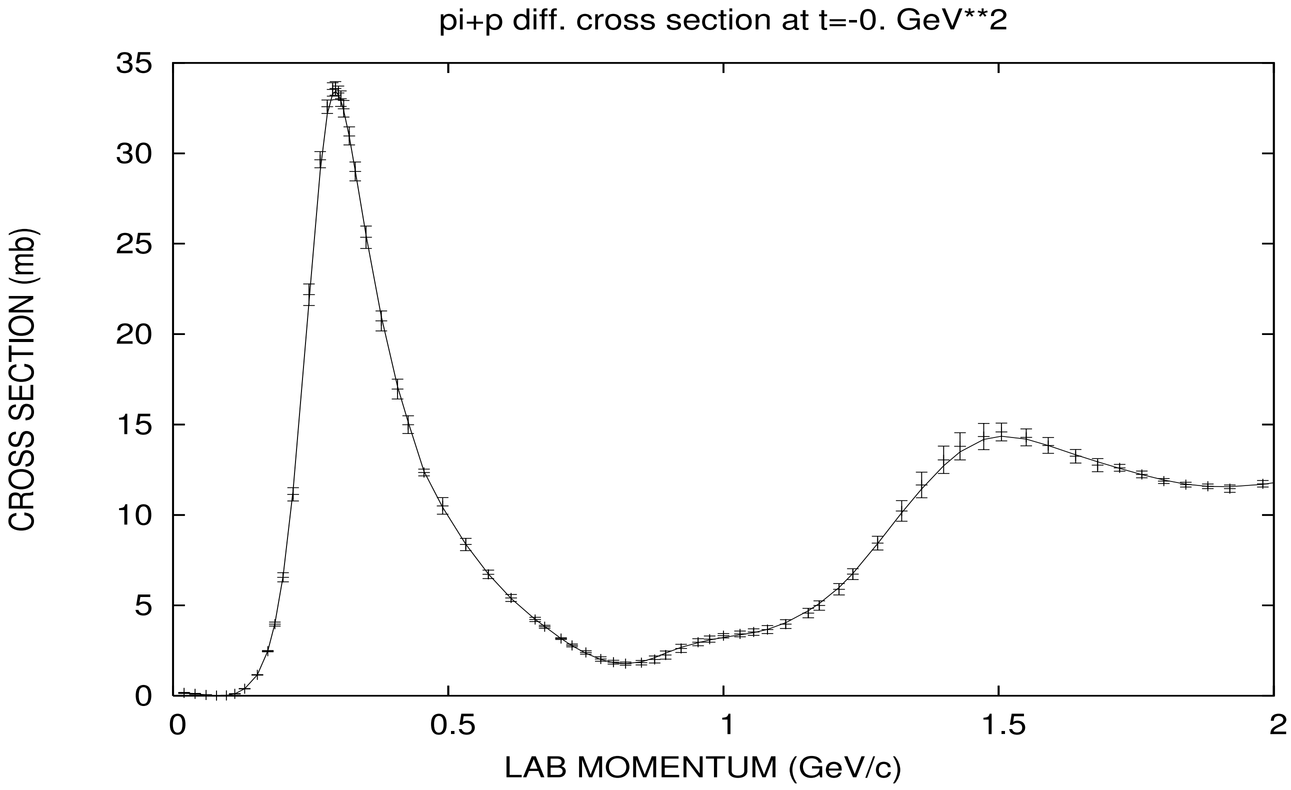
The forward solution yields e.g.



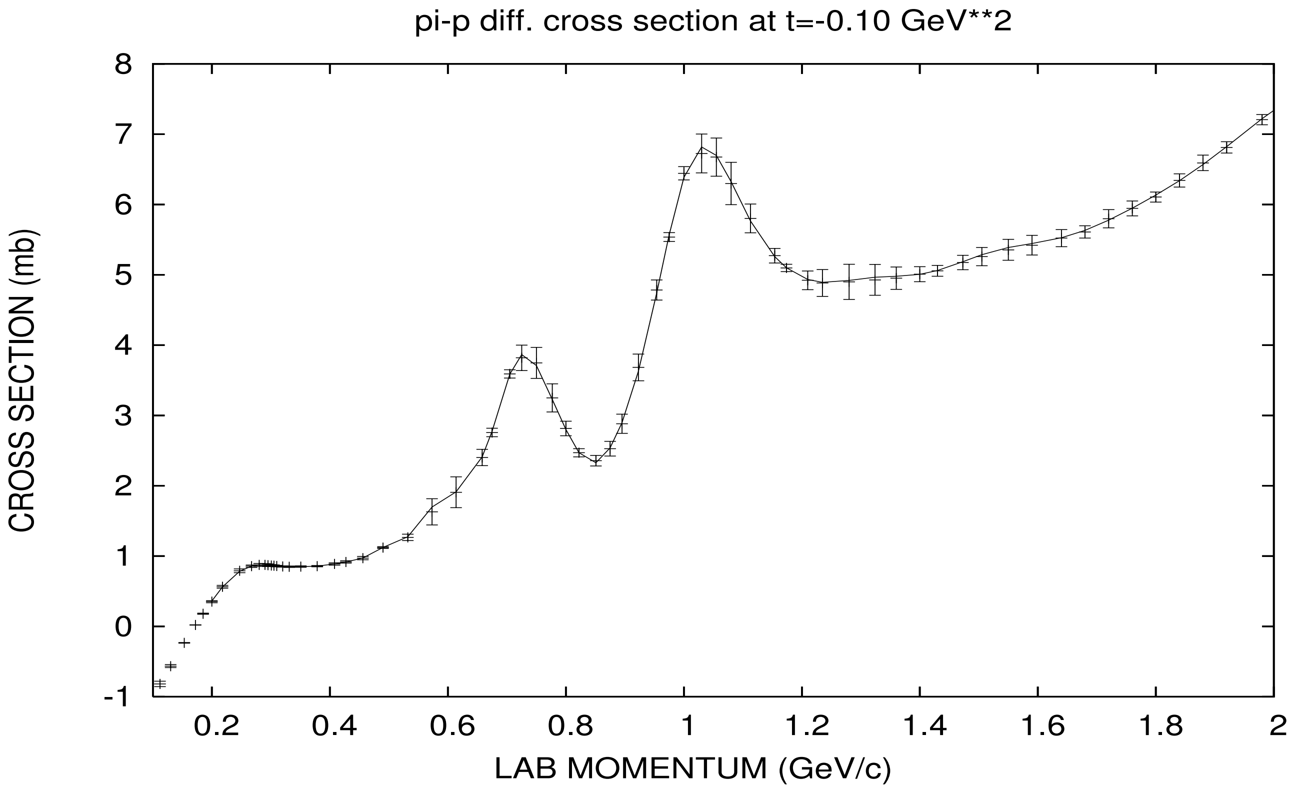
With the artificial input (average of FA02 and KA84) we have



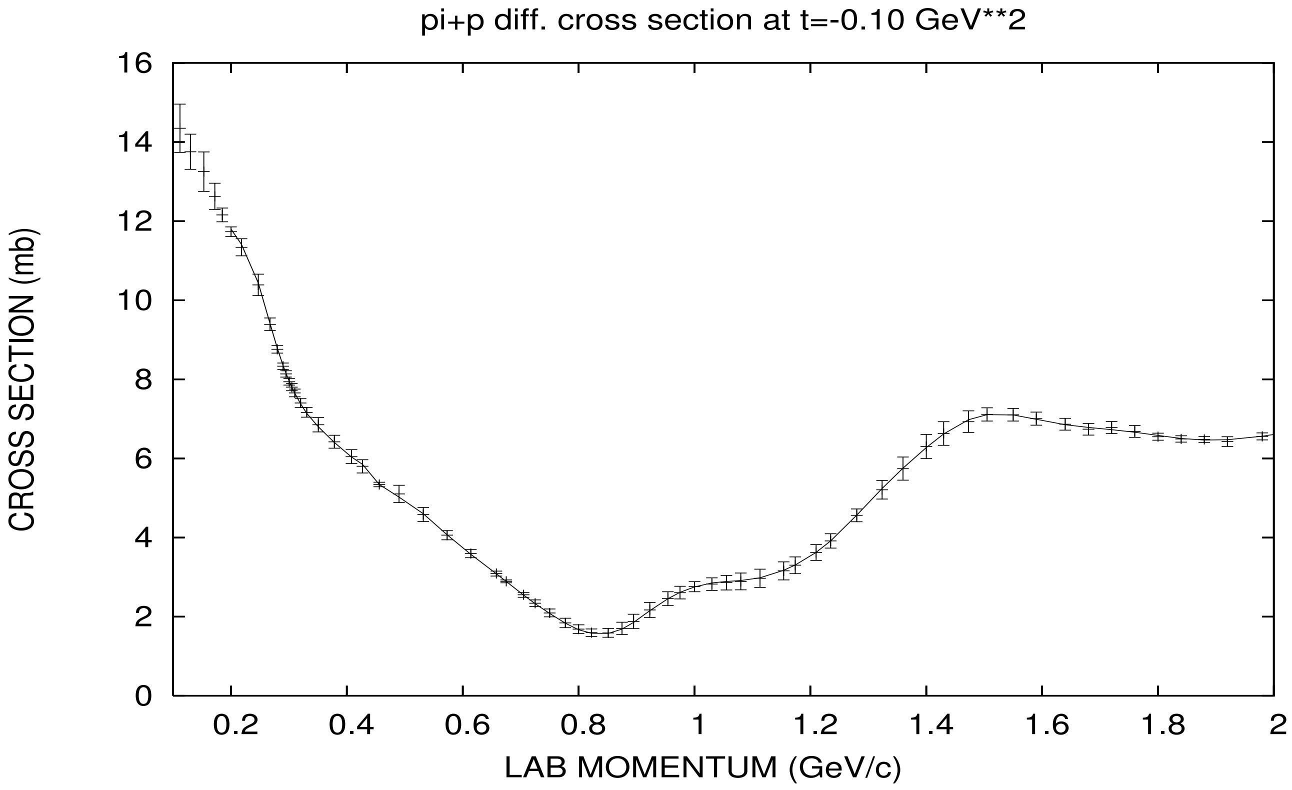
and similarly for the $\pi^+ p$ scattering

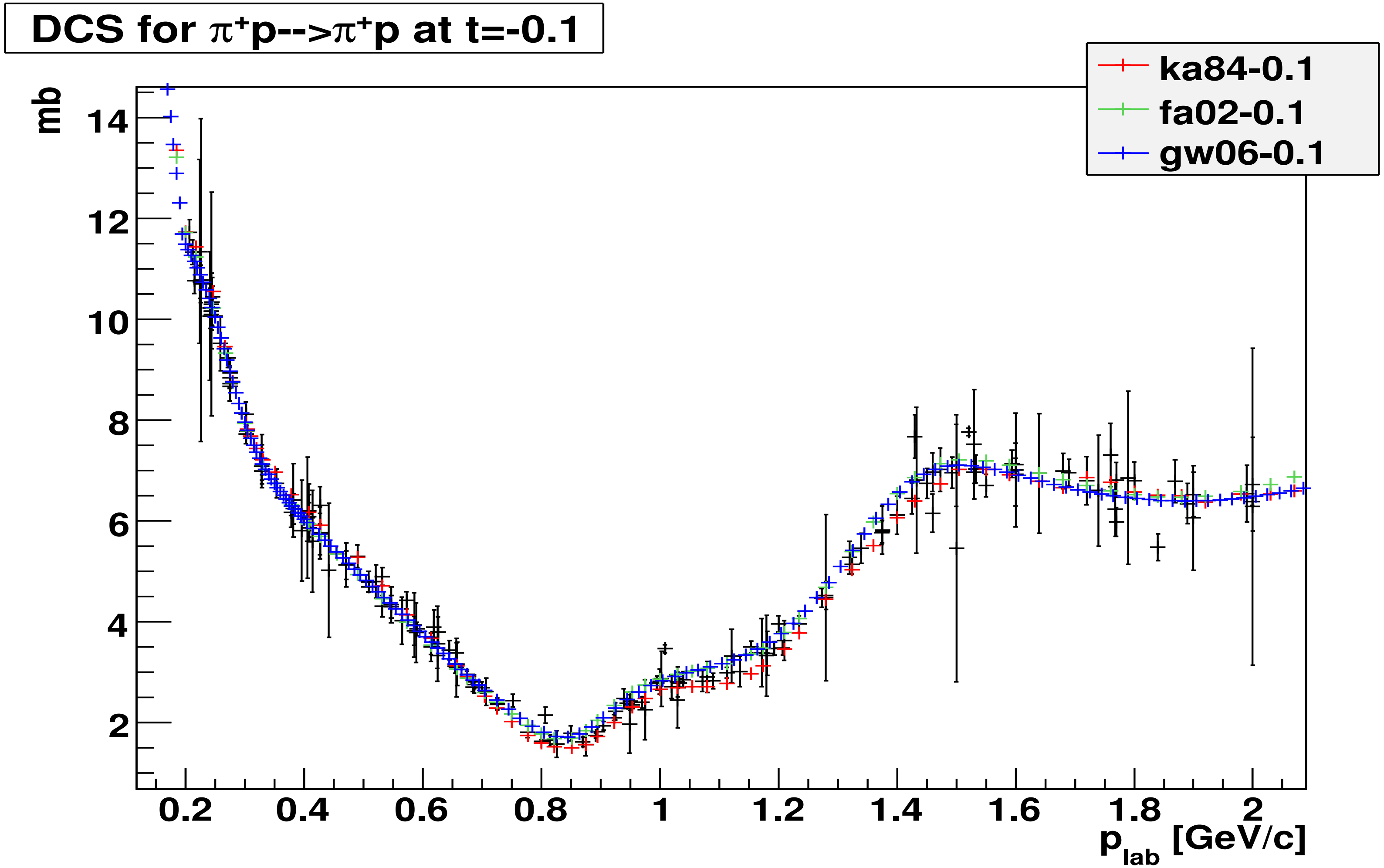


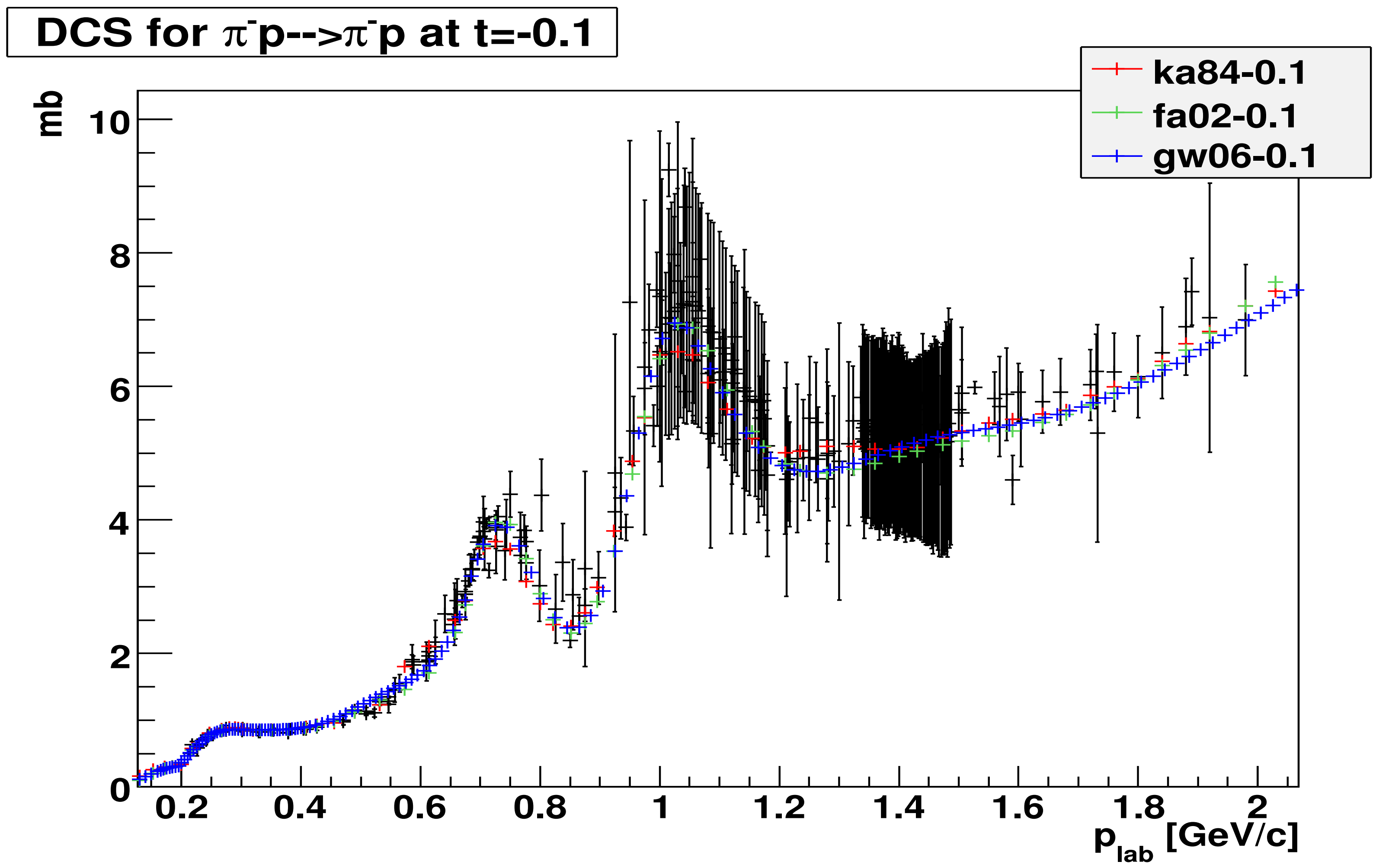
At $t = -0.1$ GeV$^2$ we have

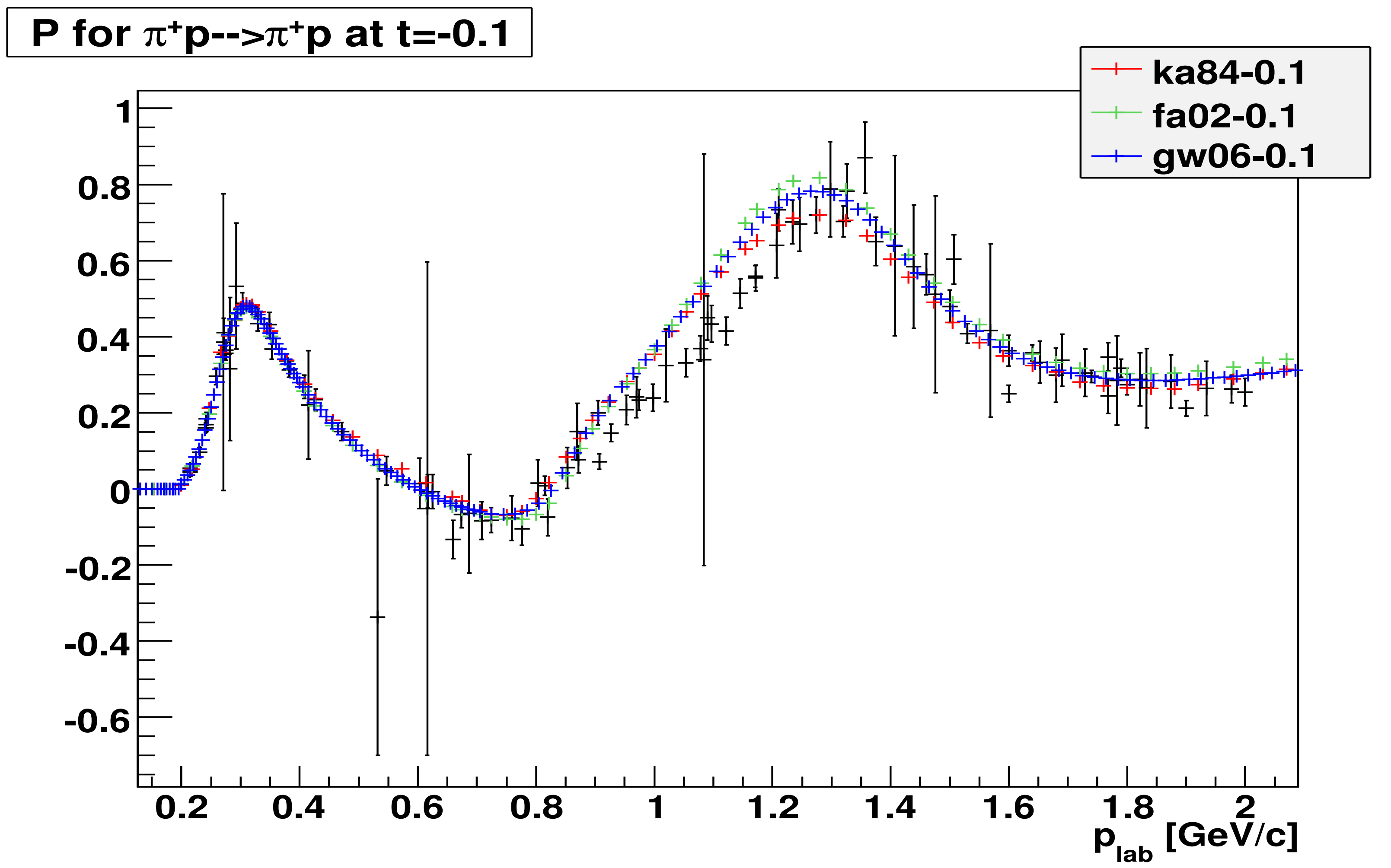


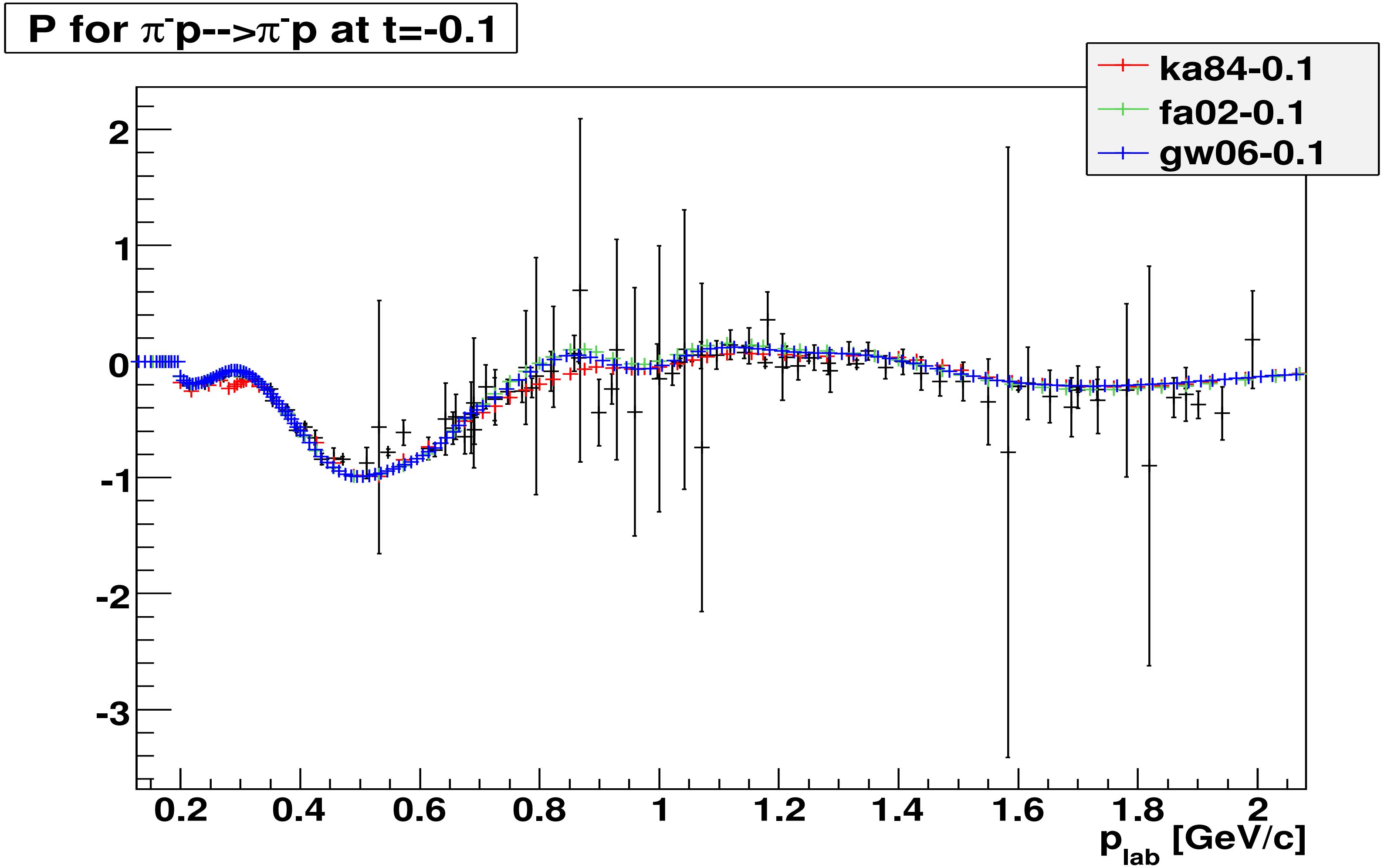
and similarly for the $\pi^+ p$ scattering



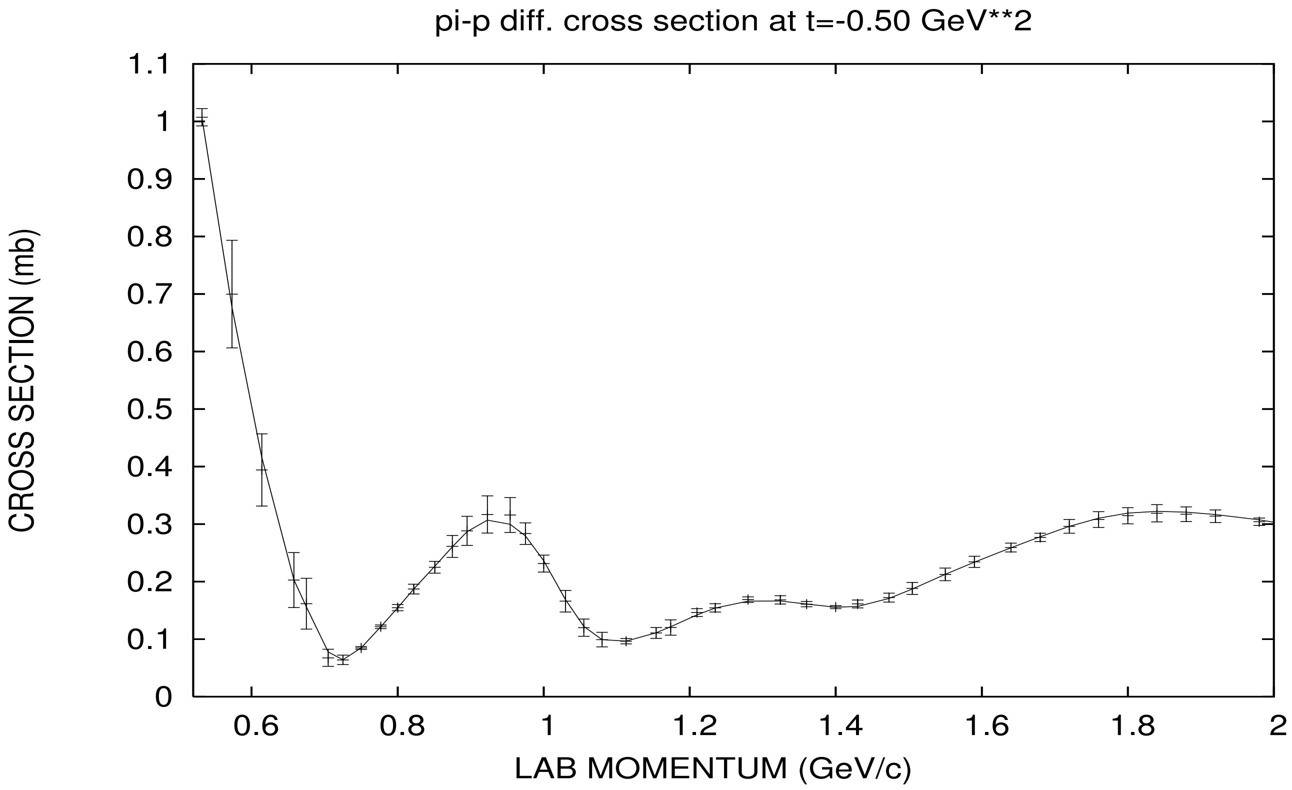

DCS for π⁺p-->π⁺p at t=-0.1
ka84-0.1
fa02-0.1
gw06-0.1
mb
14
12
10
8
6
4
2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
p_lab [GeV/c]


DCS for π⁻p-->π⁻p at t=-0.1
ka84-0.1
fa02-0.1
gw06-0.1
mb
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
p_lab [GeV/c]

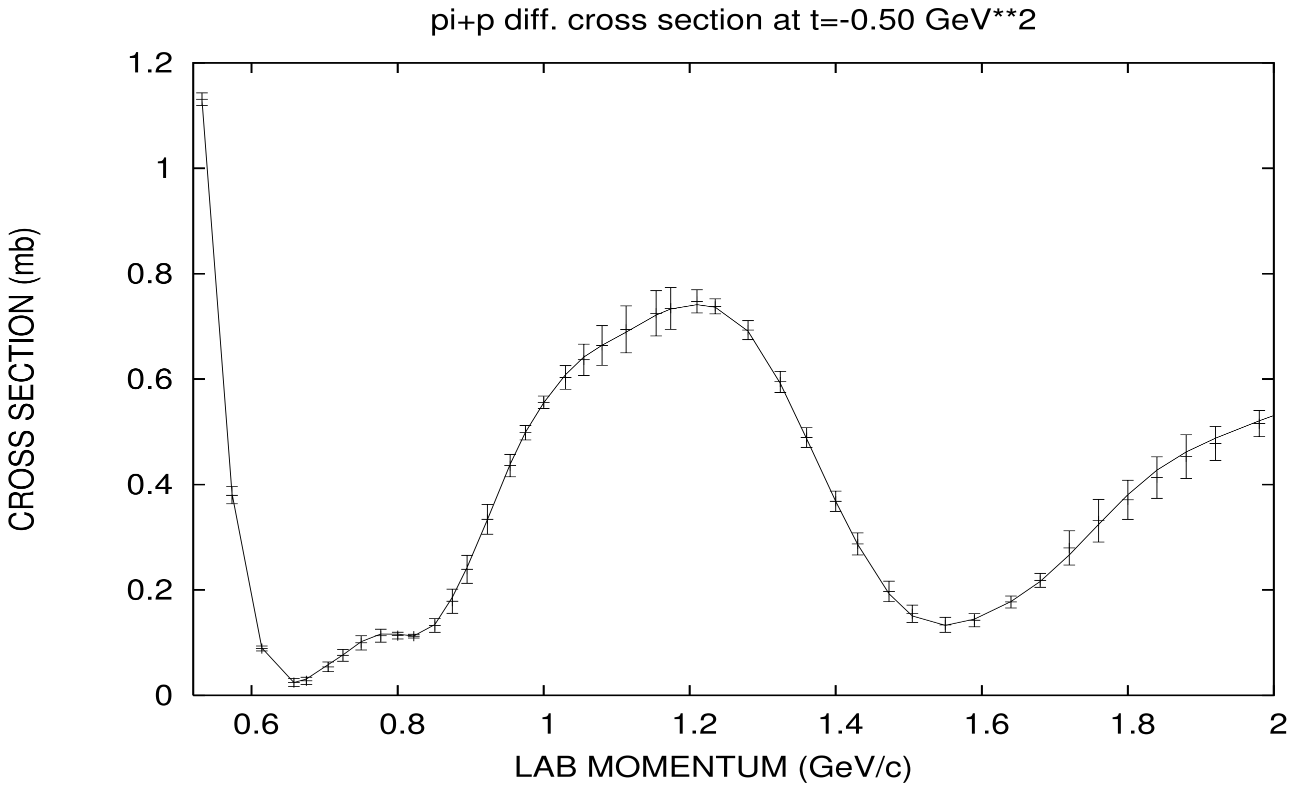

P for π⁺p-->π⁺p at t=-0.1
ka84-0.1
fa02-0.1
gw06-0.1
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
p_lab [GeV/c]

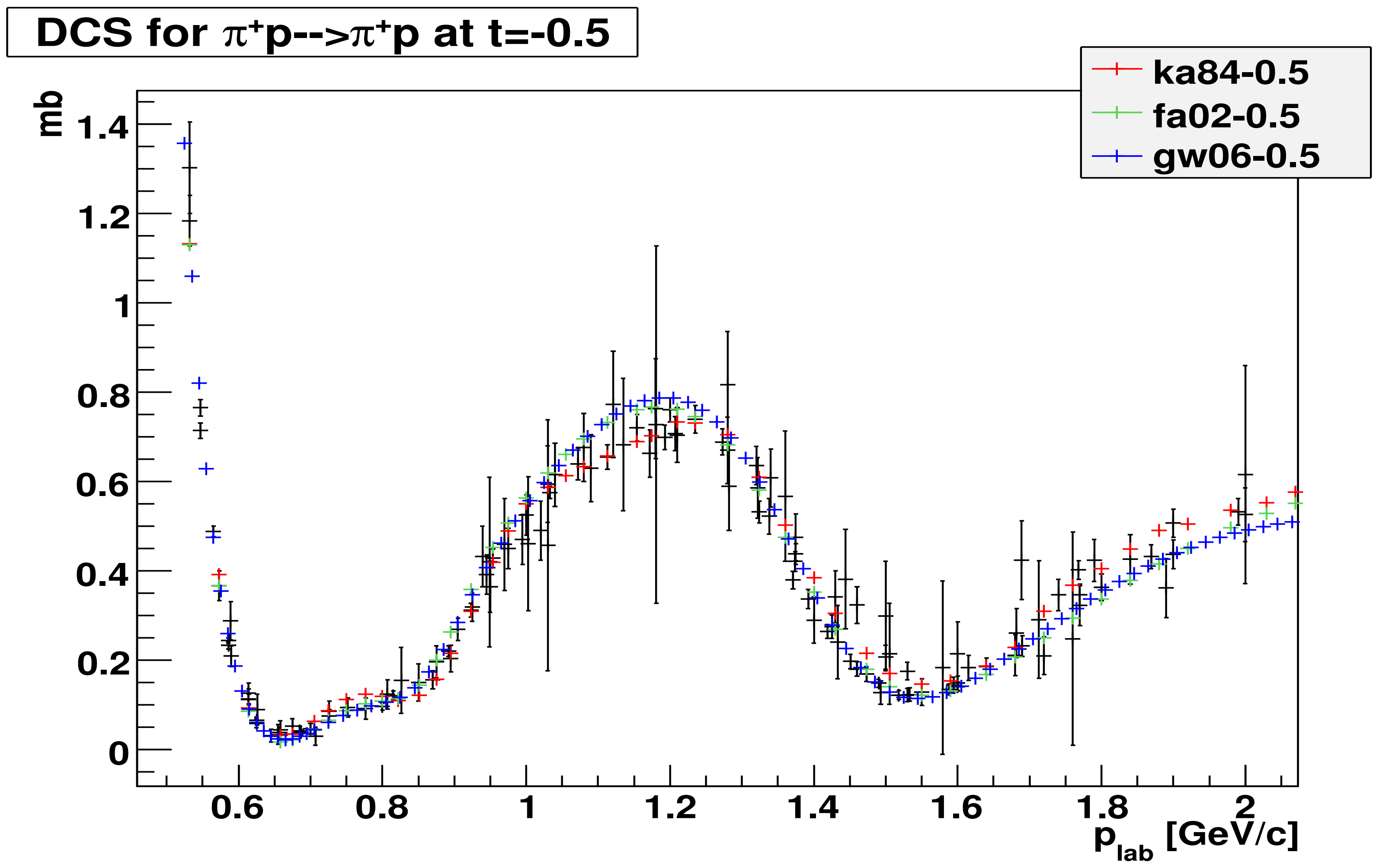

P for $\pi^- p \to \pi^- p$ at t=-0.1
ka84-0.1
fa02-0.1
gw06-0.1
2
1
0
-1
-2
-3
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
$p_{lab}$ [GeV/c]

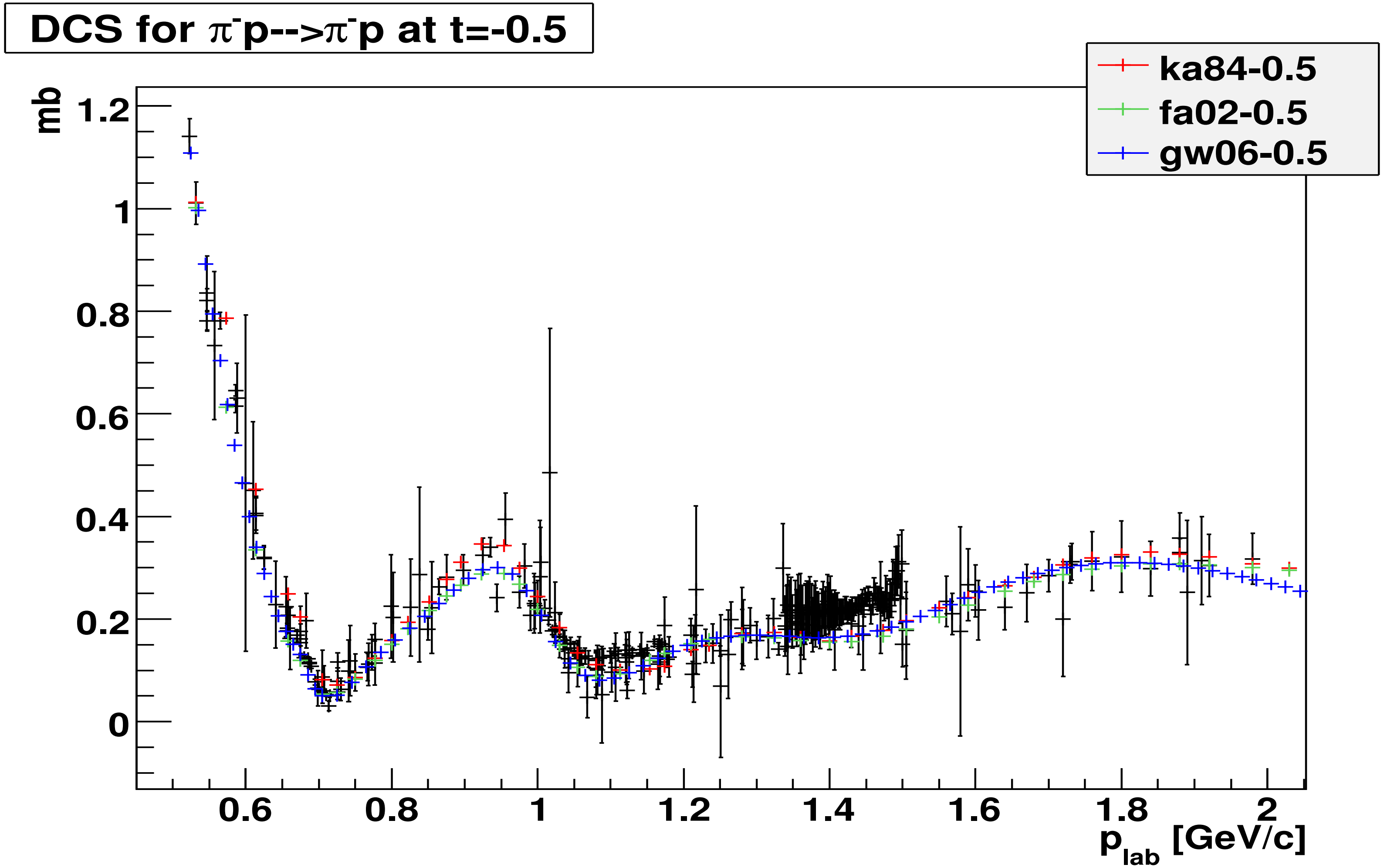
At $t = -0.5\ \mathrm{GeV}^2$ we have



and similarly for the $\pi^+p$ scattering

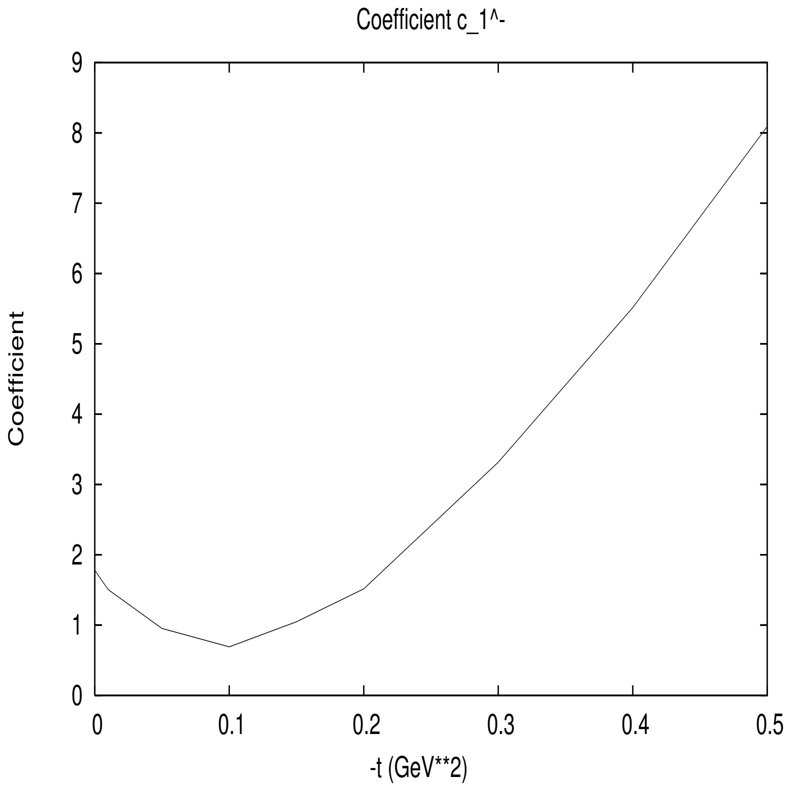


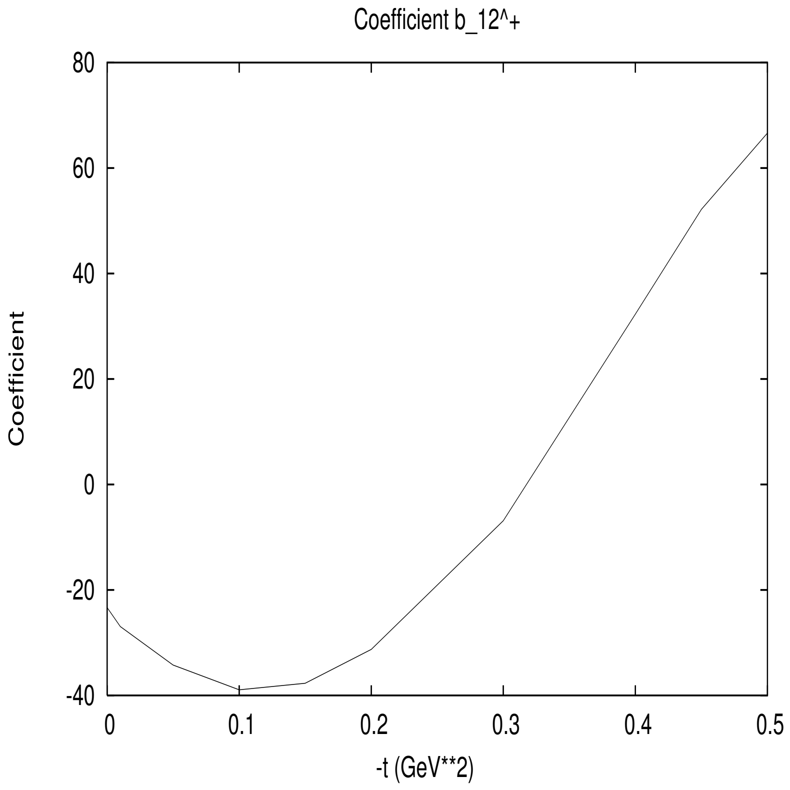

DCS for π⁺p-->π⁺p at t=-0.5
ka84-0.5
fa02-0.5
gw06-0.5
mb
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.6
0.8
1
1.2
1.4
1.6
1.8
2
p_lab [GeV/c]



DCS for π⁻p-->π⁻p at t=-0.5
ka84-0.5
fa02-0.5
gw06-0.5
mb
1.2
1
0.8
0.6
0.4
0.2
0
0.6
0.8
1
1.2
1.4
1.6
1.8
2
p_lab [GeV/c]

The $t$-dependence of the expansion coefficients:



# Conclusions

- In general the KH analysis produces a consistent set of PWA's.
- The data set has changed since 1980.
- The GWU-VPI analysis includes fixed-$t$ analyticity as well.
- At fixed-$t$ KA84 and FA02 agree surprisingly well.
- For our analysis in Helsinki we need to develop a strategy to incorporate the experimental data.