

Symmetry Energy in Nuclei

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and Neutron Rich Matter in Astrophysics
and in the Laboratory"

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Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under $n \leftrightarrow p$ interchange

An isoscalar quantity F does not change under $n \leftrightarrow p$ interchange. E.g. nuclear energy. Expansion in asymmetry $\eta = (N - Z)/A$, for smooth F , yields even terms only:

$$F(\eta) = F_0 + F_2 \eta^2 + F_4 \eta^4 + \dots$$

An isovector quantity G changes sign. Example:
 $\rho_{np}(r) = \rho_n(r) - \rho_p(r)$. Expansion with odd terms only:

$$G(\eta) = G_1 \eta + G_3 \eta^3 + \dots$$

Note: $G/\eta = G_1 + G_3 \eta^2 + \dots$

Charge invariance: invariance of nuclear interactions under rotations in n - p space



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Charge invariance:
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Isospin doublets

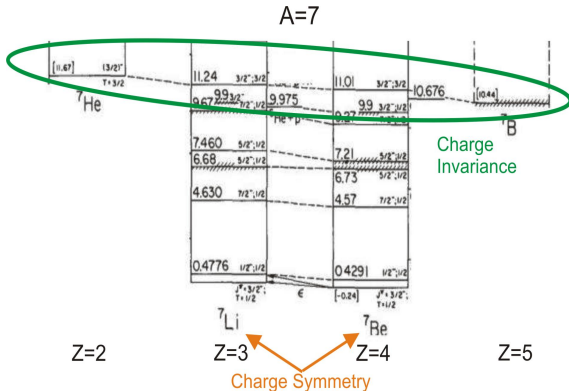
$$p : (\tau, \tau_z) = \left(\frac{1}{2}, \frac{1}{2}\right)$$

$$n : (\tau, \tau_z) = \left(\frac{1}{2}, -\frac{1}{2}\right)$$

Net isospin

$$\vec{T} = \sum_{i=1}^A \vec{\tau}_i$$

Isobars: Nuclei with the same A



$$T = \frac{3}{2}, \dots$$

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Nuclear states: $(T, T_z), \quad T \geq |T_z| = \frac{1}{2}|N - Z|$



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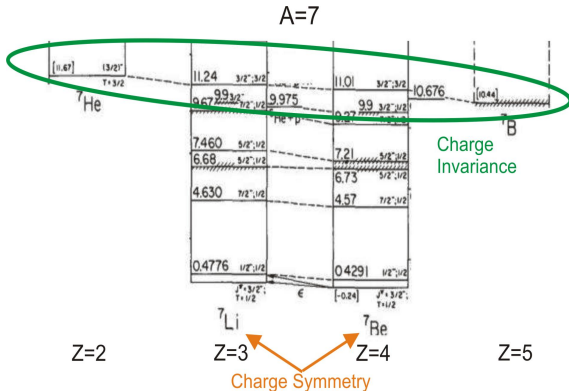
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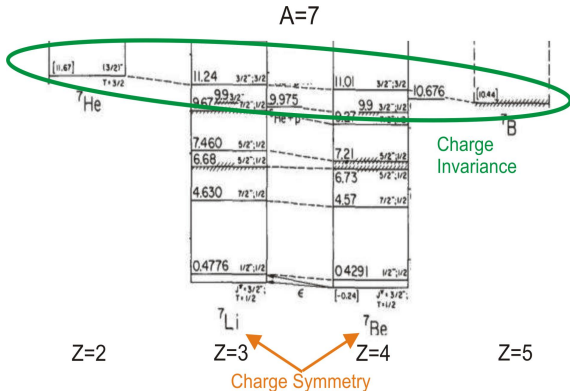
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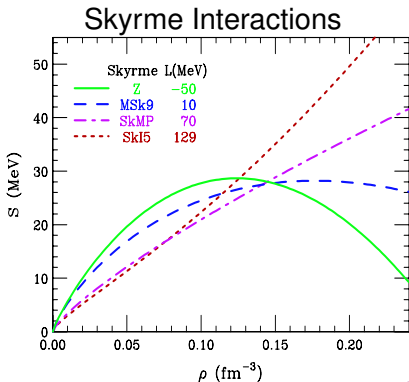
Symmetry Energy: From Finite to ∞ System

$\eta = (\rho_n - \rho_p)/\rho$ -expansion
under $n \leftrightarrow p$ symmetry

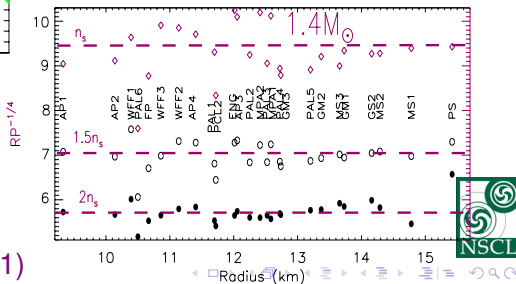
$$E(\rho_n, \rho_p) = E_0(\rho) + S(\rho) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2$$

$$S(\rho) = a_a^V + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0} + \dots$$

In neutron matter: $E \simeq E_0 + S$
& $P \simeq \rho^2 dS/d\rho \simeq L \rho^2 / (3\rho_0)$



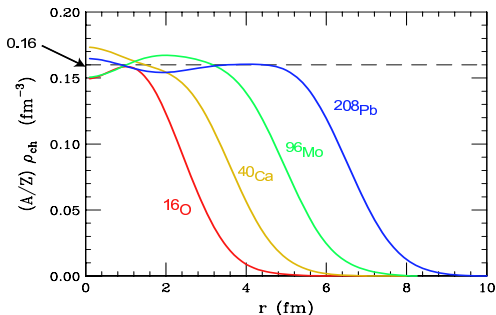
Empirical correlation for
neutron stars
 $RP^{-1/4} \approx \text{const}$
Lattimer&Prakash ApJ550(01)



Finite System

Interrelation between nucleonic densities...

Net density: $\rho(r) \stackrel{?}{=} \frac{A}{Z} \rho_p(r)$



Bethe-Weizsäcker formula:

$$E = -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_a \frac{(N - Z)^2}{A} + E_{\text{mic}}$$

$a_V = 16 \text{ MeV}$ $a_S = 18 \text{ MeV}$ $a_a = 21 \text{ MeV}$ $a_C = 0.7 \text{ MeV}$

symmetry term of volume character
mass-dependent

$a_a^V = a_a = 21 \text{ MeV}??$
 $a_a(A)??$

minimally finite system

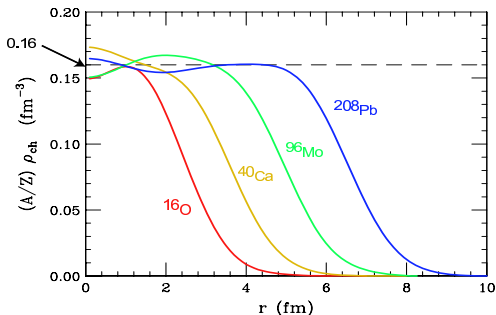
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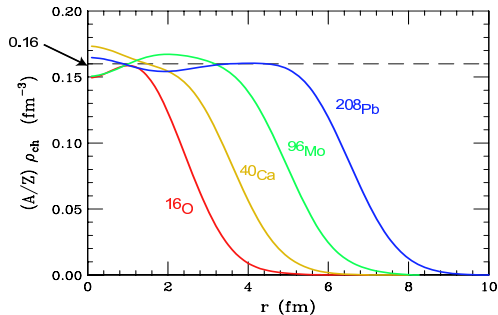
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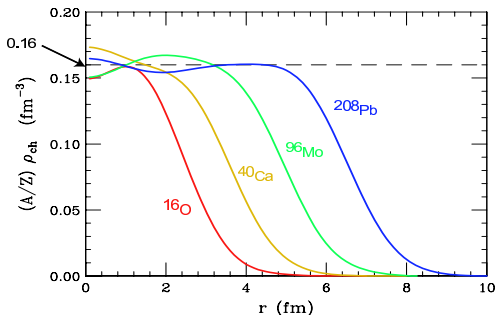
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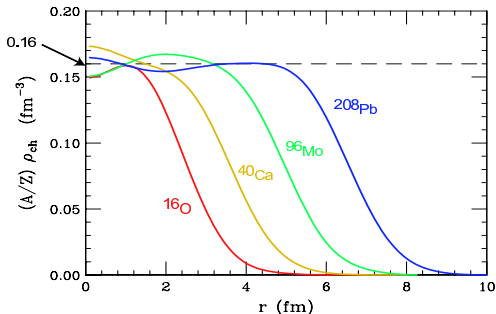
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Nucleus as Capacitor for Asymmetry

$$E = -a_v A + a_s A^{2/3} + \frac{a_a}{A} (N - Z)^2$$

$$= E_0(A) + \frac{a_a(A)}{A} (N - Z)^2$$

Capacitor analogy

$$E = E_0 + \frac{Q^2}{2C} \Rightarrow \begin{cases} Q \equiv N - Z \\ C \equiv \frac{A}{2a_a(A)} \end{cases}$$

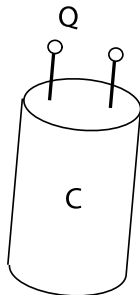
Asymmetry chemical potential

$$\mu_a = \frac{\partial E}{\partial (N - Z)} = \frac{1}{2} (\mu_n - \mu_p) = \frac{2a_a(A)}{A} (N - Z)$$

Analogy

$$V = \frac{1}{C} Q \Rightarrow V \equiv \mu_a$$

Note: for connected capacitors, charge (asymmetry) distributes itself *in proportion to capacitance*



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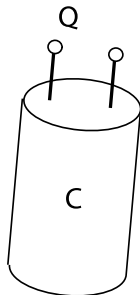
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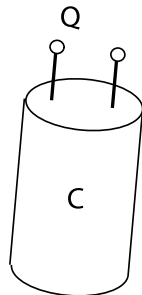
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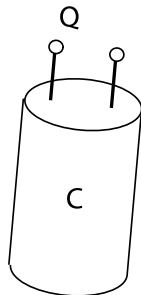
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Invariant Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar \Rightarrow weakly depends on $(N - Z)$ for given A . [Coulomb suppressed. . .]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A \rho_{np}(r)/(N - Z)$ isoscalar!
 $A/(N - Z)$ normalizing factor global. . . Similar local normalizing factor, in terms of intense quantities, $2a_a^V/\mu_a$, where $a_a^V \equiv S(\rho_0)$
 Asymmetric density (formfactor for isovector density) defined:

$$\rho_a(r) = \frac{2a_a^V}{\mu_a} [\rho_n(r) - \rho_p(r)]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on η !

In any nucleus:

$$\rho_{n,p}(r) = \frac{1}{2} \left[\rho(r) \pm \frac{\mu_a}{2a_a^V} \rho_a(r) \right]$$

where $\rho(r)$ & $\rho_a(r)$ have universal features!



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Net isoscalar density ρ usually parameterized w/Fermi function

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{d}\right)} \quad \text{with} \quad R = r_0 A^{1/3}$$

Isovector density ρ_a ?? Related to $a_a(A)$ & to $S(\rho)$!

$$2C \equiv \frac{A}{a_a(A)} = \frac{2(N-Z)}{\mu_a} = 2 \int dr \frac{\rho_{np}}{\mu_a} = \frac{1}{a_a^V} \int dr \rho_a(r)$$

In uniform matter

$$\mu_a = \frac{\partial E}{\partial(N-Z)} = \frac{2S(\rho)}{\rho} \rho_{np} \equiv \frac{\rho_{np}}{c(\rho)} \quad c(\rho) = \rho/2S(\rho) \text{ density of capacitance}$$

$$\Rightarrow \rho_a = \frac{2a_a^V}{\mu_a} \rho_{np} = \frac{a_a^V \rho}{S(\rho)} \propto c(\rho) \quad \text{asymmetry} \propto \text{capacitance}$$



n & p densities carry record of $S(\rho)$! \Rightarrow Hartree-Fock study of surface

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Nuclear Densities

$$\rho_{n,p}(r) = \frac{1}{2} \left[\rho(r) \pm \frac{\mu_a}{2a_a^V} \rho_a(r) \right]$$

Net isoscalar density ρ usually parameterized w/Fermi function

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{d}\right)} \quad \text{with} \quad R = r_0 A^{1/3}$$

Isovector density ρ_a ?? Related to $a_a(A)$ & to $S(\rho)$!

$$2C \equiv \frac{A}{a_a(A)} = \frac{2(N-Z)}{\mu_a} = 2 \int dr \frac{\rho_{np}}{\mu_a} = \frac{1}{a_a^V} \int dr \rho_a(r)$$

In uniform matter

$$\mu_a = \frac{\partial E}{\partial(N-Z)} = \frac{2S(\rho)}{\rho} \rho_{np} \equiv \frac{\rho_{np}}{c(\rho)} \quad c(\rho) = \rho/2S(\rho) \text{ density of capacitance}$$

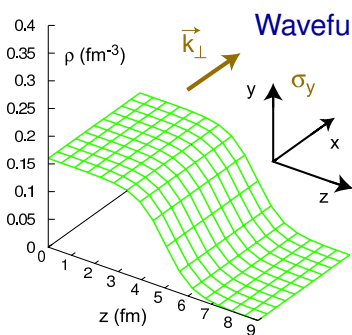
$$\Rightarrow \rho_a = \frac{2a_a^V}{\mu_a} \rho_{np} = \frac{a_a^V \rho}{S(\rho)} \propto c(\rho) \quad \text{asymmetry} \propto \text{capacitance}$$

n & p densities carry record of $S(\rho)$! \Rightarrow Hartree-Fock study of surface



Half-Infinite Matter in Skyrme-Hartree-Fock

To one side infinite uniform matter & vacuum to the other



Wavefunctions: $\Phi(\mathbf{r}) = \phi(z) e^{i\mathbf{k}_\perp \cdot \mathbf{r}_\perp}$

matter interior/exterior:

$$\phi(z) \propto \sin(k_z z + \delta(\mathbf{k}))$$

$$\phi(z) \propto e^{-\kappa(\mathbf{k})z}$$

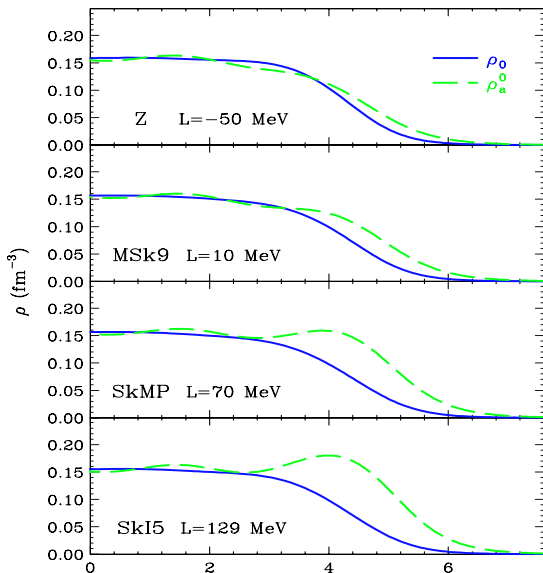
Discretization in \mathbf{k} -space. Set of 1D HF eqs solved until self-consistency for different Skyrme interactions:

$$-\frac{d}{dz} \frac{\hbar^2}{2m^*(z)} \frac{d}{dz} \phi(z) + \left(\frac{\hbar^2 k_\perp^2}{2m^*(z)} + U(z) \right) \phi(z) = \epsilon(\mathbf{k}) \phi(z)$$

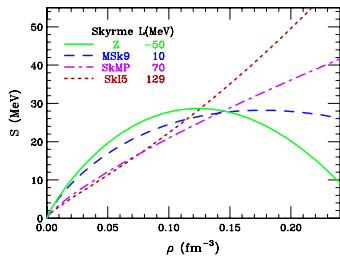
PD&Lee NP818(09)36



Isoscalar (Net) & Isovector Densities from SHF



Results for different Skyrme interactions in half- ∞ matter.

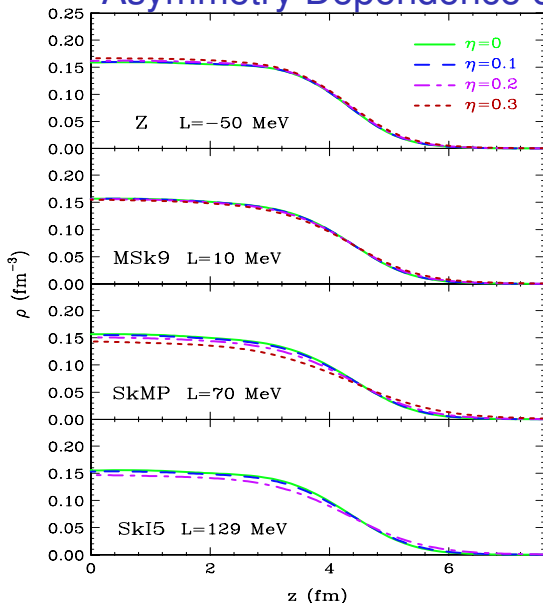


Net & isovector densities displaced relative to each other

As $S(\rho)$ changes gradually, so does the displacement.



Asymmetry Dependence of Net Density

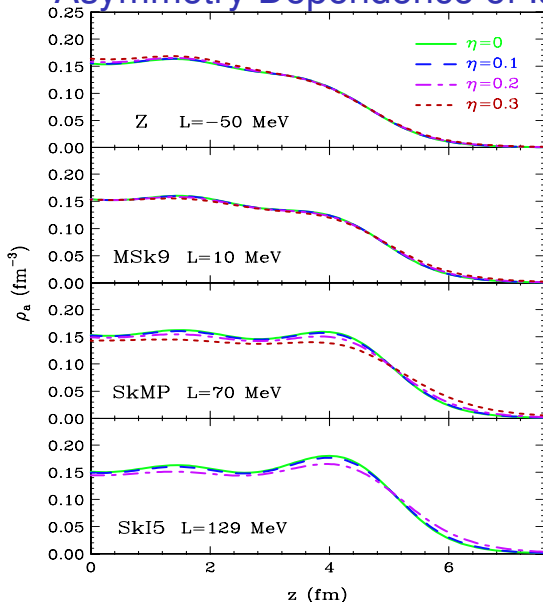


Results for different asymmetries

$$\eta = \frac{N-Z}{A}$$

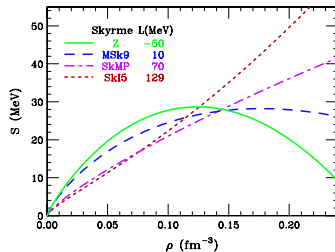


Asymmetry Dependence of Isovector Density



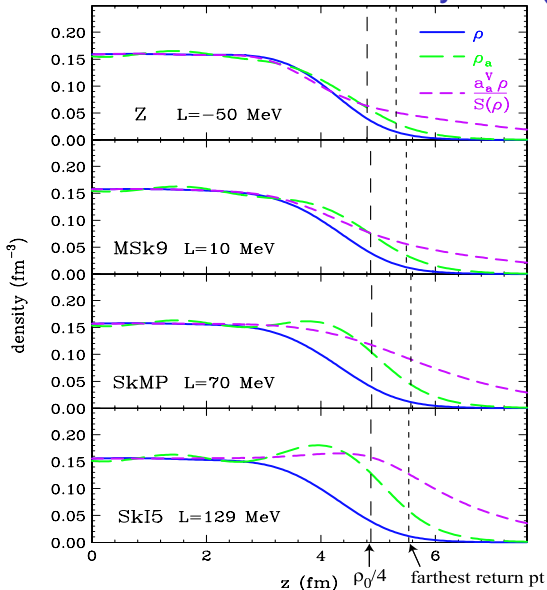
$$\rho_a = \frac{2a_a^V}{\mu_a} (\rho_n - \rho_p)$$

Results for different asymmetries



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NPA818(09)36



Sensitivity to $S(\rho)$ 

For weakly nonuniform matter, expected asymmetric density

$$\rho_a = \rho a_a^V / S(\rho)$$

Isovector density ρ_a oscillates around the expectation down to $\rho \simeq \rho_0/4$

The higher L , the lower is S in the surface & the higher the surface capacitance \Leftrightarrow the more different ρ & ρ_a .



WKB Analysis

Semiclassical wavefunction

$$\phi_{\mathbf{k}^\infty}(z) \propto \begin{cases} \sin\left(\int_z^{z_0} k_z(z') dz'\right) & \text{allowed } z < z_0 \\ \exp\left(-\int_{z_0}^z \kappa_z(z') dz'\right) & \text{forbidden } z > z_0 \end{cases}$$

Density from $\rho_\alpha(z) = \int d\mathbf{k}^\infty |\phi_{\mathbf{k}^\infty\alpha}(z)|^2$, classical return pt z_0

Findings:

At $z < z_0$ $\rho_a(z) \approx \frac{a_a^V \rho}{S(\rho)} \left(1 + \frac{\rho^{2/3}}{S(\rho)} \mathcal{F}\right)$

where $\mathcal{F}(z) \propto \sin(2k_F(z_0 - z))$,
describing Friedel oscillations around ρ/S .



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Skyrme Parameters

Name	a_V	m^*/m	K	a_a^V	L	a_S	a_a^S	ΔR
SkT	-15.40	0.602	333	24.8	28.2	14.2	17.5	0.477
SkT1	-15.98	1.000	236	32.0	56.2	18.2	14.6	0.799
SkT2	-15.94	1.000	235	32.0	56.2	18.0	14.7	0.794
SkT3	-15.94	1.000	235	31.5	55.3	17.7	15.3	0.776
SkT4	-15.95	1.000	235	35.4	94.1	18.1	11.5	0.986
SkT5	-16.00	1.000	201	37.0	98.5	18.1	10.9	1.084
SkM1	-15.77	0.789	216	25.1	-35.3	17.4	59.6	0.180
SkI1	-15.95	0.693	242	37.5	161.0	17.4	11.4	1.126
$G\sigma$	-15.59	0.784	237	31.3	94.0	16.0	10.1	0.929
$R\sigma$	-15.59	0.783	237	30.5	85.7	16.0	10.5	0.888
T	-15.93	1.000	235	28.3	27.2	17.7	22.6	0.587
Z	-15.97	0.842	330	26.8	-49.7	17.7	51.5	0.213
$Z\sigma$	-15.88	0.783	233	26.6	-29.3	17.0	46.6	0.233
$Z\sigma\sigma$	-15.96	0.775	234	28.8	-4.5	17.3	29.3	0.406



Symmetry Coefficient

$$\frac{A}{a_a(A)} = \frac{1}{a_a^V} \int d^3r \rho_a(r) = \frac{1}{a_a^V} \int d^3r \rho(r) + \frac{1}{a_a^V} \int_{\text{surface region}} d^3r (\rho_a - \rho)(r)$$

$$\simeq \frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}$$

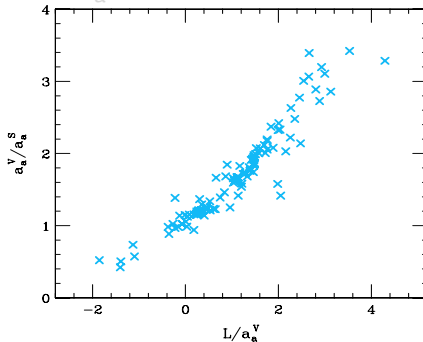
where

$$\frac{a_a^V}{a_a^S} = 4\pi r_0^2 \int dr (\rho_a - \rho)(r)$$

$$\simeq \frac{3 \Delta R}{r_0}$$

and ΔR is displacement of isovector and isoscalar surfaces.

$$\Rightarrow \frac{1}{a_a(A)} \simeq \frac{1}{a_a^V} + \frac{A^{-1/3}}{a_a^S}$$

 $a_a^S - L$ Correlation f/SHF

$L \propto \text{slope of } S(\rho) \text{ at } \rho_0$



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surface region

$$\simeq \frac{A}{a_a^V} + \frac{A^{2/3}}{a_a^S}$$

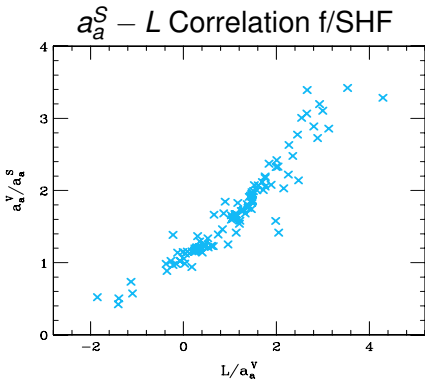
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$L \propto$ slope of $S(\rho)$ at ρ_0



Charge Invariance

? $a_a(A)$? Conclusions on sym-energy details, following E -formula fits, interrelated with conclusions on other terms in the formula: asymmetry-dependent Coulomb, Wigner & pairing + asymmetry-independent, due to $(N - Z)/A$ - A correlations along stability line [PD NPA727(03)233]!

Best would be to study the symmetry energy in isolation from the rest of E -formula! Absurd?!

Charge invariance to rescue: lowest nuclear states characterized by different isospin values (T, T_z), $T_z = (Z - N)/2$. Nuclear energy scalar in isospin space:

sym energy
$$E_a = a_a(A) \frac{(N - Z)^2}{A} = 4 a_a(A) \frac{T_z^2}{A}$$

$$\rightarrow E_a = 4 a_a(A) \frac{T^2}{A} = 4 a_a(A) \frac{T(T + 1)}{A}$$



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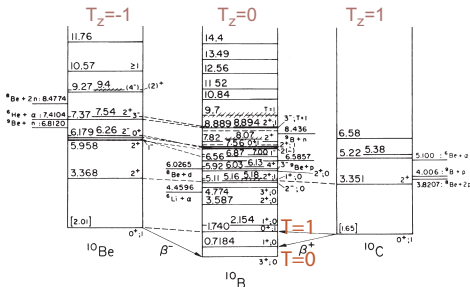


$a_a(A)$ Nucleus-by-Nucleus

$$\rightarrow E_a = 4 a_a(A) \frac{T(T+1)}{A}$$

In the ground state T takes on the lowest possible value
 $T = |T_z| = |N - Z|/2$. Through '+1' most of the Wigner term absorbed.

Formula generalized to the lowest state of a given T (e.g. Jänecke *et al.*, NPA728(03)23). Pairing depends on evenness of T .
 ?Lowest state of a given T : isobaric analogue state (IAS) of some neighboring nucleus ground-state.



Study of changes in the symmetry term possible nucleus by nucleus



IAS Data Analysis

In the same nucleus:

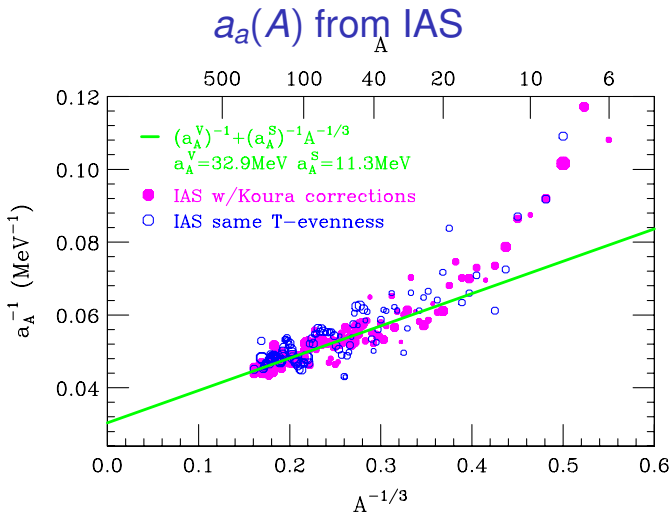
$$E_2(T_2) - E_1(T_1) = \frac{4 a_a}{A} \{ T_2(T_2 + 1) - T_1(T_1 + 1) \} \\ + E_{\text{mic}}(T_2, T_z) - E_{\text{mic}}(T_1, T_z)$$

$$a_a^{-1}(A) = \frac{4 \Delta T^2}{A \Delta E} \quad ? \quad = (a_a^V)^{-1} + (a_a^S)^{-1} A^{-1/3}$$

Data: Antony *et al.* ADNDT66(97)1

E_{mic} : Koura *et al.*, ProTheoPhys113(05)305
v Groote *et al.*, AtDatNucDatTab17(76)418
Moller *et al.*, AtDatNucDatTab59(95)185

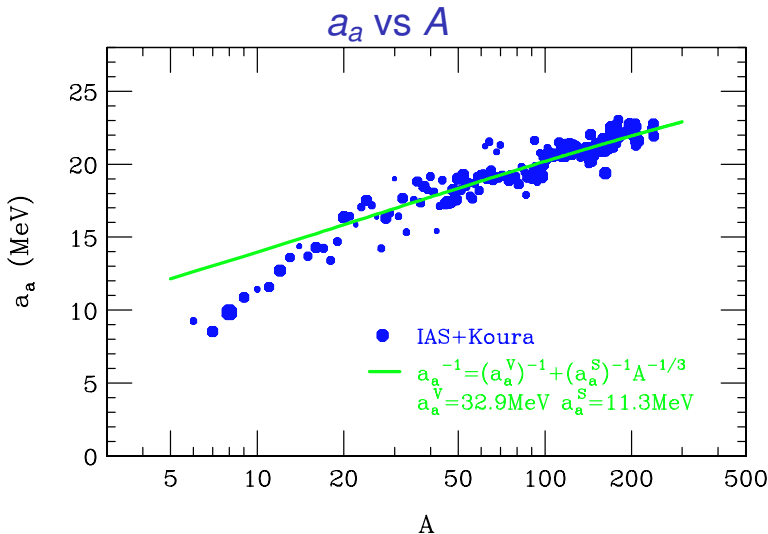




Symbol size proportional to relative significance.

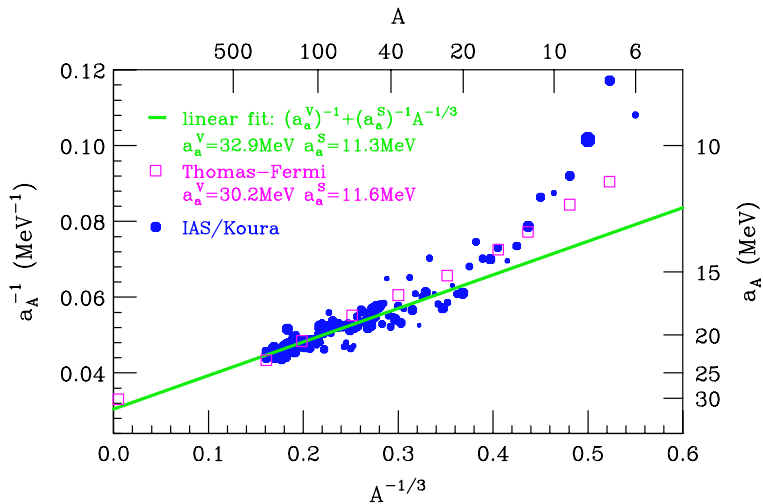
~Linear dependance from $A \gtrsim 20$ on.



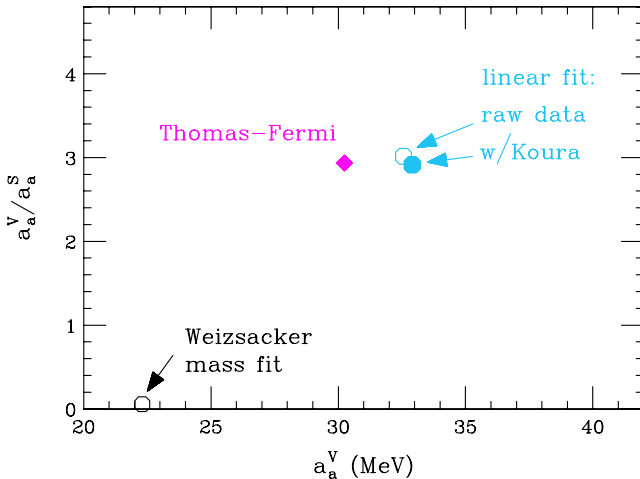


Line: best fit at $A > 20$.



Best $a(A)$ -Descriptions

Symmetry-Energy Parameters

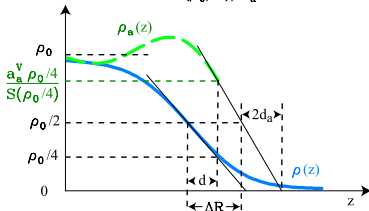
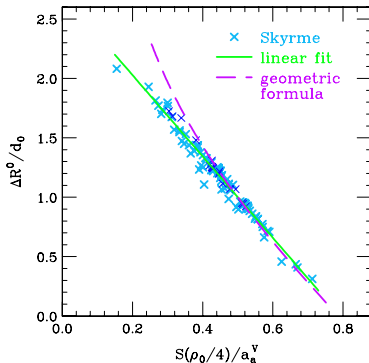
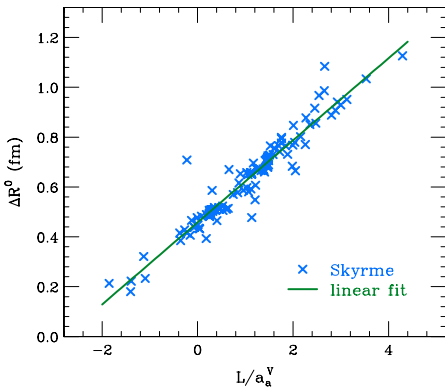


$$a_a^V = (31.5 - 33.5) \text{ MeV}, a_a^S = (9.5 - 12) \text{ MeV}, L \sim 95 \text{ MeV}$$



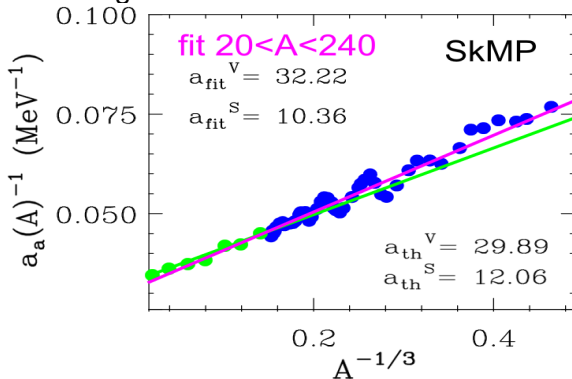
Displacement of Isovector Surface

$$\langle r^2 \rangle_n^{1/2} - \langle r^2 \rangle_p^{1/2} \approx \frac{A(N-Z)}{4NZ} \frac{\langle r^2 \rangle_a - \langle r^2 \rangle}{\langle r^2 \rangle^{1/2}}$$



Recent Developments: More SHF Calculations

Coefficient-extraction from spherical SHF results for realistic (blue) and humongous (green) (up to $A \sim 10^6$) nuclei, using P.-G. Reinhard's codes



Nuclei in Nature too small for a clean surface/volume separation!

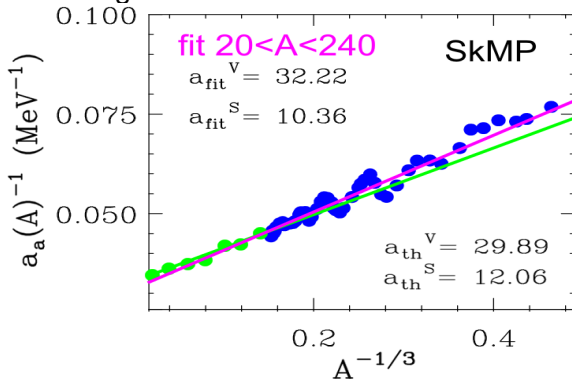
Consequence: a_a^V a bit smaller, a_a^S a bit larger and L a bit lower than from the simple analysis.

Outlier issue: limits on 'range' in the Skyrme symmetry energy



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- Weakening of the symmetry term is tied to the asymmetry skins and to the shape of $S(\rho)$.
- Skin sizes in all nuclei quantifiable in terms of single ratio, already known, $a_a^V / a_a^S \lesssim 3.0$. Corresponding $L \lesssim 95 \text{ MeV}$.
- Systematic of proton densities for one A should principally contain as much info as the skins and even more.
- 2 fundamental densities characterize nucleon distributions: isoscalar & isovector. Density surfaces are displaced from each other by $\Delta R \lesssim 0.95 \text{ fm}$ and have different diffusenesses, $d \sim 0.54 \text{ fm}$ and $d_a \sim 0.40 \text{ fm}$.
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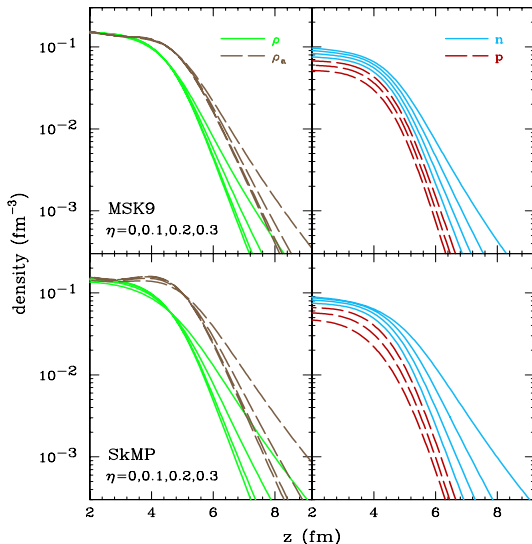


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Density Tails



Two Skyrme
interactions +
different
asymmetries



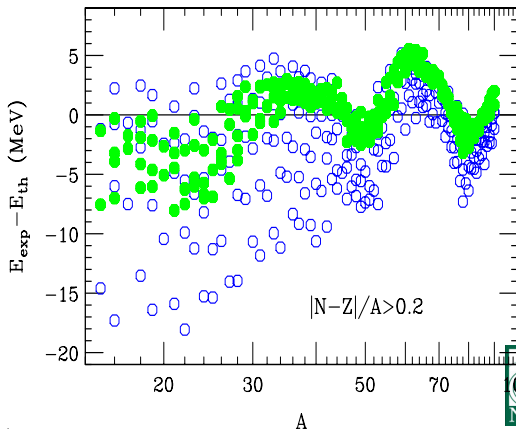
Modified Binding Formula

$$E = -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + \frac{a_a^V}{1 + A^{-1/3} a_a^V / a_a^S} \frac{(N - Z)^2}{A}$$

Energy Formula

Performance:

Fit residuals f/light asymmetric nuclei, either following the **Bethe-Weizsäcker formula** (open symbols) or the **modified formula** with $a_a^V / a_a^S = 2.8$ imposed (closed), i.e. the same parameter No.



Droplet Model

Droplet model (Myers & Swiatecki '69)

$$E = \left(-a_1 + J \bar{\delta}^2 - \frac{1}{2} K \bar{\epsilon}^2 + \frac{1}{2} M \bar{\delta}^4 \right) A$$

$$+ \left(a_2 + Q \tau^2 + a_3 A^{-1/3} \right) A^{2/3} + c_1 \frac{Z^2}{A^{1/3}} \left(1 + \frac{1}{2} \tau A^{-1/3} \right)$$

$$- c_2 Z^2 A^{1/3} - c_3 \frac{Z^2}{A} - c_4 \frac{Z^{4/3}}{A^{1/3}}$$

where

$$\bar{\epsilon} = \frac{1}{K} \left(-2a_2 A^{-1/3} + L \bar{\delta}^2 + c_1 \frac{Z^2}{A^{4/3}} \right), \quad \tau = \frac{3}{2} \frac{J}{Q} (\bar{\delta} + \bar{\delta}_s)$$

$$\bar{\delta} = \frac{I + \frac{3}{8} \frac{c_1}{Q} \frac{Z^2}{A^{5/3}}}{1 + \frac{9}{4} \frac{J}{Q} A^{-1/3}}, \quad \bar{\delta}_s = -\frac{c_1}{12J} \frac{Z}{A^{1/3}}, \quad I = \frac{N - Z}{N + Z}$$

$Q = H / (1 - \frac{2}{3} P / J)$. Expansion in asymmetry yields results consistent with current, but approach more complex...



Liquid Drop Model

The current formula:

$$E = -a_V A + a_S A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_A^V \frac{(N-Z)^2}{A} \frac{1}{1 + \frac{a_A^V}{a_A^S} A^{-1/3}}$$

Liquid drop model [LDM] (Myers & Swiatecki '66)

$$E = -a_V (1 - \kappa_V I^2) A + a_S (1 - \kappa_S I^2) A^{2/3} + a_C \frac{Z^2}{A^{1/3}} - a_4 \frac{Z^2}{A}$$

with $I = (N - Z)/A$. LDM corresponds to the expansion in the current formula:

$$\frac{1}{\frac{a_A^V}{A} + \frac{A^{2/3}}{a_A^S}} \approx \frac{a_A^V}{A} \left(1 - \frac{a_A^V}{a_A^S} A^{-1/3} \right)$$

But that expansion only accurate for $A \gtrsim 1000$, i.e. never!

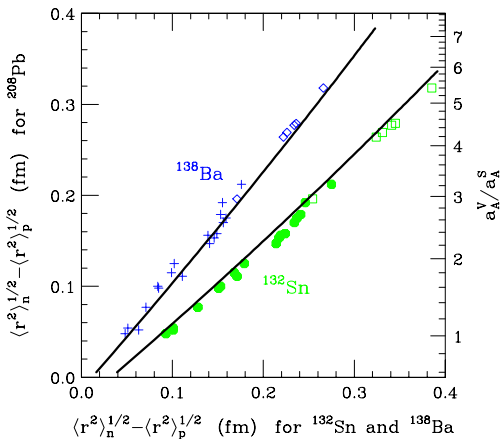


Analytic Expression for Skin Size

symmetry energy only

Coulomb correction

$$\frac{\langle r^2 \rangle_n^{1/2} - \langle r^2 \rangle_p^{1/2}}{\langle r^2 \rangle^{1/2}} = \frac{A}{6NZ} \frac{N-Z}{1 + A^{1/3} a_a^S/a_a^V} - \frac{a_C}{168a_a^V} \frac{A^{5/3}}{N} \frac{\frac{10}{3} + A^{1/3} a_a^S/a_a^V}{1 + A^{1/3} a_a^S/a_a^V}$$

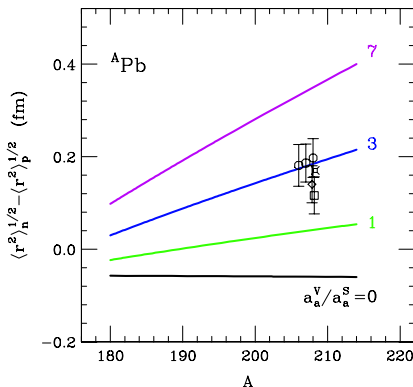
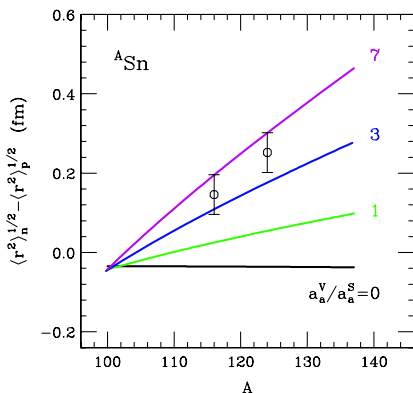


Formula (lines)
vs Typel & Brown
results (symbols)
from
nonrelativistic &
relativistic
mean-field
calculations,
PRC64(01)027302



Skin Sizes for Sn & Pb Isotopes

Lines - formula predictions, PD NPA727(03)233



Favored ratio $a_a^v/a_a^s \simeq 32.5/10.8 \simeq 3.0$



Comparison with Droplet-Model Values

