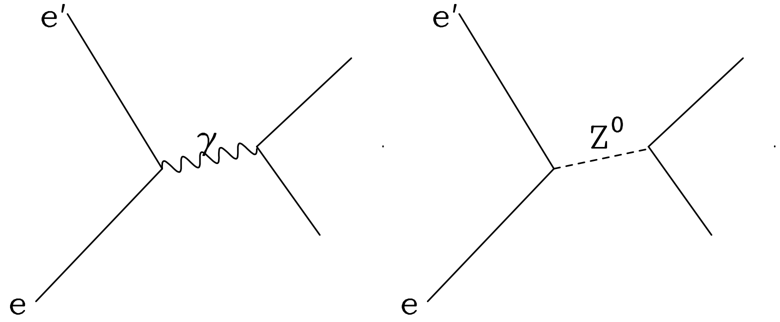
Parity-violating electron-nucleus elastic scattering

Nuclear isospin mixing and neutron distribution

Ingo Sick



PV from interference of γ and Z^0 exchange

Asymmetry defined by $\mathcal{A} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$ **the +,− indicating the $\vec{e}$ helicity.**

Motivation to study $\vec{e}$-A elastic scattering

has undergone quite some evolution

Initial emphasis: elastic scattering, T=0, N=Z nuclei

Feinberg PRD 12 (75) 3575
Walecka NPA 285 (77) 349

⇒ weak and EM form factors proportional to each other, yielding

$$\mathcal{A} = \mathcal{A}^0 \equiv \left[\frac{G\,|Q^2|}{2\pi\alpha\sqrt{2}}\right] 2sin^2\Theta_W \cong 3.22 \times 10^{-6}|Q^2| \text{ (in fm}^{-2}\text{)}$$

Main observations

$\mathcal{A} \sim Q^2$

prefactor $\sim 10^{-6}$

independent of nuclear structure

Consequence

can use PV $\vec{e} - A$ scattering to measure G, $G(Q^2)$

can use for test of SM

Early test of SM

SLAC E122 (∼1979): $\vec{e}$–^{2}H DIS

more complicated: 2 flavors, L and T

→ confirmation of SM to ±10%

→ proof of feasibility of PV e-scattering

First experiment elastic, T=0, N=Z

Bates, ^{12}C, 250MeV, ∼35°

Souder *et al.*, PRL 65 (90) 694

$\mathcal{A} \sim (1.6 \pm 0.3 \pm 0.1)\ 10^{-6}$

consistent with SM

Complications

- P-violation in nucleus
- contribution from MEC
- contribution of strange quarks
- T-mixing in nucleus

- **Parity-violation in nuclei**

  small effect for elastic scattering
  B. Serot, NP 322 (79) 408
  would be more important for inelastic scattering

- **Meson Exchange Currents**

  cancel for pure T=0, J=0
  for T≠0 and L: small
  see Musolf *et al.*, Phys. Rep. 239 (94) 1
  but at large Q^2: not entirely small

- **Strange quarks**

  became popular due to "missing spin" in e-$\vec{p}$ DIS
  $\longrightarrow$ large contribution of s-quarks??

  became major motivation for $\vec{e}-p$: SAMPLE, HAPPEX, A4, G^0
  $\vec{e}-{}^4$He: JLAB (but was never a good argument!)

- T-mixing

  focus of this talk

  early work: Donnelly, Dubach, Sick NPA 503 (89) 589

  recent work: Moreno,Sarriguren,Moya,Udias,Donnelly,Sick NPA, in print

For general case

$$\mathcal{A} = \mathcal{A}^0(1+\Gamma)$$

i.e.

$$\Gamma = \frac{1}{\beta_V^{(0)}} \frac{\tilde{F}_{C0}(q)}{F_{C0}(q)} - 1 = \Gamma^T + \Gamma^s$$

retain contributions from strangeness for comparison

Written in terms of T=0,1

$$\frac{\widetilde{F}_{C0}(q)}{F_{C0}(q)} = \frac{\beta_V^{(0)} \langle 0||M^C_{0\,(T=0)}(q)||0\rangle + \beta_V^{(1)} \langle 0||M^C_{0\,(T=1)}(q)||0\rangle}{\langle 0||M^C_{0\,(T=0)}(q)||0\rangle + \langle 0||M^C_{0\,(T=1)}(q)||0\rangle},$$

T=0,1 multipole operators in terms of 0^{th}, 1^{st} order contributions in p/m_N

$$M_J^{C\,M_J}(q\mathrm{x}) = M_J^{C\,M_J}(q\mathrm{x})_{[0]} + M_J^{C\,M_J}(q\mathrm{x})_{[1]}$$

with

$$\begin{aligned} M_J^{C\,M_J}(q\mathrm{x})_{[0]} &= \frac{\kappa}{\sqrt{\tau}} G_E(\tau) M_J^{M_J}(q\mathrm{x}) \,, \\ M_J^{C\,M_J}(q\mathrm{x})_{[1]} &= \kappa\sqrt{\tau}\, [2G_M(\tau) - G_E(\tau)]\, \Theta_J^{M_J}(q\mathrm{x}) \,, \end{aligned}$$

with usual $\kappa = q/(2m_N)$ and $\tau = Q^2/(4m_N^2)$.

The Coulomb and spin-orbit operators are given by

$$\begin{aligned} M_J^{M_J}(q\mathrm{x}) &= j_J(q\mathrm{x}) Y_J^{M_J}(\hat{\mathrm{x}}) \,, \\ \Theta_J^{M_J}(q\mathrm{x}) &= -\frac{i}{q^2}\vec{\sigma} \cdot \left[\left(\vec{\nabla} M_J^{M_J}(q\mathrm{x}) \times \vec{\nabla} \right) \right] \end{aligned}$$

with usual relation between T=0,1 and n,p M's and G's
This yields

$$\frac{\widetilde{F}_{C0}(q)}{F_{C0}(q)} = \frac{\langle 0 || \widetilde{M}^C_{0\,p}(q) || 0 \rangle + \langle 0 || \widetilde{M}^C_{0\,n}(q) || 0 \rangle}{\langle 0 || M^C_{0\,p}(q) || 0 \rangle + \langle 0 || M^C_{0\,n}(q) || 0 \rangle} \,,$$

The $\widetilde{M}$'s contain the weak form factors

$$\widetilde{G}_E = \frac{1}{2}(\beta_V^{(0)} G_E^{(0)} + \beta_V^{(s)} G_E^{(s)} + \tau_3 \beta_V^{(1)} G_E^{(1)}) \,,$$

or

$$\widetilde{G}_E = \widetilde{G}_{E_p} \frac{1}{2}(1+\tau_3) + \widetilde{G}_{E_n} \frac{1}{2}(1-\tau_3) \,,$$

with

$$\begin{aligned} \widetilde{G}_{E_p} &= \beta_V^p G_{E_p} + \beta_V^n G_{E_n} + \frac{1}{2} \beta_V^{(s)} G_E^{(s)} \,, \\ \widetilde{G}_{E_n} &= \beta_V^n G_{E_p} + \beta_V^p G_{E_n} + \frac{1}{2} \beta_V^{(s)} G_E^{(s)} \,. \end{aligned}$$

and usual SM coupling constants $\beta_V^{(0)}$=.–46, $\beta_V^{(1)}$=.54, $\beta_V^p = ..$, $\beta_V^n = ..$, β_V^s=–1

Simplifying things a bit

For strangeness contribution

assume $G_{E_p} = G_{M_p} = G_{E_n} = G_{M_n} = 0$ in weak form factor

neglect $\rho_n - \rho_p$ (OK for N=Z), use G_{E_n}=0, SO=0

$$\Gamma^{(s)} = \frac{\beta_V^{(s)}}{\beta_V^{(0)}} \frac{G_E^{(s)}}{G_E^{(0)}}$$

For T-mixing

assume $G_E^s = G_M^s = 0$, neglect SO term, set G_{E_n}=0

Find

$$\frac{\widetilde{F}_{C0}(q)}{F_{C0}(q)} = \beta_V^p + \beta_V^n \frac{\langle 0||M_{0\,n}(q)||0\rangle}{\langle 0||M_{0\,p}(q)||0\rangle}\,, \qquad \beta_V^p = 0.04, \quad \beta_V^n = -0.5$$

with

$$\langle 0||M_{0\,\xi}(q)||0\rangle \sim \int j_0(qr)\rho_\xi(r) r^2 dr\,, \qquad \xi = p, n$$

or equivalently

$$\Gamma = \frac{\beta_V^n}{\beta_V^{(0)}} \left(\frac{\langle 0||M_{0\,n}(q)||0\rangle - \langle 0||M_{0\,p}(q)||0\rangle}{\langle 0||M_{0\,p}(q)||0\rangle} \right)$$

depends on FT of n,p-densities ..... in PWBA

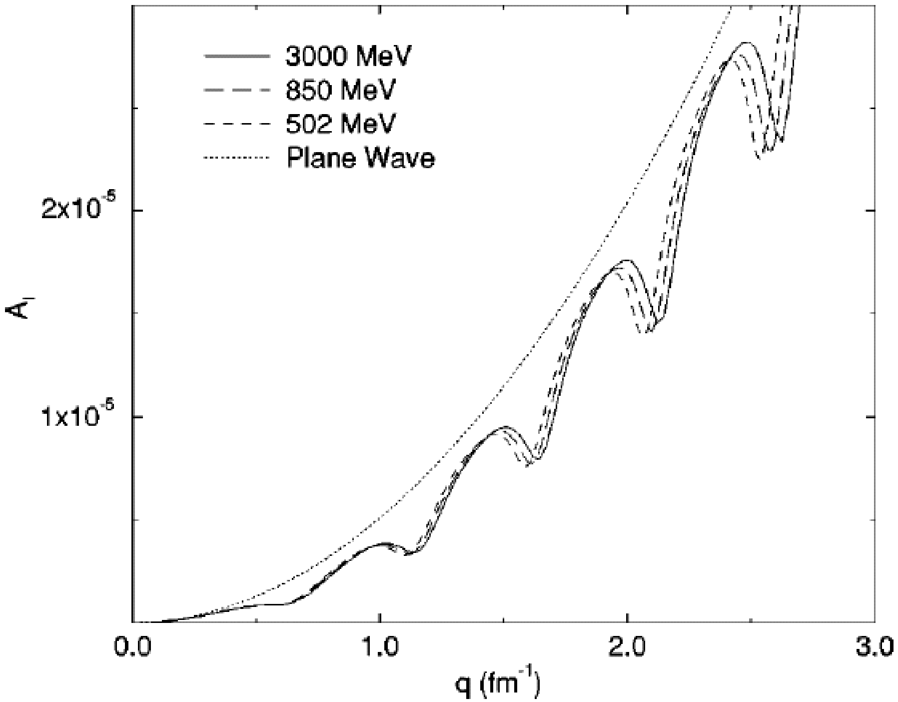
Coulomb distortion

for light N=Z nuclei only important in diffraction-minimum
for heavy nuclei must include

Horowitz, PRC 57 (98) 3430
accounts for distortion in $^{208}Pb(\vec{e}, e)$, PREX

Moreno *et al.*, Nucl.Phys. A, in print
account for distortion for p- and s/d-shell nuclei

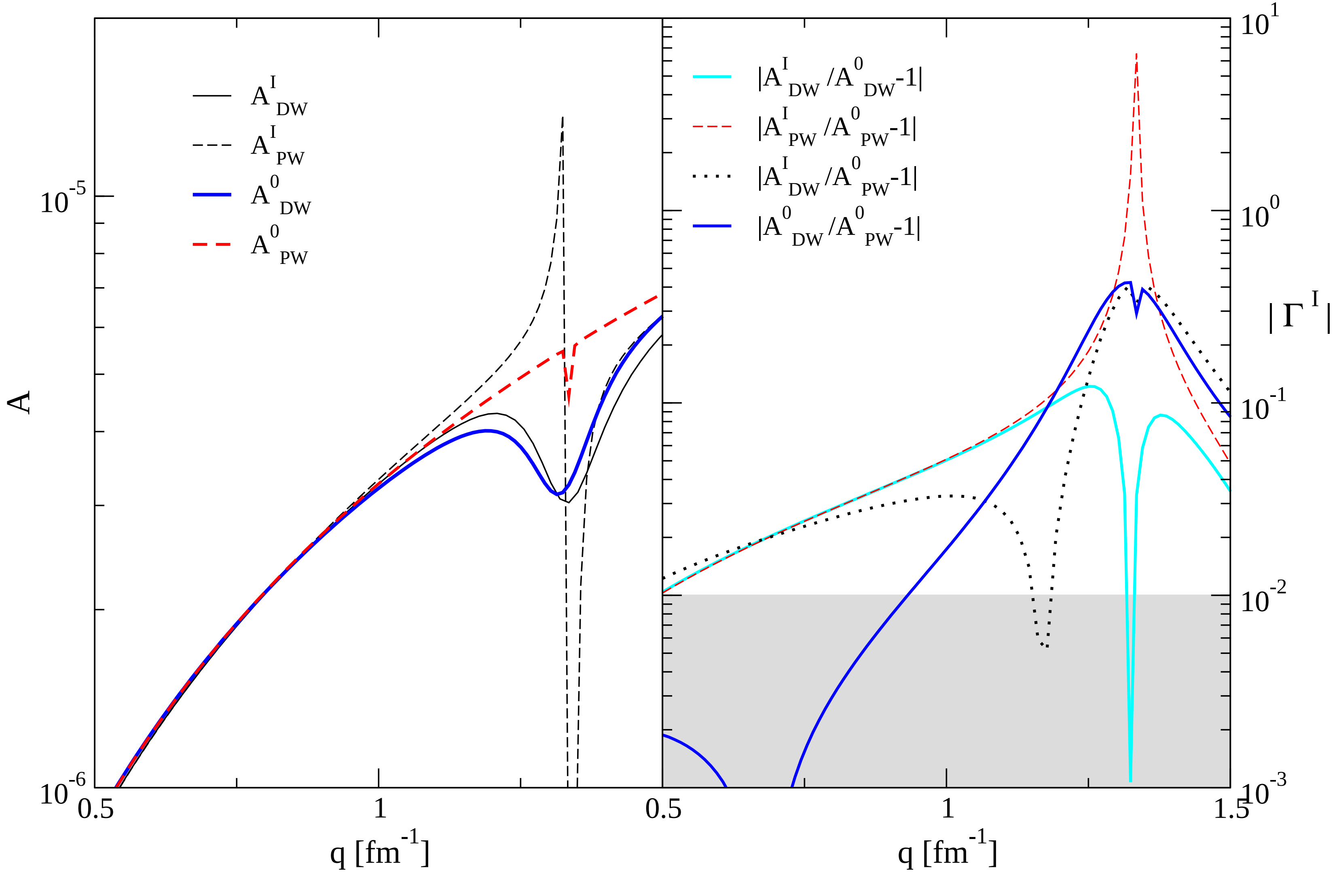
both use solution of Dirac equation



$^{208}Pb(\vec{e}$,e)
Horowitz, PW vs. Dirac

Distortion for s/d-shell nuclei

calculated for E=1GeV



T-mixing in nuclei

sources: $m_n \neq m_p$, $V_{nn} \neq V_{np} \neq V_{pp}$, Coulomb

Coulomb dominates for $Z \gg 1$

notoriously difficult problem, T-impurities still rather uncertain

accessible in T-forbidden E1-decays in N=Z nuclei

accessible via isobaric multiplet mass equations

main complication for shell model calculations:

structure of T=1, 0^+ states, mixing with nearby states

Role in parity-violating (e,e)

Donnelly, Dubach, Sick, NPA 503 (89) 589

simple 2-level model involving lowest 0^+ T=1 state

use $2\hbar\omega$ matrix elements of Dubach+Haxton, WS radial wave functions

adopt $\langle 0^+ \; T = 1 | H_{CSB} | 0^+ \; T = 0 \rangle$ = 300keV, fairly uncertain

Moreno *et al.* (show one example)

Hartree-Fock + BCS calculation, axially symmetric deformation, SLy4 force

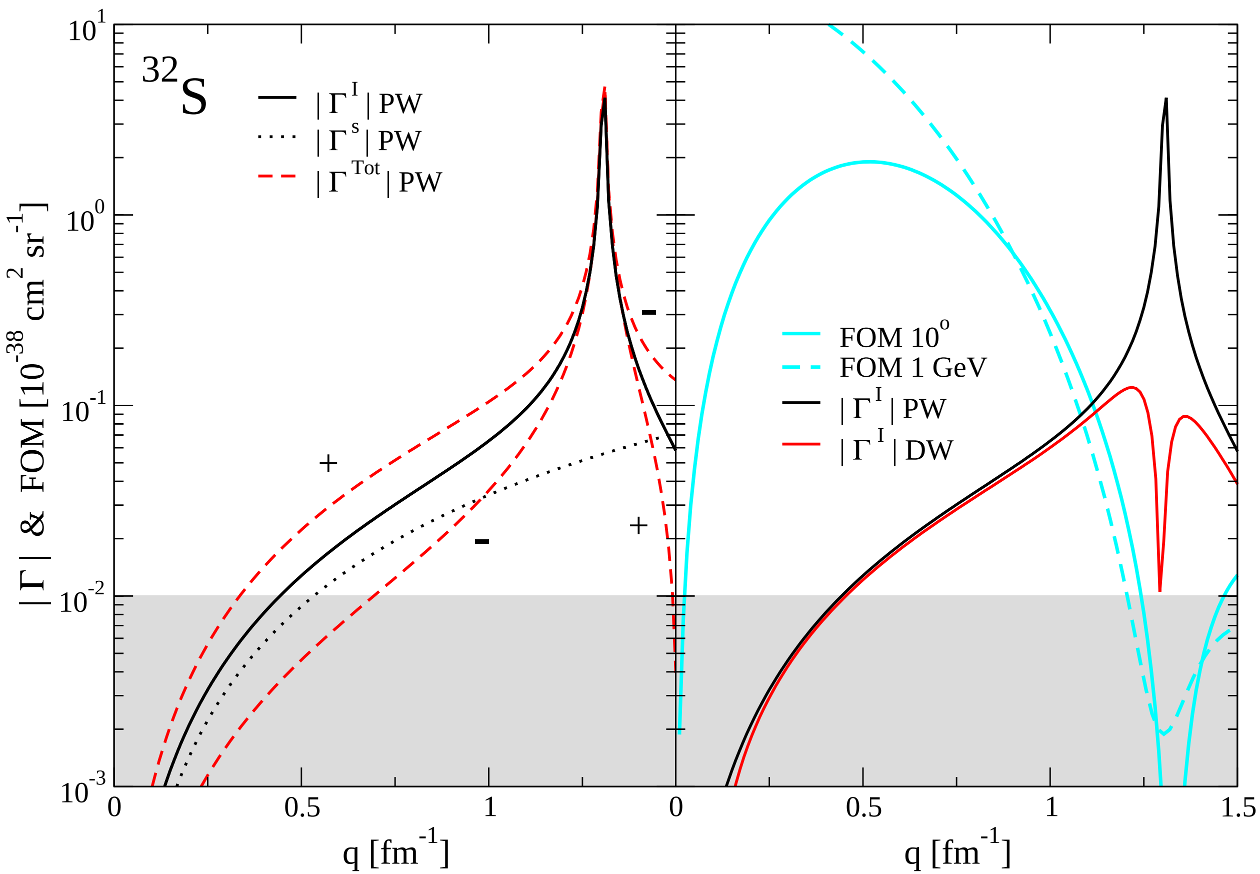
$\rightarrow$ Coulomb included non-perturbatively

For numerical calculation of F's use:

Hoehler, $G_E^{(s)} = \rho_s \tau G_D^V \xi_E^{(s)}$, $G_M^{(s)} = \mu_s G_D^V$, with $G_D^V = (1 + 4.97\tau)^{-2}$, $\xi_E^{(s)} = (1 + 5.6\tau)^{-1}$

use upper limit $\rho_s = \pm$ 1.5 (HAPPEX-He) and μ_s –0.31 (Musolf, effect minor)

Asymmetries



$$\mathcal{F} = \frac{d\sigma}{d\Omega}\mathcal{A}^2$$

Observations

- T-admixture important on 10%-level
- effects larger than found by DDS (non-perturbative, off-diagonal terms *e.g.* the T=1, $0d_{5/2} - 1d_{5/2}$ matrix elements)
- subtleties of nuclear dynamics don't matter too much
- effects increase with larger Z
- effects larger than upper limit of s-contribution

T-mixing for very light nuclei?

not negligible in ^{4}He either, although $Z\alpha << 1$
see Viviani *et al.*, PRL 99 (07) 112002

Sources of T-admixtures in ^{4}He

- admixture in nucleon, see Kubis, Lewis PRC 74 (06) 015204
- in ^{4}He, both due to Coulomb and isospin-breaking effects in V_{NN} (give contributions of similar size)

HAPPEX-He, Q^2=0.077 $\mathcal{A}_{PV} = [6.40 \pm 0.23 \pm 0.12]$ ppm

implies contribution of s-quarks, T≠0

$$\Gamma = -2\frac{F^{(1)}(q)}{F^{(0)}(q)} - \frac{2G_E^{\not T} - G_E^s}{(G_E^p + G_E^n)/2} = 0.010 \pm 0.038$$

after adding correction for ISB in ^{4}He ($F^{(1)}$) and N ($G_E^{\not T}$)

$$G_E^s = 0.001 \pm 0.016$$

i.e. the ISB correction explains the Γ found by HAPPEX
(which is very small, and only 1/4 of the exp. error)

Conclusion:

T-admixtures are important, since s-contribution very small

And how about the proton?

HAPPEX for Q^2=0.11 finds

$$G_E^s + 0.09 G_M^s = 0.007 \pm 0.011 \pm 0.004 \pm 0.005(\text{FF})$$

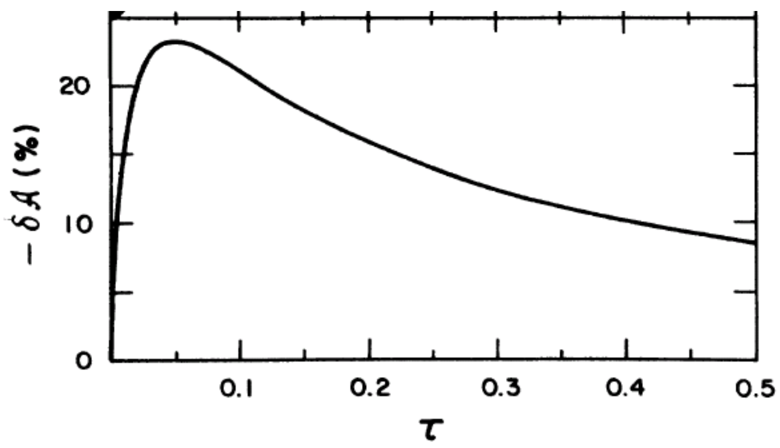
Kubis+Lewis PRC 74 (06) 015204 calculate T-mixing
using ChPT, resonance saturation and $\rho^0\omega$ mixing → LE-constants
also discuss other models

Their result $G^{u,d} = 0.004 \ldots 0.009$ shows:
T-mixing contribution is at level of exp. uncertainty

⇒ "T-mixing precludes significant improvement of measurements" (HAPPEX publication)

note: T-mixing explains much of NuTeV anomaly

Mixing was a worry of DDS in 88 already in:
"Determination of the neutron electric
form factor from H($\vec{e}$,e)H"
possible since effect of G_{en} in $\mathcal{A}_p$ large, see ⟹



main worry: role of mixing of J=1/2, T=3/2 state with πN continuum channels = ?

Similar size of s-quark and T-violation contribution for small Z:

is this an accident? **it can be qualitatively understood!**

$s\bar{s}$ **components come with**

energy denominator of ∼1GeV

(strong) coupling constant of order ∼1

T-admixtures come with

energy denominator of ∼20MeV

(EM) coupling constant of order Zα ∼0.02 (for small Z)

similar $\frac{strength}{energy}$ ⇒ similar overall contribution (for very small Z)

⇒ can hardly separate strangeness from T-mixing contributions!

⇒ T-mixing received too little attention in the past

DDS (1989), somewhat prophetical:

"In fact, with only a modest improvement in our present confidence in the SM, PV electron scattering might prove to be a viable (albeit experimentally difficult and expensive) probe of the isospin structure of nuclei"

.... indeed, but the "difficult and expensive" unfortunately was correct too

Much better situation for Z≫1: T-mixing = dominating effect ⇒ can exploit

Nuclei with N $\neq$ Z

stay with p, n - rather than $T = 0, 1$ - notation

repeat eqs. from above

$$\frac{\widetilde{F}_{C0}(q)}{F_{C0}(q)} = \beta_V^p + \beta_V^n \frac{\langle 0||M_{0\,n}(q)||0\rangle}{\langle 0||M_{0\,p}(q)||0\rangle}\,,$$

$$\boxed{\beta_V^p = 0.04, \quad \beta_V^n = -0.5}$$

or equivalently

$$\Gamma = \frac{\beta_V^n}{\beta_V^{(0)}} \left(\frac{\langle 0||M_{0\,n}(q)||0\rangle - \langle 0||M_{0\,p}(q)||0\rangle}{\langle 0||M_{0\,p}(q)||0\rangle} \right)$$

with

$$\langle 0||M_{0\,\xi}(q)||0\rangle \sim \int j_0(qr)\rho_\xi(r) r^2 dr \sim F_\xi(q)\,, \qquad \xi = p, n$$

Accident of SM

PV coupling to neutrons 10 times larger than to protons!

$\Rightarrow$ non-PV (e,e) sees basically protons

$\Rightarrow$ PV (e,e) sees basically neutrons!

Observations first made by Donnelly, Dubach, Sick, 1989:

- PV scattering can be used to determine ρ_n (wanted for long time!)
- PV experiment on ^{208}Pb is practical

Proposal of DDS (1989): measure $\mathcal{A}$ at *one* q-value

where still mainly sensitive to rms_n
where figure-of-merit highest
$\rightarrow q \sim 0.5\text{fm}^{-1}$

Case initially considered (pre-JLab)

300MeV, 17°
2 symmetrical, large-Ω spectrometers
resolution $\ll$3MeV
$\delta\mathcal{A}/\mathcal{A} \sim 6\% \Rightarrow \delta\text{rms}_n/\text{rms}_n \sim 1\%$ (PWIA!)
need rather $\delta\mathcal{A}/\mathcal{A} \sim 3\%$ when distortion accounted for
see PREX proposal

Present plans after optimization for JLab, PREX experiment

1GeV, 5°, same $q \sim 0.5$ fm^{-1}
2 standard hall-A spectrometers
septum magnets to reach small angle

Effects potentially complicating interpretation

see Horowitz, Pollock, Souder, Michaels, PRC 63 (01) 025501
s-admixture, parity admixture, MEC, dispersion corrections, inelastic , ...

find that all are small

the case where only *one* physical effect matters $\Rightarrow$ clean measurement

Main progress of last years: technology of $(\vec{e}, e)$

- high polarization, high intensity, reliable $\vec{e}$-beams
- reduction of false asymmetries to extremely low levels
- precise beam polarimetry

$\Rightarrow$ experiment to measure $\rho_n(r)$ is indeed feasible

$\Rightarrow$ PREX experiment in Hall A JLab, to run in 2010

I'm looking forward to see the results!

Do we need a point in first diffraction maximum?

how strict is correlation $r_n \leftrightarrow \mathcal{A}$ at $\theta \sim 5°$?

explore by fitting theoretical $\rho(r)$ with 2pF, extract c, z (units: fm)

3 theoretical calculations

calculation	nucleon	c	z	rms
Gogny	n	6.6855	0.5255	5.5344
	p	6.6050	0.4630	
Skyrme	n	6.6531	0.5267	5.5125
	p	6.5428	0.4386	
RFHNC	n	6.9500	0.5297	5.7317
	p	6.5580	0.4762	

Find: z_n fairly close, probably accidental.
z_p have fair dispersion
z_p-z_n can be quite large

Explore: effect of Δz for $\mathcal{A}$ =constant

Δz=0.02fm corresponding to variation z_n

$\rightarrow \Delta r_n \sim 0.1\%$

Δz=0.07fm corresponding to variation z_n-z_p

$\rightarrow \Delta r_n \sim 0.4\%$ $\rightarrow$ model-dependence of r_n small enough
agrees with Horowitz *et al.*