An extended liquid drop approach

Symmetry energy, charge radii and neutron skins

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Motivation

1. Provide nuclear input for test of standard model in apv
   charge and neutron radii
2. Determine the symmetry energy in nuclear matter

Nuclear structure and Atomic parity violation

$$H = \frac{G}{2\sqrt{2}} \int d^3r[-N\rho_n(r) + Z(1-4\sin^2\theta_W)\rho_p(r)]\psi^\dagger\gamma_5\psi$$

$$<\psi_s|\gamma_5|\psi_p> = iC_Z^{sp}\mathcal{N}[u_1^s(r)u_2^p(r) - u_2^s(r)u_1^p(r)]/r^2$$

factorize m.e.: $H_{if} \sim \frac{G_F}{2\sqrt{2}} C_{if}\mathcal{N}Q_W$ $\qquad \gamma = \sqrt{1-(\alpha Z)^2}$

electr overlap for point nucleus $\qquad \mathcal{N} = \psi_s^\dagger(0)\gamma_5\psi_p(0) \sim R_p^{2\gamma-2}$

Nucleus: $Q_w = -Nq_n + Zq_p(1-4\sin^2\theta_W) + \Delta Q_{new}$

q_i : convolution of electr. wf with finite nucleus; expand in $(Z\alpha)^n$

$$q_p = 1 - 0.26(Z\alpha)^2 + ...$$

$$q_n = 1 - \frac{3}{70}(Z\alpha)^2(1+5(\frac{R_n}{R_p})^2) + ...$$ (+ corrections to sharp radius)

why large Z? since $H_{pv} \sim Z^3$ enhancement (+rel. corr)

aim: for Ra (Z=88), for 0.1% measurement (to go after ΔQ_{new})

need: $R_p \approx 1\%$, $(R_n - R_p)/R_p \approx 25\%$ (at present no direct exp info)

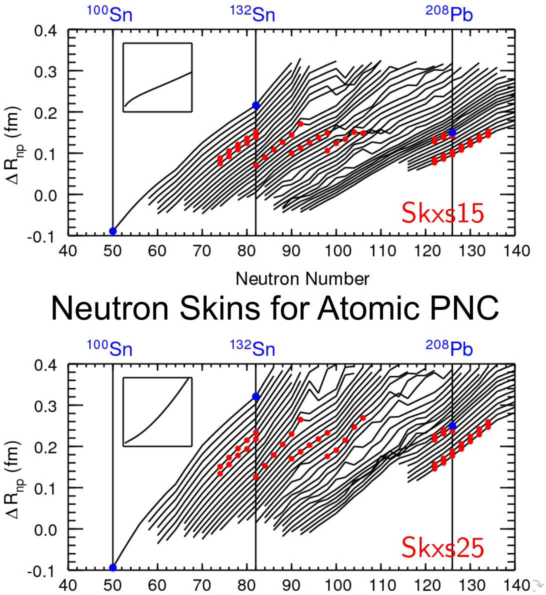
Let us see whether we can meet these requirements

from Brown et al PRC76,2009
based upon spherical Skyrme EDF

ΔR depends on Skxs(15..25)
idea: PREX@Jlab will fix that

Questions

- Irregularities due to high spin intruder orbitals ($i13/2$)
- Effect of deformation?
- Are slopes really constant?
- Does not fit exp R_c very well (except ^{208}Pb)



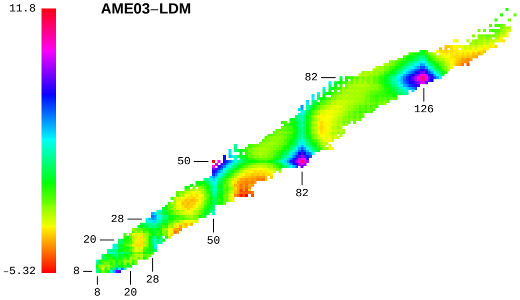
content

- Extended Liquid Drop Model see Pawel Danielewicz' talk
  surface symmetry energy
  LDM adapted shell corrections
- Applications
  - symmetry energy for $A \longrightarrow \infty$
  - charge radii and isotope shifts
  - neutron skin prediction
  - atomic parity violation: nuclear corrections to weak charge measurements

Conventional Bethe-von Weizsäcker (liquid drop) formula

$$E_A = -a_B A + a_{surf} A^{2/3} + S_{vol}(N-Z)^2/A + a_C \frac{Z^2}{A^{1/3}} + E_{pair}$$

- Incomplete form of symmetry energy
  Need to introduce volume and surface symmetry energy
- Coulomb need to be refined ($R_c(N,Z) \neq r_0 A^{1/3}$)
- shell corrections need to be added



Extended Liquid Drop Model

- Improved BW: $E_{sym} = E(vol, surf)$

  Decompose asymmetry $N - Z = N_s - Z_s + N_v - Z_v$ surface+ volume

  $$E^A_{vol} = a_B A + S_{vol}\frac{(N_v - Z_v)^2}{A}$$
  $$E^A_{surf} = E^0_{surf} + S_{surf}(N_s - Z_s)^2/A^{2/3}$$

  minimize under fixed $N - Z$:

  $$\frac{N_s - Z_s}{N - Z} = \frac{1}{1 + y^{-1}A^{1/3}} \qquad y \equiv S_{vol}/S_{surf}$$

  $$E_A = -a_B A + a_{surf} A^{2/3} + \frac{S_v}{1 + yA^{-1/3}}(N - Z)^2/A + \ldots$$

Extended Liquid Drop Model

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  $$\frac{N_s - Z_s}{N - Z} = \frac{1}{1 + y^{-1}A^{1/3}} \qquad y \equiv S_{vol}/S_{surf}$$

  $$E_A = -a_B A + a_{surf} A^{2/3} + \frac{S_v}{1 + yA^{-1/3}}(N - Z)^2/A + \ldots$$

- LDM yields also relation between skin and S_s, S_v $(N_s - Z_s \ll A)$

  $$\frac{N}{N_v} = \left(\frac{R_n}{R_0}\right)^3 \rightarrow \frac{R_n - R_0}{R_0} \approx \frac{N_s}{3N}, \qquad \frac{R_p - R_0}{R_0} \approx \frac{Z_s}{3Z},$$

  $$\frac{R_n - R_p}{R} = \frac{A(N_s - Z_s)}{6NZ} \approx \frac{A}{6NZ}\frac{N - Z - \frac{a_C}{12}ZA^{2/3}/S_v}{1 + A^{1/3}/y}$$

  Coulomb term: for $N = Z$ $R_p > R_n$

Myers-Swiatecki, Ann Phys 55 (1969)395

Danielewicz, NPA 727(2003)233; Steiner et al, Phys. Rep. 411,325

differ in the choice of condition $N_s + Z_s = 0$, or $Z_s = 0$

several methods have been proposed, Strutinsky, Koura......

here we adapt microscopic mass model of Duflo-Zuker
rms dev $\approx$ 500 keV, (14-28 parameters)

idea: count number of valence particles $(n_v,\ z_z)$
with respect to closed shells $\Delta E_{sh} = \Delta E(n_v, z_v)$

Example: monopole force in single j-shell
with degeneracy $D_j = 2j + 1$, seniority $v = 0$

$$E_{pair}(n_v) = \frac{g}{D} n_v(D - n_v + 2) \equiv g' n_v.\bar{n}_v + \frac{2g}{D} n_v$$

$\downarrow$

$\bar{n}_v \equiv D - n_v$ =number of holes

absorb in core

$$E_{\rm shell}(N,Z) = a_1 S_2 + a_2 (S_2)^2 + a_3 S_3 + a_{\rm np} S_{\rm np}$$

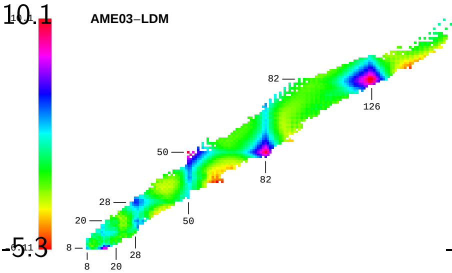
$$\begin{aligned}
S_2 &= \frac{n_{\rm v}\bar{n}_{\rm v}}{D_n} + \frac{z_{\rm v}\bar{z}_{\rm v}}{D_z}, \qquad \xrightarrow{n_v << D} n_v + z_v \\
S_3 &= \frac{n_{\rm v}\bar{n}_{\rm v}(n_{\rm v} - \bar{n}_{\rm v})}{D_n} + \frac{z_{\rm v}\bar{z}_{\rm v}(z_{\rm v} - \bar{z}_{\rm v})}{D_z}, \\
S_{\rm np} &= \frac{n_{\rm v}\bar{n}_{\rm v} z_{\rm v}\bar{z}_{\rm v}}{D_n D_z},
\end{aligned}$$

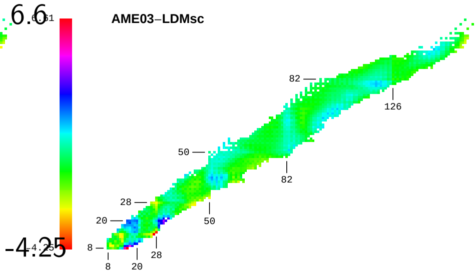
with $\bar{n}_{\rm v} \equiv D_n - n_{\rm v}$

Magic numbers: 6, 14, 28, 50, 82, 126, 184

similar to terms in microscopic mass formula of Duflo and Zuker
they refer to S_3 as "monopole drift" (changes sign midshell)

examples





without shell corr

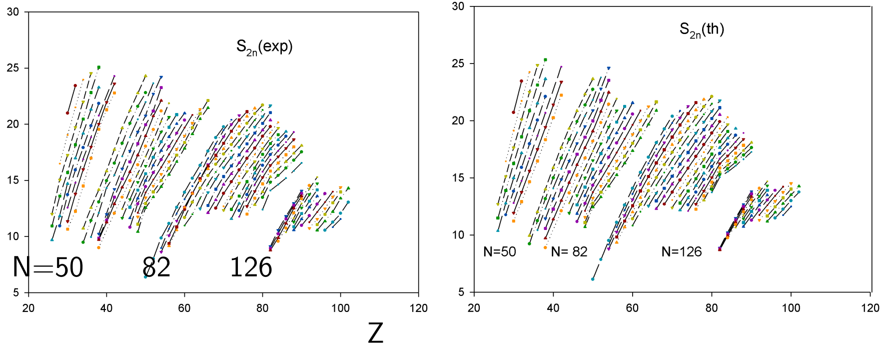
rms deviation 2.4 MeV

with shell corr

0.790 MeV (mostly due to light A)

Illustration: neutron separation energies

Often convenient to consider relative quantities,
e.g. neutron separation energies $S_{2n} = E(N,Z) - E(N-2,Z)$ vs Z



relevance: extrapolation,

Determination of Symmetry energy

$$E_{\text{LDM}} \approx -a_B A + a_{surf} A^{2/3} + \frac{(N-Z)^2}{A}\frac{S_v}{1+yA^{-1/3}} + a_C \frac{Z(Z-1)}{A^{1/3}} + E_{pair} + E_{shell}$$

aim: determine $S_v, y = \frac{S_v}{S_s}$ in isolation of other LDM parameters

$$\frac{1}{2}\left(\frac{dE}{dN} - \frac{dE}{dZ}\right) = \frac{N-Z}{A} S_A + a_c \frac{Z}{A^{1/3}} + \delta E_{shell}$$

$$\downarrow$$

$$\mu_a = \frac{1}{2}(\mu_n - \mu_p) = \frac{1}{2}[B(N-1,Z) - B(N,Z-1)]$$ (isovector chem. pot.)

invert $(N \neq Z)$ $S_A = \frac{A}{N-Z}(\mu_a - \delta E_C - \delta E_{shell})$ independent of $a_B,\ a_{surf}$

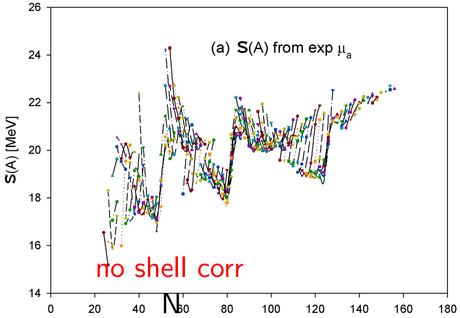
subtlety: SU(2): $(N-Z)^2 = 4T_z^2 = 4T(T+1)$

suggests $N - Z \rightarrow N - Z + 1$ partially absorbs Wigner energy

Symmetrize removal and addition $\mu_a = \frac{1}{4}[B(N-1,Z) - B(N,Z-1) + B(N,Z+1) - B(N+1,Z)]$

Symmetry energy from exp n-p sep energies

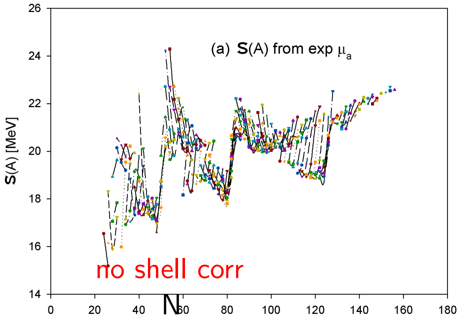
$$S_A = \frac{A}{N-Z}(\mu_a + \delta E_C)$$

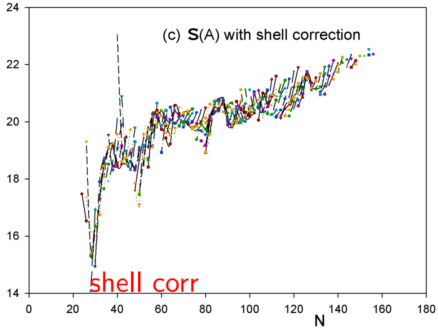


Note: the regularity of shell effects (up/down sloping)

$$S_A = \frac{A}{N-Z}(\mu_a + \delta E_C)$$

$$S_A = \frac{A}{N-Z}(\mu_a + \delta E_C + \delta E_{shell})$$





Note: the regularity of shell effects (up/down sloping)

particle-particle: $E_{sh} \sim n_v + z_v - [(n_v+1) + (z_v-1)] = 0$

particle-hole (or h-p) : $E_{sh} \sim \pm(n_v - z_v - [(n_z+1) - (z_v-1)]) = \pm 2$

problem at $N = 40, \ Z = 38$ (midshell in 28-50)

Symmetry energy for nuclear matter

$S_A(N,Z) = \frac{S_v}{1+yA^{-1/3}}$, $y = S_v/S_s$

fit in isolation of other parameters

Leads to 2-par fit: $S_A^{-1} = \frac{1+yA^{-1/3}}{S_v}$

extrapolation to $A \to \infty$

fit values

$S_v = 32.5 \pm 1.8$, $y = 2.95 \pm .4$ (sh.c)

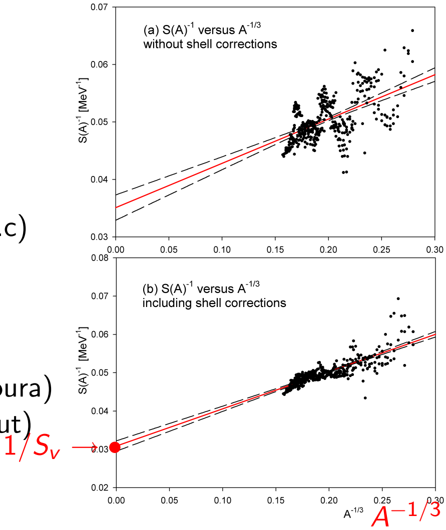
$S_v = 28.5$ MeV (no sh corr)

compare with Danielewicz

using IAS and sh corr from Koura

$S_v = 32.8$ y=2.8 with sh corr (Koura)

$S_v = 29 \pm 2$, $y = 2.4 \pm 0.4$ (without)

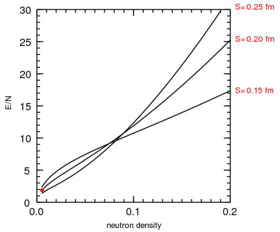


Note correlation between S_v and y

Symmetry energy $S(\rho)$

Can be converted to nuclear matter $S(\rho)$:
$S(\rho_0) \equiv S_v$ and S_s related to some $S(\rho < \rho_0)$

Using Thomas-Fermi: $E_a = \frac{\mu_a^2}{4} \int dr \frac{\rho(r)}{S}$
$S_v/S_s = \frac{3}{r_0} \int dr \frac{\rho(r)}{\rho_0} (\frac{S_v}{S(\rho)} - 1)$



simplest case: take $S(\rho) = S_v.(\rho/\rho_0)^\gamma$

note $\gamma = 1 \rightarrow S(\rho) =$ constant $\rightarrow S_s = \infty$

we find $\gamma \approx 0.7 \pm 0.1$

Danielewicz $\gamma = 0.65 \pm 0.1$ soft EOS

radii

express in terms of iso-scalar/vector

$$R_i(N,Z) = R_0(N,Z) \pm \frac{N-Z}{2A} R_1(N,Z) \quad \text{(i=n,p)}$$

mass radius: $R_0 = r_0 A^{1/3} + a A^{-2/3} + c\frac{(N-Z)^2}{A^2}$, in practice $c \approx 0$

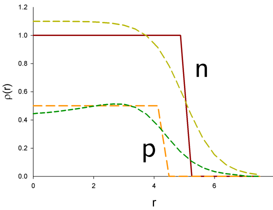
isovector radius $R_1 = b$

$R_{0,1}$ depend only weakly on $N-Z$

by charge symmetry: $R_p(N,Z)$ determines R_n

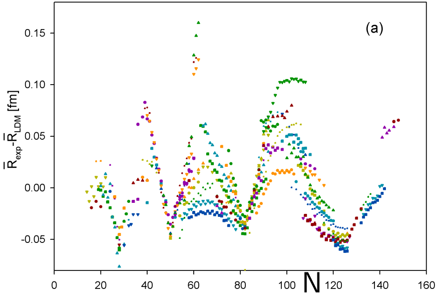
however there are Coulomb effects

1. in case of sharp radius:
   $\frac{\delta R_c}{R_0} = -\frac{a_c}{144 S_v} \frac{A^{8/3}}{NZ(1+y^{-1}A^{1/3})}$
   i.e. for $N = Z$ $\quad R_p > R_n$
2. polarization correction (if R is converted to rms radius): since $\rho_p(r) \neq \rho_n(r)$ in interior



charge radii, shell corrections

Closed shells: Strong binding ↔ small R
near closed shells: decrease of binding, increase of R
midshell: deformation: increase of binding, increase R



rms dev= 0.036 fm

charge radii, shell corrections

Closed shells: Strong binding $\leftrightarrow$ small R
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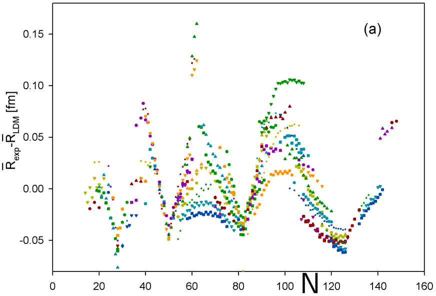

rms dev= 0.036 fm

Try: $\delta R_{shell}/R = a_2(n_v\bar{n}_v + z_v\bar{z}_v) + a_{pn}(n_v\bar{n}_v.z_v\bar{z}_v)$

$R_c(N,Z) = R_0(A) + \frac{N-Z}{A}R_1 + \delta R_c + \delta R_{shell}$

charge radii, shell corrections

Closed shells: Strong binding $\leftrightarrow$ small R

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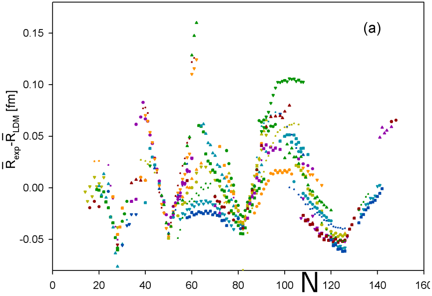
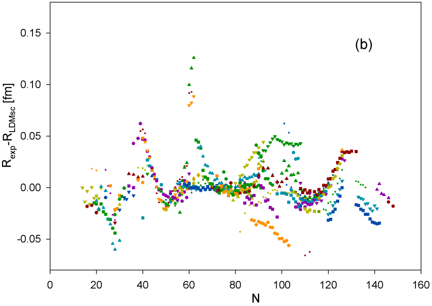




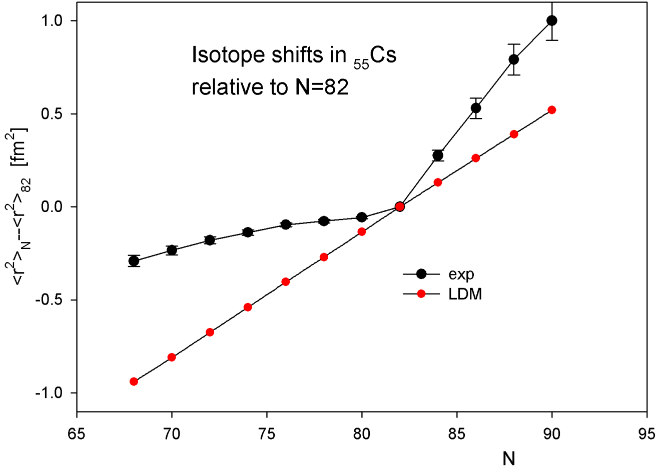
rms dev= 0.036 fm 0.021 fm (only 5 par's!)

Try: $\delta R_{shell}/R = a_2(n_v \bar{n}_v + z_v \bar{z}_v) + a_{pn}(n_v \bar{n}_v . z_v \bar{z}_v)$

$R_c(N,Z) = R_0(A) + \frac{N-Z}{A} R_1 + \delta R_c + \delta R_{shell}$

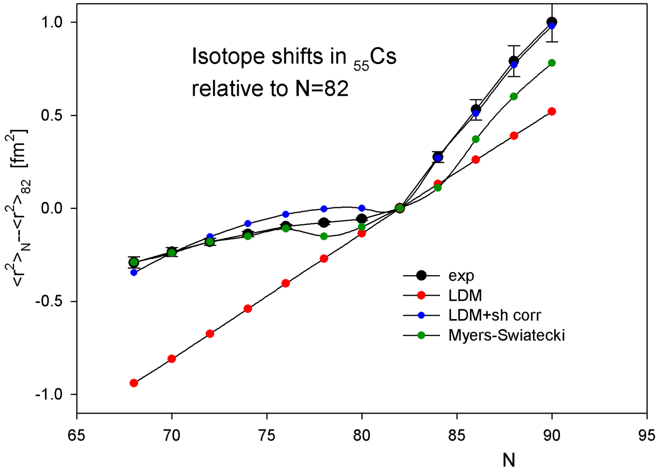
some data (Angeli) have large error bars (mix of stat, syst, theor errors)

isotope shifts for Cs



isotope shifts $\langle r^2 \rangle_N - \langle r^2 \rangle_{N=82}$

isotope shifts for Cs



isotope shifts $\langle r^2 \rangle_N - \langle r^2 \rangle_{N=82}$

charge radii, deformation

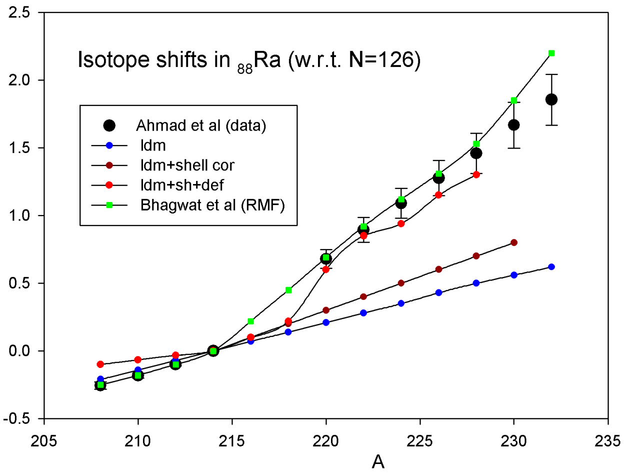
Away from closed shells with say $n_v,\ z_v \leq 4$
deformation of the ground state occurs:
increase of binding, increase of radius

$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta))$
then $R^2 = R^2_{spher}(1 + \frac{5}{4\pi}\beta_2^2 + ..)$

Simplest: take β_2 from exp BE(2) or Q-moments
We are working on a dynamic approach using a collective H to describe excited states.

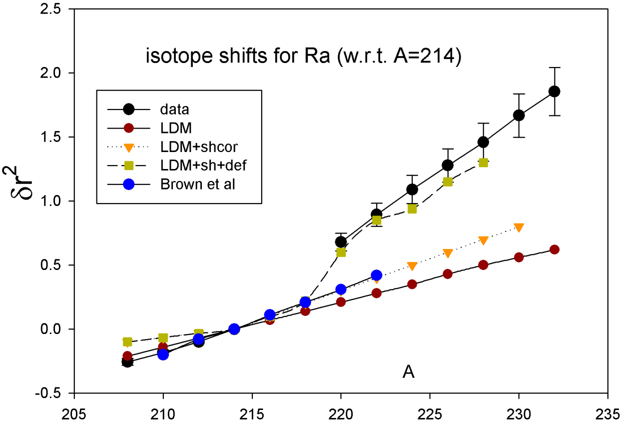
isotope shifts for Ra

isotope shifts $\langle r^2 \rangle_N - \langle r^2 \rangle_{N=126}$



isotope shifts for Ra

isotope shifts $\langle r^2 \rangle_N - \langle r^2 \rangle_{N=126}$



Deformation effect is substantial, absent in spherical EDF

neutron skin

options:

1. from isovector term in R: $\Delta R = \frac{N-Z}{A} R_1 + \delta R_C$
   e.g. ^{208}Pb: $\Delta R = 0.18 \pm 0.03$ fm $\downarrow$
   reduction of skin by 30%

2. from exp μ_a
   (i) $\frac{R_n - R_p}{R} = \frac{A(N_S - Z_S)}{6NZ} \approx \frac{A}{6NZ} \frac{N-Z-a_c Z A^{2/3}/S_v}{1+A^{1/3}/y}$

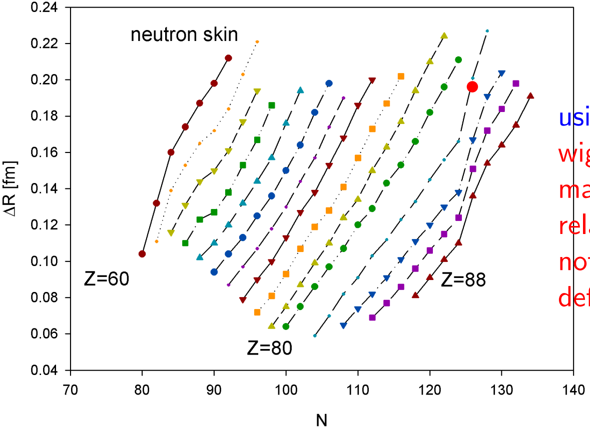
   (ii) $\mu_a(N,Z) = \frac{2(N-Z)}{A} \frac{S_s A^{1/3}}{1+A^{1/3}/y} - \frac{5a_c}{6} \frac{Z}{A^{1/3}}$

   from (i)+(ii)

$$\frac{\Delta \bar{R}}{\bar{R}_0} = \frac{\mu_a}{12S_s} \frac{A^{5/3}}{NZ} + \frac{5a_c}{72S_s} \frac{A^{4/3}}{N}$$

take μ_a from exp, shell effects implicitly included
overall uncertainty in ΔR from error in S_s (15%)
in ratio $\frac{\Delta R_{N'}}{\Delta R_N}$ S_s drops out

Results for ΔR



using μ_a from exp

wiggles mostly occur at magic numbers

related to loss of binding

not an effect of deformation!

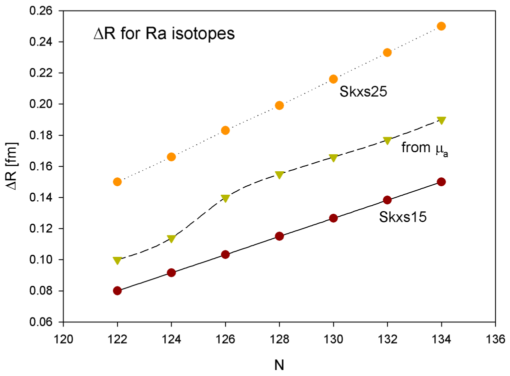
^{208}Pb exp: $\Delta R = 0.20 \pm 0.04 \pm 0.05$ fm (anti-protonic atoms)

present: method(1): 0.18 fm; method(2): 0.20 fm

Brown: 0.15- 0.25 fm, Piekarewicz 0.22 fm, vanGiai 0.21 fm

Results for ΔR in Radium

comparison with Brown et al.



uncertainty in ΔR solely due to error in S_s, $\approx 15\%$?

isotopic chains

Nuclear effects for single atom

$$S = \frac{\text{real nucleus}}{\text{constant density model}} = 1 + (Z\alpha)^2(-\tfrac{1}{2}\delta_c - \tfrac{116}{525}\delta_{np} + ...)$$

$$\delta_c = R_c^2/R_0^2 - 1, \qquad \delta_{pn} = R_n^2/R_c^2 - 1.......$$

To eliminate the atomic uncertainties use isotopic ratio

ratio $\mathcal{R} = \frac{E'_{PNC}}{E_{PNC}} = \frac{Q'_W}{Q_W}(\frac{R'_p}{R_p})^{2\gamma-2}$

$\sim \frac{N'}{N}(\frac{R'_p}{R_p})^{2\gamma-2}(1 + f_n(\frac{R_n}{R_p}) - f_n(\frac{R'_n}{R'_p}))$

$$S \rightarrow S_{\Delta N} = (Z\alpha)^2 \frac{N}{\Delta N}(-\tfrac{1}{2}\Delta\delta_c + \tfrac{116}{525}\Delta\delta_{pn} + ...)$$

Summary

- Extended LDM for masses and radii with simple shell corrections
- Symmetry energy $S_v = 32 \pm 2$MeV, $S_v/S_s = 3.0 \pm 0.3$
  Agrees with Danielewicz
- Radii R_c can be described by similar shell corrections; for a quantitative fit need deformation
- neutron skin predicted from n-p separation energies, weak shell effects
  weak disagreement with Brown et al