Neutron Star Masses and Radii

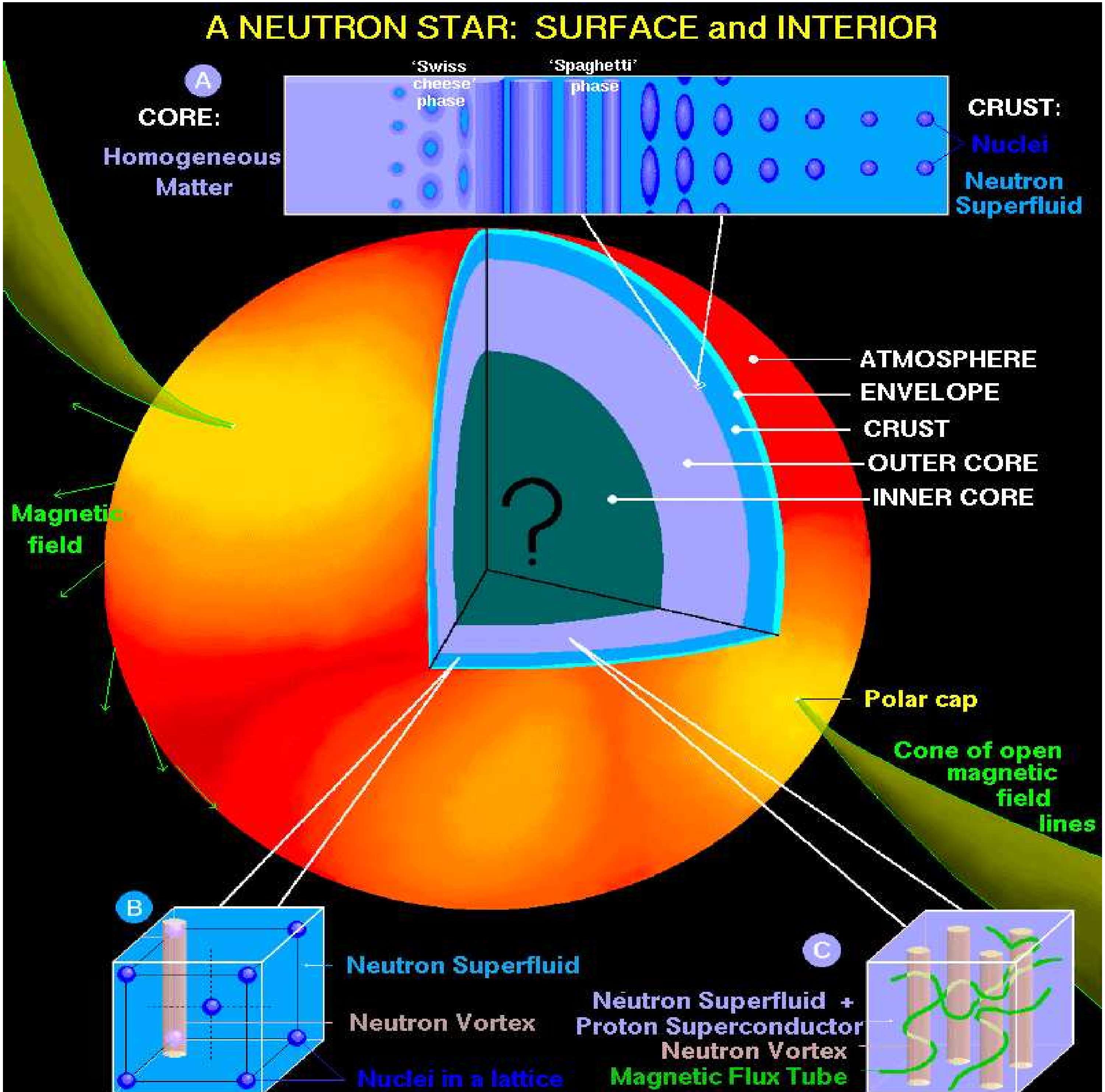
James M. Lattimer

lattimer@astro.sunysb.edu

Department of Physics & Astronomy
Stony Brook University

Neutron Stars and the Equation of State

- Extreme Properties
- Pulsar Constraints – Rotation and Mass
- Pressure–Radius Correlation and Nuclear Symmetry Energy
- Neutron Star Cooling and Nuclear Symmetry Energy
- Radius Constraints From X-Ray Bursts in Binaries
- Inverting the TOV Equations



Credit: Dany Page, UNAM

Neutron Star Structure

TOV equations of relativistic hydrostatic equilibr/ium:

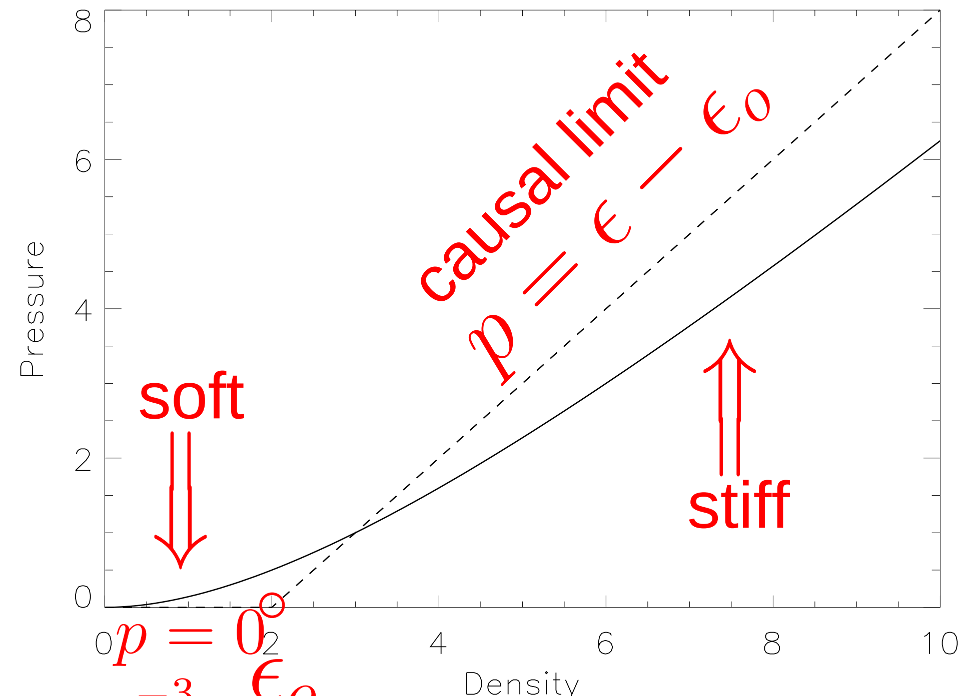
$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi p r^3)(\epsilon + p)}{r(r - 2Gm/c^2)} \qquad \frac{dmc^2}{dr} = 4\pi\epsilon r^2$$

Extreme compactness:

Koranda, Stergioulas & Friedman (1997)

ϵ_0 is the only EOS parameter

TOV solutions scale with ϵ_0



$$M_{max} = 4.2\left(\frac{\epsilon_s}{\epsilon_0}\right)^{1/2} \mathrm{M}_\odot$$

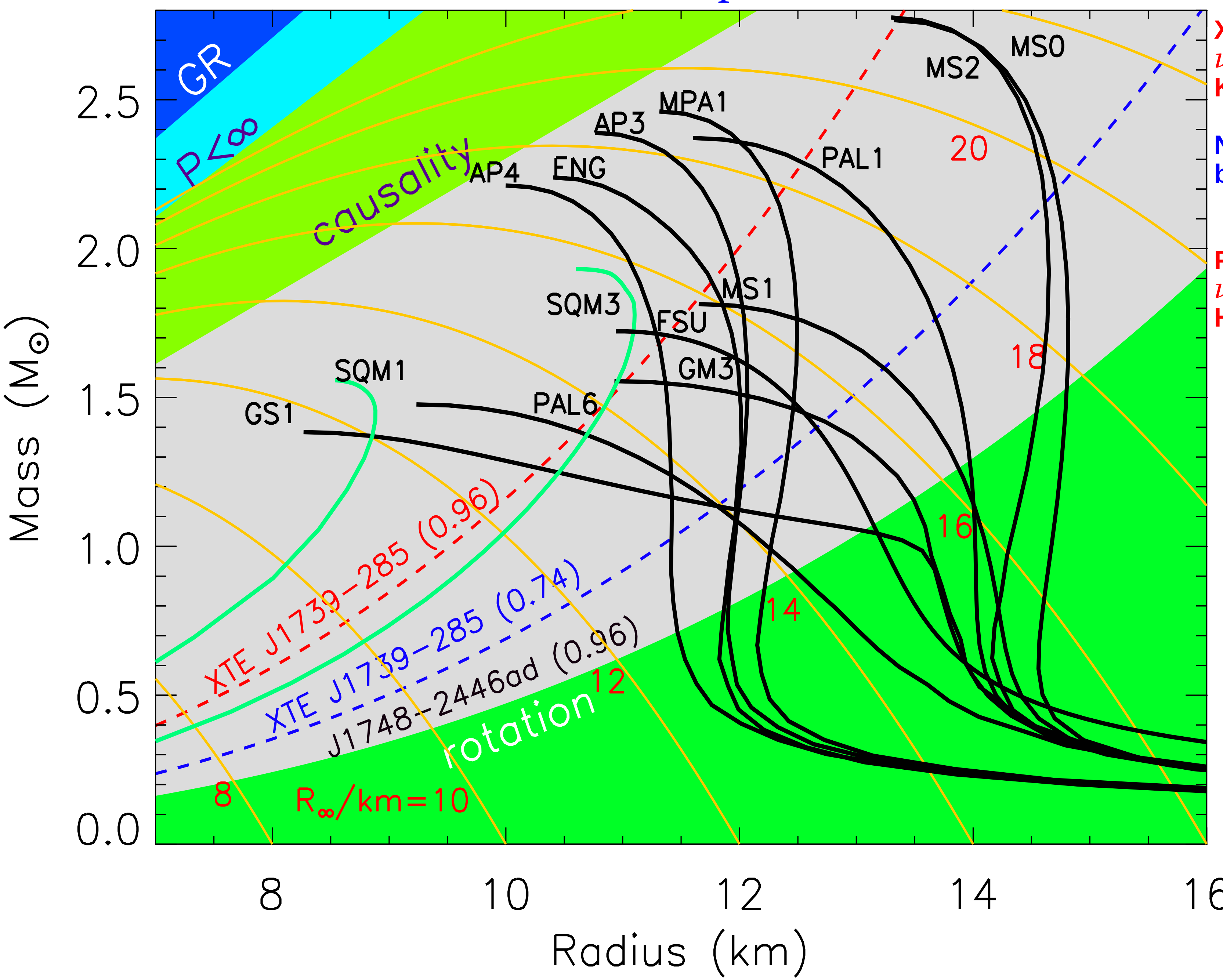
$$R_{min} = 2.9GM/c^2 = 4.3\frac{M}{\mathrm{M}_\odot}\ \mathrm{km}$$

$$\epsilon_{central} < 4.5 \times 10^{15}\left(\frac{\mathrm{M}_\odot}{M_{largest}}\right)^2 \mathrm{g\ cm^{-3}}$$

$$P_{min} \simeq 0.74\left(\frac{\mathrm{M}_\odot}{M_{sph}}\right)^{1/2}\left(\frac{R_{sph}}{10\ \mathrm{km}}\right)^{3/2} \mathrm{ms}$$

$$P_{min} \simeq 0.96 \pm 0.03\left(\frac{\mathrm{M}_\odot}{M_{sph}}\right)^{1/2}\left(\frac{R_{sph}}{10\ \mathrm{km}}\right)^{3/2} \mathrm{ms} \qquad \text{empirical}$$

Constraints from Pulsar Spins



XTE J1739-285
$\nu = 1122$ Hz
Kaaret et al. 2006

Not confirmed to be a rotation rate

PSR J1748-2446ad
$\nu = 716$ Hz
Hessels et al. 2006

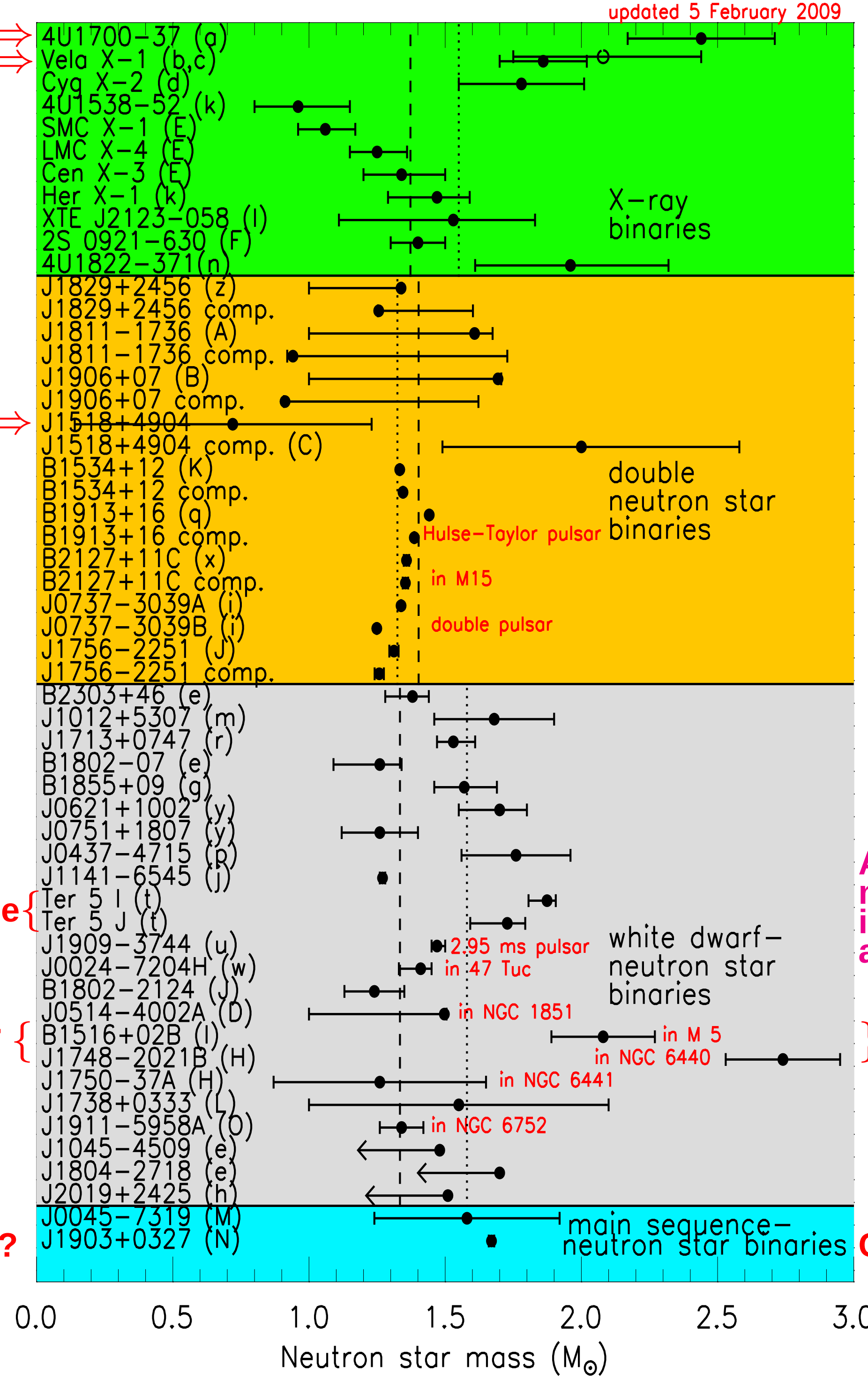
Black hole? ⇒
Firm lower mass limit? ⇒

$M < 1.17$ M$_\odot$ (95%) ⇒

$M > 1.68$ M$_\odot$, 95% confidence {

Freire et al. 2007 {

The new standard 1.67 M$_\odot$?



Although simple average mass of w.d. companions is 0.27 M$_\odot$ larger, weighted average is 0.08 M$_\odot$ smaller

} w.d. companion? statistics?

Champion et al. 2008

Neutron Star Matter Pressure and the Radius

$$p \simeq K\epsilon^{1+1/n}$$

$$n^{-1} = d\ln p/d\ln\epsilon - 1 \sim 1$$

$$R \propto K^{n/(3-n)} M^{(1-n)/(3-n)}$$

$$R \propto p_*^{1/2}\epsilon_*^{-1} M^0$$

$$(1 < \epsilon_*/\epsilon_0 < 2)$$

Wide variation:

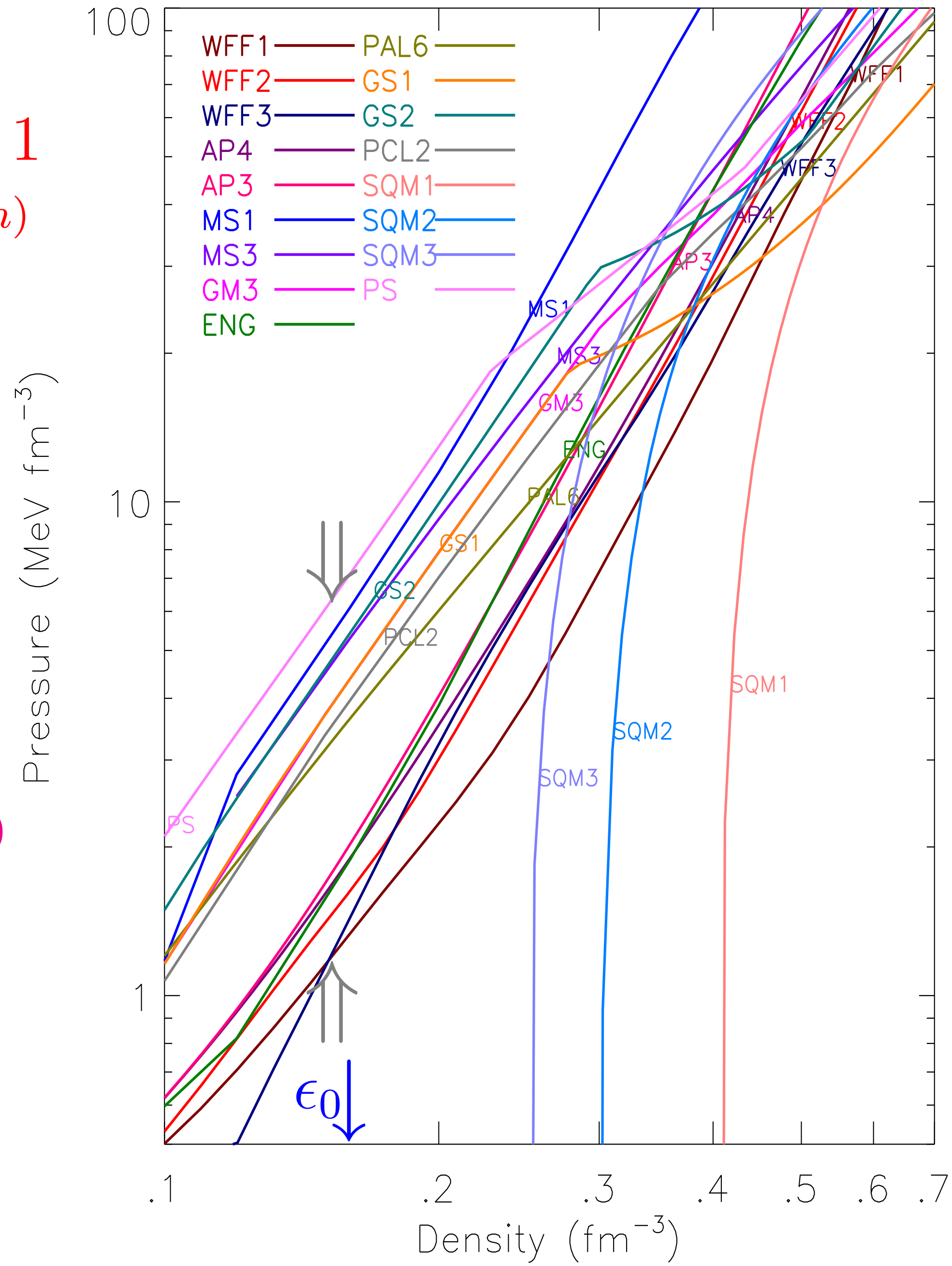
$$1.2 < \frac{p(\epsilon_0)}{\text{MeV fm}^{-3}} < 7$$

GR phenomenological result (Lattimer & Prakash 2001)

$$R \propto p_*^{1/4}\epsilon_*^{-1/2}$$

follows from Buchdahl's analytic TOV solution

$$p_* = n^2\frac{dE_{sym}}{dn} = \frac{n^2 L}{3n_s}$$



Possible Kinds of Observations

- Maximum and Minimum Mass (binary pulsars)
- Minimum Rotational Period*
- Radiation Radii or Redshifts from X-ray Thermal Emission*
- Crustal Cooling Timescale from X-ray Transients*
- X-ray Bursts from Accreting Neutron Stars*
- Seismology from Giant Flares in SGR's*
- Neutron Star Thermal Evolution (URCA or not)*
- Moments of Inertia from Spin-Orbit Coupling*
- Neutrinos from Proto-Neutron Stars (Binding Energies, Neutrino Opacities, Radii)*
- Redshifts from Pulse Shape Modulation*
- Gravitational Radiation from Neutron Star Mergers* (Masses, Radii from tidal Love numbers)

* Significant dependence on symmetry energy

Potentially Observable Quantities

- Apparent angular diameter from flux and temperature measurements

  $\beta \equiv GM/Rc^2$

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2\beta}} = \sqrt{\frac{F_\infty}{\sigma}}\frac{1}{f_\infty^2 T_\infty^2}$$

- Redshift

$$z = (1-2\beta)^{-1/2} - 1$$

- Eddington flux

$$F_{EDD} = \frac{GMc}{\kappa c^2 D^2}(1-2\beta)^{1/2}$$

- Crust thickness

$$\frac{m_b c^2}{2}\ln\mathcal{H} \equiv h_t = \int_0^{p_t}\frac{dp}{n} = \mu_{n,t} - \mu_{n,t}(p=0)$$

$$\frac{\Delta}{R} \equiv \frac{R-R_t}{R} = \frac{(\mathcal{H}-1)(1-2\beta)}{\mathcal{H}+2\beta-1} \simeq (\mathcal{H}-1)\left(\frac{1}{2\beta}-1\right).$$

- Moment of Inertia

$$I \simeq (0.237 \pm 0.008) MR^2(1 + 2.84\beta + 18.9\beta^4)\ \mathrm{M}_\odot\ \mathrm{km}^2$$

- Crustal Moment of Inertia

$$\frac{\Delta I}{I} \simeq \frac{8\pi}{3}\frac{R^6 p_t}{IMc^2}$$

- Binding Energy

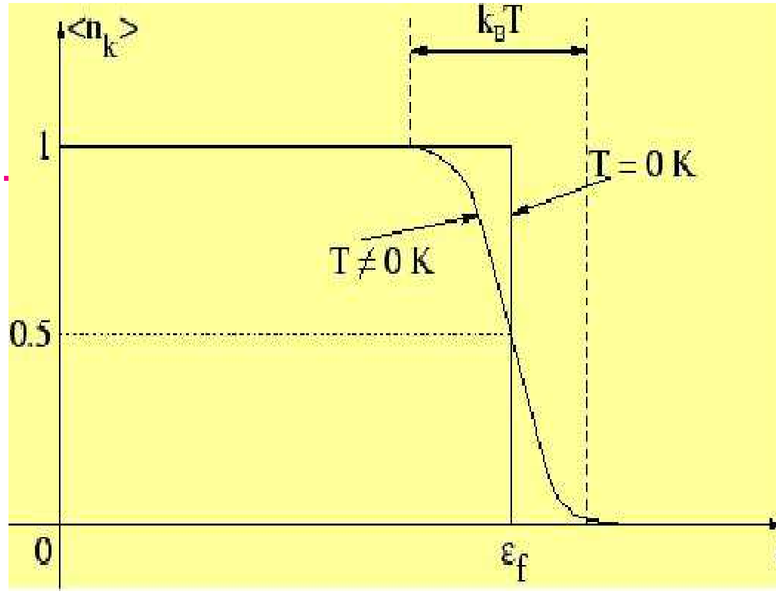
$$\mathrm{B.E.} \simeq (0.60 \pm 0.05)\frac{\beta}{1-\beta/2}$$

Neutron Star Cooling

Gamow & Schönberg proposed the direct Urca process: nucleons at the top of the Fermi sea beta decay.

$$n \to p + e^- + \nu_e\,, \qquad p \to n + e^+ + \bar{\nu}_e$$

Energy conservation guaranteed by beta equilibrium
$\mu_n - \mu_p = \mu_e$
Momentum conservation requires $|k_{Fn}| \le |k_{Fp}| + |k_{Fe}|$.
Charge neutrality requires $k_{Fp} = k_{Fe}$,
therefore $|k_{Fp}| \ge 2|k_{Fn}|$.
Degeneracy implies $n_i \propto k_{Fi}^3$, thus $x \ge x_{DU} = 1/9$.
With muons $(n > 2n_s), x_{DU} = \frac{2}{2+(1+2^{1/3})^3} \simeq 0.148$
If $x < x_{DU}$, bystander nucleons needed:
modified Urca process is then dominant.



$$(n,p) + n \to (n,p) + p + e^- + \nu_e, \qquad (n,p) + p \to (n,p) + n + e^+ + \bar{\nu}_e$$

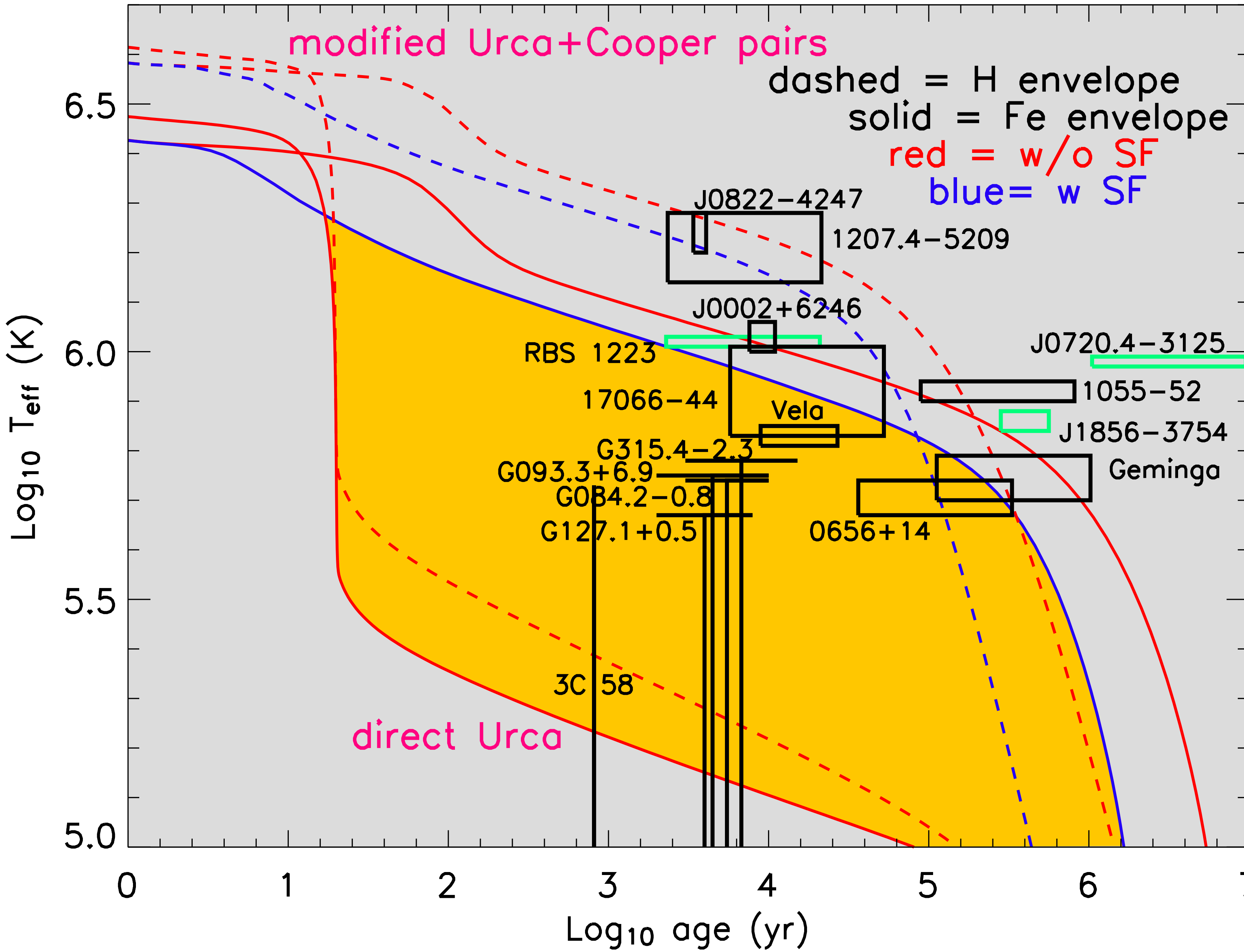
Neutrino emissivities:

$$\dot{\epsilon}_{MURCA} \simeq \left(\frac{T}{\mu_n}\right)^2 \dot{\epsilon}_{DURCA}\,.$$

Beta equilibrium composition:

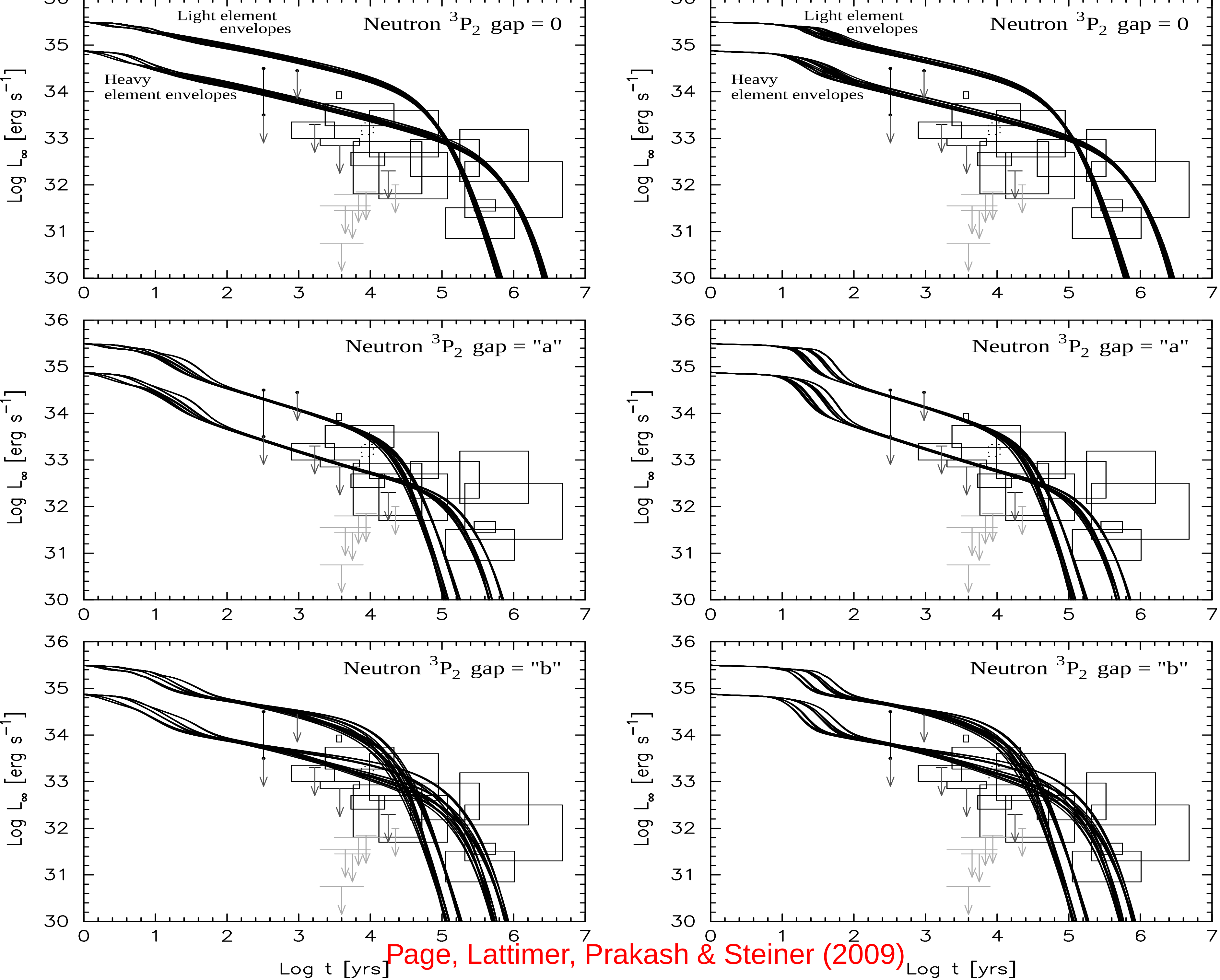
$$x_\beta \simeq (3\pi^2 n)^{-1} \left(\frac{4E_{sym}}{\hbar c}\right)^3 \simeq 0.04 \left(\frac{n}{n_s}\right)^{0.5-2}\,.$$

Neutron Star Cooling



Page, Lattimer, Prakash & Steiner (2004)

Minimal Cooling Paradigm



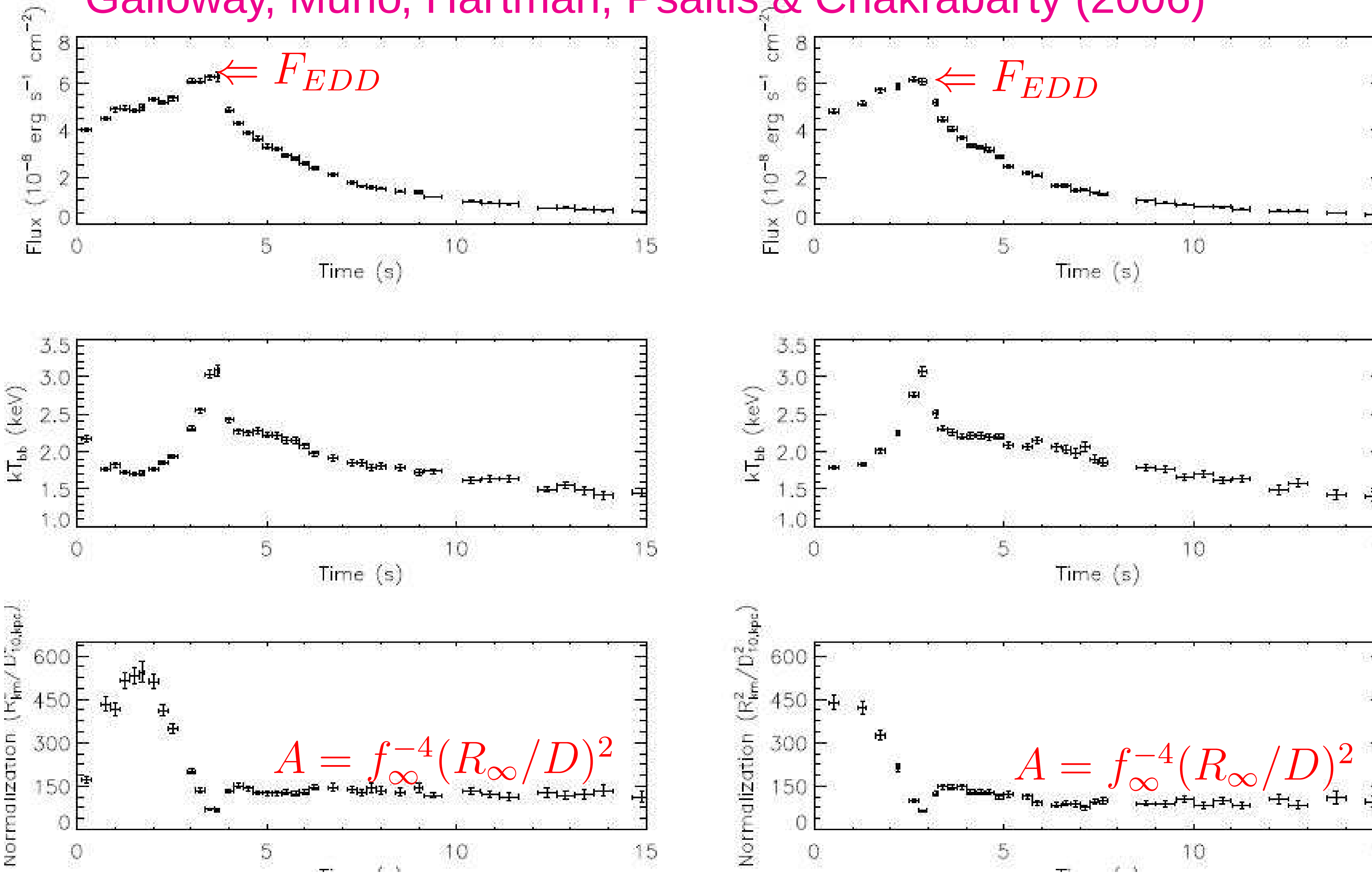
Page, Lattimer, Prakash & Steiner (2009)

Minimal Cooling Paradigm

- If some observations are inconsistent with the MCP, then according to Sherlock Holmes, rapid cooling must occur for these exceptions.
- All sources are consistent with the MCP only IF
  - tight conditions are placed on the magnitude and density dependence of the neutron ^{3}P$_2$ gap, AND
  - some neutron stars have heavy Z envelopes and others have light Z envelopes, AND
  - ALL core-collapse supernova remnants with no observable thermal emission contain black holes.
- Highly suggestive that rapid cooling occurs in some neutron stars (of higher masses?)
- A possible constraint on $E_{sym}(n)$, *i.e.*, it's not supersoft?

Cooling Following An X-Ray Burst

Galloway, Muno, Hartman, Psaltis & Chakrabarty (2006)



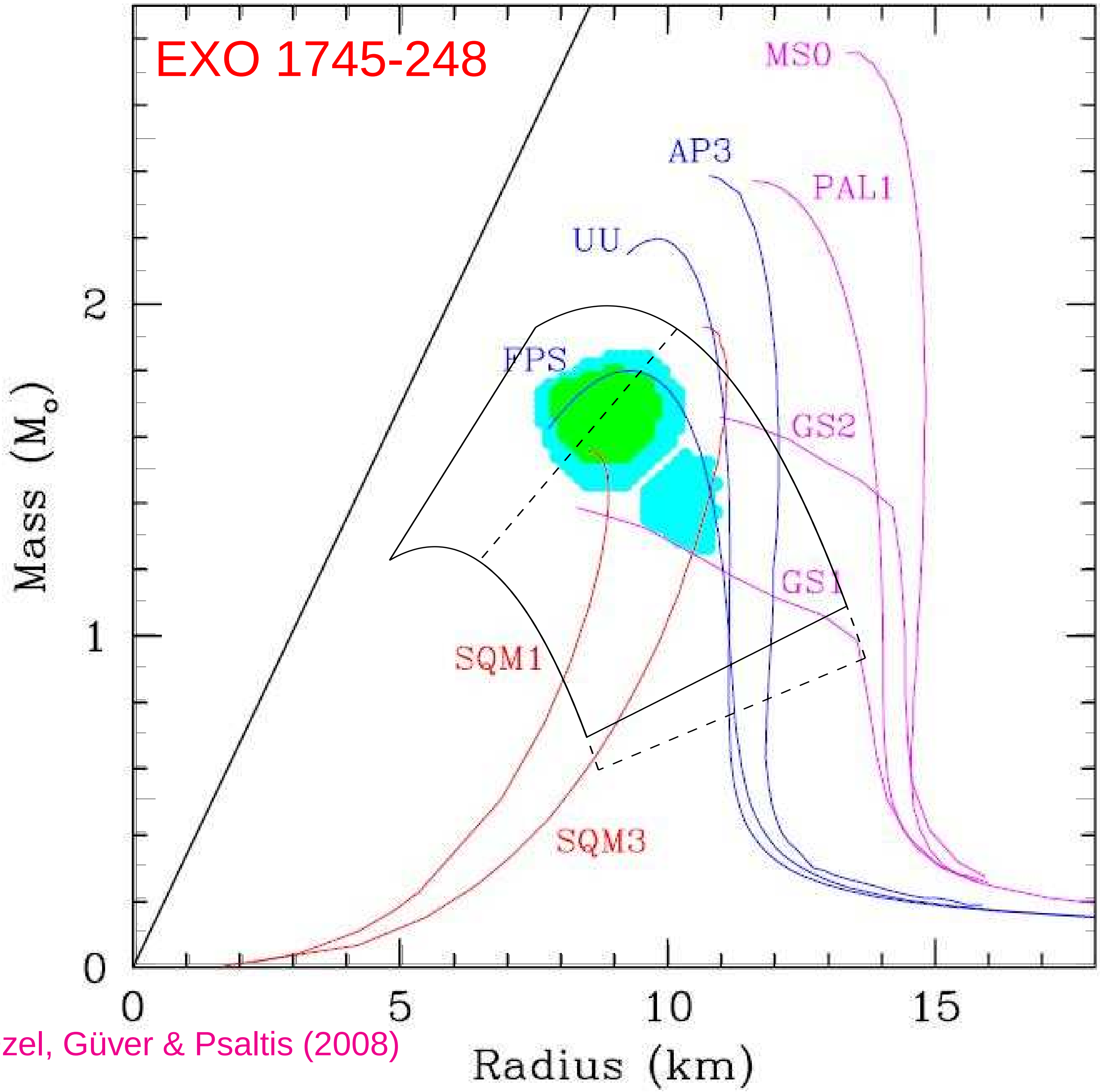
EXO 1745-248

Analysis

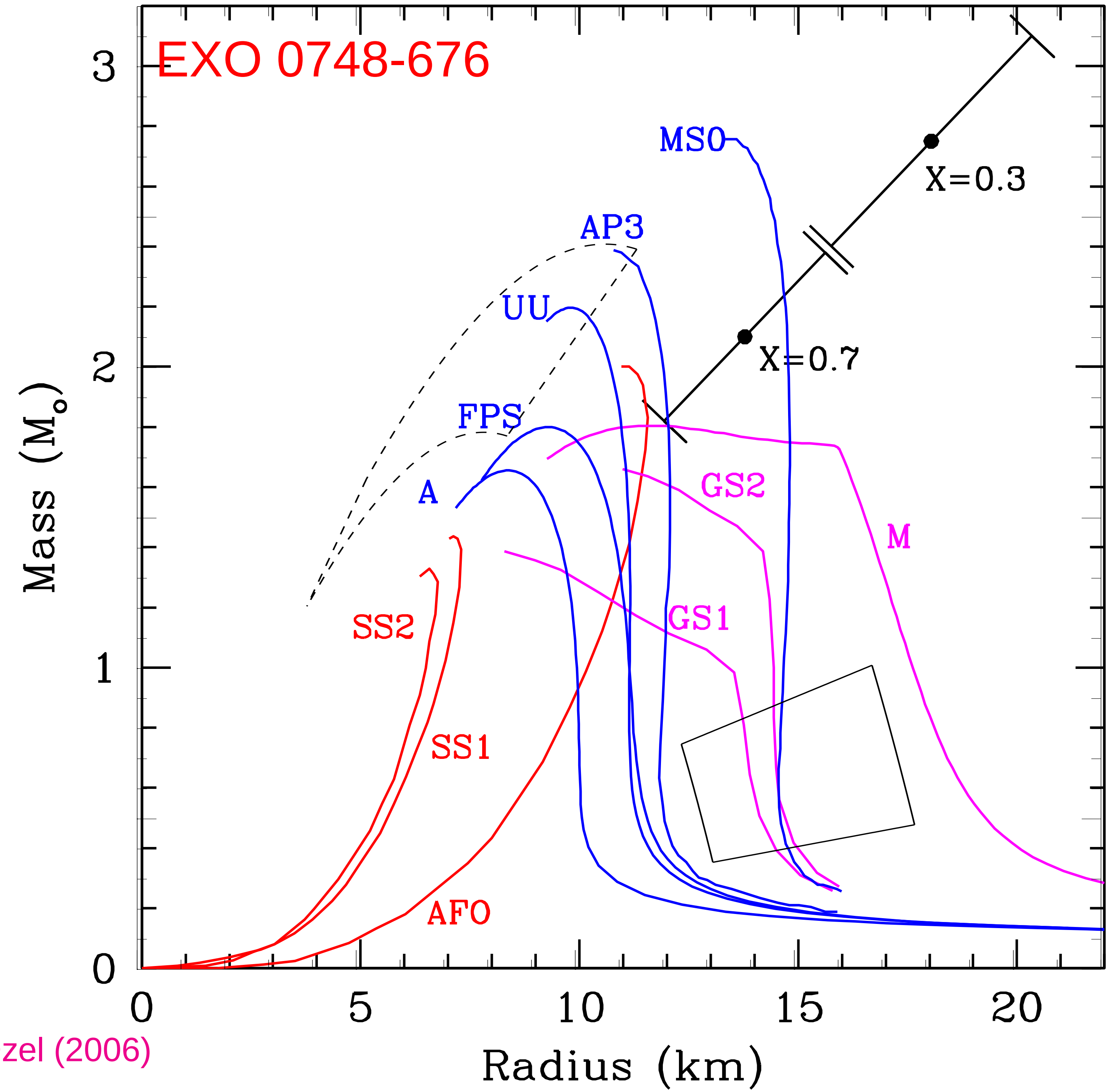
$$F_{EDD} = \frac{GMc}{\kappa D^2}\sqrt{1-\frac{2GM}{Rc^2}}, \qquad A = \left(\frac{R_\infty}{D}\right)^2 f_c^{-4}$$

$$\alpha \equiv \frac{F_{EDD}\kappa D}{c^3 f_c^2 \sqrt{A}} = \beta(1-2\beta) \le \frac{1}{8}, \qquad \beta = \sqrt{1-\frac{2GM}{Rc^2}}$$

	EXO 0748-676	EXO 1745-248	4U 1608-52
F_{EDD} (10^{-8} erg/cm^2/s)	2.25 ± 0.23	6.25 ± 0.20	25.42 ± 0.65
A (km/kpc)^2	1.14 ± 0.10	1.16 ± 0.13	3.246 ± 0.024
D (kpc)	7.7 ± 0.9	5.9 ± 0.5	5.2 ± 0.7
α	0.055 ± 0.011	0.117 ± 0.018	0.152 ± 0.26
β	0.065 ± 0.015, 0.39 ± 0.06	0.31 ± 0.06, 0.19 ± 0.06	0.25
R_∞ (km)	16.1 ± 1.6	12.5 ± 1.9	18.4 ± 3.1
R (km)	17.1 ± 1.7, 8.6 ± 0.9	7.7 ± 1.2, 9.8 ± 1.6	13.1 ± 2.2
M ($M_\odot$)	0.75 ± 0.08, 2.27 ± 0.23	1.62 ± 0.25, 1.27 ± 0.19	2.2 ± 0.4
Ozel et al.			
R (km)	13.8 ± 1.8	8.5 ± 1.2	9.8 ± 1.2
M ($M_\odot$)	2.1 ± 0.3	1.62 ± 0.12	1.84 ± 0.09



Özel, Güver & Psaltis (2008)



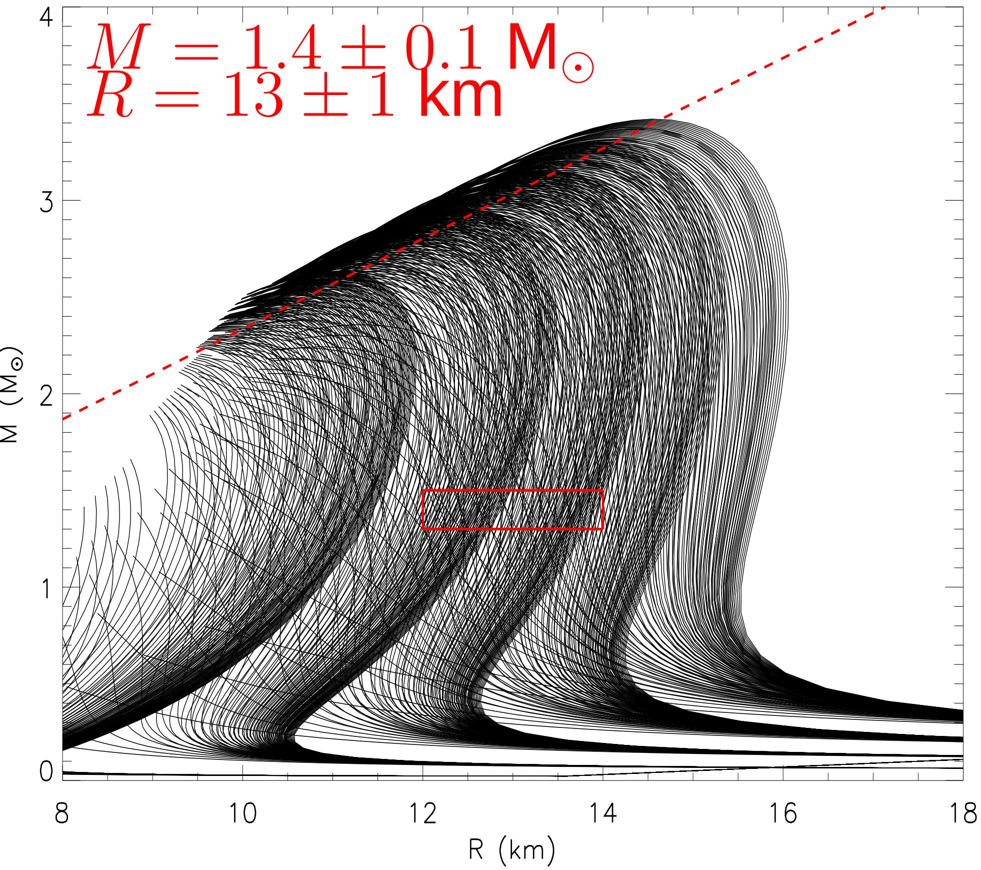
EXO 0748-676
MS0
X=0.3
AP3
UU
X=0.7
FPS
A
GS2
M
SS2
GS1
SS1
AFO
Mass (M$_\odot$)
Radius (km)
0
1
2
3
0
5
10
15
20

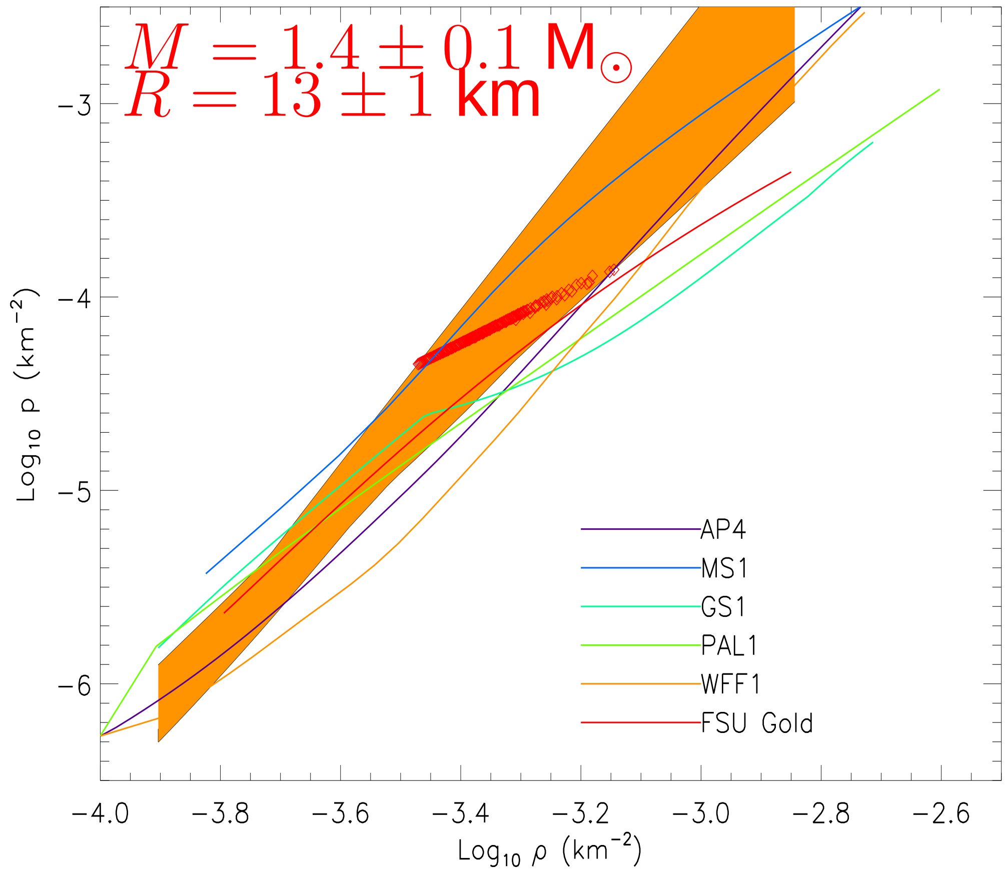
Özel (2006)

TOV Inversion

How would a simultaneous $M - R$ determination constrain the EOS? Each M-R curve specifies a unique $p - \rho$ relation.

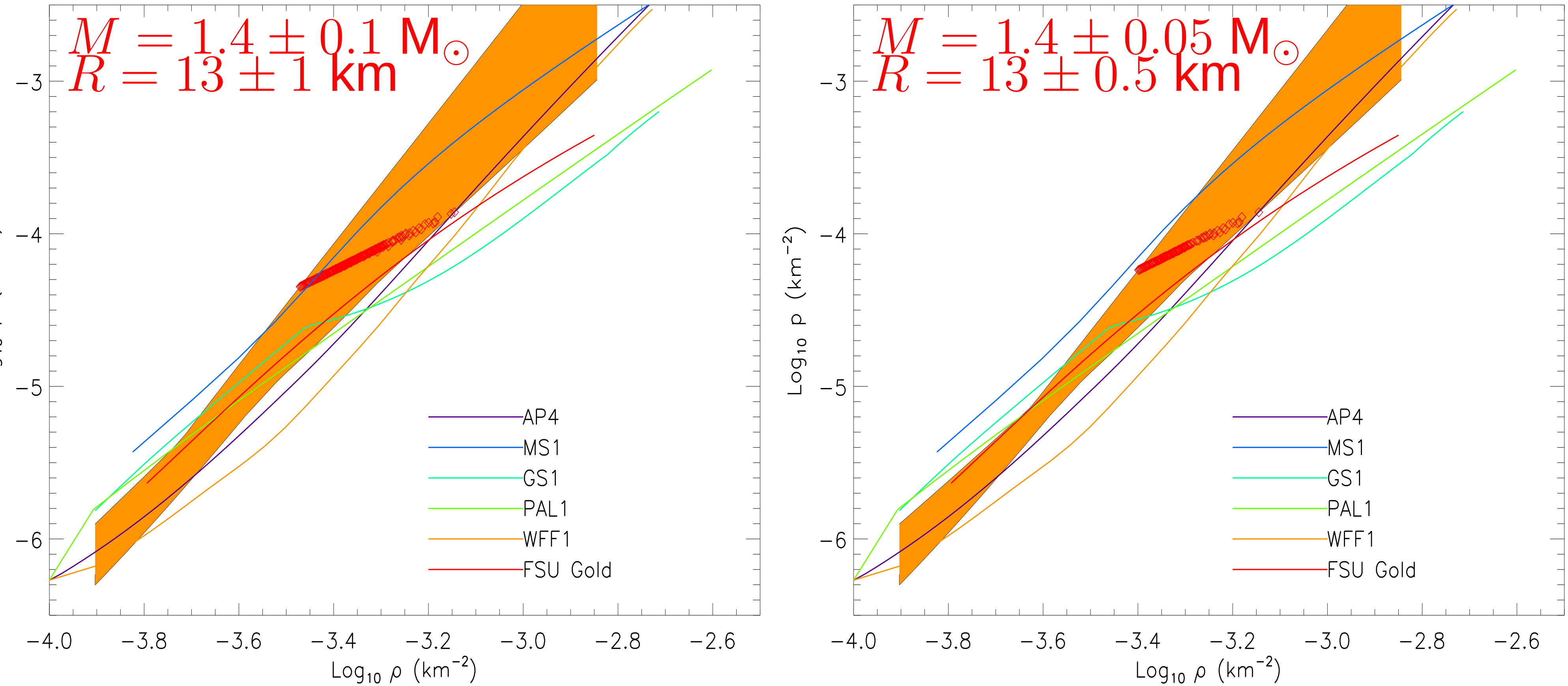
- Generate physically reasonable $M - R$ curves and the $p - \rho$ relations that they specify.
- Generate arbitrary $p - \rho$ relations and compute $M - R$ curves from them; select those $M - R$ curves passing within the error box.





TOV Inversion (cont.)

Dependence on measurement errors



The current uncertainty in the subnuclear EOS introduces significant width to the inferred high-density pressure-density relation.

Conclusions

- **Neutron stars are a powerful laboratory to constrain dense matter physics, especially the symmetry energy and composition at supranuclear densities.**
- **Many aspects of neutron star structure depend on specific equation of state parameters or their density dependence in a model-independent fashion.**
- **Increasing evidence supports the existence of massive neutron stars ($M \gtrsim 1.7$ $M_\odot$), constraining exotic matter.**
- **Many kinds of observations are now available to constrain neutron star radii, although no reliable measures yet exist.**
- **An accurate, simultaneous mass and radius measurement from even one neutron star would provide a significant constraint.**