Non-perturbative improvement of stout-smeared three flavour clover fermions

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- $O(a)$ Improvement
- The SLiNC action
- Schrödinger functional
- c_{sw}
- κ_c
- Finite size effects
- Some preliminary results
- Conclusions

$O(a)$ Improvement

- Gluon action has $O(a^2)$ corrections
- Naive fermion action has $O(a^2)$ corrections, but
  - Introduces 'doubling problem' [and loss of chiral invariance]
    'Cure'

    $$\text{action} = \text{naive} + \text{Wilson mass term}$$

    but has $O(a)$ corrections, so eg

    $$\frac{m_H}{m_{H'}} = \# + \# \, a$$

- Symanzik:
  - Systematic improvement to $O(a^n)$ — $n = 2$
    Add basis of irrelevant operators and tune coefficients to remove completely $O(a^{n-1})$ effects — Asymptotic series ??
  - Restrict to on-shell $\Rightarrow$
    equation of motion reduce the set of operators
    - in action
    - in matrix elements
- $O(a)$ improvement $\Rightarrow$ only one additional operator in action required

$$\mathcal{L}_{addit} \propto a c_{sw}(g_0^2) \sum \overline{\psi} \sigma_{\mu\nu} F_{\mu\nu} \psi = \text{clover term}$$

If can improve one on-shell quantity to $O(a^2)$:

- Fixes $c_{sw}(g_0^2)$
- Then all other physical on-shell quantities are automatically improved to $O(a^2)$, ie

$$\frac{m_H}{m_{H'}} = \# + \# \, a^2$$

Matrix Elements:

- Require additional $O(a)$ operators, for example

$$\begin{aligned} \mathcal{A}_\mu &= (1 + b_A a m_q)(A_\mu + c_A a \partial_\mu^{LAT} P) \, , \\ \mathcal{P} &= (1 + b_P a m_q) P \end{aligned}$$

with

$$A_\mu = \overline{q} \gamma_\mu \gamma_5 q \, , \qquad P = \overline{q} \gamma_5 q$$

PCAC:

- Find quark mass m_q^{WI} from *PCAC* relation

$$m_q^{WI} = \frac{\langle \partial_0^{LAT}(A_4(x_0) + c_A a \partial_0^{LAT} P(x_0)) O \rangle}{2\langle P(x_0) O \rangle}$$

- If improved then

$$m_q^{\mathcal{R}} = \underbrace{\frac{Z_A(1 + b_A a m_q)}{Z_P(1 + b_P a m_q)}}_{\text{numerical factor}} m_q^{WI} + O(a^2)$$

- Choosing different { boundary conditions, O } gives different determinations of quark mass $m_q^{(i)\,WI}$, $i = 1, 2 \ldots$
- If improved then errors in quark masses are $O(a^2)$.
So find improvement coefficients, $c_{sw}, \ldots$, by determining point where

$$m_q^{(1)\,WI} = m_q^{(2)\,WI}$$

SLiNC fermions $2+1$ flavours

Stout LinkNon-perturbative Clover = SLiNC

$$S_F = \sum_x \Big\{ \kappa \overline{\psi}(x) \tilde{U}_\mu(x) [\gamma_\mu - 1] \psi(x + a\hat{\mu}) - \kappa \overline{\psi}(x) \tilde{U}^\dagger_\mu (x - a\hat{\mu}) [\gamma_\mu + 1] \psi(x - a\hat{\mu})$$
$$+ \overline{\psi}(x)\psi(x) + \tfrac{1}{2} a c_{sw}(g_0^2) \overline{\psi}(x) \sigma_{\mu\nu} F_{\mu\nu}(x) \psi(x) \Big\}$$

- The hopping terms use a stout smeared link ('fat link')

Dirac kinetic term and Wilson mass term

$$\tilde{U}_\mu = \exp\{iQ_\mu(x)\}\, U_\mu(x)$$
$$Q_\mu(x) = \frac{\alpha}{2i} \left[VU^\dagger - UV^\dagger - \tfrac{1}{3} \mathrm{Tr}(VU^\dagger - UV^\dagger) \right]$$

V_μ is the sum of all staples around U_μ

- Clover term built from thin links
  (already length $4a$ do not want fermion matrix too extended)

Why stout smearing?

- Need smearing at present lattice spacings — to avoid near first-order phase transition
- Analytic
  - can take derivative (so HMC force well defined)
  - perturbation expansions

eg QCDSF: arXiv:0807.0345

VWI quark mass

- loss of chiral invariance means that a critical $\kappa_c(c_{sw})$ has to be determined (for a given c_{sw}).
- then if rescale quark fields $q \to q/\sqrt{2\kappa}$ and define

$$m_q = \frac{1}{2a}\left(\frac{1}{\kappa} - \frac{1}{\kappa_c(c_{sw})}\right)$$

which is also multiplicatively renormalisable, so $m_q \propto m_q^{\mathcal{R}} \propto m_q^{WI}$

To complete action:

- Gluon action: Symanzik tree–level (plaquette + rectangle)

$$S_G = \frac{6}{g_0^2}\left\{c_0 \sum_{Plaquette} \frac{1}{3}\mathrm{Re\,Tr}(1 - U_{Plaquette}) + c_1 \sum_{Rectangle} \frac{1}{3}\mathrm{Re\,Tr}(1 - U_{Rectangle})\right\}$$

with

$$\beta = \frac{6c_0}{g_0^2} = \frac{10}{g_0^2} \qquad \text{with} \quad c_0 = \frac{20}{12}\,,\ c_1 = -\frac{1}{12}$$

Finding suitable $m_q^{(1)\,WI} = m_q^{(2)\,WI}$

Choosing different { boundary conditions, O } gives different determinations of quark mass $m_q^{(i)\,WI}$, $i = 1, 2 \ldots$

ALPHA Collab.: achieve this by means of 'Schrödinger Functional'

Dirichlet boundary conditions on time boundaries

- $i = 1$ lower boundary $x_0 = -a, 0$
- $i = 2$ upper boundary $x_0 = T - a, T$

- Gluons fields fixed (at $x_0 = -a, 0$ and $T - a, T$)
  - equivalent to constant chromo-electric background field
  - Can simulate with $m_q \sim 0$ with no zero mode problems
- Quark fields fixed (at $x_0 = 0$ and $T - a$)
  - 'sinks/sources' $\rho, \overline{\rho}$ ($\rightarrow 0$)
  - for correlation functions can then choose, eg

$$O^{(i)} = \sum_{\vec{y},\vec{z}} \left(-\frac{\delta}{\delta\rho^{(i)}(\vec{y})} \right) \gamma_5 \left(\frac{\delta}{\delta\overline{\rho}^{(i)}(\vec{z})} \right) \Bigg|_{\rho=0=\overline{\rho}}$$

so can look at PCAC behaviour at different distances from boundary

In a little more detail

$$m_q^{(i)\,WI} = r^{(i)}(x_0) + c_A\, s^{(i)}(x_0) \qquad i = 1, 2$$

with

$$r^{(i)}(x_0) = \frac{\partial_0^{LAT} f_A^{(i)}(x_0)}{2 f_P^{(i)}(x_0)} \qquad s^{(i)}(x_0) = a\,\frac{\partial_0^{2\,LAT} f_P^{(i)}(x_0)}{2 f_P^{(i)}(x_0)}$$

where

$$f_A^{(1)}(x_0) = -\frac{1}{n_f^2 - 1}\langle A_0(x_0) O^{(1)}\rangle \qquad f_P^{(1)}(x_0) = -\frac{1}{n_f^2 - 1}\langle P(x_0) O^{(1)}\rangle$$

$$f_A^{(2)}(T - x_0) = +\frac{1}{n_f^2 - 1}\langle A_0(x_0) O^{(2)}\rangle \qquad f_P^{(2)}(T - x_0) = -\frac{1}{n_f^2 - 1}\langle P(x_0) O^{(2)}\rangle$$

Redefine quark mass (slightly, coincides to $O(a^2)$ in improved theory) to eliminate (unknown) c_A:

$$M^{(i)}(x_0, y_0) = r^{(i)}(x_0) + \widehat{c}_A s^{(i)}(x_0) \qquad \widehat{c}_A = -\frac{r^{(1)}(y_0) - r^{(2)}(y_0)}{s^{(1)}(y_0) - s^{(2)}(y_0)}$$

Define improvement when

$$(M, \Delta M) = (0, 0) \qquad \text{giving} \quad c_{sw}^*\,, \kappa_c^*\ \ldots$$

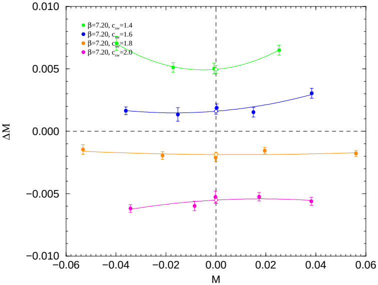
where

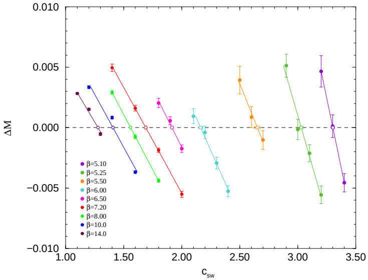
$$M \equiv M^{(1)}(T/2, T/4) \qquad \Delta M \equiv M^{(1)}(3T/4, T/4) - M^{(2)}(3T/4, T/4)$$

Programme:

- Chroma – R. Edwards and B. Joó, arXiv:hep-lat/0409003
  [for Bagel additions P.A. Boyle,
  http://www.ph.ed.ac.uk/~ paboyle/bagel/Bagel.html]
- SF details follow T. Klassen, arXiv:hep-lat/9705025
- Practically:
  - 'Mild smearing' $\alpha = 0.1$
  - $8^3 \times 16$ lattices
  - T. Kaltenbrunner initiated investigation

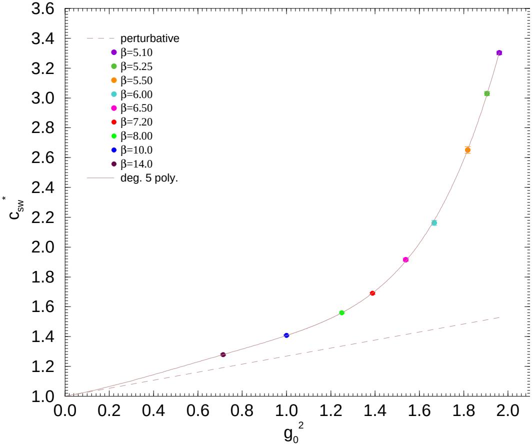
c_{sw}^*





$M = 0$ gives
$\Delta M(c_{sw}, \kappa_c(c_{sw}))$

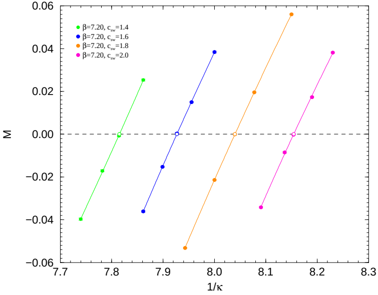
$\Delta M = 0$ gives
c_{sw}^*

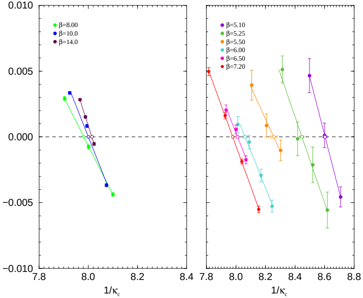


c_{SW}^*

$c_{SW}^{TI} = \frac{u_0^S}{u_0^4}$ cf $c_{SW}^{TI} = \frac{1}{u_0^3}$

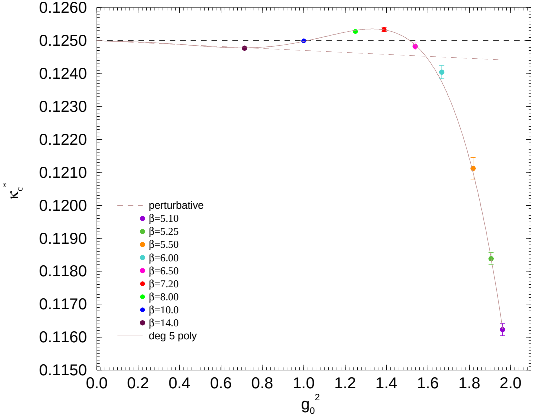
κ_c^*





$M = 0$ gives $\kappa_c(c_{sw})$

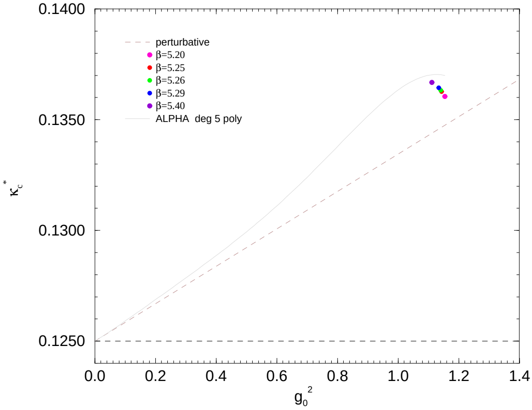
$\Delta M = 0$ gives κ_c^*



0.1260
0.1250
0.1240
0.1230
0.1220
0.1210
0.1200
0.1190
0.1180
0.1170
0.1160
0.1150
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
κ*c
g0²
perturbative
β=5.10
β=5.25
β=5.50
β=6.00
β=6.50
β=7.20
β=8.00
β=10.0
β=14.0
deg 5 poly

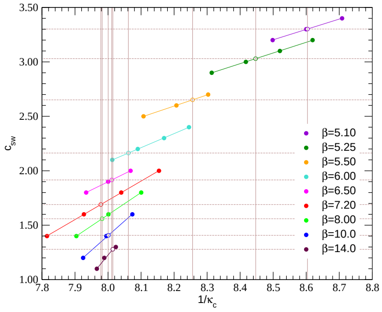
κ_c^*

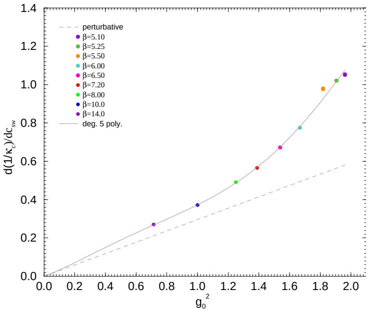
Not unexpected behaviour:



cf κ_c^* for $n_f = 2$

Consistency check - $\left.\frac{\partial(1/\kappa_c)}{\partial c_{sw}}\right|_{c_{sw}^*}$



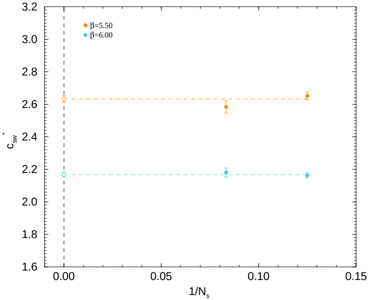


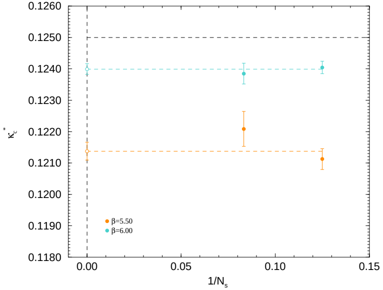
Finite size effects

$$\mathcal{Q} = \mathcal{Q}(a) + q_L \underbrace{\left(c^*_{sw}(g_0, L/a) - c^*_{sw}(g_0, \infty)\right)}_{c^*_{sw}(g_0, L/a) = c^*_{sw}(g_0,\infty) + c_L \frac{a}{L} + c_\Lambda a\Lambda_{QCD} + O(a^2)} + O(a^2) + \ldots$$

$$= \mathcal{Q}(a) + q_L c_L \frac{a}{L} a\Lambda_{QCD} + O(a^2)$$

$$\frac{a}{L} = \begin{cases} O(a) & L = \text{const. ie `constant physics'} \\ \frac{1}{N_s} & L = aN_s \end{cases}$$

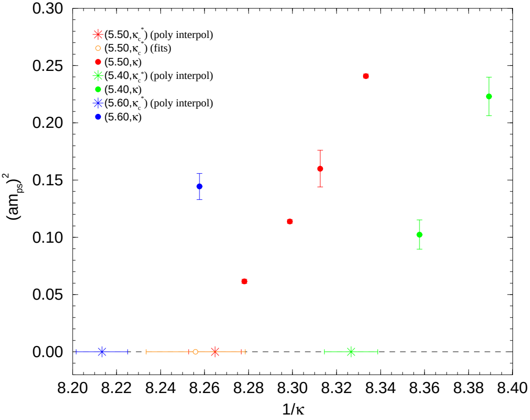




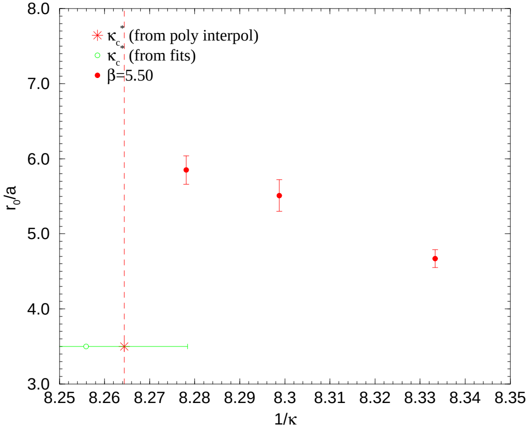
No systematic trend

What is a sensible region to work in

Short runs (3-flavour) on $16^3 \times 32$ lattices at $\beta = 5.50$ $[5.40, 5.60]$



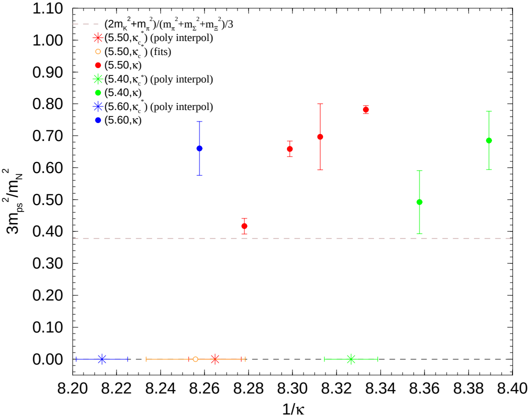
Scale - or why this is a sensible region to work in



- $(r_0/a)(5.50)|_{cl} \sim 6.2$ with scale $r_0 = 0.50\,\text{fm}$ gives $a \sim 0.08\,\text{fm} \sim (2.4\,\text{GeV})^{-1}$
- smallest pseudoscalar mass $\sim 600\,\text{MeV}$, $m_{ps}L \sim 4$

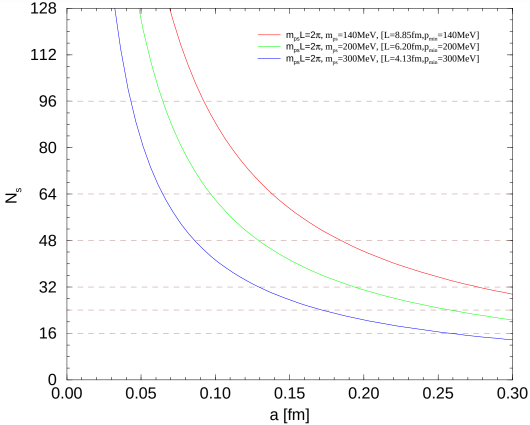
Tuning the strange quark mass – '*muds*'-trajectory

$$m_u + m_d + m_s \equiv 2m_l + m_s = \text{const.} \Longleftrightarrow 2m_K^2 + m_{ps}^2 = \text{const.}$$



A long way to go:

$p_{\min} \equiv \frac{2\pi}{L_s} = m_{ps}$



Either:

- Small a with 'large' $m_{\rm ps}$
  no continuum extrapolation but chiral extrapolation
- 'Coarse' a with $m_{ps} \sim m_\pi$
  no chiral extrapolation but continuum extrapolation
- Mixture

Conclusions

- $O(a)$ improvement works for (stout) smeared actions
- Typical clover results obtained
    - as a increases need a significant $c_{sw} \gg c_{sw}^{tree} \equiv 1$ for $O(a)$ improvement
    - Seeking/have found a region where $a \sim 0.05 - 0.1\,\text{fm}$