

Current status of higher loop LPT

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Outline

Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

Introduction

Diagrammatic approach

Numeric stochastic perturbation theory

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Outline

Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

Introduction

Diagrammatic approach

Numeric stochastic perturbation theory

Example: Ghost propagator

Outline

Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

Introduction

Diagrammatic approach

Numeric stochastic perturbation theory

Example: Ghost propagator

Outline

Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

Introduction

Diagrammatic approach

Numeric stochastic perturbation theory

Example: Ghost propagator

Acknowledgments

Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

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Thanks also for discussions in the QCDSF collaboration

Introduction

Introduction

Diagrammatic approach

Numeric stochastic perturbation theory

Example: Ghost propagator

- ▶ Higher loop corrections to renormalization factors and/or improvement coefficients in gauge-fermionic actions under consideration
- ▶ use of **D**igrammatic **A**pproach and **N**umeric **S**tochastic **P**erturbation **T**heory

Introduction

Introduction

Diagrammatic approach

Numeric stochastic perturbation theory

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- ▶ use of **D**igrammatic **A**pproach and **N**umeric **S**tochastic **P**erturbation **T**heory

DA vs. NSPT

Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

- ▶ **DA:**
 - direct access to the finite and divergent expressions for the quantity under investigation order by order of LPT
 - complicated Feynman rules, complicated integrals beyond one loop
- ▶ **NSPT:**
 - coupled system of equations for the fields decomposed with respect to their perturbative order
 - various extrapolations and limits have to be taken, treatment of singularities (logs) are difficult

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Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

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Feynman rules

Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

General aspect for higher loops: FR become
“exponential” complicated

Example: 4-gluon vertex, which is part of many two-loop diagrams

gggg-vertex (I)

$$\begin{aligned}
 & -g^2 \text{fc}[a1, a2, ee] \text{fc}[a3, a4, ee] \\
 & \left(\frac{2}{3} \cos\left[\frac{ak4}{2}, \mu3\right] \text{dd}[\mu1, \mu2] \text{dd}[\mu1, \mu4] \left(-\sin\left[\frac{ak1}{2}, \mu3\right] \left(\cos\left[-\frac{ak2}{2}, \mu3\right] + i \sin\left[-\frac{ak2}{2}, \mu3\right] \right) + \right. \right. \\
 & \quad \left. \left(\cos\left[-\frac{ak1}{2}, \mu3\right] + i \sin\left[-\frac{ak1}{2}, \mu3\right] \right) \sin\left[\frac{ak2}{2}, \mu3\right] \right) \sin\left[\frac{ak3}{2}, \mu1\right] + \right. \\
 & \quad \left. \frac{1}{3} \text{dd}[\mu1, \mu2] \text{dd}[\mu1, \mu3] \text{dd}[\mu1, \mu4] \left(-\sin\left[\frac{ak1}{2}, t\right] \left(\cos\left[-\frac{ak2}{2}, t\right] + i \sin\left[-\frac{ak2}{2}, t\right] \right) + \right. \right. \\
 & \quad \left. \left(\cos\left[-\frac{ak1}{2}, t\right] + i \sin\left[-\frac{ak1}{2}, t\right] \right) \sin\left[\frac{ak2}{2}, t\right] \right) \left(-\sin\left[\frac{ak3}{2}, t\right] \right. \\
 & \quad \left. \left(\cos\left[-\frac{ak4}{2}, t\right] + i \sin\left[-\frac{ak4}{2}, t\right] \right) + \left(\cos\left[-\frac{ak3}{2}, t\right] + i \sin\left[-\frac{ak3}{2}, t\right] \right) \sin\left[\frac{ak4}{2}, t\right] \right) - \\
 & \quad \left. \frac{2}{3} \cos\left[\frac{ak3}{2}, \mu4\right] \text{dd}[\mu1, \mu2] \text{dd}[\mu1, \mu3] \left(-\sin\left[\frac{ak1}{2}, \mu4\right] \left(\cos\left[-\frac{ak2}{2}, \mu4\right] + i \sin\left[-\frac{ak2}{2}, \mu4\right] \right) + \right. \right. \\
 & \quad \left. \left(\cos\left[-\frac{ak1}{2}, \mu4\right] + i \sin\left[-\frac{ak1}{2}, \mu4\right] \right) \sin\left[\frac{ak2}{2}, \mu4\right] \right) \sin\left[\frac{ak4}{2}, \mu1\right] + \frac{2}{3} \cos\left[\frac{ak2}{2}, \mu1\right] \\
 & \quad \text{dd}[\mu2, \mu3] \text{dd}[\mu2, \mu4] \sin\left[\frac{ak1}{2}, \mu2\right] \left(-\sin\left[\frac{ak3}{2}, \mu1\right] \left(\cos\left[-\frac{ak4}{2}, \mu1\right] + i \sin\left[-\frac{ak4}{2}, \mu1\right] \right) + \right. \\
 & \quad \left. \left(\cos\left[-\frac{ak3}{2}, \mu1\right] + i \sin\left[-\frac{ak3}{2}, \mu1\right] \right) \sin\left[\frac{ak4}{2}, \mu1\right] \right) + \\
 & \quad \text{dd}[\mu1, \mu3] \text{dd}[\mu2, \mu4] \left(\cos\left[\frac{1}{2} a (k1 - k3), \mu2\right] \cos\left[\frac{1}{2} a (k2 - k4), \mu1\right] - \right. \\
 & \quad \left. \frac{4}{3} \sin\left[\frac{ak1}{2}, \mu2\right] \sin\left[\frac{ak2}{2}, \mu1\right] \sin\left[\frac{ak3}{2}, \mu2\right] \sin\left[\frac{ak4}{2}, \mu1\right] \right) - \frac{2}{3} \cos\left[\frac{ak1}{2}, \mu2\right] \text{dd}[\mu1, \mu3] \\
 & \quad \text{dd}[\mu1, \mu4] \sin\left[\frac{ak2}{2}, \mu1\right] \left(-\sin\left[\frac{ak3}{2}, \mu2\right] \left(\cos\left[-\frac{ak4}{2}, \mu2\right] + i \sin\left[-\frac{ak4}{2}, \mu2\right] \right) + \right. \\
 & \quad \left. \left(\cos\left[-\frac{ak3}{2}, \mu2\right] + i \sin\left[-\frac{ak3}{2}, \mu2\right] \right) \sin\left[\frac{ak4}{2}, \mu2\right] \right) - \\
 & \quad \left. \text{dd}[\mu1, \mu4] \text{dd}[\mu2, \mu3] \left(\cos\left[\frac{1}{2} a (k2 - k3), \mu1\right] \cos\left[\frac{1}{2} a (k1 - k4), \mu2\right] - \right. \right. \\
 & \quad \left. \left. \frac{4}{3} \sin\left[\frac{ak1}{2}, \mu2\right] \sin\left[\frac{ak2}{2}, \mu1\right] \sin\left[\frac{ak3}{2}, \mu1\right] \sin\left[\frac{ak4}{2}, \mu2\right] \right) \right) \left. \right)
 \end{aligned}$$

Figure: gggg vertex

gggg-vertex (II)

$$\begin{aligned}
 & +g^2 \text{fc}[a1, a4, ee] \text{fc}[a2, a3, ee] \\
 & \left(\frac{2}{3} \cos\left[\frac{ak4}{2}, \mu1\right] \text{dd}[\mu2, \mu4] \text{dd}[\mu3, \mu4] \sin\left[\frac{ak1}{2}, \mu4\right] \left(\sin\left[\frac{ak2}{2}, \mu1\right] \right. \right. \\
 & \quad \left. \left. \left(\cos\left[-\frac{ak3}{2}, \mu1\right] + i \sin\left[-\frac{ak3}{2}, \mu1\right] \right) - \left(\cos\left[-\frac{ak2}{2}, \mu1\right] + i \sin\left[-\frac{ak2}{2}, \mu1\right] \right) \sin\left[\frac{ak3}{2}, \mu1\right] \right) + \right. \\
 & \quad \left. \frac{1}{3} \text{dd}[\mu1, \mu2] \text{dd}[\mu1, \mu3] \text{dd}[\mu1, \mu4] \left(\sin\left[\frac{ak2}{2}, t\right] \left(\cos\left[-\frac{ak3}{2}, t\right] + i \sin\left[-\frac{ak3}{2}, t\right] \right) - \right. \right. \\
 & \quad \left. \left. \left(\cos\left[-\frac{ak2}{2}, t\right] + i \sin\left[-\frac{ak2}{2}, t\right] \right) \sin\left[\frac{ak3}{2}, t\right] \right) \left(-\sin\left[\frac{ak1}{2}, t\right] \right) \right. \\
 & \quad \left. \left(\cos\left[-\frac{ak4}{2}, t\right] + i \sin\left[-\frac{ak4}{2}, t\right] \right) + \left(\cos\left[-\frac{ak1}{2}, t\right] + i \sin\left[-\frac{ak1}{2}, t\right] \right) \sin\left[\frac{ak4}{2}, t\right] \right) - \\
 & \quad \frac{2}{3} \cos\left[\frac{ak1}{2}, \mu4\right] \text{dd}[\mu1, \mu2] \text{dd}[\mu1, \mu3] \left(\sin\left[\frac{ak2}{2}, \mu4\right] \left(\cos\left[-\frac{ak3}{2}, \mu4\right] + i \sin\left[-\frac{ak3}{2}, \mu4\right] \right) - \right. \\
 & \quad \left. \left(\cos\left[-\frac{ak2}{2}, \mu4\right] + i \sin\left[-\frac{ak2}{2}, \mu4\right] \right) \sin\left[\frac{ak3}{2}, \mu4\right] \right) \sin\left[\frac{ak4}{2}, \mu1\right] - \\
 & \quad \text{dd}[\mu1, \mu2] \text{dd}[\mu3, \mu4] \left(\cos\left[\frac{1}{2} a (k1 - k2), \mu4\right] \cos\left[\frac{1}{2} a (-k3 + k4), \mu1\right] - \right. \\
 & \quad \left. \frac{4}{3} \sin\left[\frac{ak1}{2}, \mu4\right] \sin\left[\frac{ak2}{2}, \mu4\right] \sin\left[\frac{ak3}{2}, \mu1\right] \sin\left[\frac{ak4}{2}, \mu1\right] \right) + \\
 & \quad \text{dd}[\mu1, \mu3] \text{dd}[\mu2, \mu4] \left(\cos\left[\frac{1}{2} a (k1 - k3), \mu4\right] \cos\left[\frac{1}{2} a (-k2 + k4), \mu1\right] - \right. \\
 & \quad \left. \frac{4}{3} \sin\left[\frac{ak1}{2}, \mu4\right] \sin\left[\frac{ak2}{2}, \mu1\right] \sin\left[\frac{ak3}{2}, \mu4\right] \sin\left[\frac{ak4}{2}, \mu1\right] \right) - \frac{2}{3} \cos\left[\frac{ak3}{2}, \mu2\right] \text{dd}[\mu1, \mu3] \\
 & \quad \text{dd}[\mu1, \mu4] \sin\left[\frac{ak2}{2}, \mu1\right] \left(-\sin\left[\frac{ak1}{2}, \mu2\right] \left(\cos\left[-\frac{ak4}{2}, \mu2\right] + i \sin\left[-\frac{ak4}{2}, \mu2\right] \right) + \right. \\
 & \quad \left. \left(\cos\left[-\frac{ak1}{2}, \mu2\right] + i \sin\left[-\frac{ak1}{2}, \mu2\right] \right) \sin\left[\frac{ak4}{2}, \mu2\right] \right) + \frac{2}{3} \cos\left[\frac{ak2}{2}, \mu3\right] \text{dd}[\mu1, \mu2] \\
 & \quad \text{dd}[\mu1, \mu4] \sin\left[\frac{ak3}{2}, \mu1\right] \left(-\sin\left[\frac{ak1}{2}, \mu3\right] \left(\cos\left[-\frac{ak4}{2}, \mu3\right] + i \sin\left[-\frac{ak4}{2}, \mu3\right] \right) + \right. \\
 & \quad \left. \left. \left(\cos\left[-\frac{ak1}{2}, \mu3\right] + i \sin\left[-\frac{ak1}{2}, \mu3\right] \right) \sin\left[\frac{ak4}{2}, \mu3\right] \right) \right) \right)
 \end{aligned}$$

Figure: gggg vertex

gggg-vertex (III)

$$\begin{aligned}
 & -g^2 \text{fc}[a1, a3, ee] \text{fc}[a2, a4, ee] \\
 & \left(\frac{2}{3} \cos\left[\frac{ak4}{2}, \mu2\right] \text{dd}[\mu1, \mu3] \text{dd}[\mu1, \mu4] \sin\left[\frac{ak2}{2}, \mu1\right] \left(-\sin\left[\frac{ak1}{2}, \mu2\right] \right. \right. \\
 & \quad \left. \left. \left(\cos\left[-\frac{ak3}{2}, \mu2\right] + i \sin\left[-\frac{ak3}{2}, \mu2\right] \right) + \left(\cos\left[-\frac{ak1}{2}, \mu2\right] + i \sin\left[-\frac{ak1}{2}, \mu2\right] \right) \sin\left[\frac{ak3}{2}, \mu2\right] \right) + \right. \\
 & \quad \left. \frac{1}{3} \text{dd}[\mu1, \mu2] \text{dd}[\mu1, \mu3] \text{dd}[\mu1, \mu4] \left(-\sin\left[\frac{ak1}{2}, t\right] \left(\cos\left[-\frac{ak3}{2}, t\right] + i \sin\left[-\frac{ak3}{2}, t\right] \right) + \right. \right. \\
 & \quad \left. \left. \left(\cos\left[-\frac{ak1}{2}, t\right] + i \sin\left[-\frac{ak1}{2}, t\right] \right) \sin\left[\frac{ak3}{2}, t\right] \right) \left(-\sin\left[\frac{ak2}{2}, t\right] \right. \right. \\
 & \quad \left. \left. \left(\cos\left[-\frac{ak4}{2}, t\right] + i \sin\left[-\frac{ak4}{2}, t\right] \right) + \left(\cos\left[-\frac{ak2}{2}, t\right] + i \sin\left[-\frac{ak2}{2}, t\right] \right) \sin\left[\frac{ak4}{2}, t\right] \right) - \right. \\
 & \quad \left. \frac{2}{3} \cos\left[\frac{ak2}{2}, \mu4\right] \text{dd}[\mu1, \mu2] \text{dd}[\mu1, \mu3] \left(-\sin\left[\frac{ak1}{2}, \mu4\right] \left(\cos\left[-\frac{ak3}{2}, \mu4\right] + i \sin\left[-\frac{ak3}{2}, \mu4\right] \right) + \right. \right. \\
 & \quad \left. \left. \left(\cos\left[-\frac{ak1}{2}, \mu4\right] + i \sin\left[-\frac{ak1}{2}, \mu4\right] \right) \sin\left[\frac{ak3}{2}, \mu4\right] \right) \sin\left[\frac{ak4}{2}, \mu1\right] + \frac{2}{3} \cos\left[\frac{ak3}{2}, \mu1\right] \right. \\
 & \quad \left. \text{dd}[\mu2, \mu3] \text{dd}[\mu3, \mu4] \sin\left[\frac{ak1}{2}, \mu3\right] \left(-\sin\left[\frac{ak2}{2}, \mu1\right] \left(\cos\left[-\frac{ak4}{2}, \mu1\right] + i \sin\left[-\frac{ak4}{2}, \mu1\right] \right) + \right. \right. \\
 & \quad \left. \left. \left(\cos\left[-\frac{ak2}{2}, \mu1\right] + i \sin\left[-\frac{ak2}{2}, \mu1\right] \right) \sin\left[\frac{ak4}{2}, \mu1\right] \right) + \right. \\
 & \quad \left. \text{dd}[\mu1, \mu2] \text{dd}[\mu3, \mu4] \left(\cos\left[\frac{1}{2} a (k1 - k2), \mu3\right] \cos\left[\frac{1}{2} a (k3 - k4), \mu1\right] - \right. \right. \\
 & \quad \left. \left. \frac{4}{3} \sin\left[\frac{ak1}{2}, \mu3\right] \sin\left[\frac{ak2}{2}, \mu3\right] \sin\left[\frac{ak3}{2}, \mu1\right] \sin\left[\frac{ak4}{2}, \mu1\right] \right) - \frac{2}{3} \cos\left[\frac{ak1}{2}, \mu3\right] \text{dd}[\mu1, \mu2] \right. \\
 & \quad \left. \text{dd}[\mu1, \mu4] \sin\left[\frac{ak3}{2}, \mu1\right] \left(-\sin\left[\frac{ak2}{2}, \mu3\right] \left(\cos\left[-\frac{ak4}{2}, \mu3\right] + i \sin\left[-\frac{ak4}{2}, \mu3\right] \right) + \right. \right. \\
 & \quad \left. \left. \left(\cos\left[-\frac{ak2}{2}, \mu3\right] + i \sin\left[-\frac{ak2}{2}, \mu3\right] \right) \sin\left[\frac{ak4}{2}, \mu3\right] \right) - \right. \\
 & \quad \left. \text{dd}[\mu1, \mu4] \text{dd}[\mu2, \mu3] \left(\cos\left[\frac{1}{2} a (-k2 + k3), \mu1\right] \cos\left[\frac{1}{2} a (k1 - k4), \mu3\right] - \right. \right. \\
 & \quad \left. \left. \frac{4}{3} \sin\left[\frac{ak1}{2}, \mu3\right] \sin\left[\frac{ak2}{2}, \mu1\right] \sin\left[\frac{ak3}{2}, \mu1\right] \sin\left[\frac{ak4}{2}, \mu3\right] \right) \right) \right)
 \end{aligned}$$

Figure: gggg vertex

gggg-vertex (IV)

$$\begin{aligned}
 & + \frac{4}{3} g^2 \left(dd[\mu_1, \mu_2] dd[\mu_1, \mu_3] dd[\mu_1, \mu_4] \sin\left[\frac{ak_1}{2}, t\right] \sin\left[\frac{ak_2}{2}, t\right] \sin\left[\frac{ak_3}{2}, t\right] \sin\left[\frac{ak_4}{2}, t\right] - \right. \\
 & \quad dd[\mu_2, \mu_3] dd[\mu_2, \mu_4] \sin\left[\frac{ak_1}{2}, \mu_2\right] \sin\left[\frac{ak_2}{2}, \mu_1\right] \sin\left[\frac{ak_3}{2}, \mu_1\right] \sin\left[\frac{ak_4}{2}, \mu_1\right] + \\
 & \quad dd[\mu_1, \mu_2] dd[\mu_3, \mu_4] \sin\left[\frac{ak_1}{2}, \mu_3\right] \sin\left[\frac{ak_2}{2}, \mu_3\right] \sin\left[\frac{ak_3}{2}, \mu_1\right] \sin\left[\frac{ak_4}{2}, \mu_1\right] + \\
 & \quad dd[\mu_1, \mu_3] dd[\mu_2, \mu_4] \sin\left[\frac{ak_1}{2}, \mu_2\right] \sin\left[\frac{ak_2}{2}, \mu_1\right] \sin\left[\frac{ak_3}{2}, \mu_2\right] \sin\left[\frac{ak_4}{2}, \mu_1\right] - \\
 & \quad dd[\mu_1, \mu_2] dd[\mu_1, \mu_3] \sin\left[\frac{ak_1}{2}, \mu_4\right] \sin\left[\frac{ak_2}{2}, \mu_4\right] \sin\left[\frac{ak_3}{2}, \mu_4\right] \sin\left[\frac{ak_4}{2}, \mu_1\right] + \\
 & \quad dd[\mu_1, \mu_4] dd[\mu_2, \mu_3] \sin\left[\frac{ak_1}{2}, \mu_2\right] \sin\left[\frac{ak_2}{2}, \mu_1\right] \sin\left[\frac{ak_3}{2}, \mu_1\right] \sin\left[\frac{ak_4}{2}, \mu_2\right] - \\
 & \quad dd[\mu_1, \mu_3] dd[\mu_1, \mu_4] \sin\left[\frac{ak_1}{2}, \mu_2\right] \sin\left[\frac{ak_2}{2}, \mu_1\right] \sin\left[\frac{ak_3}{2}, \mu_2\right] \sin\left[\frac{ak_4}{2}, \mu_2\right] - \\
 & \quad \left. dd[\mu_1, \mu_2] dd[\mu_1, \mu_4] \sin\left[\frac{ak_1}{2}, \mu_3\right] \sin\left[\frac{ak_2}{2}, \mu_3\right] \sin\left[\frac{ak_3}{2}, \mu_1\right] \sin\left[\frac{ak_4}{2}, \mu_3\right] \right) \\
 & \left(dc[a_1, a_4, ee] dc[a_2, a_3, ee] + dc[a_1, a_3, ee] dc[a_2, a_4, ee] + \right. \\
 & \quad dc[a_1, a_2, ee] dc[a_3, a_4, ee] + \frac{1}{N_{color}} 2 (SUNDelta[a_1, a_4] SUNDelta[a_2, a_3] + \\
 & \quad \left. SUNDelta[a_1, a_3] SUNDelta[a_2, a_4] + SUNDelta[a_1, a_2] SUNDelta[a_3, a_4]) \right)
 \end{aligned}$$

Figure: gggg vertex

Momentum integration

$$\mathcal{I}(p) = \int_{-\pi}^{\pi} \frac{d^D k_1}{(2\pi)^D} \frac{d^D k_2}{(2\pi)^D} \frac{\text{Polynomial}(\widehat{k}_1, \widehat{k}_2)}{\widehat{k}_1^{2n_1} \widehat{k}_2^{2n_2} (\widehat{k}_1 - \widehat{k}_2)^{2n_3} (\widehat{k}_1 - p)^{2n_4} (\widehat{k}_2 - p)^{2n_5}}$$

$$\widehat{k} = 2 \sin(k/2)$$

Kawai approach ($V \rightarrow \infty$), modified by Alles et al. (1997):

- ▶ isolation of potential divergent terms
- ▶ +/- of equivalent (exact calculable) divergent terms
- ▶ calculation of the pole ($\frac{1}{D-4}$)- terms \rightarrow must cancel
 $\rightarrow \log^n(a^2 p^2)$ -terms remain
- ▶ calculation of finite differences numerically

Momentum integration

$$\mathcal{I}(p) = \int_{-\pi}^{\pi} \frac{d^D k_1}{(2\pi)^D} \frac{d^D k_2}{(2\pi)^D} \frac{\text{Polynomial}(\widehat{k}_1, \widehat{k}_2)}{\widehat{k}_1^{2n_1} \widehat{k}_2^{2n_2} (\widehat{k}_1 - \widehat{k}_2)^{2n_3} (\widehat{k}_1 - p)^{2n_4} (\widehat{k}_2 - p)^{2n_5}}$$

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$$\mathcal{I}(p) = \int_{-\pi}^{\pi} \frac{d^D k_1}{(2\pi)^D} \frac{d^D k_2}{(2\pi)^D} \frac{\text{Polynomial}(\widehat{k}_1, \widehat{k}_2)}{\widehat{k}_1^{2n_1} \widehat{k}_2^{2n_2} (\widehat{k}_1 - \widehat{k}_2)^{2n_3} (\widehat{k}_1 - p)^{2n_4} (\widehat{k}_2 - p)^{2n_5}}$$

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Momentum integration

$$\mathcal{I}(p) = \int_{-\pi}^{\pi} \frac{d^D k_1}{(2\pi)^D} \frac{d^D k_2}{(2\pi)^D} \frac{\text{Polynomial}(\widehat{k}_1, \widehat{k}_2)}{\widehat{k}_1^{2n_1} \widehat{k}_2^{2n_2} \widehat{(k_1 - k_2)}^{2n_3} \widehat{(k_1 - p)}^{2n_4} \widehat{(k_2 - p)}^{2n_5}}$$

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Momentum integration - CSM

Introduction

Diagrammatic
approach

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stochastic
perturbation
theory

Example: Ghost
propagator

Lüscher-Weisz approach (coordinate space method)

- ▶ Representation of momentum dependent quantities as Fourier-transforms of corresponding coordinate space quantities
- ▶ Evaluation of finite integrals as sums over the whole lattice ... (!)
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Momentum integration - CSM

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Momentum integration - CSM

Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

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Momentum integration - CSM

Typical integral

$$\begin{aligned} E(p) &= \int_{-\pi}^{\pi} \frac{d^4 k}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{\widehat{k}^2 (\widehat{k+p})^2 \widehat{q}^2 (\widehat{k+q})^2} \\ &= E_s(p) + E_r(p) + \left(P_2 + \frac{2}{(4\pi)^2} \right) A(p) \end{aligned}$$

with

$$E_r(p) = -0.000002546129 + \mathcal{O}(p^2)$$

$$E_s(p) = \frac{1}{(4\pi)^4} \left(\frac{1}{2} \ln^2(p^2) - \ln(p^2) \right) - \frac{1}{(4\pi)^2} P_3 + \mathcal{O}(p^2)$$

$$A(p) = \frac{1}{(4\pi)^2} \left(-\ln(p^2) + 2 \right) + P_2 + \mathcal{O}(p^2)$$

$$P_2 = 0.0240131811\dots$$

$$P_3 = 0.0346724321\dots$$

Higher loops (2-loop)

There are few 2-loop calculations on the lattice - mainly by the group around H. Panagopoulos

- ▶ up to now: quark self energy, point operators
- ▶ difficulties: generation of all diagrams, grouping diagrams for cancellation of singularities, momentum integration
- ▶ **FeynArts** - a program for generating the corresponding topologies
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Numeric stochastic perturbation theory (NSPT)

Complementary approach to perturbative calculations

- ▶ Use **Langevin equation** as basis of **stochastic quantisation** (Parisi and Wu, 1981)
- ▶ Powerful numerical approach for higher order calculations: **Numerical stochastic perturbation theory** (NSPT)
- ▶ omit details ... (cf. to talks of Chr. Torrero at the last SFB meetings)

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Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

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NSPT - conceptual problems

- ▶ In order to compare with infinite volume results or to be used in renormalization of observables: three limits have to be taken
 - ▶ **Limit $\tau \rightarrow 0$** : discrete Langevin time step $\rightarrow 0$
 - ▶ **Limit $V \rightarrow \infty$** : Large volumes needed
 - ▶ **Limit $a \rightarrow 0$** : Expansion in ap
- ▶ Logs in NSPT, finite size effects
 - ▶ "logs" on the finite lattice \neq logs on the infinite lattice
 - ▶ higher order terms involve higher powers of logs \rightarrow difficult to project out without prior knowledge of the log-structure
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Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

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Ghost propagator in Landau gauge

$$G^{ab}(x-y) = G(x-y) \delta^{ab} = \left\langle M^{-1}_{xy}{}^{ab} \right\rangle_U$$

In Landau gauge $\Delta \cdot A \equiv \sum_{\mu} \partial_{\mu}^L A_{\mu}(x) = 0$ we have

$$M = -\nabla \cdot D(U) = -D(U) \cdot \nabla.$$

Expanding the matrix in terms of the gauge fields $A^{(n)}$ one easily finds

$$[M^{(0)}]^{-1} = M^{(0)}$$

$$[M^{(l)}]^{-1} = -[M^{(0)}]^{-1} \sum_{j=0}^{l-1} M^{(l-j)} [M^{(j)}]^{-1}$$

Ghost self energy: continuum diagrams

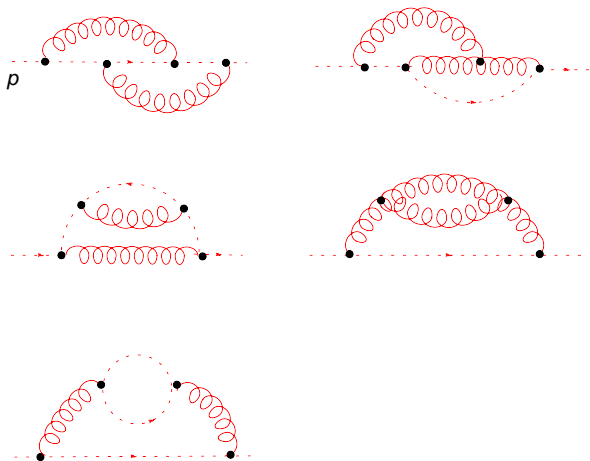


Figure: Ghost self energy: two-loop continuum diagrams

Ghost self energy: add. lattice diagrams

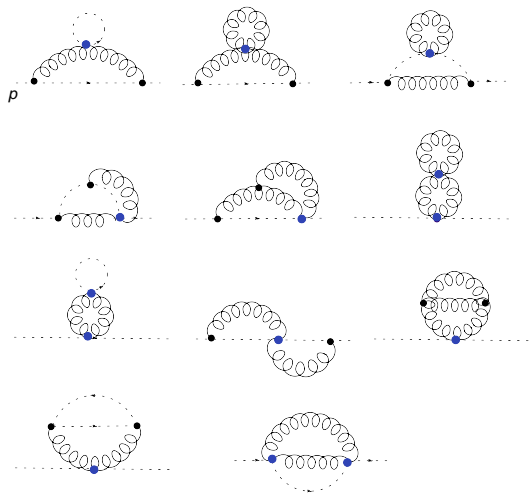


Figure: Ghost self energy: two-loop add. lattice diagrams

$\Pi_c(p)$: Landau gauge, $N_f = 0$

We know from two-loop **continuum** perturbation theory

$$(D = 4 - 2\epsilon)$$

(cf. also Davydychev et al. (1998))

$$\begin{aligned} \frac{1}{p^2} \Pi_c^{\text{cont}}(p^2, \mu^2) &= 1 + N_c \frac{g^2}{(4\pi)^{D/2}} (p^2/\mu^2)^{-\epsilon} \left(\frac{3}{4} \frac{1}{\epsilon} + 1 \right) \\ &\quad + N_c^2 \frac{g^4}{(4\pi)^D} (p^2/\mu^2)^{-2\epsilon} \left(\frac{35}{32} \frac{1}{\epsilon^2} + \frac{125}{32} \frac{1}{\epsilon} + \frac{957}{64} - \frac{9}{16} \zeta_3 \right) \end{aligned}$$

On the **lattice** we expect the following form

$$\begin{aligned} \frac{1}{p^2} \Pi_c^{\text{latt}}(p^2, a^2) &= 1 + N_c \frac{g^2}{(4\pi)^2} \left(\frac{3}{4} \log(a^2 p^2) + c_{10} \right) \\ &\quad + N_c^2 \frac{g^4}{(4\pi)^4} \left(\frac{35}{8} \log^2(a^2 p^2) + c_{21} \log(a^2 p^2) + c_{20} \right) \end{aligned}$$

Using the relation

$$\Pi^{\text{latt}}(p^2, a^2) = p^2 / J(p^2, a^2)$$

we look for the dressing function $J(p^2, a^2)$ ($\beta \propto g^{-2}$):

$$\begin{aligned} J^{\text{2-loop}}(p^2, a^2, \beta) &= 1 + \frac{1}{\beta} \left(J_{1,1} \log(a^2 p^2) + J_{1,0} \right) + \\ &\quad \frac{1}{\beta^2} \left(J_{2,2} \log^2(a^2 p^2) + J_{2,1} \log(a^2 p^2) + J_{2,0} \right) \end{aligned}$$

Treating the logs

- ▶ Two sources of lattice artifacts
 - ▶ **Hypercubic symmetry**

$$F_{latt}(ap) = f_0 + f_1 p^2 + f_2 a^2 p^4 / p^2 + f_3 a^4 p^4 + f_4 a^4 (p^2)^2 + f_5 a^4 p^6 / p^2 + \dots$$

$$(p^n = \sum_{i=1,4} p_i^n)$$

- ▶ **Finite size effects**

Computing on a finite lattice of size L - correction of order $\mathcal{O}(p^{(L)}L)^n$

$$p_i^{(L)} = 2\pi n_i / L, \quad n_i = 0, \dots, L-1$$

→ $(p^{(L)}L)^n$ independent of L

Introduction

Diagrammatic
approachNumeric
stochastic
perturbation
theoryExample: Ghost
propagator

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Introduction

Diagrammatic
approachNumeric
stochastic
perturbation
theoryExample: Ghost
propagator

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Treating the logs: first approach

Algorithm (I)

- ▶ Taking near diagonal data
- ▶ Subtracting the log(s) with coefficient(s) known from $(V = \infty)$ -LPT
- ▶ Extrapolation to $a^2 p^2 \rightarrow 0$ for every L , ($L = 8, 10, 12, 14$): $c_0(L)$
- ▶ Extrapolating $c_0(L) \rightarrow c_0(\infty)$

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Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

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Introduction

Diagrammatic
approach

Numeric
stochastic
perturbation
theory

Example: Ghost
propagator

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Treating the logs: F. DiRenzo

Algorithm (II), functional form of $(p^{(L)}L)$ -dependence assumed, linear log \leftrightarrow one-loop

- ▶ $F^{latt, V=\infty}(a, p, \log p) = F_0 + \gamma \log a^2 p^2 + \mathcal{O}(a^2 p^2)$
- ▶ $F^{latt, NSPT}(a, p, \log p, L)$ contains FS and HS effects
- ▶ $F^{latt, NSPT}(a, p, \log p, L) \rightarrow$

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- ▶ Fit: $\tilde{F}^{latt}(a, p, L) = F_0^{NSPT} + c_1 a^2 p^2 + c_2 a^2 p^4 / p^2 + c_3 a^4 (p^2)^2 + c_4 a^4 p^4 + c_5 a^4 p^6 / p^2 + c_6 / (n_1^2 + n_2^2 + n_3^2 + n_4^2) + c_7 / (n_1^4 + n_2^4 + n_3^4 + n_4^4)$

$$(p = p^{(L)} = \frac{2\pi}{L}(n_1, n_2, n_3, n_4))$$

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Problems:

- ▶ Selection of p^2 -range
- ▶ Rating of the fit quality χ^2

Results for the ansatz

$$J^{2\text{-loop}}(p^2, a^2, \beta) = 1 + \frac{1}{\beta} (J_{1,1} \log(a^2 p^2) + J_{1,0}) + \frac{1}{\beta^2} (J_{2,2} \log^2(a^2 p^2) + J_{2,1} \log(a^2 p^2) + J_{2,0})$$

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 $V \rightarrow \infty$: 0.525314, NSPT (I): 0.5246(22), NSPT (II): 0.5239(1)

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 $V \rightarrow \infty$: 0.525314, NSPT (I): 0.5246(22), NSPT (II): 0.5239(1)
- ▶ $J_{2,0}$:
 $V \rightarrow \infty$: — — — , NSPT (I): 1.4737(12), NSPT (II): 1.4272(1)

Treating the logs: F. DiRenzo

Problems:

- ▶ Selection of p^2 -range
- ▶ Rating of the fit quality χ^2

Results for the ansatz

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Possible next applications

- ▶ Diagrammatic approach - one-loop
 - ▶ Only for essentially new problems
 - ▶ Lattice OPE - Wilson coefficients: rather complex
- ▶ Diagrammatic approach - two-loop
 - ▶ Selected two-point functions: ghost self energy, quark bilinears
 - ▶ Three-point functions seem to be too complicated
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