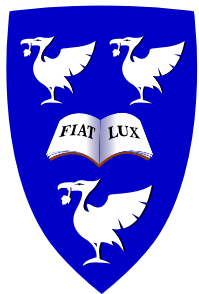


# The hadronic light-by-light contribution to the anomalous magnetic moment of the muon: a lattice approach

Paul Rakow for QCDSF



UNIVERSITY OF  
LIVERPOOL



# QCDSF

M Göckeler, M Gürtler, R Horsley, H Perlt,  
K Petrov, D Pleiter, PR, G Schierholz, ...

Berlin, DESY, Edinburgh, Leipzig,  
Liverpool, Munich, Regensburg

# Introduction

- Muon anomalous magnetic moment,  $g - 2$
- Vacuum polarisation contribution to  $g - 2$
- Light-by-Light: Introduction
- Light-by-Light: method
- Light-by-Light results (intermediate).
- Conclusions

# Muon Magnetic Moment

Consider a classical charged body. If it rotates it has angular momentum

$$\mathbf{J} = m \langle r_m^2 \rangle \omega$$

and a magnetic moment

$$\mathbf{M} = \frac{1}{2} q \langle r_q^2 \rangle \omega$$

If the mass-radius  $r_m$  and charge-radius  $r_q$  are the same,

$$\mathbf{M} = \frac{q}{2m} \mathbf{J}$$

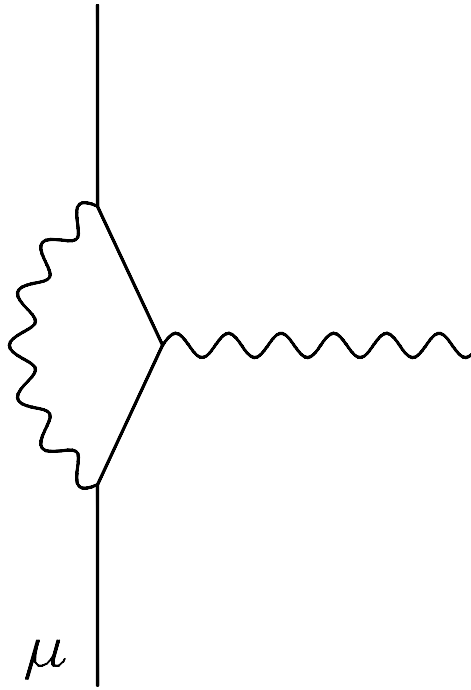
# Muon Magnetic Moment

If you solve the Dirac equation in a magnetic field, you find that a fermion has a magnetic moment **twice** as big as this classical model:

$$\mathbf{M} = g \frac{q}{2m} \mathbf{J}$$

with  $g = 2$  from the (tree-level) Dirac equation.

# Muon Anomalous Magnetic Moment



Quantum corrections change the result:

$$a_{\mu} \equiv (g - 2)/2 = \frac{\alpha_e}{2\pi} + \dots$$

$a_{\mu}$  is called the anomalous magnetic moment.

# Muon Anomalous Magnetic Moment

Can be measured to great accuracy

Solve Dirac equation in mag field, have many exact degeneracies.

When  $g \neq 2$ , splittings  $\propto g - 2$ .

So  $g - 2$  is directly measurable.

# Muon Anomalous Magnetic Moment

Can be measured to great accuracy

$$a_\mu = 116\,592\,082(55) \times 10^{-11}$$

Can also be calculated:

5 loop QED

2 loop electro-weak

hadronic effects

Possible mismatch,  $2.7 \sigma$ .

Main theory uncertainty, hadronic effects.

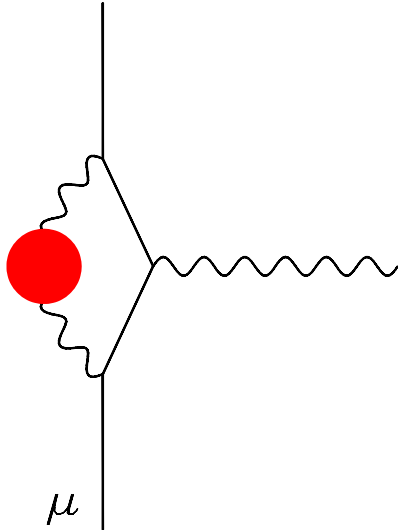
# Muon Anomalous Magnetic Moment

Is this mismatch of  $2.7 \sigma$  a sign of new physics? Expect

$$\sim \frac{m_\mu^2}{M_{new}^2}$$

Or is it a problem on the theory side?

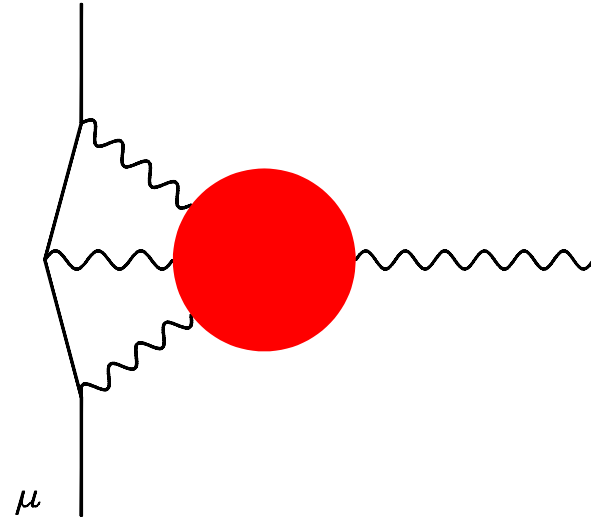
# Hadronic Effects



Vacuum Polarisation

$$\alpha_e^2$$

$$6934(63) \times 10^{-11}$$



Light-by-Light

$$\alpha_e^3$$

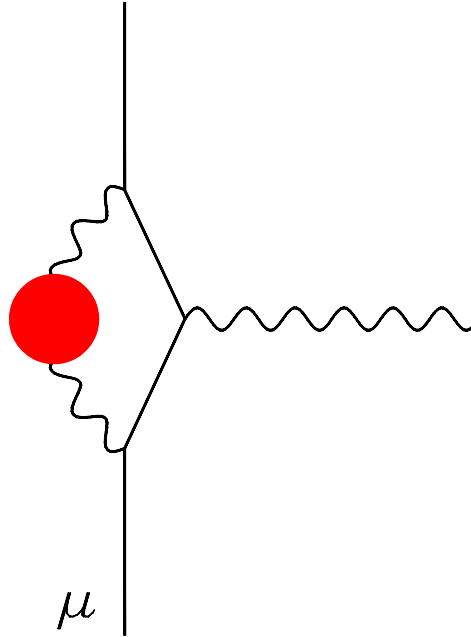
$$-98(1) \times 10^{-11}$$

$$\alpha_e^3$$

$$136(25) \times 10^{-11}$$

(Melnikov + Vainshtein)

# Vacuum Polarisation

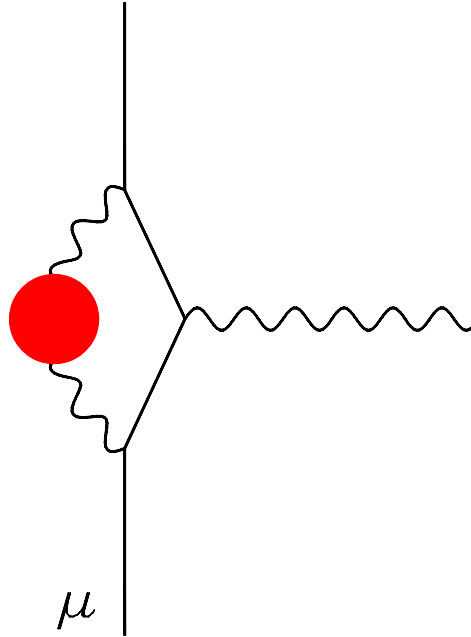


Related to cross-section

$$e^+ e^- \rightarrow \text{hadrons}$$

via dispersion relation and electromagnetic kernel.

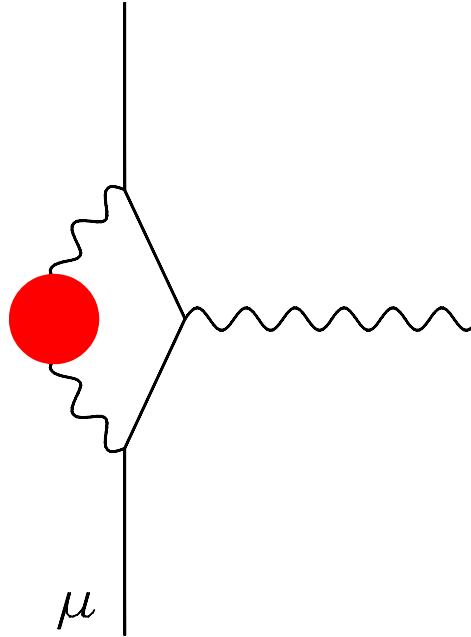
# Vacuum Polarisation



Calculating the vacuum polarisation in lattice gauge theory.

$$\langle J_\mu(0) J_\nu(x) \rangle$$

# Vacuum Polarisation

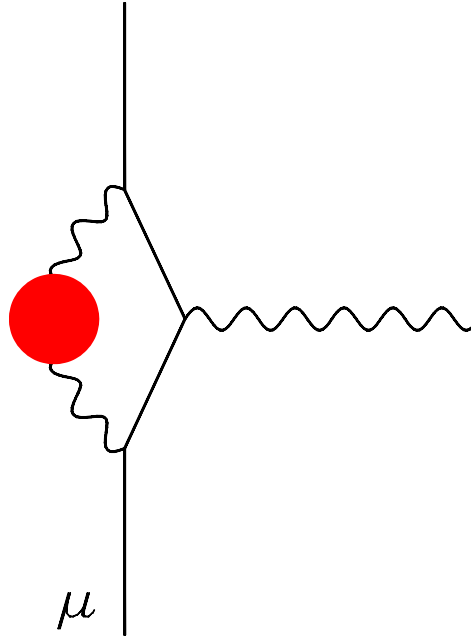


Blum

QCDSF

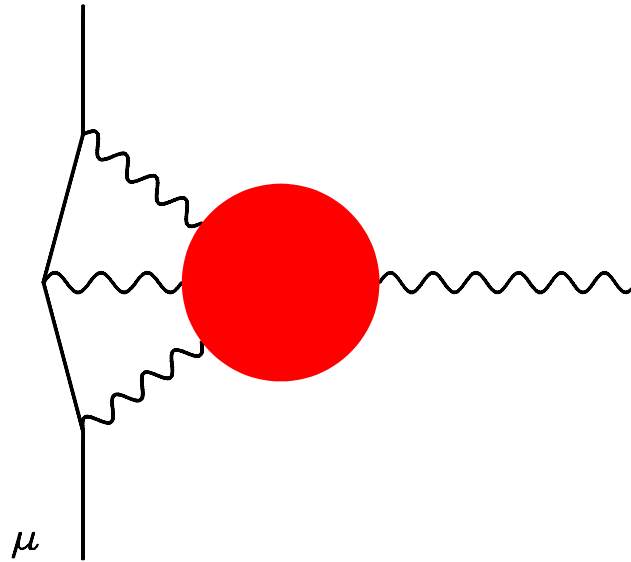
Renner

# Vacuum Polarisation



Dominated by photon momenta  $\sim m_\mu$ , needs extrapolation to small  $Q$ , large distance.

# Light-by-Light

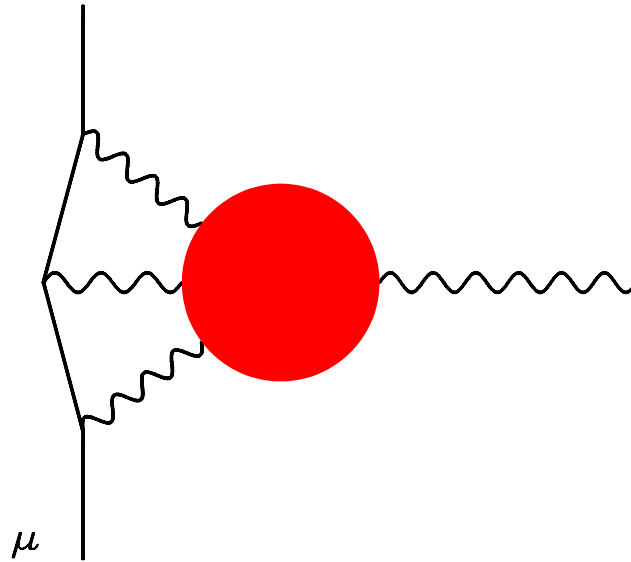


Called light-by-light because ‘blob’ represents photon-photon scattering.

Light-by-light contribution: Lattice could be useful. (Current uncertainty from phenomenology is 20% or more.)

Lattice proposal: Hayakawa, Blum, Izubuchi, Yamada; Lattice 2005, Dublin Lattice 2008

# Light-by-Light



For light-by-light ‘blob’, we need QCD contribution to

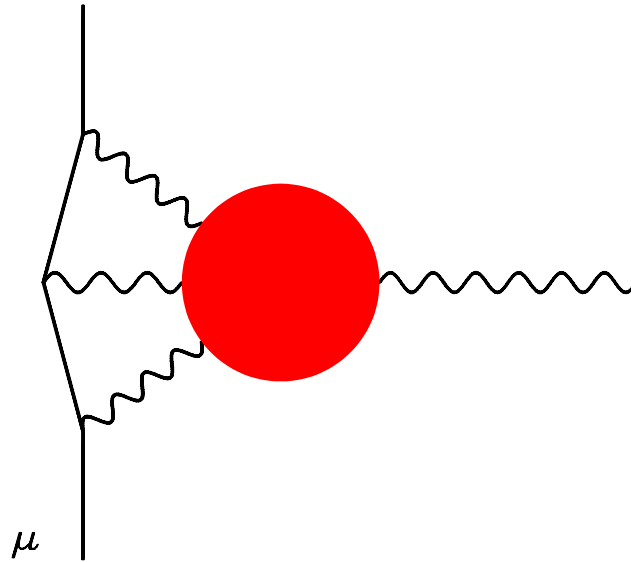
$$\langle J_{\nu_1}(x_1) J_{\nu_2}(x_2) J_{\nu_3}(x_3) J_{\mu}(x_q) \rangle_{con}$$

or its Fourier transform.

Rest of calculation by QED perturbation theory.

More complicated than  $\langle JJ \rangle$ , which just depends on a single momentum.

# Light-by-Light



For a given  $q$  the light-by-light amplitude depends on two internal photon momenta (eg  $k_1, k_2$ ). Final momentum fixed by momentum conservation.

Our plan is to measure  $\langle JJJJ \rangle$  for enough values of the photon momentum to be able to calculate the light-by-light contribution to  $g - 2$ . ("Direct Method")

# Light-by-Light: Results

Calculations in Jülich.

Clover fermions, two-flavour simulation.

$24^3 \times 48$  lattice. Currently 24 configurations measured.

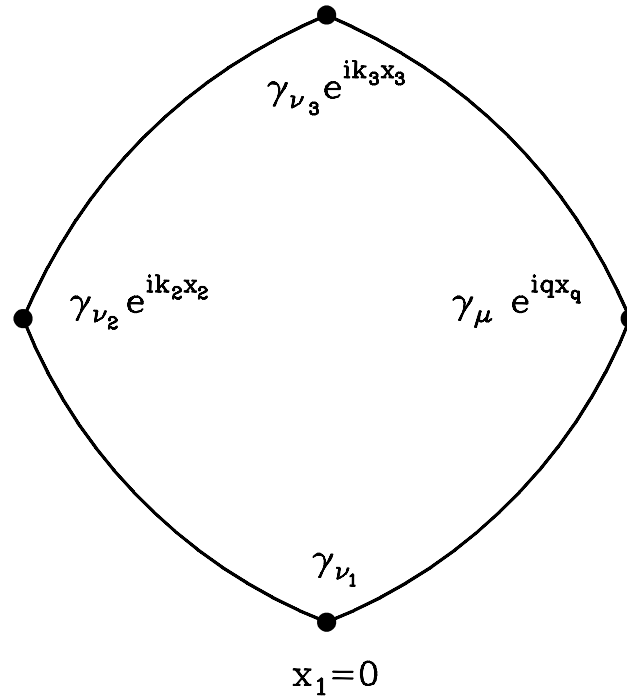
$$\beta = 5.40, \quad \kappa = 0.13625$$

$$a = 0.07 \text{ fm}, \quad a^{-1} = 2.75 \text{ GeV}$$

Local current  $J_\mu(x) = \bar{\psi}(x)\gamma_\mu\psi(x)$

Disconnected contributions neglected.

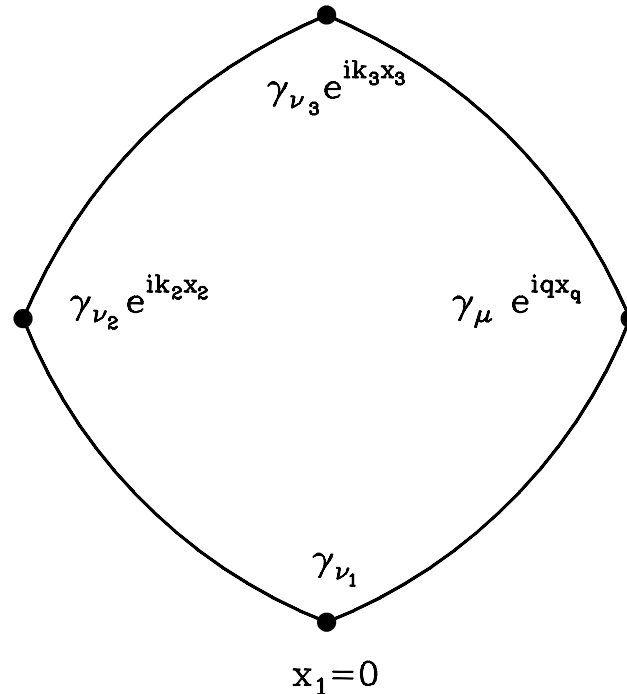
# Light-by-Light: Results



$$\langle J_{\nu_1}(k_1) J_{\nu_2}(k_2) J_{\nu_3}(k_3) J_{\mu}(q) \rangle$$

Now have measurements of the 4-current tensor for all polarisations and a respectable range of momenta.

# Light-by-Light: Results

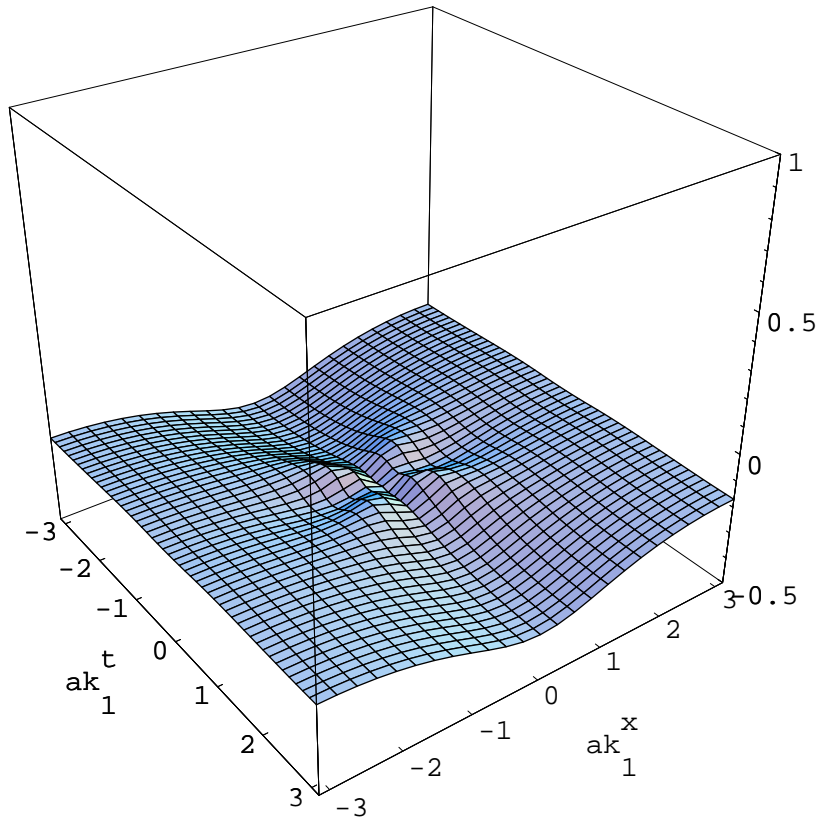


$$\langle J_{\nu_1}(k_1) J_{\nu_2}(k_2) J_{\nu_3}(k_3) J_{\mu}(q) \rangle$$

Picture: Fix polarisations;  $q$  and  $k_2$ ; plot as a function of  $k_1$ . (Have all  $k_1$ , but just plot as function of  $x$  and  $t$  components).

# Light-by-Light: Results

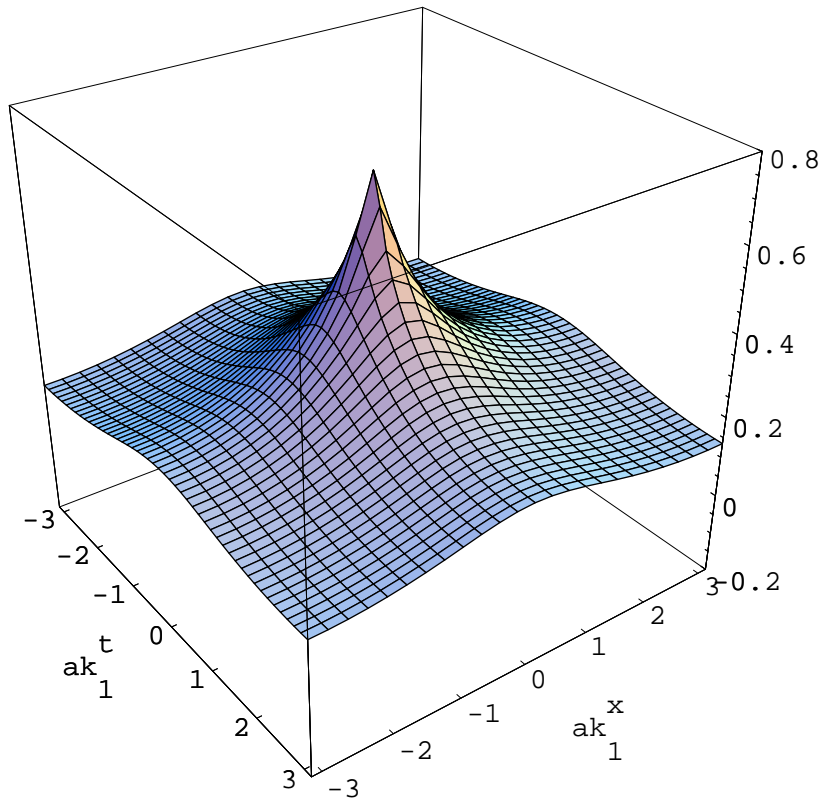
Parallel polarisations: | | | |



Statistical errors, a few %.

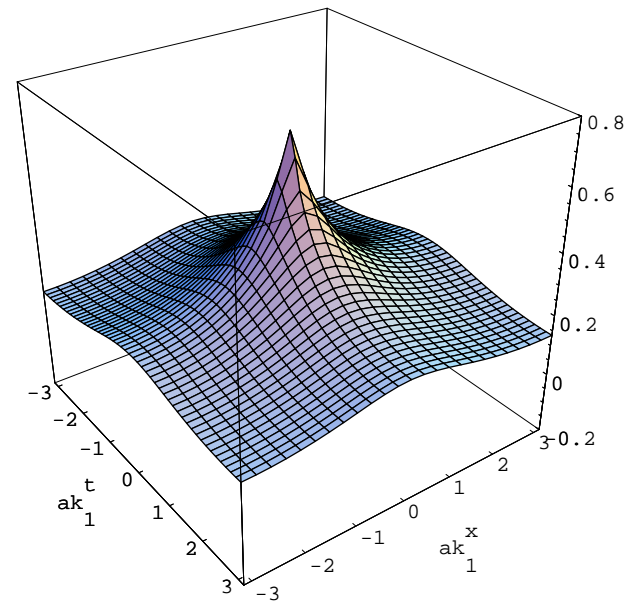
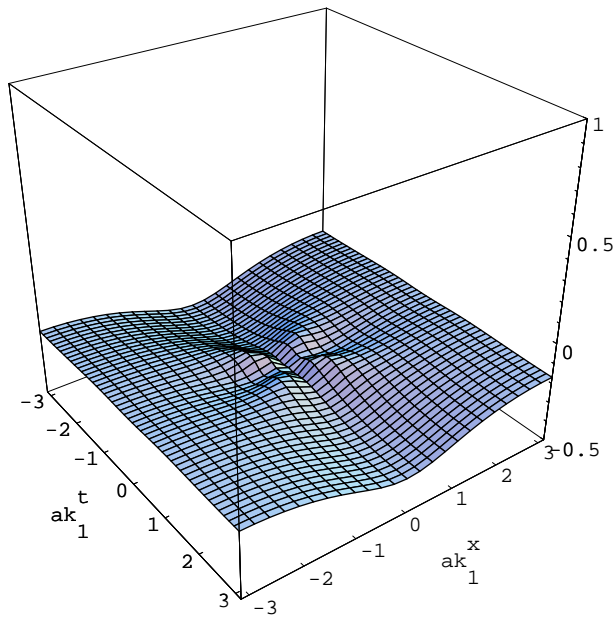
# Light-by-Light: Results

Crossed polarisations: | — | —



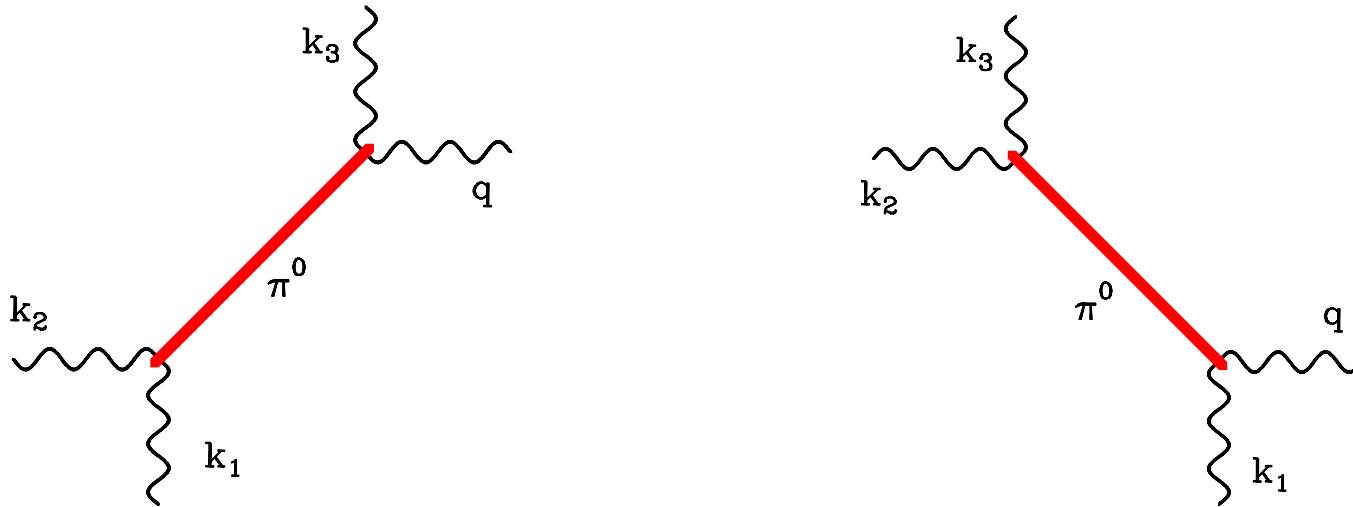
# Light-by-Light: Results

Why the dramatic difference between crossed polarisation and parallel?



# Light-by-Light: Results

$\pi^0$  decays to two cross-polarised photons (coupling to  $F^{\mu\nu} F_{\mu\nu}^*$ ).



Possible in crossed case, not in parallel case.

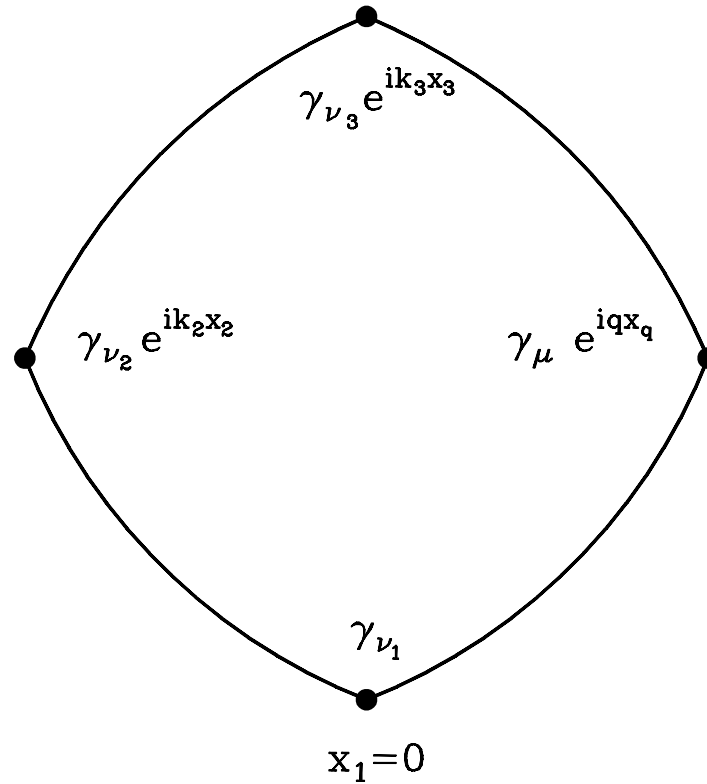
# Light-by-Light: Results

Look at same data in position space.

**Parallel polarisation:** Almost a  $\delta$  function.

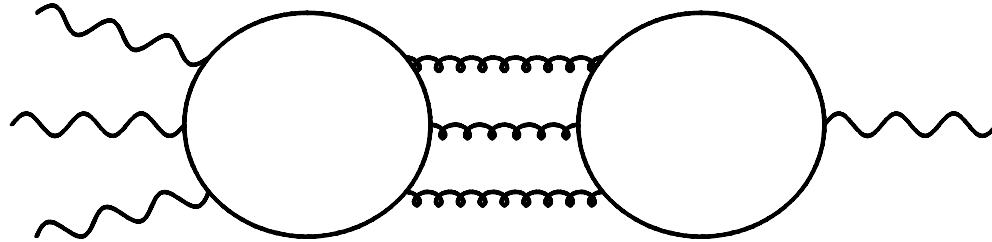
**Crossed polarisation:** Long tail, matches known pion mass.

# Light-by-Light: Fermion Lines



Not quite full: Current calculation only considers the diagrams with all 4 photons attached to the same quark line,  $\sum_f e_f^4$

# Light-by-Light: Fermion Lines



Flavour  $SU(3)$  suppresses most other diagrams, because there are quark loops with a single photon attached.

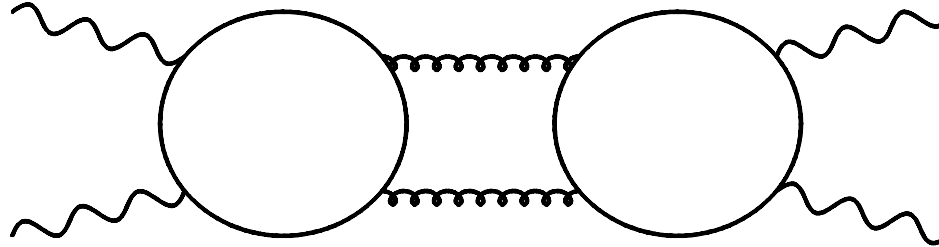
$$\left( \sum_{f'} e_{f'}^3 \right) \times \left( \sum_f e_f \right)$$

$$e_u + e_d + e_s = \frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$$

$\sum_f e_f$  argument, or **V-spin**.

Argument suppresses all 3-line and 4-line diagrams, some 2-line diagrams.

# Light-by-Light: Fermion Lines

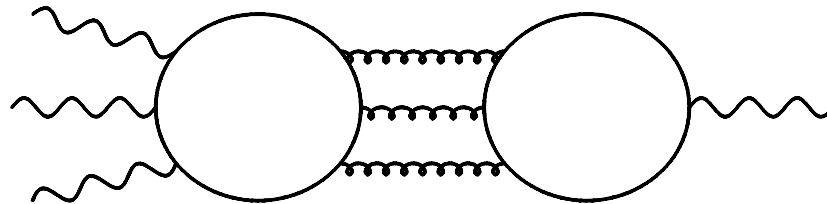
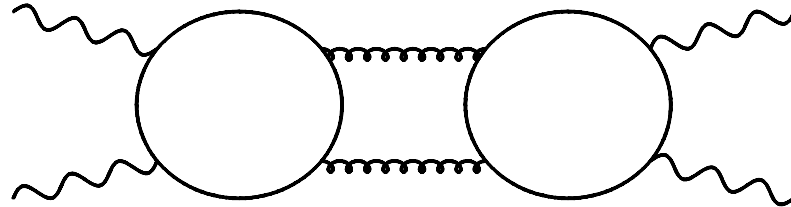


However there are diagrams with 2 photons on one quark line, and the other two on another quark line, which we are missing, and which aren't suppressed by interference between flavours.

$$\left( \sum_f e_f^2 \right)^2$$

Some hope from large  $N_c$ . Should think about ways to include neglected diagrams.

# Light-by-Light: Fermion Lines



Can write down a 'direct' method that produces all two-fermion-line diagrams at a cost-per-configuration about the same as the one-fermion-line contribution.

# Extrapolations

Even after measuring  $\langle JJJJ \rangle$  will be faced with extrapolations before we have a physical number.

- Lattice spacing to zero.
- Sea quark mass to  $m_u, m_d, m_s$ .
- Momentum extrapolations to  $k \sim m_\mu$ .

# Conclusions

- Have interesting results for the hadronic tensor  $\langle JJJJ \rangle$ .
- Statistical Errors seem small (a few percent).
- $\pi^0$  contribution dramatic.
- Working on convolution of  $\langle JJJJ \rangle$  tensor with QED part to give  $g - 2$ .
- Currently just looking at the single quark-line connected contribution. Is this sufficient to give a useful result?
- Should try to find ways to include the other diagrams – two-quark-line diagrams feasible, all others suppressed by flavour SU(3).
- Limitations in computer time still mean that lattice results need to be extrapolated to physical points.
- Making good progress.