A specific lattice artefact in non-perturbative renormalization of operators

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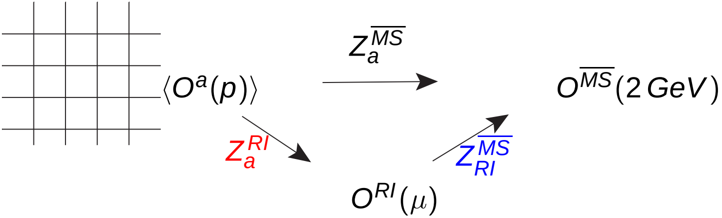
Outline

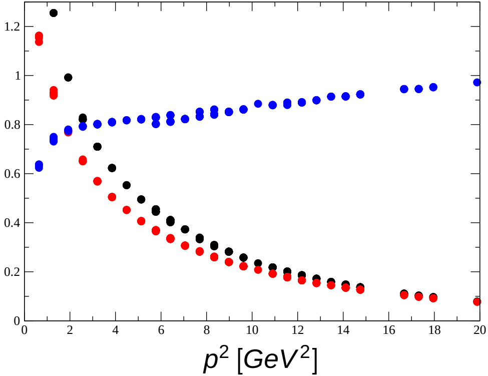


Martinelli et al., 1995; Gockeler et al., 1999
Franco and Lubicz, 1998; Chetyrkin and Retey, 2000

General idea

$$
\begin{aligned}
O_{ren}|_{\mu^2} &\doteq O^{bare}_{free}|_{\mu^2} = O^{a}_{free}|_{\mu^2} \\
O_{ren} &= Z^{RI} O^{a} \\
Z^{RI} O^{a}|_{\mu^2} &\doteq O^{a}_{free}|_{\mu^2}
\end{aligned}
$$

Example: Renormalization of the propagator



$$\psi_{ren} = Z_q^{-1/2} \psi$$
$$S = \langle \psi \bar{\psi} \rangle$$

function of $S(p)$
function of $S_{free}(p)$
$Z(p)$

The RI scheme

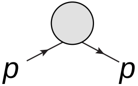
Fix the gauge to Landau gauge: $\mathrm{tr}\left(\sum_{x,\rho}(U_\rho(x) + U_\rho^\dagger(x)\right)$

$$
\begin{aligned}
O_\Gamma &= \overline{\psi}\Gamma\psi \\
G_\Gamma(p,U) &= \frac{1}{V}\sum e^{-ip(x-y)}\psi_x O_\Gamma(U)\overline{\psi}_y \\
&= \frac{1}{V}\sum e^{-ip(x-y)} S(U)_{xz}\,\Gamma\, S(U)_{zy} \\
\Lambda_\Gamma(p) &= \langle S(p,U)\rangle^{-1}\langle G_\Gamma(p,U)\rangle\langle S(p,U)\rangle^{-1}
\end{aligned}
$$

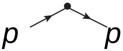
The RI scheme

$$Z_\Gamma Z_q \, \Lambda_\Gamma^{Born}(p)|_{p^2=\mu^2} \simeq (\Lambda_\Gamma(p))_{free} \, |_{p^2=\mu^2}$$

$Z_\Gamma Z_q \times$



$\simeq$



$$\begin{aligned} \Lambda_\Gamma(p)_{free} &= \langle S(p,U_0)\rangle^{-1}\langle G_\Gamma(p,U_0)\rangle\langle S(p,U_0)\rangle^{-1} \\ &= S_{free}^{-1} G_{free} S_{free}^{-1} \\ &= S_{free}^{-1} S_{free} \Gamma S_{free} S_{free}^{-1} = \Gamma \end{aligned}$$

$$Z_\Gamma Z_q \, \mathrm{tr}\left(\Lambda_\Gamma(p)\Gamma^{-1}\right)|_{p^2=\mu^2} = \mathrm{tr}(\mathbb{I})$$

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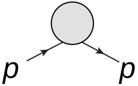

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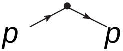

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$$Z_\Gamma Z_q\, \mathrm{tr}\left(\Lambda_\Gamma(p)\Gamma^{-1}\right)|_{p^2=\mu^2} = \mathrm{tr}(\mathbb{I})$$

The origin of the artefact

$$M_{4\times 4} = a_0\mathbb{I} + i\,a_\mu\gamma_\mu + a_5\gamma_5 + \dots$$

$$a_0 = \frac{1}{4}\mathrm{tr}(M)\,, \quad a_\mu = \frac{-i}{4}\mathrm{tr}(\gamma_\mu M)\dots$$

Massless case

$$D_{cont}^{-1}(p) = S_{cont}(p) = \gamma_\mu p_\mu f(p^2)$$

$$S_{lat}(p) = \langle S_{lat}(p,U)\rangle = b_0(p)\mathbb{I} + i\,b_\mu(p)\gamma_\mu$$

The origin of the artefact

$$\begin{aligned} M_{4\times 4} &= a_0 \mathbb{I} + i\, a_\mu \gamma_\mu + a_5 \gamma_5 + \ldots \\ a_0 &= \frac{1}{4} \mathrm{tr}(M)\,, \quad a_\mu = \frac{-i}{4} \mathrm{tr}(\gamma_\mu M) \ldots \end{aligned}$$

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The origin of the unit matrix part

$$
\begin{aligned}
D\gamma_5 + \gamma_5 D &= a\, D\gamma_5 D\, 2\kappa \\
\gamma_5 S_{xy}(U) + S_{xy}(U)\gamma_5 &= a\, \gamma_5\, \delta_{xy}\, 2\kappa
\end{aligned}
$$

Averaging over the configurations U and using

$$
\begin{aligned}
S_{xy} &= (b_0)_{xy} + i\gamma_\mu (b_\mu)_{xy} \\
(b_0)_{xy} &= a\, \kappa\, \delta_{xy}\ , \quad b_0(p) = a\, \kappa
\end{aligned}
$$

Off-shell propagators and Green's functions need $\mathcal{O}(a)$ corrections!

The origin of the unit matrix part

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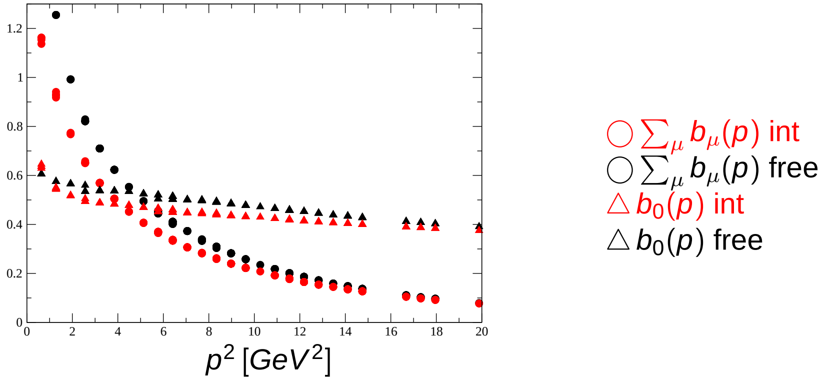
Off-shell propagators and Green's functions need $\mathcal{O}(a)$ corrections!

Demonstration of the effect on the propagator $S(p)$



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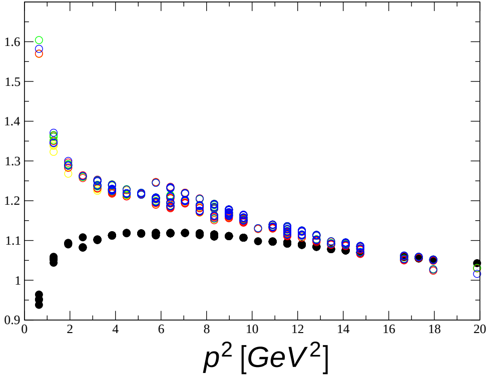

$b_0(p)_{free}/b_0(p)_{int}$

$b_\mu(p)_{free}/b_\mu(p)_{int}$

For $p^2 > 4[GeV^2]$:

$1 - cos(\theta(p)) \lesssim 10^{-5}$

$\theta(p) \lesssim 0.3°$

References

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Proposed solution

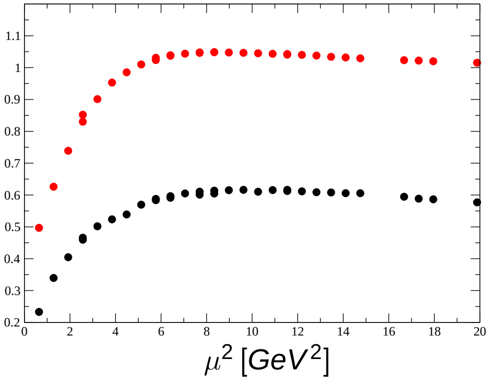
$$
\begin{aligned}
\overline{S}(p) &\doteq S(p) - b_0\mathbb{I} = S(p) - \frac{1}{4}\mathrm{tr}_{\mathrm{D}}\left(S(p)\right)\mathbb{I} \\
\overline{S}(p) &\doteq i\, b_\mu(p)\gamma_\mu \\
b_\mu(p) &= \frac{-i}{4}\mathrm{tr}_{\mathrm{D}}(S\,\gamma_\mu) \\
\overline{S}_{xy}(U) &\doteq S_{xy}(U) - (b_0)_{xy}\mathbb{I} = i\,(b_\mu(U))_{xy}\gamma_\mu
\end{aligned}
$$

Proposed solution

$$\hat{G}_\Gamma(p,U) = \frac{1}{V}\sum e^{-ip(x-y)}\overline{S}_{xz}(U)\,\Gamma\,\overline{S}_{zy}(U)$$
$$\hat{\Lambda}(p) = \overline{S}(p)^{-1}\langle\hat{G}_\Gamma(p,U)\rangle\,\overline{S}(p)^{-1}$$

$$(Z_\Gamma Z_q)\,(\mu^2)\,\mathrm{tr}\left(\hat{\Lambda}_\Gamma(p)\Gamma^{-1}\right)|_{p^2=\mu^2} = \mathrm{tr}(\mathbb{I})$$

Example: $Z_s Z_q$



$$O = \overline{\psi}\psi\,, \quad \Gamma = 1$$

$Z_s Z_q$ unsubtracted

$Z_s Z_q$ subtracted

Summary

- The $\mathcal{O}(a)$ effects need to be subtracted from off-shell quantities.
- The technique we suggest is simple and applicable to all kinds of Diracoperators.
- However, it does not remove all $\mathcal{O}(a)$ effects from non-Ginsparg-Wilson fermions.