Confinement in $(4+1)$ dimensions

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Motivation: why study $(4+1)d$ YM on the lattice?

- Connect with existing extra-dim. phenomenology: based on pert.th. or string

  Non-perturbative extra-dimensions ?

  - $d > 4$: non-renormalizable $\longrightarrow$ no continuum limit

    Observables depend on details of UV completion

  - Consider $(4+1)d$ YM as effective theory with cutoff Λ of "TOE"

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    Here, **Kaluza-Klein** $SU(2)$: $\mathcal{S}_E = \int \mathrm{d}^4x \int_0^{L_5} \mathrm{d}x_5 \; \frac{1}{2}\mathrm{Tr}\, F_{MN}^2$

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- Domain-wall fermions, $4d$ localization,..

Farakos et al, Ejiri et al, Irges & Knechtli, Beard, Wiese et al.

Compact $U(1)$ in 4 dimensions

- Two phases:

  Strong coupling: confining $\longleftrightarrow$ Weak coupling: Coulomb

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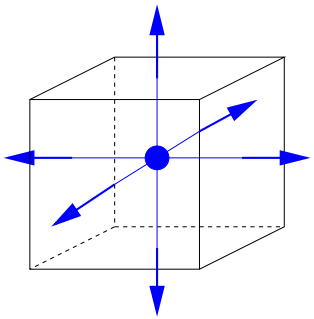
Periodic action $\cos\theta_P \rightarrow$ magnetic monopoles

$\theta_P \in [-\pi, +\pi]$, $\sum_6 \theta_P = 2k\pi$

gauge-invariant, codim 3

conserved current $\rightarrow$ *monopole loops*

free en. per unit current $\equiv$ monopole *mass*



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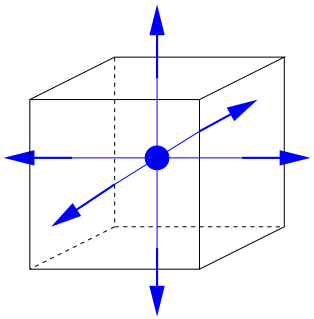
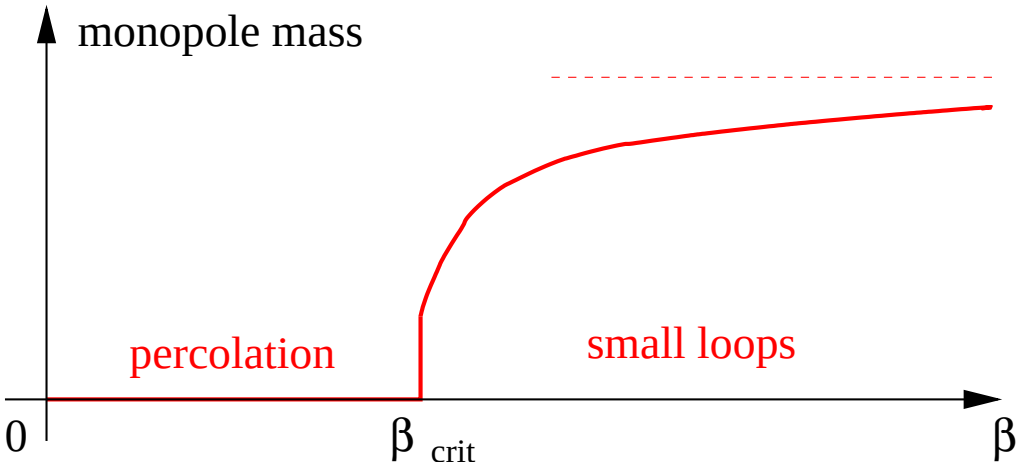
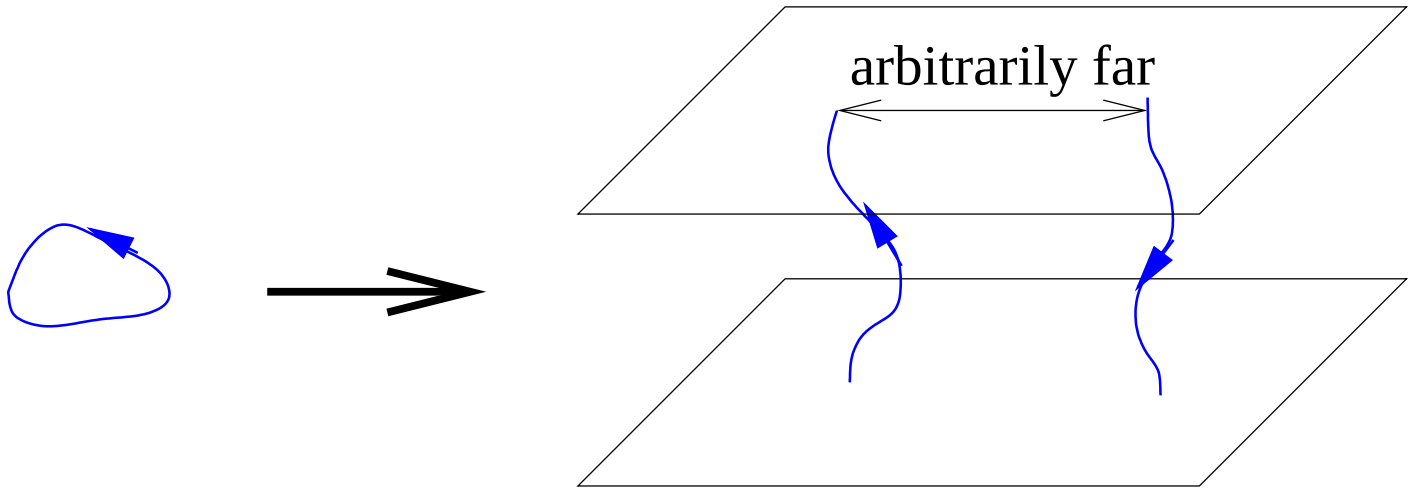




Polley & Wiese, PdF & Vettorazzo

Compact $U(1)$ at finite temperature in Coulomb phase

- Probability $\sim \exp(-m_{\mathrm{Monopole}}/T)$ to find *wrapping* monopole loop



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- Flux of wrapping monopole disorders spatial Wilson loop ($\leftrightarrow$ non-wrapping)

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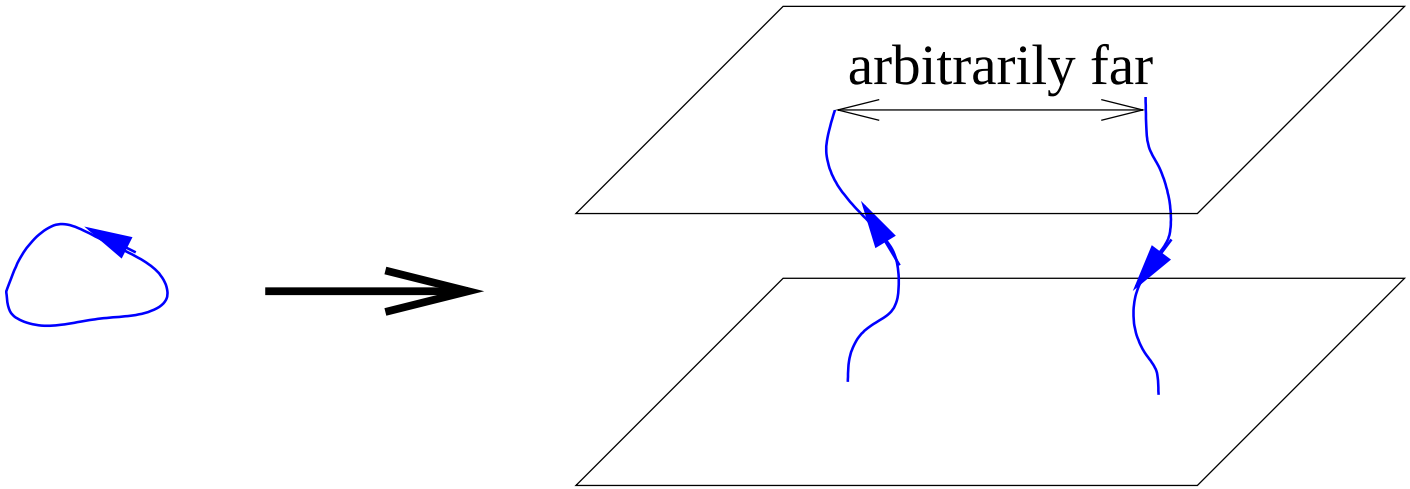
Correlation length grows **exponentially** with size of compact dimension



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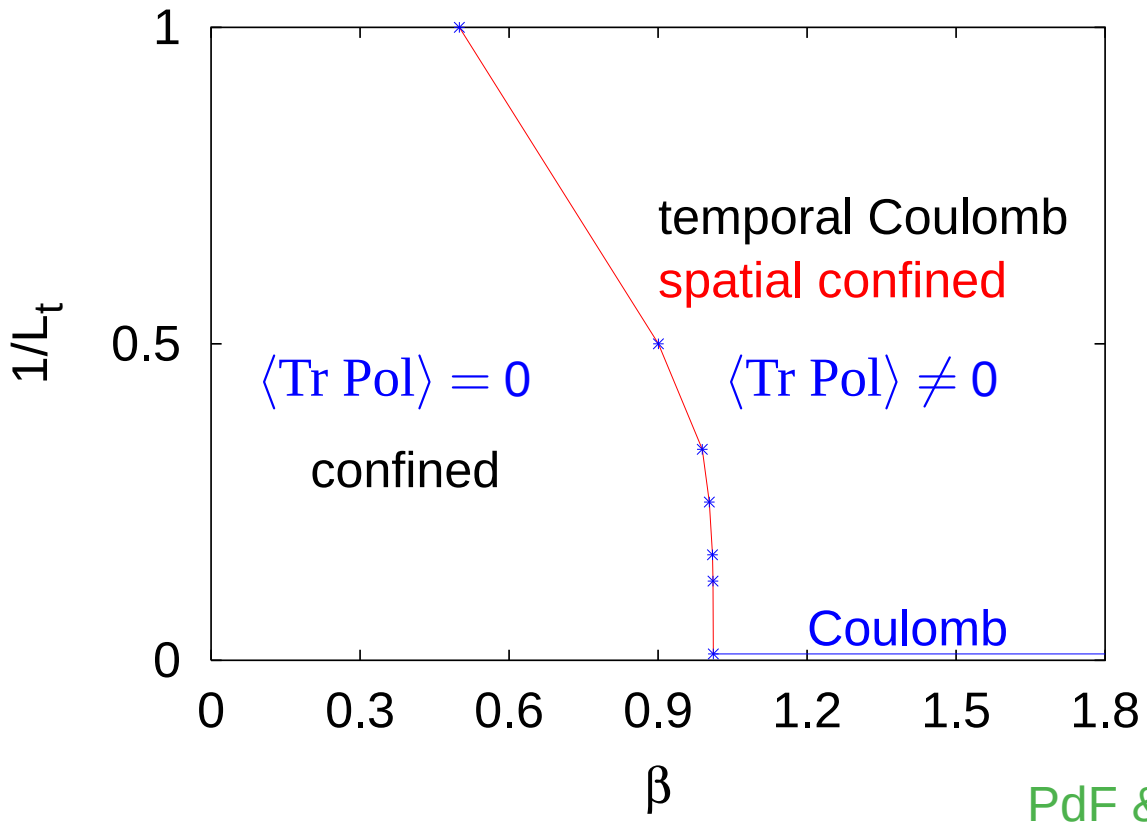


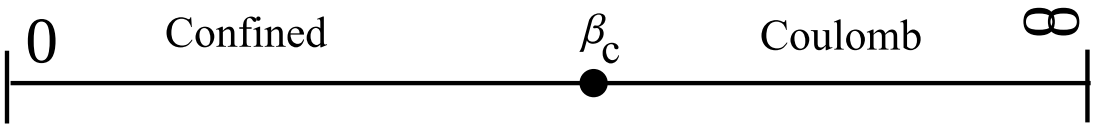
PdF & Vettorazzo

$(4+1)d$ $SU(2)$ on $N_s^4 \times N_5$ lattice

$$S_L = \frac{\beta_5}{\gamma} \sum_{1 \le M < N \le 4} \left[1 - \frac{1}{2}\mathrm{ReTr}\, P_{MN}(x)\right] + \gamma\beta_5 \sum_{M=1}^{4} \left[1 - \frac{1}{2}\mathrm{ReTr}\, P_{M5}(x)\right]$$

- $\gamma = 1$ on L^5 lattice: 1rst-order transition — Creutz 1979
- Weak coupling is Coulomb for $d > 4$: $F_{\mu_0\nu_0} = \frac{2\pi}{L^2} \Rightarrow S_L \sim L^{d-4} \overset{L\to\infty}{\longrightarrow} +\infty$

checked numerically — Beard et al



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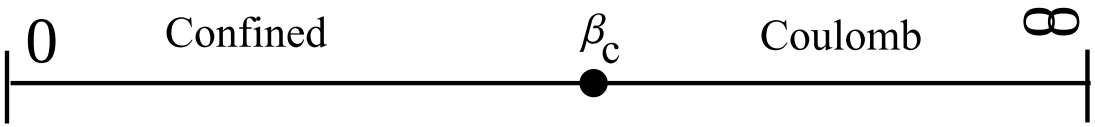


Vary L_5 in Coulomb phase:

- cf. $(3+1)d$ $U(1)$: $\langle P_5 \rangle \neq 0$ as soon as $L_5 < \infty$

  (deconfinement when compact dim. < corr. length)
- We take continuum limit in 5th direction: $N_5 \to \infty$, $\frac{N_5}{\gamma}$ fixed $\leftrightarrow$ Farakos et al

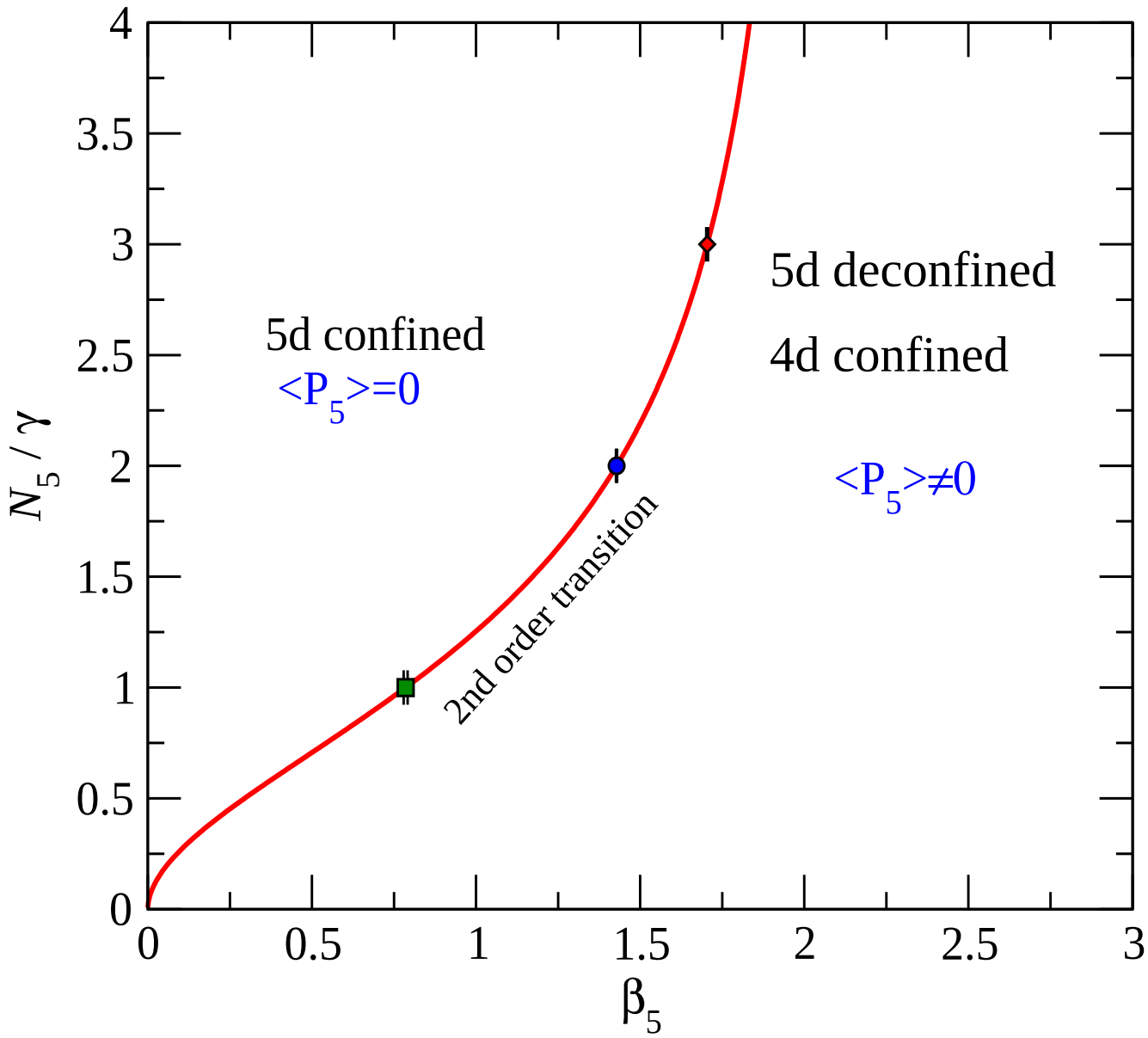
Advantages: - Continuum $5d$ YM theory defined by 2 dimless ratios $(g_5^2\Lambda, L_5\Lambda)$

$\longrightarrow$ Lattice theory defined by 2 parameters:

$$\beta_5 \equiv \frac{4a}{g_5^2}, \qquad \widetilde{N}_5 \equiv \frac{N_5}{\gamma} \sim \frac{L_5}{a} + \ldots$$

- Remove discretization error from 5th direction

Phase diagram



Dimensional reduction (in deconfined phase)

- Zero modes at large distances ($\Delta x \gg L_5$) described by a 4d continuum theory:

$$S_{\rm eff} = \frac{1}{g_4^2(L_5)} \int {\rm d}^4x \left[\tfrac{1}{2} {\rm Tr} F_{\mu\nu}^2 + {\rm Tr}\left[D_\mu A_5\right]^2 + m_5^2 {\rm Tr} A_5^2 + \lambda {\rm Tr} A_5^4 \ldots \right]$$

- *4d* YM + adjoint Higgs: renormalizable continuum theory

  $\longrightarrow$ We know something about it:

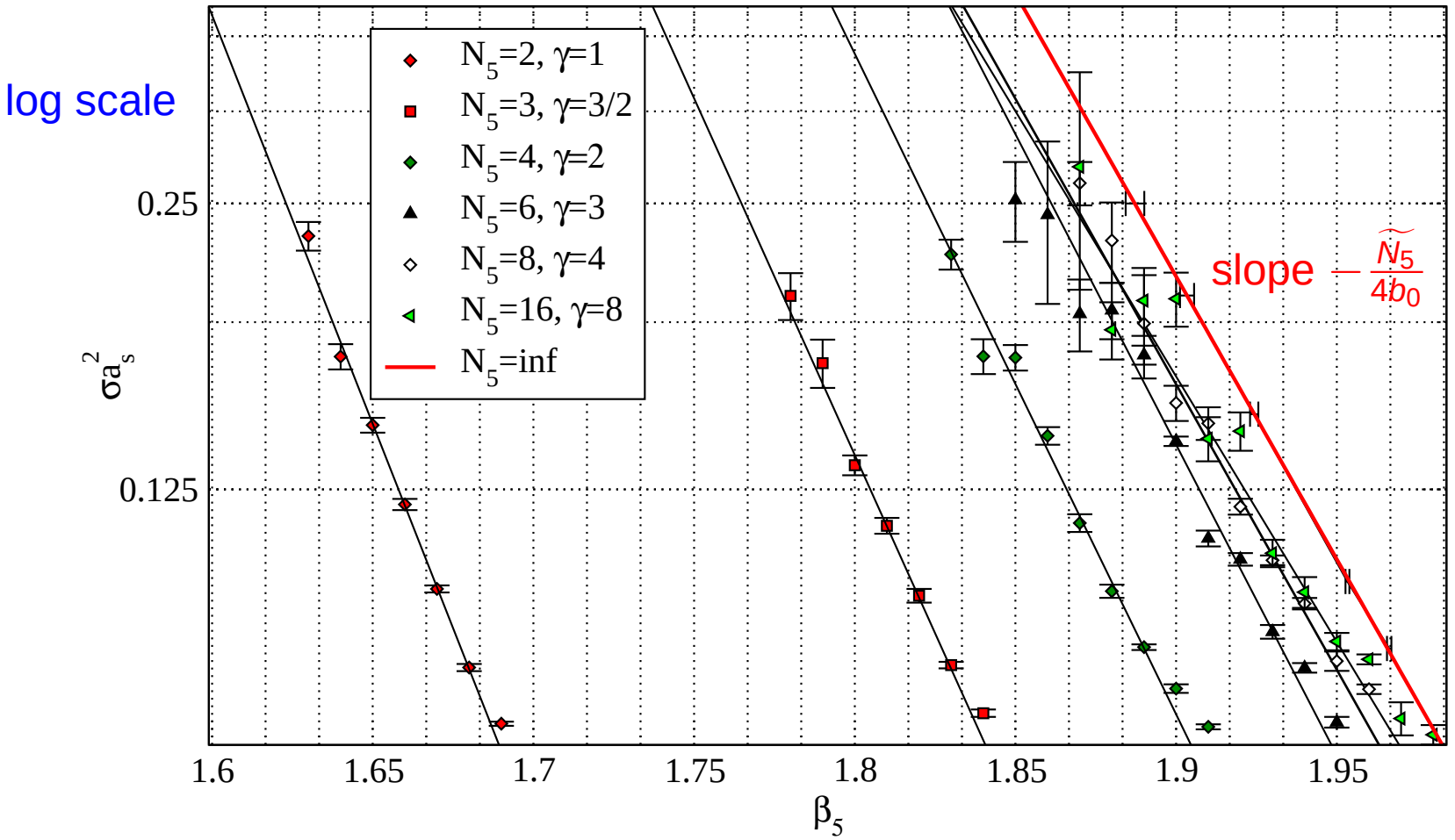
  - Confinement: $\sigma \neq 0$
  - Asymptotic freedom: $\sigma \sim \frac{1}{{L_5}^2} \exp\left[-\frac{1}{b_0} \frac{1}{g_4^2(L_5)} \right]$

  For a start, match in PT (tree-level): $\quad g_4^2(L_5) \sim \frac{g_5^2}{L_5} = \frac{4}{\beta_5 \widetilde{N}_5}$

$$\longrightarrow \sigma a^2 \sim \frac{a^2}{L_5^2} \exp\left[-\frac{1}{2b_0} \frac{1}{g_4^2(L_5)} \right] \sim \frac{1}{\widetilde{N}_5^2} \exp\left[-\frac{1}{4b_0} \widetilde{N_5} \times \beta_5 \right]$$

Measured $4d$ string tension

Dim. Red., tree-level PT: $\sigma a^2 \sim \frac{1}{\widetilde{N}_5^2} \exp\left[-\frac{1}{4b_0}\widetilde{N}_5 \times \beta_5\right]$



σa^2 versus β_5, $\widetilde{N}_5 = N_5/\gamma = 2$ fixed, $V = 6^4 \times N_5$

"Inverse" dimensional reduction

- The $4d$ correlation length $\xi_{4d} = \frac{1}{\sqrt{\sigma}}$ increases *exponentially* with the size of the compact direction cf. $(3+1)d$ $U(1)$

$$\frac{L_5}{a} = \widetilde{N}_5 \qquad \frac{\xi_{4d}}{a} \sim \widetilde{N}_5 \exp(+\frac{1}{2b_0}\widetilde{N}_5\beta_5)$$

$\longrightarrow$ Dimensional reduction to $4d$ at **LARGE** $\widetilde{N}_5 = L_5/a$!

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- Increasing the scale separation $L_5/a = \widetilde{N}_5$ makes the 5th dimension exponentially small in $4d$ units Beard, Wiese et al

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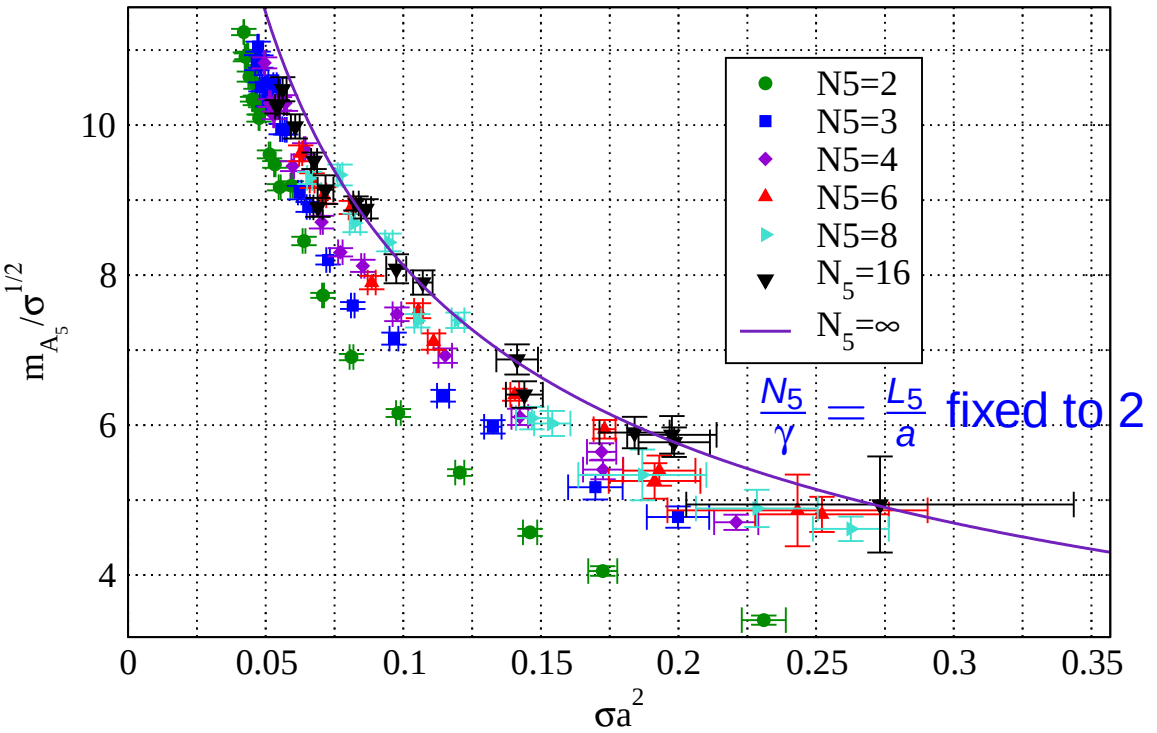
- Consequences:
  - Can take $4d$ continuum limit at *fixed* β_5 by increasing $\widetilde{N}_5$ Wiese
  - If a/ξ_{4d} is given, the extent of 5th dim. is **bounded**

  eg. for $a/\xi_{4d} = M_{\rm Planck}^{-1}/\text{GeV}^{-1}$: $\beta_5 > \beta_5^{\rm crit} \to L_5 \lesssim 10 M_{\rm Planck}^{-1}$

Predict the scale of New Physics from LHC Higgs-like results

- Effective $4d$ theory has 2 parameters m_5 and λ, which depend on L_5 and "scale of New Physics" a
  - Leading order PT: $m_5 \sim \frac{1}{L_5}\sqrt{\frac{g_5^2}{L_5}}, \qquad \lambda \sim \frac{g_5^4}{L_5^2}$
  - Determined non-perturbatively on the lattice

Example: m_5 versus a, keeping $\frac{L_5}{a}$ fixed



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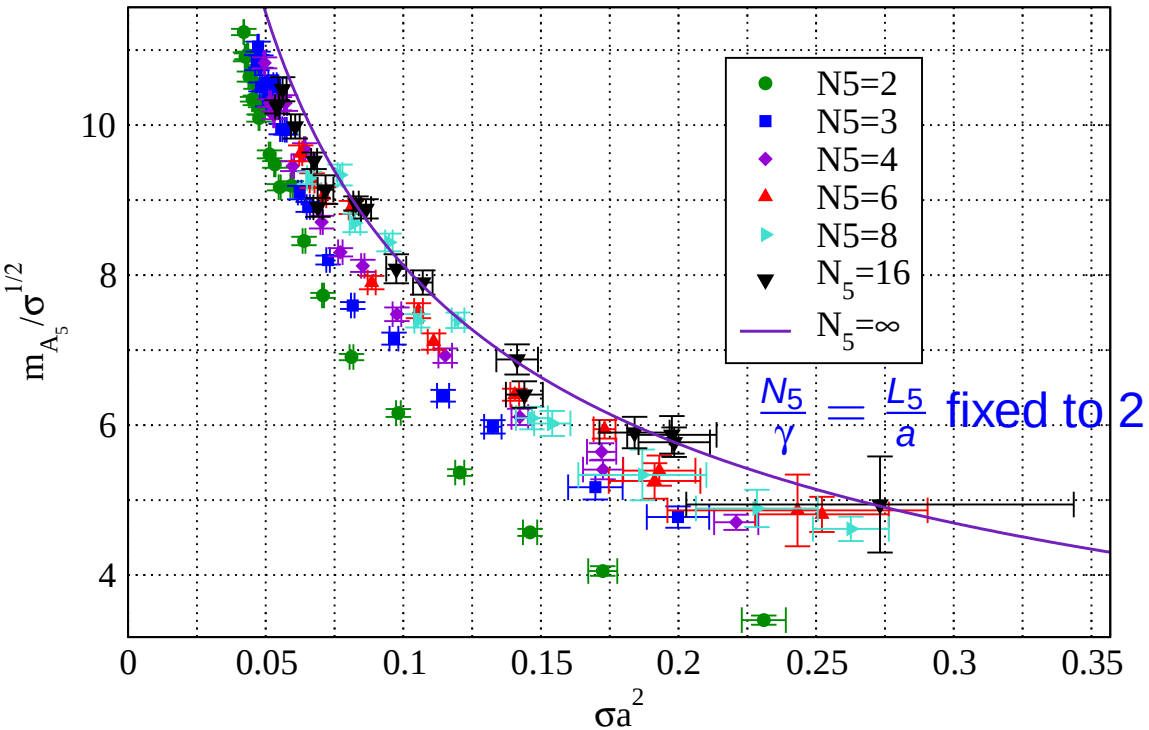
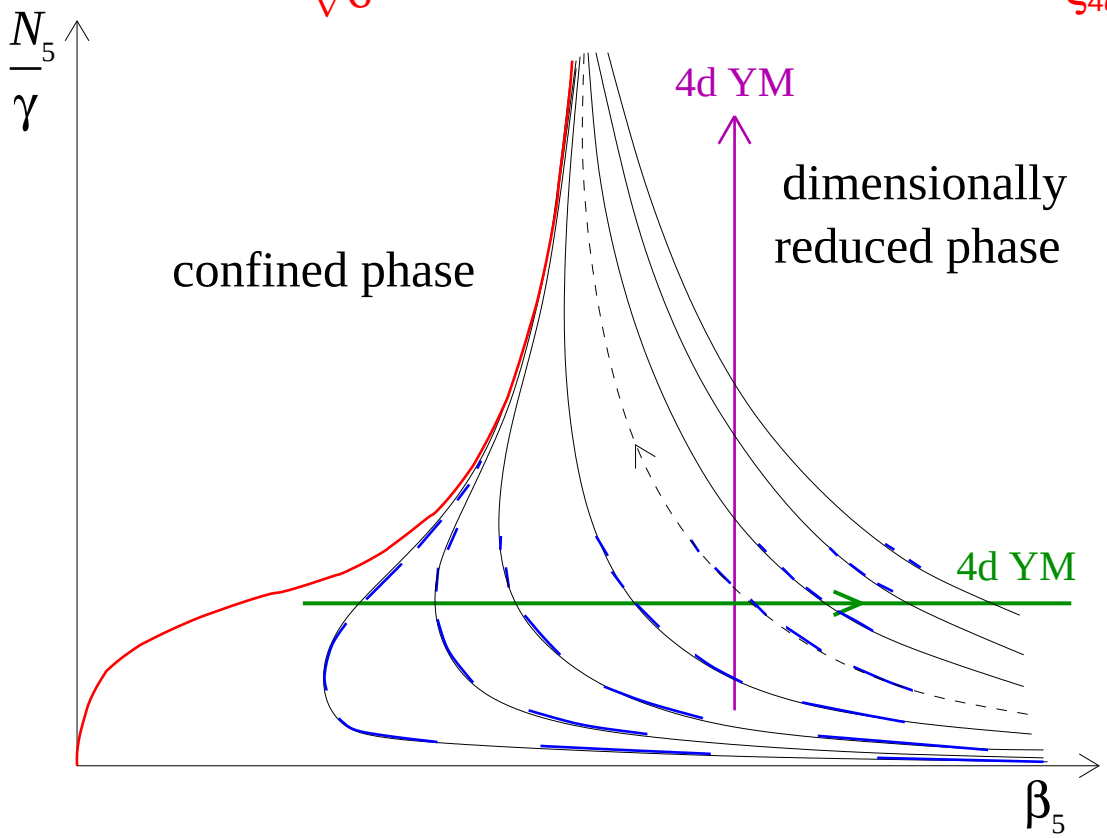


- IF LHC sees *adjoint* Higgs with mass m_5 and self-coupling λ,
  THEN predict L_5 and a

Continuum limits: all $4d$

$$\frac{m_5}{\sqrt{\sigma}} \sim \frac{1}{\sqrt{\beta_5 \widetilde{N}_5}} \exp\left[+\frac{1}{2b_0}\widetilde{N}_5\beta_5\right]$$

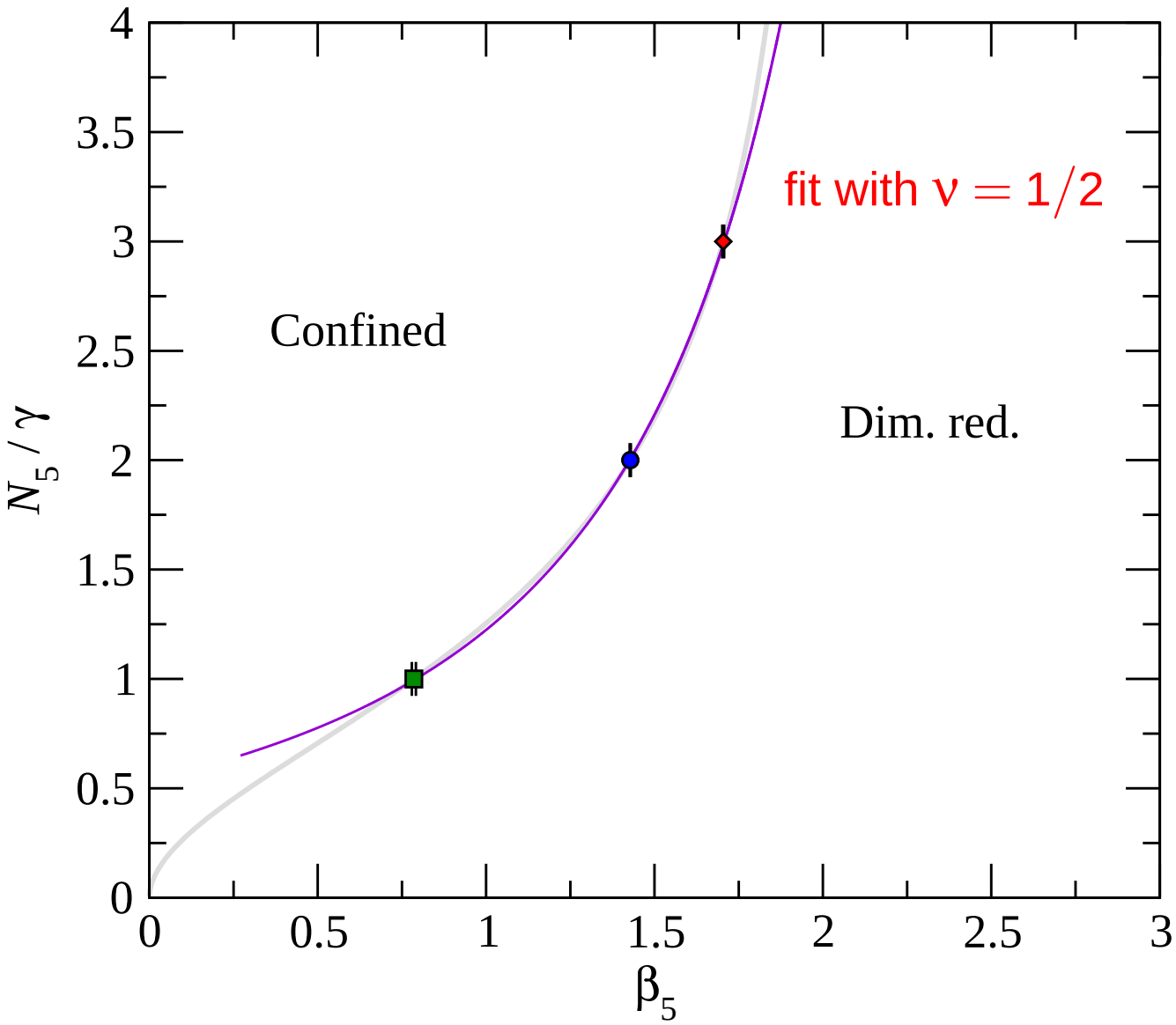
- Fix $\widetilde{N}_5$, let $\beta_5 \to \infty$: $\sigma a^2 \to 0$, $\frac{m_5}{\sqrt{\sigma}} \to \infty$, ie. $4d$ YM continuum limit
- Fix β_5, let $\widetilde{N}_5 \to \infty$: $4d$ **YM continuum limit also**
- Lines of constant physics: $\frac{m_5}{\sqrt{\sigma}}$ fixed $\quad a \searrow \leftrightarrow \beta_5 \searrow$, and $\frac{L_5}{\xi_{4d}} \to 0$, ie. $4d$



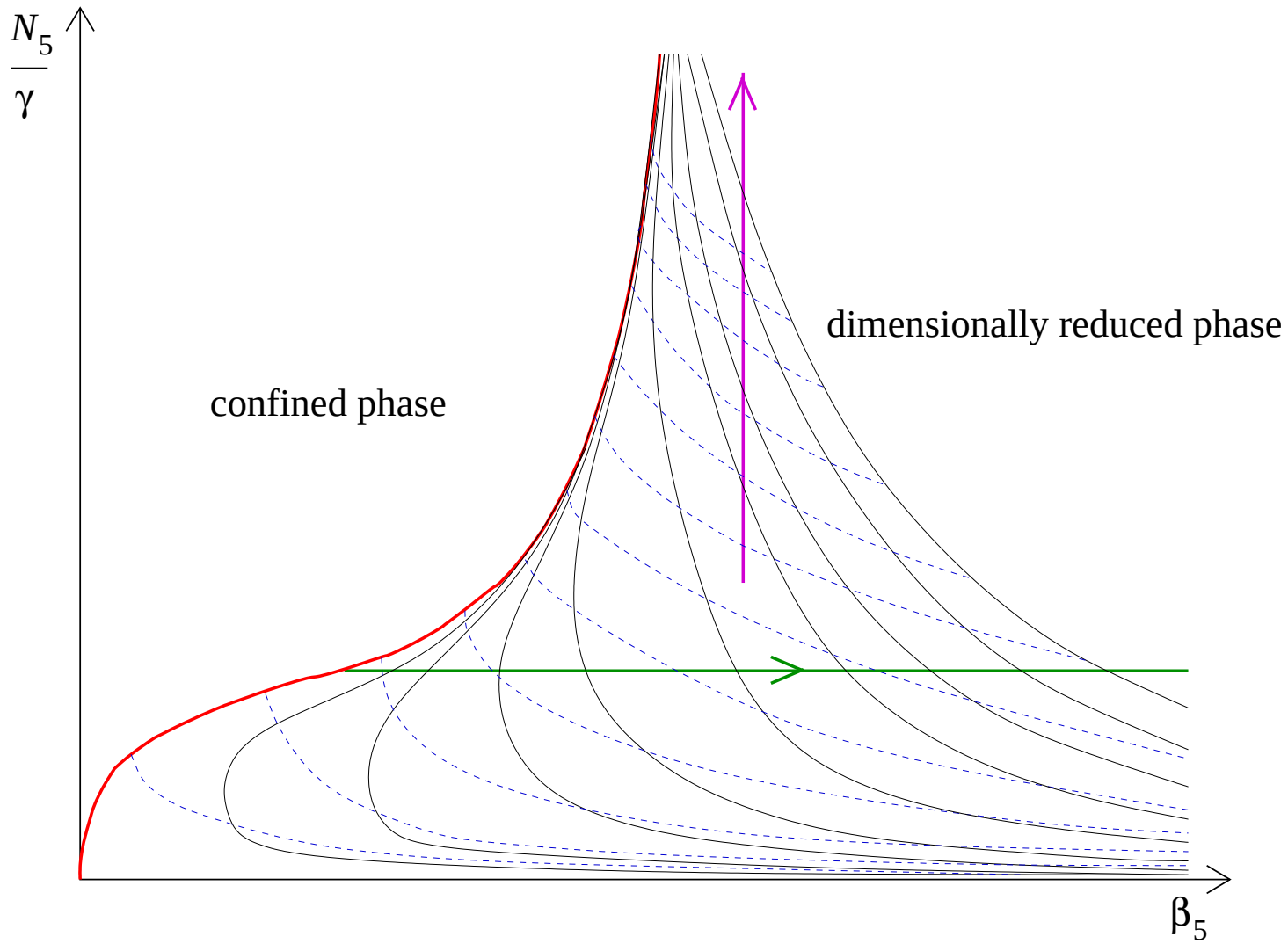
Conclusions

- $(3+1)d$ compact $U(1)$: playground for thermal monopoles
- $(4+1)d$ $SU(2)$: $\xi \sim \exp(+\frac{1}{2b_0 g_5^2} L_5)$ similar to $(3+1)d$ $U(1)$
  confinement by codim 4 objects of mass $\sim \frac{1}{2b_0 g_5^2}$??
- $(4+1)d$ $SU(2)$: various $4d$ continuum limits (YM, YM + adj. Higgs)
- Natural scale hierarchy $a \ll L_5 \ll \sigma^{-\frac{1}{2}}$
  - $5d$ non-renormalizable, but can push a to $M_{\rm Planck}^{-1}$
  - constraint $\frac{L_5}{a} \leq f(\sigma a^2)$
- Adjoint Higgs (m_5, λ) $\longleftrightarrow$ scales (a, L_5)
- Outlook: other boundary conditions (esp. orbifold), fermions,..

Backup: second-order transition



Backup: curves of constant lattice spacing



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