Hadrons as Holograms

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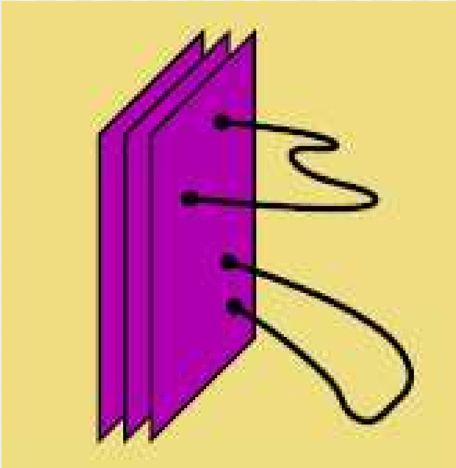
HF, T. Frederico, M. Beyer, JHEP 07 (2007) 077, IJMPE 16 (2007) 2794,
… + W. de Paula, PRD 79 (2009) 075019;
HF, E. Klempt, PLB 679 (2009) 77;
HF, PRD 78 (2008) 025071

- Gauge/string correspondence
- Meson & baryon spectra, Regge trajs.
- Dynamical AdS/QCD
- Hadron correlators, espec. glueballs
- Wrapping up...

Gauge/string correspondence in a (tiny) nutshell

2 equivalent descriptions of $E \ll 1/l_s$ excitations of $N_c \gg 1$ D3-branes:

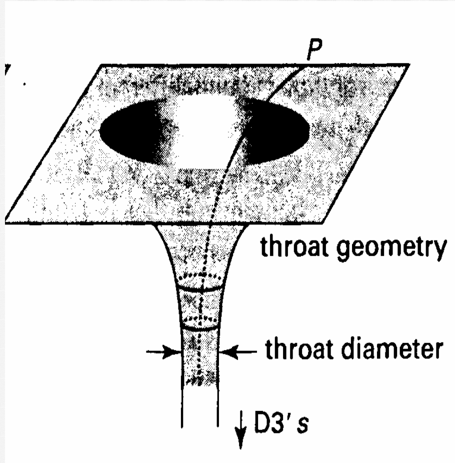
1) Open m=0 string modes on D3s :



$G_{(10)} \sim g^2 l_s^8 \to 0$:
little distortion of metric: (almost) flat
closed strings & bulk fields decouple,
+ open string modes ($m \neq 0$ inaccessible) →

(S)U(N_c) (S)Yang-Mills fields on D3s, strongly coupled

2) SUGRA soln. sourced by branes:



metric has `throat´, ends at horizon,
in far region: flat, gravity decoupled,
+ near horizon: fin. E excits. redshifted
at ∞: → metric curved →

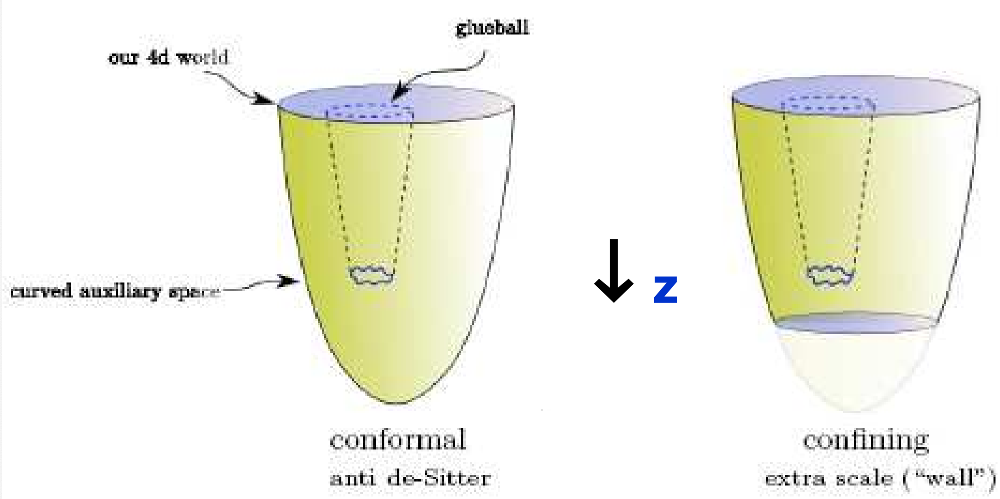
Graviton- & other excits. in $AAdS_5 \times X^5$, weakly coupled

$AAdS_5$ bulk backgrounds expected for QCD

Minimal features of bulk geometry:

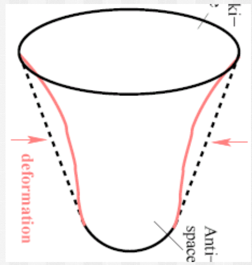
- **Poincaré invariant boundary**
- **Conf. sym. broken at $p \sim \Lambda_{YM}$**
- **Confinement: `IR wall´ in z**

→ IR-deformed $AdS_5 \times X_5$:



$$ds^2 = g_{MN}(x)\, dx^M dx^N = e^{2A(z)} \frac{R^2}{z^2} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right) + R^2 ds^2_{X_5}$$

IR deform. A(z) with $A(z \to 0) \to 0 \Rightarrow$ in UV (ie. for $z \to 0$) metric remains conformal



+ dilaton $\Phi(z)$ in `soft wall´ + axions, tachyons, gauge fields + ...

Hadron dual-modes: rad. & orbital excitations in AAdS$_5$

Example: baryon bulk modes:

$$\Psi(x,z) = \left[\frac{1+\gamma^5}{2} f_+(z) + \frac{1-\gamma^5}{2} f_-(z)\right] \Psi_{(4)}(x),$$

Dual to QCD baryon interpolator:

$$\mathcal{O}_{B,\bar{\tau}=L+3} = qD_{\{\ell_1} \ldots D_{\ell_q} q D_{\ell_{q+1}} \ldots D_{\ell_m\}} q$$

→ **iterated 5d Dirac equation:**
with (Polchinski, Strassler) bc:

$$f_i(z) \overset{z\to 0}{\longrightarrow} z^{\tau_i}, \qquad \tau_i = \Delta_i - \sigma_i$$

$$\left[\partial_z^2 + 4\left(A' - \frac{1}{z}\right)\partial_z + 2\left(A'' + \frac{1}{z^2}\right) + 4\left(A' - \frac{1}{z}\right)^2 - \left(m_{5,B}R\frac{e^A}{z}\right)^2 \mp m_{5,B}R\frac{e^A}{z}\left(A' - \frac{1}{z}\right) + M^2\right] f_\pm(z) = 0$$

⇒ **Sturm-Liouville problem:**

$$\left[-\partial_z^2 + V_{B,\pm}(z)\right]\psi_\pm(z) = M_B^2 \psi_\pm(z)$$

$$V_{B,\pm}(z) = m_{5,B}R\frac{e^A}{z}\left[\pm\left(A' - \frac{1}{z}\right) + m_{5,B}R\frac{e^A}{z}\right]$$

Analog. for spin-0/1 mesons:

$$V_S(z) = \frac{3}{2}\left[A'' + \frac{3}{2}A'^2 - 3\frac{A'}{z} + \frac{5}{2}\frac{1}{z^2}\right] + m_{5,S}^2 R^2 \frac{e^{2A}}{z^2},$$
$$V_V(z) = \frac{3}{2}\left[-A'' + \frac{3}{2}A'^2 - 3\frac{A'}{z} + \frac{1}{2}\frac{1}{z^2}\right] + m_{5,V}^2 R^2 \frac{e^{2A}}{z^2}$$

AdS/CFT dictionary sets bcs. ⇒

$$m_{5,S}^2 R^2 = \bar{\tau}_M(\bar{\tau}_M - 4) = L^2 - 4,$$
$$m_{5,V}^2 R^2 = \bar{\tau}_M(\bar{\tau}_M - 4) + 3 = L^2 - 1,$$
$$m_{5,B} R = \bar{\tau}_B - 2 = L + 1,$$

Trajectory structure in the light hadron spectrum

Simplest dual models, e.g. Polchinski-Strassler's 'hard wall':

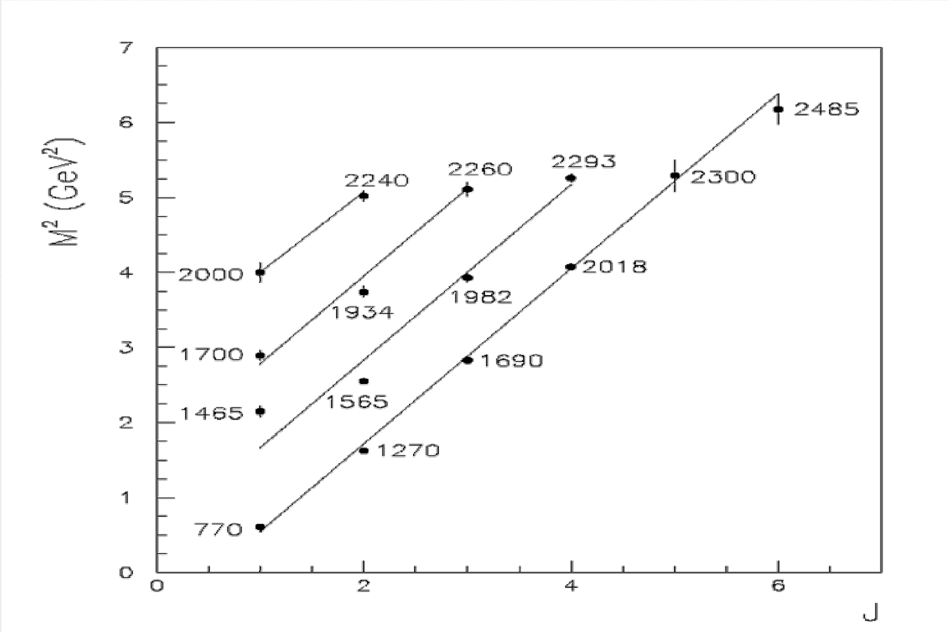
$$e^{2A^{(\mathrm{hw})}(z)} = \theta\left(z_m - z\right), \qquad z_m \simeq \Lambda_{\mathrm{QCD}}^{-1},$$

predict: **$M^2 \sim N^2, L^2, J^2$**

but experiment finds:

$$M^2 = M_0^2 + W\left(N + L\right)$$

$W = 1.14 \pm 0.013\ \mathrm{GeV}^2$	mesons
$W = 1.081 \pm 0.035\ \mathrm{GeV}^2$	baryons



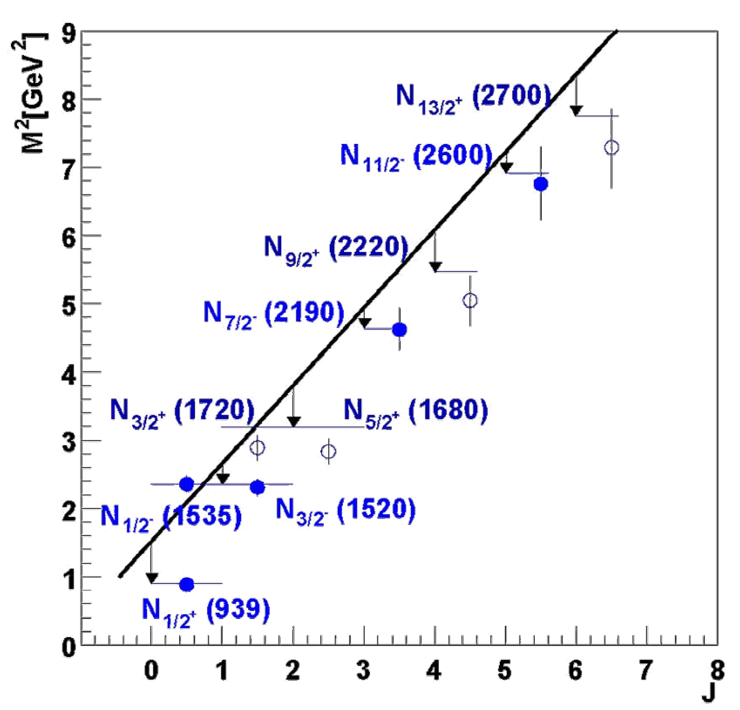
~ **universal slopes!**

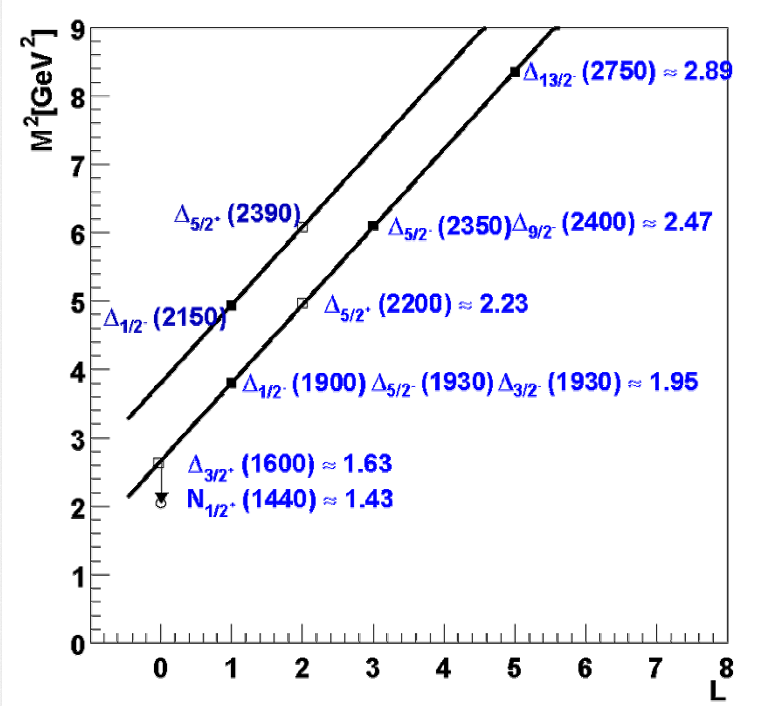
Karch et al.: dilaton 'soft wall':

⇒ **linear trajs. but only for mesons!**

$$A^{(\mathrm{sw})}(z) \equiv 0, \qquad \Phi^{(\mathrm{sw})}(z) = \lambda^2 z^2.$$

Linear trajectories in the baryon spectra





How to obtain observed meson & baryon trajs. with universal slope?
Addtl. bulk fields necessary, or 5d metric background sufficient?

(Affirmative) answer constructively: assume only background metric, check whether A can be constructed such that desired spectra arise!

Construction of metric for univ. lin. trajectory spectra

Lin. Trajectories require oscillator-type Sturm-Liouville potentials

Univ. slopes can be obtained from AdS$_5$ 1/z^2 potentials via: $\bar{\tau}_i \to \bar{\tau}_i + \lambda^2 z^2.$

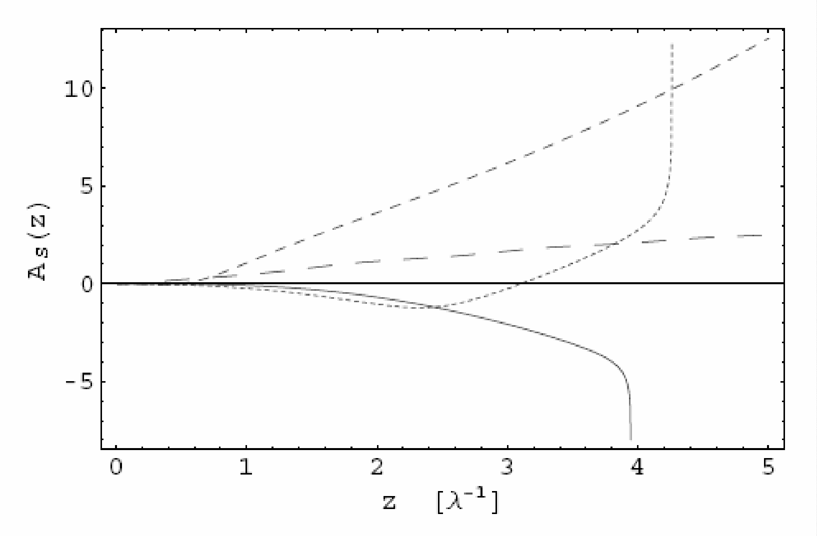
$$V_M^{(\mathrm{LT})}(z) = \left[\left(\lambda^2 z^2 + L\right)^2 - \frac{1}{4}\right]\frac{1}{z^2}$$

$$V_{B,\pm}^{(\mathrm{LT})}(z) = \left\{(L+1)(L+1\mp 1) + \left[2(L+1)\pm 1\right]\lambda^2 z^2 + \lambda^4 z^4\right\}\frac{1}{z^2}.$$

Derive dual geometry by equating to gen.-A potentials ⇒ ODEs for A(z),
∃ unique warp factor solutions A(z):

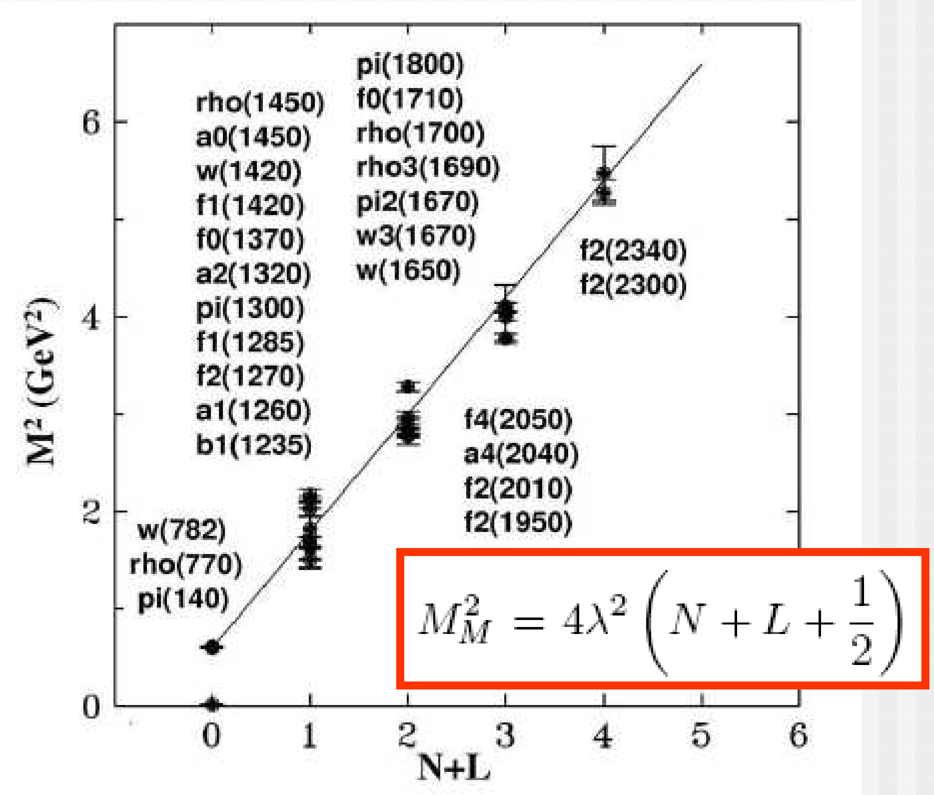
$$A_S(z) = \frac{1}{2}\ln\left[1 + \sum_{n=1}^{\infty} A_n(L)\left(\frac{\lambda^2 z^2}{2}\right)^n\right]$$

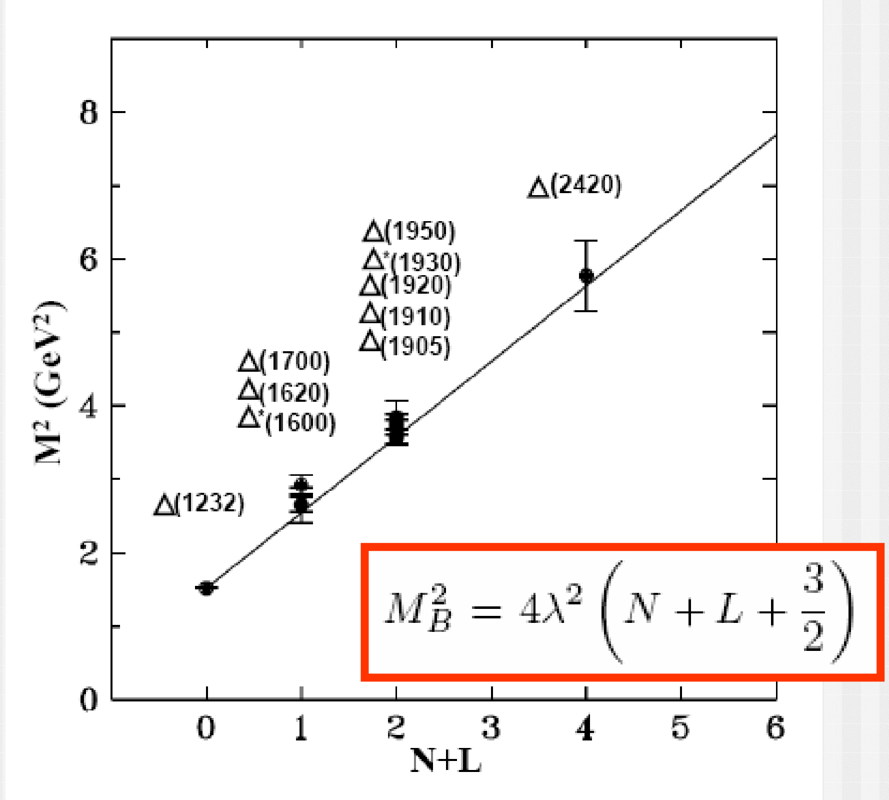
$$A_B(z) = \ln\left(1 + \frac{\lambda^2 z^2}{L+1}\right)$$



Resulting spectra and properties

Dual eigenmodes and mass spectra (obtained analytically):





- **Lin. trajectories of universal slope also in baryon sector!**
- **Encoded via AdS$_5$ IR deform. (orbital flucts. backreacted)**
- **Dual confinement signatures (metric singularities)**
- **New relations between slopes and ground state masses!**

Diquark correlations in light baryons holographically

Nucleon excitations improved by incl. diquark-content dependent anomalous dims of interpolators

$$\eta_t(x) = 2\left[\eta_{pd}(x) + t\,\eta_{sd}(x)\right]$$

$$\eta_{pd} = \varepsilon_{abc}(u_a^T C d_b)\gamma^5 u_c$$

$$\eta_{sd} = \varepsilon_{abc}(u_a^T C\gamma^5 d_b) u_c$$

⇒ **string-mode mass corrections:**

$$\Delta m_5^{(\kappa_{gd})} = \gamma_t(\kappa_{gd}) = \frac{\Delta M^2_{\kappa_{gd}}}{4\lambda^2 R}$$

dep. on "good-diquark fraction" κ !

$$M^2_{N,L} = M^{(ms)2}_{N,L+\Delta m_5 R} = 4\lambda^2\left(N + L + \frac{3}{2}\right) + \Delta M^2_{\kappa_{gd}}$$

L, N	κ_{gd}	Resonance					Pred.
0, 0	$\frac{1}{2}$	$N(940)$				input:	**0.94**
0, 0	0	$\Delta(1232)$					1.27
0, 1	$\frac{1}{2}$	$N(1440)$					1.40
1, 0	$\frac{1}{4}$	$N(1535)$	$N(1520)$				1.53
1, 0	0	$N(1650)$	$N(1700)$	$N(1675)$			1.64
1, 0	0	$\Delta(1620)$	$\Delta(1700)$		$L, N = 0, 1$:	$\Delta(1600)$	1.64
2, 0	$\frac{1}{2}$	$N(1720)$	$N(1680)$		$L, N = 0, 2$:	$N(1710)$	1.72
2, 0	0	$N(1900)$	$N(1990)$	$N(2000)$	$\Delta(1910)$	$\Delta(1920)$	1.92
2, 0	0	$\Delta(1905)$	$\Delta(1950)$	$\Delta(1900)^*$	$\Delta(1940)^*$	$\Delta(1930)^*$	1.92
0, 3	$\frac{1}{2}$	$N(2100)$					2.03
3, 0	$\frac{1}{4}$	$N(2070)$	$N(2190)$	$L, N = 1, 2$:	$N(2080)$	$N(2090)$	2.12
3, 0	0	$N(2200)$	$N(2250)$	$\Delta(2223)$	$\Delta(2200)$	$L, N = 1, 2$:	2.20
						$\Delta(2150)$	
4, 0	$\frac{1}{2}$	$N(2220)$					2.27
4, 0	0	$\Delta(2390)$	$\Delta(2300)$	$\Delta(2420)$	$L, N = 3, 1$:	$\Delta(2400)$	2.43
						$\Delta(2350)$	
5, 0	$\frac{1}{4}$	$N(2600)$					2.57
6, 0	$\frac{1}{2}$	$N(2700)$					2.71
6, 0	0	$\Delta(2950)$			$L, N = 5, 1$:	$\Delta(2750)$	2.84

$$\Delta M^2_{\kappa_{gd}} = -2\left(M^2_\Delta - M^2_N\right)\kappa_{gd}$$

Manifestly simple spectrum, reprod. all 48 observed N res. better than quark models !

Dynamical AdS/QCD

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{|g|}\left(-R + \frac{1}{2}g^{MN}\partial_M\Phi\partial_N\Phi - V(\Phi)\right),$$

$\phi = \phi(z)$ and g_{AAdS5} → Einstein-dilaton eqs.:

$$6A'^2 - \tfrac{1}{2}\Phi'^2 + e^{-2A}V(\Phi) = 0,$$
$$3A'' - 3A'^2 - \tfrac{1}{2}\Phi'^2 - e^{-2A}V(\Phi) = 0,$$
$$\Phi'' - 3A'\Phi' - e^{-2A}\frac{dV}{d\Phi} = 0$$

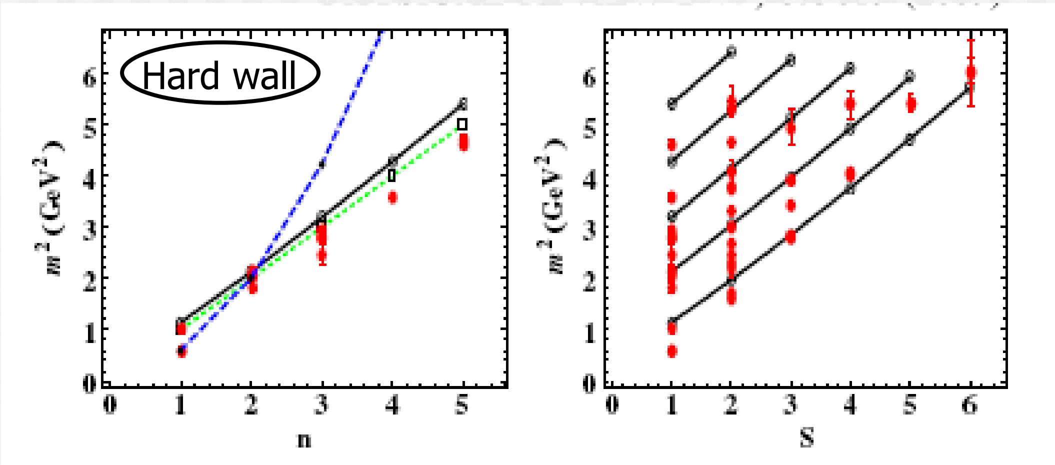
Find UV-conformal solution for ϕ, $V(\phi)$ with area-law gen. warp factor

$$A(z) = \ln z + C(z)$$

$$C(z) = \frac{1+\sqrt{3}}{2S+\sqrt{3}-1}\frac{(z\Lambda_{QCD})^2}{1+e^{(1-z\Lambda_{QCD})}}$$

→ Mass spectra

$$m^2_{n,S} \simeq \tfrac{1}{10}(11n + 9S - 9),$$



Pert. Log. Corrections can be included in C…

Glueball correlators in "hard- and soft-wall" duals

Hard-wall correlator (IR brane at $z=z_m$):

$$\hat{\Pi}\left(Q^2\right) = \frac{R^3}{8\kappa^2}Q^4\left[2\frac{K_1\left(Qz_m\right)}{I_1\left(Qz_m\right)} - \ln\left(\frac{Q^2}{\mu^2}\right)\right]$$

$$\frac{R^3}{\kappa^2} = \frac{2\left(N_c^2-1\right)}{\pi^2}.$$

Dilaton soft-wall correlator ($\phi = \lambda^2 z^2$):

$$\hat{\Pi}\left(Q^2\right) = -\frac{2R^3}{\kappa^2}\lambda^4\left[1+\frac{Q^2}{4\lambda^2}\left(1+\frac{Q^2}{4\lambda^2}\right)\psi\left(\frac{Q^2}{4\lambda^2}\right)\right]$$

Spectral representation:

$$\hat{\Pi}\left(Q^2\right) = \int_{m_1^2}^{\infty} ds \frac{\rho\left(s\right)}{s+Q^2}$$

with spect. density ρ:

⇒ **both correlators have $\rho \geq 0$, mass gap, zero-width Gb poles ↔ large N_c:**

Hard wall:

$$\rho\left(s\right) = \frac{R^3}{2\kappa^2 z_m^2}s^2\sum_{n=1}^{\infty}\frac{\delta\left(s-m_n^2\right)}{J_0^2\left(j_{1,n}\right)}$$

⇒

$$m_n^{(\mathrm{N})} = \frac{j_{1,n}}{z_m},$$

lin. "pomeron" trajectory!

Soft wall:

$$\rho\left(s\right) = \frac{\lambda^2 R^3}{2\kappa^2}s\left(s-m_0^2/2\right)\sum_{n=0}^{\infty}\delta\left(s-m_n^2\right).$$

⇒

$$m_n^2 = 4\left(n+2\right)\lambda^2$$

Hard-wall content: small-instanton contributions

At large Q ≫ z_m^{-1} npert. Q dependence becomes exponential:

$$\hat{\Pi}^{(np)}\left(Q^2\right) \equiv \frac{R^3}{4\kappa^2}\frac{K_1\left(Qz_m\right)}{I_1\left(Qz_m\right)}Q^4$$
$$\overset{Qz_m \gg 1}{\longrightarrow} \frac{4}{\pi}\left[1+\frac{3}{4}\frac{1}{Qz_m}+O\left(\frac{1}{\left(Qz_m\right)^2}\right)\right]Q^4 e^{-2Qz_m}$$

→ **No power corrections** (expected to be weak from QCD-OPE!)!
→ **LET** trivially satisfied (no anomaly, no gluon condensate...)

Compare with small-scale QCD instanton contribution to OPE:

$$\hat{\Pi}^{(I+\bar{I})}\left(Q^2\right) \overset{Q\bar{\rho}\gg 1}{\longrightarrow} 2^4 5^2 \pi \zeta \bar{n}\left(Q\bar{\rho}\right)^3 e^{-2Q\bar{\rho}}$$

⇒ $$\bar{\rho} \simeq z_m, \qquad \bar{n} \simeq \frac{3}{2^4 5^2 \pi^2 \zeta}\frac{1}{z_m^4},$$ $$\rho^{(\mathrm{hw})} \le z_m \sim \mu^{-1}$$

Crucial **small-instanton contribs**. (semi-quant.) reproduced!

Soft-wall dynamcis: power corrections, UV gluon mass

Asymptotic expansion for $Q^2 \gg \lambda^2 \cong \Lambda^2$:

$$\hat{\Pi}\left(Q^2\right) = -\frac{2R^3}{\kappa^2}\lambda^4\left[1+\frac{Q^2}{4\lambda^2}\left(1+\frac{Q^2}{4\lambda^2}\right)\left(\ln\frac{Q^2}{4\lambda^2}-\frac{2\lambda^2}{Q^2}-\sum_{n=1}^{\infty}\frac{B_{2n}}{2n}\left(\frac{4\lambda^2}{Q^2}\right)^{2n}\right)\right]$$

$$= -\frac{2}{\pi^2}Q^4\left[\ln\frac{Q^2}{\mu^2}+\frac{4\lambda^2}{Q^2}\ln\frac{Q^2}{\mu^2}+\frac{2^2 5}{3}\frac{\lambda^4}{Q^4}-\frac{2^4}{3}\frac{\lambda^6}{Q^6}+\frac{2^5}{15}\frac{\lambda^8}{Q^8}+\dots\right].$$

Contains all OPE power corrections:

$$\left\langle G^2\right\rangle \simeq -\frac{10}{3\pi^2}\lambda^4,$$

$$\left\langle gG^3\right\rangle \simeq \frac{4}{3\pi^2}\lambda^6,$$

$$\left\langle G^4\right\rangle \simeq -\frac{8}{15\pi^3\alpha_s}\lambda^8.$$

Even addtl. term expected in QCD from UV-ren., gluon mass, 2d-nonlocal cond. :

$$\hat{\Pi}^{(\mathrm{CNZ})}\left(Q^2\right) = -\frac{2}{\pi^2}Q^4\ln\frac{Q^2}{\mu^2}\left(1+6\frac{\bar{\lambda}^2}{Q^2}+\dots\right)$$

$$\bar{\lambda}^2 \simeq \frac{2}{3}\lambda^2$$

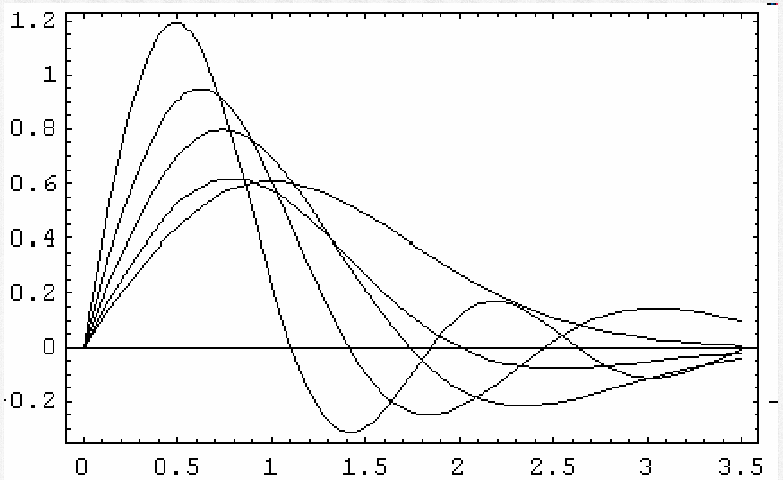
Leading coeff. has sign opposite to QCD, LET → artefact of strong-coupl. UV!

Decay constants

Solve bulk field eq. for norm. eigenmodes:

$$\psi_n(z) = N_n \hat{\varphi}(m_n, z)$$

(for plot mult. by sqrt of int. measure)



For dilaton soft wall:

$$f_n^{(\mathrm{sw})} = 4 I_n^{-1/2} \frac{\lambda^3 R^{3/2}}{m_n^2 \kappa} = \frac{1}{\sqrt{2}} \sqrt{\frac{n+1}{n+2}} \frac{\lambda R^{3/2}}{\kappa} \quad \Rightarrow \quad f_S^{(\mathrm{sw})} \simeq 0.3 \text{ GeV}.$$

Hard wall (expected to represent more relevant Gb physics):

$$f_n = \lim_{\varepsilon \to 0} \frac{R^3}{\kappa m_n^2} \frac{\psi_n'(\varepsilon)}{\varepsilon^3} = \frac{N_n}{2} \frac{R^3}{\kappa} m_n^2 \quad \Rightarrow \quad f_S^{(\mathrm{hw})} \simeq 0.8 - 0.9 \text{ GeV},$$

Consistent with ILM, IOPE sum rules & lattice results

Summary and conclusions

Hadron spectra:

- **Linear Regge-type trajectories of universal slope for mesons AND baryons**
- **Minimal 'metric soft wall': encoded solely via IR deformed AdS_5 metric**
- **New relations between trajectory slopes and ground-state masses**

- **Diquark effects can enter via anom. dims. of baryon interpolators**
- **Decisively improve predictions for light baryon spectra!**

- **Backreacted 5d Einstein-dilaton solution: generates area law**
- **Vector meson excitations with linear trajectories dynamically!**

Hadron correlators:

Detailed, quantitative testing ground → generates new insights:

→ **In comparison with QCD-OPE: map impact of strongly-coupled UV**
→ **Hard/soft wall duals contain complementary gauge/QCD physics**
→ **(beyond spectra) Predictions of glueball decay constants, coupls.**
→ **Suggests systematic bottom-up improvement strategies**