Physical charges in QED and QCD

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- With Martin Lavelle, David McMullan,
  0907.4071 [hep-th], JHEP 0703:044 (2007)

  and Tom Heinzl, Kurt Langfeld, Wolfgang Lutz
  Phys.Rev.D77:054501 (2008), Phys.Rev.D78:034504 (2008).

Outline

1. Physical charges are dressed charges.
   - The dressing approach.
   - Exact dressing for minimal energy states.

2. States in finite volumes.
   - Compact descriptions of charge neutral states.
   - Inter–'quark' potential.

3. Gribov copies and confinement.
   - Effect of Gribov copies on dressings.
   - Dressings $\Longrightarrow$ no coloured particles.

Fermions, photons and charges.

- Lagrangian fermions ψ are not gauge invariant.
- Even asymptotically.
- Dressing: describe physical charges as composites,

$$\Psi = h^{-1}[A]\psi \ , \qquad \text{e.g.} \quad \Psi = \exp\left[ie\frac{\nabla.\boldsymbol{A}}{\nabla^2}\right]\psi \ .$$

- A minimal requirement of h^{-1} is gauge invariance:

$$\psi \to U^{-1}\psi \implies h^{-1}[A] \to h^{-1}[A]U \ .$$

- Related approaches: Cornwall '79, Cahill & Stump '79, Shabanov & Prokhorov '92, Lunev '94, Neeman & Sijaki '95...

A simple model

- Bottom up approach: start with a simple gauge theory.
- U(1) gauge fields with heavy matter: Georgi: PLB 1990

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\, Q^\dagger v^\mu (\partial_\mu + ieA_\mu) Q \; .$$

- Matter particles have fixed velocity v^μ.
- Toy model for QED.

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- Matter particles have fixed velocity v^μ.
- Toy model for QED.
- Exact. Nontrivial because of gauge invariance.
- Now extract dressing for minimal energy states.

Hamiltonian picture

- Work in the Hamiltonian picture: Talks by J. Greensite, V. Nair, H. Reinhardt,...

$$i\partial_t|\,\Psi\,\rangle = H|\,\Psi\,\rangle\ , \qquad \text{Schrödinger equation,}$$

$$\nabla.\boldsymbol{E}|\,\Psi\,\rangle = -e\gamma Q^\dagger Q|\,\Psi\,\rangle\ , \qquad \text{Gauss's law.}$$

- Construct the ground state for a moving charge.

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- Construct the ground state for a moving charge.

1. Start with the vacuum. $\Psi_0[\boldsymbol{A}_T]$
2. Add a particle: $\boldsymbol{x}_t \equiv \boldsymbol{x}_0 + \boldsymbol{v}t$. $Q^\dagger(\boldsymbol{x}_t)\Psi_0[\boldsymbol{A}_T]$
3. Apply Gauss $\Longrightarrow$ gauge invariance. $\vdots$
4. Solve Schrödinger equation and minimise. $\vdots$

The ground state of a moving charge

$$\Psi = \exp\left[\underbrace{ie\,\chi_{\boldsymbol{v}}^{T}\cdot \boldsymbol{A}(\boldsymbol{x}_t)}_{\text{Schrödinger}} + \underbrace{ie\frac{\nabla.\boldsymbol{A}}{\nabla^2}(\boldsymbol{x}_t)}_{\text{Gauss}}\right]\underbrace{Q^{\dagger}(\boldsymbol{x}_t)}_{\text{matter}}\underbrace{\Psi_0[\boldsymbol{A}_T]}_{\text{vacuum}}$$

- Dressed: 'photon' cloud around the matter.
- Neither matter or cloud individually gauge invariant or observable.
- Together, they describe a gauge invariant, physical charge.
- This charge is nonlocal: Ferrari, Picasso & Strocchi, CMP '74, INC '77

$$\frac{\nabla.\boldsymbol{A}}{\nabla^2}(\boldsymbol{x}_t) \equiv -\frac{1}{4\pi}\int \mathrm{d}^3\boldsymbol{x}\ \nabla_j\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{x}_t|}\right)\boldsymbol{A}_j(\boldsymbol{x})$$

- An immediate consequence of Gauss's law.

Nonlocality, locality, observables

- Recall: QED charge operator on a gauge invariant state:

$$\mathcal{Q}\,|\,\Psi\,\rangle = \int_{S_\infty} \mathrm{ds}.\boldsymbol{E}\,|\,\Psi\,\rangle\ .$$

- Charges in gauge theories must be nonlocal.

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- Recall: QED charge operator on a gauge invariant state:

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- Charges in gauge theories must be nonlocal.
- Our ground state has good, local observables:

$$\langle \boldsymbol{E}(\boldsymbol{z},t)\rangle_\Psi = -\frac{e\gamma}{4\pi}\frac{\boldsymbol{z}-\boldsymbol{x}_t}{\left[|\boldsymbol{z}-\boldsymbol{x}_t|^2+\gamma^2(\boldsymbol{v}.(\boldsymbol{z}-\boldsymbol{x}_t))^2\right]^{3/2}}\ ,$$

$$\langle \boldsymbol{B}(\boldsymbol{z},t)\rangle_\Psi = \boldsymbol{v}\times\langle \boldsymbol{E}(\boldsymbol{z},t)\rangle_\Psi\ .$$

- Correct fields of a charge moving along $\boldsymbol{x}_t = \boldsymbol{x}_0 + \boldsymbol{v}t$.
- Static case: Dirac's static electron. Dirac: CJP 1955

Charges at asymptotic and short times

- Fields: spread through all of space. (Gauss again.)
- Describes an asymptotic charge: has had time to spread out.

  Kulish & Faddeev: TMP '70, Horan, Lavelle, McMullan: JMP '00
- Good IR properties, useful in S–matrix calculations.

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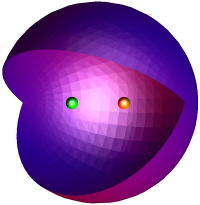
- Good IR properties, useful in S–matrix calculations.
- Now consider: $e^- \; e^+$ pair created in collision.
- At finite time after creation, fields occupy a compact region of space.
- Gauss's law should allow 'compact' pairs.



Asymptotic → compact dressings

- A single charge cannot be made compact.
- Can solve for neutral states, e.g. $e^+ \, e^-$ pair.
- Ground state of two static (asymptotic) charges at $\pm\boldsymbol{a}$:

$$\frac{1}{4\pi}\int \mathrm{d}^3\boldsymbol{x} \; \nabla_j\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{a}|}-\frac{1}{|\boldsymbol{x}+\boldsymbol{a}|}\right)\boldsymbol{A}_j(\boldsymbol{x}) \; .$$

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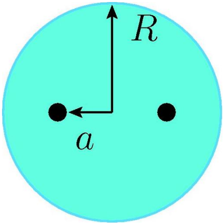
- A finite volume dressing can be written: A.I, M.L., D.M., 0907.4071

$$\frac{1}{4\pi}\int_V \mathrm{d}^3\boldsymbol{x}\ \nabla_j\left(\frac{1}{|\boldsymbol{x}-\boldsymbol{a}|}-\frac{1}{|\boldsymbol{x}+\boldsymbol{a}|}+f_V(\boldsymbol{x})\right)\boldsymbol{A}_j(\boldsymbol{x})\ .$$

- V: compact volume. Gauge invariance $\implies f_V$.

Asymptotic → compact dressings

- Dressing and fields are confined to the interior of V.
  - Example: take V to be a ball, radius R.
  - Charges sat at $\pm\boldsymbol{a}$, with $|\boldsymbol{a}| < R$.
  - Have solved for f_V:



$$f_V(r,\theta) = \frac{4}{R}\sum_{l=0}^{\infty}\left(\frac{l+1}{2l+1}\right)\left(\frac{ar}{R^2}\right)^{2l+1} P_{2l+1}(\cos\theta)\ .$$

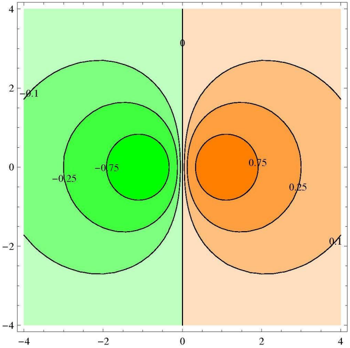
- Now compare scalar potential and electric field with asymptotic charges.

A.I, M.L., D.M., 0907.4071

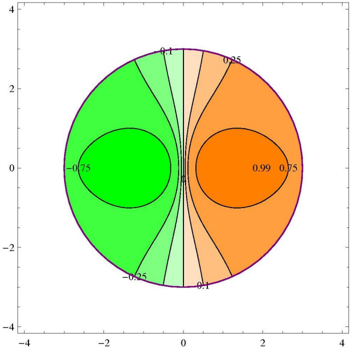
Asymptotic versus compact

- Compare scalar potential around charges:

↓ Asymptotic



↓ Compact

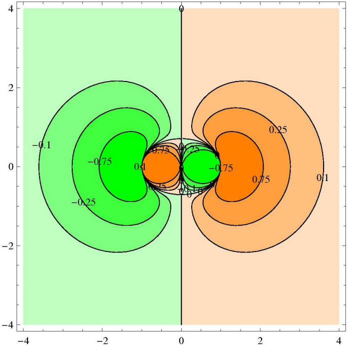


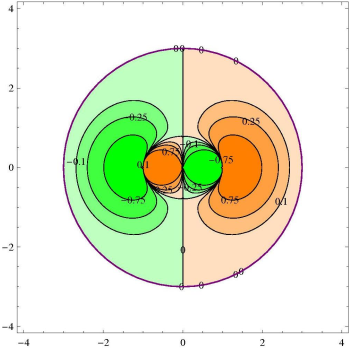
- Distortion of the scalar potential is clear to see.

Asymptotic versus compact

- Compare (radial) electric fields around charges:

↓ Asymptotic ↓ Compact





- Fields similar far from boundary.
- Compact $\boldsymbol{E}$–field vanishes on boundary (total charge $= 0$).

Inter–charge potential: stringy

- Recall: gauge invariant pairs created using Wilson lines:

$$| \Psi \rangle = \psi^\dagger(\boldsymbol{x}) \exp \left[ie \int_{\boldsymbol{y}}^{\boldsymbol{x}} \mathrm{d}\boldsymbol{z}.\boldsymbol{A} \right] \psi(\boldsymbol{y}) | 0 \rangle$$

- Inter–charge potential in this state:

$$V = e^2 \delta^2(0) |\boldsymbol{x} - \boldsymbol{y}| \ .$$

- Linear rising, divergent coefficient.
- An infinitely excited, unphysical state.

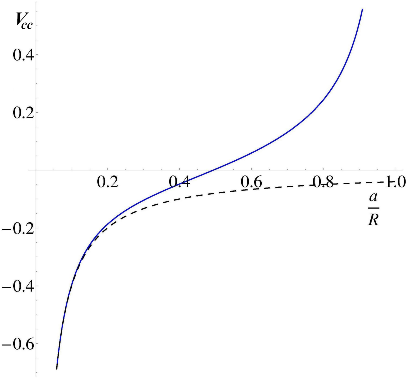
Prokhorov et al.: TMP '93, Haagensen & Johnson '97, Heinzl et al.: PRD '08

- Compare with the potential between compact charges:

Inter–charge potential: compact

- Compact: the inter–charge potential is:

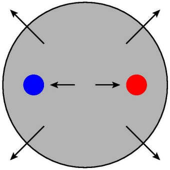
$$V_{cc} := -\frac{e^2}{4\pi}\frac{1}{|2\boldsymbol{a}|} + \frac{e^2}{2\pi}\left(\frac{|\boldsymbol{a}|^2 R}{R^4 - |\boldsymbol{a}|^4} + \frac{1}{R}\tanh^{-1}\frac{|\boldsymbol{a}|^2}{R^2}\right)$$



- $a \ll R$: Coulomb.
- $V_{cc} >$ Coulomb: excited.
- Finitely excited.
- $R \to \infty$ gives Coulomb.
- Ball swells in time, and state relaxes to Coulomb.

Pair creation and annihilation

- Creation: A.I, M.L., D.M., 0907.4071
  - Back to moving charges.
  - Reach of $\boldsymbol{E}$, $\boldsymbol{B}$ fields constrained by causality.
  - Shortly after creation, confined to a ball.



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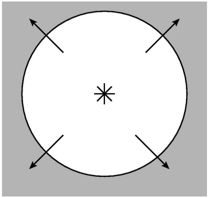

- Annihilation:
  - Matter annihilates.
  - Dressing continues evolving under photon H.
  - Cavity expands out from annihilation point, erasing fields.
  - Eventually, all observers agree: charges have vanished.

Colour, confinement, Gribov

- Turn now to QCD.
- Gribov copies and confinement.

- Recall: transformation law for a single quark dressing:

$$\underbrace{q}_{\text{quark?}} = \underbrace{h^{-1}[A]\psi}_{\text{dressed}} \implies h^{-1}[A^U] = h^{-1}[A]U \ .$$

- Look at quark dressings from gauge fixing conditions.
- Coloured quarks in perturbation theory ...
- ... and beyond.

Dressings from gauge fixing

- Choose a gauge fixing condition $\chi(A) = 0$.
- Given A, find the transformation $h[A]$ such that

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- Provided χ is a good gauge fixing:
  Uniqueness $\implies A^{h[A]} = A^{U\,h[A^u]}$,

$$h[A] = U h[A^U] \implies h^{-1}[A^U] = h^{-1}[A]U.$$

- Note: if $\chi(A) = 0$, $h[A] = \mathbb{1}$.

Charges and confinement

- Example: Coulomb gauge condition. M.L., D.M. Phys. Rept. '97

$$\nabla_j(h^{-1}A_j h + \frac{1}{g}h^{-1}\nabla_j h) = 0 \, .$$

- Perturbative solutions → static quark dressing. M.L.'s talk

$$\Psi = \exp\left[g\frac{\nabla.\boldsymbol{A}^a}{\nabla^2}T^a + \mathcal{O}(g^2)\right]\psi$$

- Gauge invariant, coloured quarks may be constructed in perturbation theory.

- This relies on having a good gauge fixing.

Gribov ambiguity

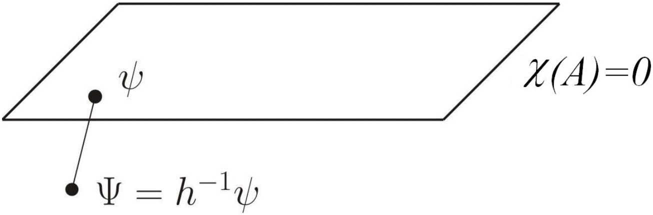
- Nonperturbatively, there is no perfect gauge fixing.
- There are Gribov copies. Gribov: NPB 1978, Singer: CMP 1978
- Copies spoil any construction of a coloured state.
- We will now see this explicitly.
- So what goes wrong?
- Recall: if $\chi(A) = 0$, $h[A] = \mathbb{1}$.

Non–perturbative colour

- $\Psi = h^{-1}\psi$

- Ψ is a dressed charge. It is gauge invariant.
- Dressing is unity in 'defining gauge'.

Non–perturbative colour



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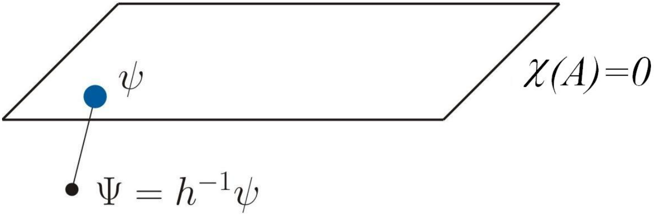
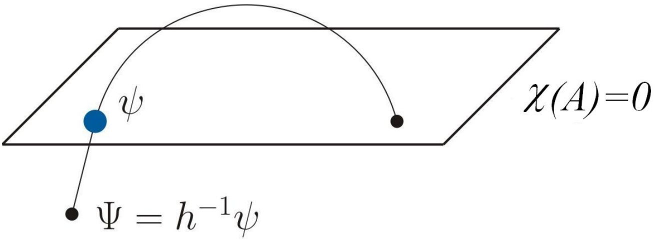


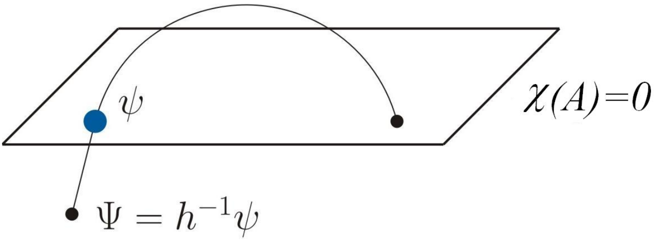
- Ψ is a dressed charge. It is gauge invariant.
- Dressing is unity in ‘defining gauge’.
- Undress the charge by going to this gauge.
- We can arrange for ψ to be blue here.

Non–perturbative colour



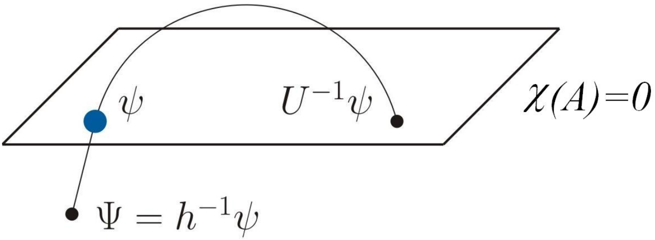
- This colour will be preserved along gauge orbits. . .
- . . . at least perturbatively.
- ∃ transformations which bring us back into χ gauge.
- These are the transformations between Gribov copies.

Non–perturbative colour



- We have constructed explicit examples. A.I., M.L, D.M. JHEP 07
- Both large and small gauge transformations.
- Coupling dependence of $\boldsymbol{A}$ is explicitly nonperturbative.
- What happens when we make such a transformation?

Non–perturbative colour



- Matter field is sensitive to these transformations: $\psi \to U^{-1}\psi$.
- The dressing is not: $h^{-1} = \mathbb{1} \to \mathbb{1}$.
- In general $[\mathcal{Q}, U^{-1}\psi] \neq [\mathcal{Q}, \psi]$.
- Ψ has a gauge dependence. It loses its colour.

From Gribov to confinement

- Coloured charges exist in perturbation theory.
- But they pick up a gauge dependence nonperturbatively.
- This dependence arises from Gribov copies.
- Thus, cannot be associated with physical states.

- Gribov ambiguity $\Longrightarrow$ no isolated coloured charges: Physical states must be white.
- Gribov ambiguity $\Longrightarrow$ confinement.

Conclusions

- Dressing emerges naturally in gauge theory.
- Physical states are dressed states.

- Ground states describe asymptotic charges.
- First compact description of a charge zero state.
- Creation and annihilation.

- Dressings make Gribov's role in confinement explicit.
- There can be no physical colour charges.