Infra-Red Problems and a Response

Martin Lavelle

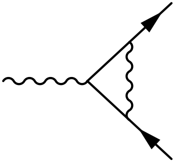
QCD-TNT, Trento 2009

Emili Bagan, Tom Heinzl, Anton Ilderton, Paul Jameson, Kurt Langfeld,
Wolfgang Lutz, David McMullan, Tom Steele, Shogo Tanimura

Outline

- Failure of the Bloch-Nordsieck & Lee-Nauenberg approaches to the IR problem.
- Constructing charges in gauge theory.
- The inter-quark potential: perturbation theory and the lattice.

Coulomb Scattering



Regularised by:
$D = 4 + 2\epsilon_{\text{IR}}$ and $m \neq 0$.

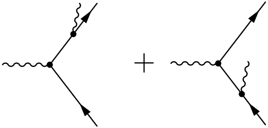
- F_2 IR finite and safe, but...
- soft and collinear divergences in F_1:

$$\frac{1}{\epsilon}\,, \qquad \frac{1}{\epsilon}\ln(m)\,, \qquad \ln^2(m)\,, \qquad \ln(m)\,.$$

- How can it be made IR finite?

The Bloch-Nordsieck Response

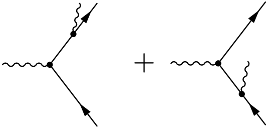
Include real, soft emission (up to a resolution Δ) but not absorbtion:



- eikonal approximation
- square and add to virtual cross-section

The Bloch-Nordsieck Response

Include real, soft emission (up to a resolution Δ) but not absorbtion:



- eikonal approximation
- square and add to virtual cross-section

Compare IR divergences in cross-sections:

- Virtual

$$-\frac{A}{\epsilon}\,, \qquad -\frac{B}{\epsilon}\ln(m)\,, \qquad C\ln^2(m)\,, \qquad F\ln(m)\,.$$

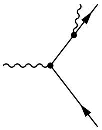
- Emission

$$+\frac{A}{\epsilon}\,, \qquad +\frac{B}{\epsilon}\ln(m)\,, \qquad -C\ln^2(m)\,, \qquad G\ln(m)\,.$$

$F \neq -G$, what kills these logs?

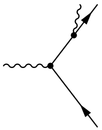
Bloch-Nordsieck Extended to Collinear?

Include (semi-hard) emission collinear with outgoing electron (up to an angular resolution δ):



Bloch-Nordsieck Extended to Collinear?

Include (semi-hard) emission collinear with outgoing electron (up to an angular resolution δ):



- Virtual + soft emission leave:

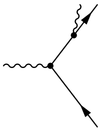
$$-\ln(m)\times\left[\frac{3}{4}-\ln\left(\frac{E}{\Delta}\right)\right]$$

- Semi-hard emission fails:

$$+\frac{1}{2}\ln(m)\times\left[\frac{3}{4}-\ln\left(\frac{E}{\Delta}\right)\right]$$

Bloch-Nordsieck Extended to Collinear?

Include (semi-hard) emission collinear with outgoing electron (up to an angular resolution δ):



- Virtual + soft emission leave:

$$-\ln(m) \times \left[\frac{3}{4} - \ln\left(\frac{E}{\Delta}\right)\right]$$

- Semi-hard emission fails:

$$+\frac{1}{2}\ln(m) \times \left[\frac{3}{4} - \ln\left(\frac{E}{\Delta}\right)\right]$$

So would it work if add semi-hard absorbtion? Lee-Nauenberg 64

Be Careful: include soft resolution Δ

- Semi-hard emission really generates

$$\frac{1}{2}\ln(m)\times\left[\frac{3}{4}-\ln\left(\frac{E}{\Delta}\right)-\frac{\Delta}{E}+\frac{1}{4}\frac{\Delta^2}{E^2}\right]$$

Be Careful: include soft resolution Δ

- Semi-hard emission really generates

$$\frac{1}{2}\ln(m)\times\left[\frac{3}{4}-\ln\left(\frac{E}{\Delta}\right)-\frac{\Delta}{E}+\frac{1}{4}\frac{\Delta^2}{E^2}\right]$$

- However, in eikonal dropped $\not{k}$ in $\not{p}+\not{k}+m$ in numerator.
- Reinstate sub-eikonal: soft finite but kills these collinear logs off.
- They are artefact of energy integral divide:

$$\underbrace{\int_{\Delta}^{E}}_{semihard}+\underbrace{\int_{0}^{\Delta}}_{soft}$$

What Should be Added to the Virtual Cross-Section?

- If try to separate BN (soft) and LN (collinear) include:
  - soft emission;
  - semi-hard emission;
  - semi-hard absorbtion;
  - soft absorbtion – but only sub-eikonal terms in integrals which do not generate a soft divergence! Inconsistent!

What Should be Added to the Virtual Cross-Section?

- If try to separate BN (soft) and LN (collinear) include:
  - soft emission;
  - semi-hard emission;
  - semi-hard absorbtion;
  - soft absorbtion – but only sub-eikonal terms in integrals which do not generate a soft divergence! Inconsistent!
- Or, more in spirit of LN, include
  - all degenerate indistinguishable processes!
  - Including initial and final soft and collinear.
  - Soft absorbtion generates soft infra-red divergences (eikonal): what cancels them?

What Should be Added to the Virtual Cross-Section?

- If try to separate BN (soft) and LN (collinear) include:
  - soft emission;
  - semi-hard emission;
  - semi-hard absorbtion;
  - soft absorbtion – but only sub-eikonal terms in integrals which do not generate a soft divergence! Inconsistent!
- Or, more in spirit of LN, include
  - all degenerate indistinguishable processes!
  - Including initial and final soft and collinear.
  - Soft absorbtion generates soft infra-red divergences (eikonal): what cancels them?

Look at Lee-Nauenberg paper again…

ML, McMullan, JHEP '06

Soft Divergences in the LN Spirit ($m \neq 0$)

LN: all degeneracies … virtual, emission and absorbtion.

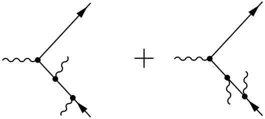
No cancellation: $-\frac{1}{\epsilon}+\frac{1}{\epsilon}+\frac{1}{\epsilon}$.

Soft Divergences in the LN Spirit ($m \neq 0$)

LN: all degeneracies … virtual, emission and absorbtion.

No cancellation: $-\frac{1}{\epsilon} + \frac{1}{\epsilon} + \frac{1}{\epsilon}$.

Add all diagrams with emission and absorbtion

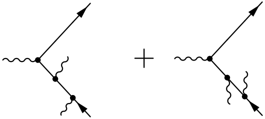


Soft Divergences in the LN Spirit ($m \neq 0$)

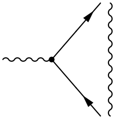
LN: all degeneracies … virtual, emission and absorbtion.

No cancellation: $-\frac{1}{\epsilon}+\frac{1}{\epsilon}+\frac{1}{\epsilon}$.

Add all diagrams with emission and absorbtion



To get cross-section at order e^4 need interference with a disconnected photon!

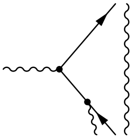


Connected interference terms.

Lee-Nauenberg; Muta-Nelson; de Calan-Valent; Bergere-Szymanowski; Ito; Akhoury-Sotiropoulos-Zakharov; …

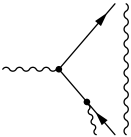
Still Need a Bit More. . .

Absorbtion plus a disconnected photon: diagrams like

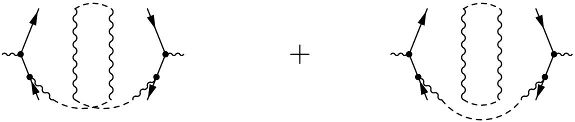


Still Need a Bit More...

Absorbtion plus a disconnected photon: diagrams like



These yield



Only use connected contribution.

$$-1+1+1-2+1=0$$

Soft divergences then sum to zero:

virtual	emit	absorb	emit & abs.	abs. plus disconn.
$-\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	$-2\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$

$$-1+1+1-2+1=0$$

Soft divergences then sum to zero:

virtual	emit	absorb	emit & abs.	abs. plus disconn.
$-\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	$-2\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$

- Arbitrary choices? Why not emission plus a disconnected photon line?
- Why stop here? Can have more than one disconnected photon line?

Beyond Truncation

Idea: I. Ito 83, Akhoury-Sotiropoulos-Zakharov 97

Write cross sections as product:

disconnected loops × sum of connected (interference) probabilities

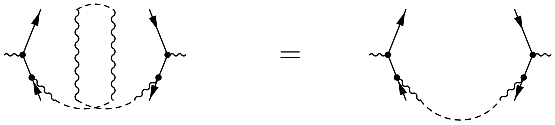
Ito, ASZ argue sum of connected probabilities could be IR finite

$$\sum_{mn}(e + m \text{ soft photons} \rightarrow e + n \text{ soft photons})$$

At order e^4 need: virtual loop; 1 emission; 1 absorbtion; 1 emission with 1 absorbtion.

Non-Convergent Series

- Connected interference from diagram with disconnected line yields same integral as diagram without:

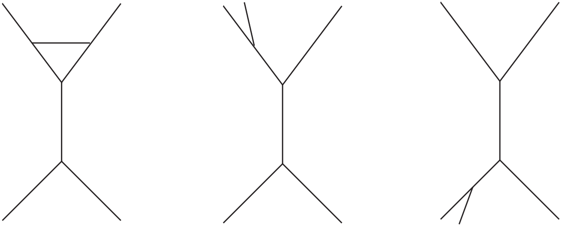


- Combinatorical factors *all* reduce to 1 for connected parts.
- Series do not converge! E.g., for soft absorbtion with n disconnected photons get:

n disconnected photons:	0	1	2	3	…
IR divergence:	$+\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	$+\frac{1}{\epsilon}$	…

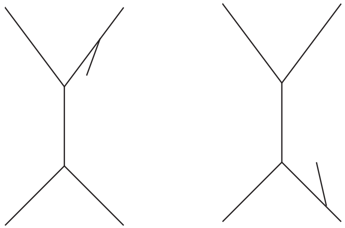
Taking LN Seriously

Consider massless ϕ^3 in $D = 6$: asymptotically free, collinear divergences but no soft divergences. ML, D. McMullan, T.G. Steele, in prep.



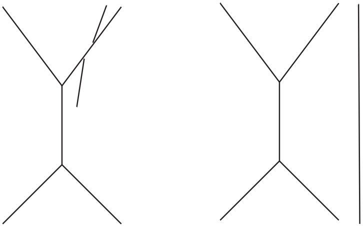
- in $\overline{\text{MS}}$ scheme appears to cancel, but…
  …there are more diagrams

Additional Collinear Divergent Diagrams



- Experiment has energy resolution, Δ
- Soft collinear absorbtion on outgoing lines
- Soft collinear emission from incoming lines
- Generate $\Delta \ln(m)$ divergences
- Not normally considered; cannot cancel with virtual loops.

Disconnected?



- Divergences of form

$$|T_0|^2 \left[\alpha \left(\frac{\Delta}{\boldsymbol{k}} \right)^2 \ln(m^2) \right] .$$

- A further sum of diagrams.
- All too simplistic? E.g., need different initial and final resolutions.
- Chance of cancellation seems very remote.

What Do We Learn?

- Saying use Bloch-Nordsieck (only final) for soft and LN (initial and final) for collinear does not work.

What Do We Learn?

- Saying use Bloch-Nordsieck (only final) for soft and LN (initial and final) for collinear does not work.
- Infinitely many diagrams at fixed order of perturbation theory: series does not converge.
- Different sorts of divergences with different series of divergences.

What Do We Learn?

- Saying use Bloch-Nordsieck (only final) for soft and LN (initial and final) for collinear does not work.
- Infinitely many diagrams at fixed order of perturbation theory: series does not converge.
- Different sorts of divergences with different series of divergences.

? Can one extract the electron charge radius (from F_1)?

? Should depend on resolution of experiment but be IR finite

What Do We Learn?

- Saying use Bloch-Nordsieck (only final) for soft and LN (initial and final) for collinear does not work.
- Infinitely many diagrams at fixed order of perturbation theory: series does not converge.
- Different sorts of divergences with different series of divergences.

? Can one extract the electron charge radius (from F_1)?

? Should depend on resolution of experiment but be IR finite

- Details of initial states? (Needed fine tuning for limited cancellations above.)

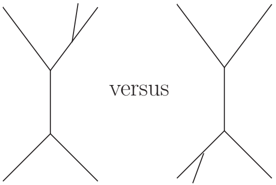
What Do We Learn?

- Saying use Bloch-Nordsieck (only final) for soft and LN (initial and final) for collinear does not work.
- Infinitely many diagrams at fixed order of perturbation theory: series does not converge.
- Different sorts of divergences with different series of divergences.

? Can one extract the electron charge radius (from F_1)?

? Should depend on resolution of experiment but be IR finite

- Details of initial states? (Needed fine tuning for limited cancellations above.)



Origin of the IR Problem

- At asymptotic times $\mathcal{H}_{\text{int}} \to 0$ slowly

Dollard 64, Kulish-Faddeev 70

- Ignoring this leads to IR divergences
- Moral: do not set $e \to 0$ in asymptotic states
- Fermion not gauge invariant at large time: $\psi \to e^{ie\Lambda}\psi$
- UV need renormalised fields
  IR need physical fields

Charges in Gauge Theories

- In QED and QCD matter fields ψ cannot be identified with observables – not gauge invariant

$$\psi \rightarrow U^{-1}\psi$$

Charges in Gauge Theories

- In QED and QCD matter fields ψ cannot be identified with observables – not gauge invariant

$$\psi \to U^{-1}\psi$$

- 'Dress' matter field with the gauge field, Dirac 1958

$$\Psi := h^{-1}[A]\,\psi.$$

- Gauge invariance implies

$$h^{-1}[A^U] = h^{-1}[A]U.$$

- What are the dressings? ML, D.McMullan, Phys. Rep. C 97

Charges in QED

$$\Psi = \exp\left[-\, ie\, \frac{\partial_i A_i}{\nabla^2} \right] \psi \quad \text{is the static electron.}$$

- Locally gauge invariant. Dirac 1958
- Commutator $[E_i^a(x), A_j^b(y)]_{\mathrm{et}} = i\delta^{ab}\delta(\underline{x} - \underline{y}) \;\Rightarrow$ Coulomb field:

$$E_j\, \Psi |\, 0\, \rangle = -\frac{e}{4\pi} \frac{r_j}{r^2}\, \Psi |\, 0\, \rangle$$

Charges in QED

$$\Psi = \exp\left[-\, ie \frac{\partial_i A_i}{\nabla^2} \right] \psi \quad \text{is the static electron.}$$

- Locally gauge invariant. Dirac 1958
- Commutator $[E_i^a(x), A_j^b(y)]_{\text{et}} = i\delta^{ab}\delta(\underline{x} - \underline{y}) \;\Rightarrow$ Coulomb field:

$$E_j \, \Psi | \, 0 \, \rangle = -\frac{e}{4\pi} \frac{r_j}{r^2} \, \Psi | \, 0 \, \rangle$$

- Improved infra-red: soft divergences cancel to all orders in on-shell Green's functions and S-matrix elements

Bagan, ML, McMullan, Ann. Phys. 2000

Choices?

- The static dressed state has Coulomb potential between charges.
- Stringy state: link by Polyakov line?

- Problem: in QED string state has electric potential energy:

$$V(x-y) \sim e^2 |\underline{x} - \underline{y}| \delta^2(0)$$

Confining potential with a divergent coefficient!

- Infinitely excited state.

Haagensen, Johnson, 97

Gauge Invariant Dressing in QCD

- The minimal static dressing in QED:

$$h^{-1} = \exp(-ie\chi) , \quad \text{with } \chi = \partial_i A_i / \nabla^2$$

- Transform QCD into Coulomb (arbitrary order in g).

In QCD we write

$$\exp(-ie\chi) \Rightarrow \exp(g\chi^a T^a) \equiv h^{-1}$$

with $g\chi^a T^a = (g\chi_1^a + g^2\chi_2^a + g^3\chi_3^a + \cdots)T^a$

The dressing gauge argument $\Rightarrow$

$$\chi_1^a = \frac{\partial_j A_j^a}{\nabla^2} ; \quad \chi_2^a = f^{abc}\frac{\partial_j}{\nabla^2}\left(\chi_1^b A_j^c + \frac{1}{2}(\partial_j \chi_1^b)\chi_1^c\right)$$

Perturbation Theory

- Leading order Coulombic:

$$V(r) = -\frac{g^2 C_F}{4\pi r}$$

- NLO:

$$V(r) = -\frac{g^2 C_F}{4\pi r}\left[1 + \frac{g^2}{4\pi}\frac{C_A}{2\pi}\left(4 - \frac{1}{3}\right)\log(\mu r)\right]$$

Compare with the one-loop beta function

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left[4 - \frac{1}{3}\right]$$

- The dominant antiscreening contribution comes from longitudinal glue (minimal dressing) and the screening part from gauge invariant glue (an additional dressing)

Overlaps

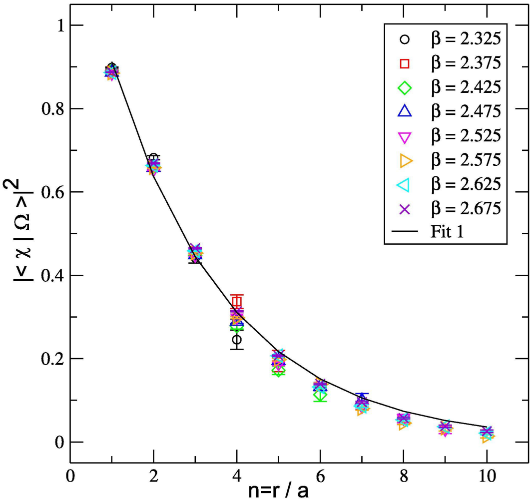
Trial quark anti-quark state, separation r. At large T

$$\langle\, \mathrm{trial}\, | e^{-HT} |\, \mathrm{trial}\, \rangle = |\langle \mathrm{trial} | \Omega \rangle|^2 \, e^{-V(r)T}$$

- Measure overlap $|\langle \mathrm{trial} | \Omega \rangle|^2$ with ground state $|\Omega\rangle$
- SU(2) Yang-Mills, 20^4 lattices, Wilson and improved actions.

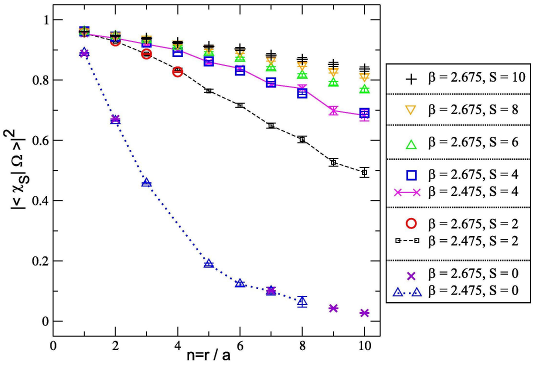
Heinzl, Ilderton, Langfeld, ML, Lutz, McMullan, PRD 08

Axial Trial State, $|\chi\rangle$



- Overlap drops exponentially as n increases
- Agrees with analytic calculation
- Continuum limit more sensitive to string UV artefacts

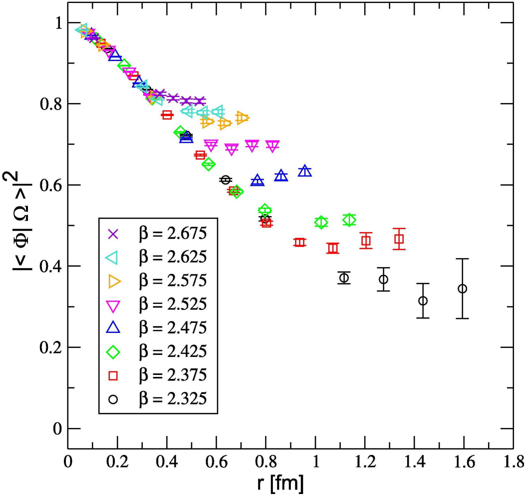
Smeared Results



- For fixed smearing overlap decreases for finer lattices
- To maintain overlap must increase smearing.

Coulomb State Overlap

- Better overlap for finer lattice spacing

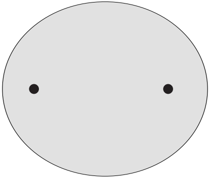


Summary

- Physical, IR finite and IR safe quantities. LN failure.
- Gauge invariance essential in QCD!
- On-shell IR structures support the construction. What about emission (IR safety)?
- Potential: perturbative and lattice investigations support relevance of these physical states.

Extensions

- Stringy state too thin, but Coulomb too ‘fat’. Can we do better?



- Could expect non-perturbative obstruction to ability to construct physical quark states
- Both addressed in Anton Ilderton’s talk on Friday

Ilderton, ML, McMullan, JHEP07, hep-th 09