How many degrees of freedom has the gluon ?

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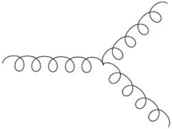
Trento, September 2009

Outline

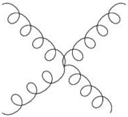
V. Mathieu, N. Kochelev, V. Vento
"The Physics of Glueballs"
Int. J. Mod. Phys. E**18**, 1 (2009)
arXiv:0810.4453 [hep-ph]

QCD = Gauge theory with the color group $\mathcal{SU}(3)$

$$\mathcal{L}_{QCD} = -\frac{1}{4}\mathrm{Tr}\, G_{\mu\nu}G^{\nu\mu} + \sum \bar{q}(\gamma^\mu D_\mu - m)q$$
$$G_{\mu\nu} = \partial_\mu A_\mu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$



Quark = Fundamental representation **3**
Gluon = Adjoint representation **8**
= Massless spin 1 → 2 polarization states

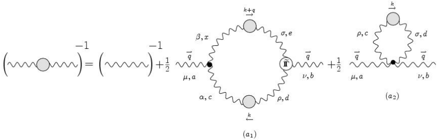


Gluon coupling → Bound states: Glueballs

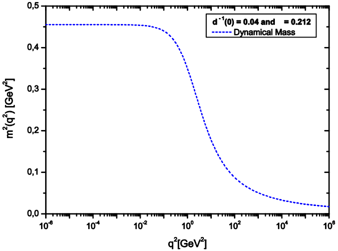
Gluon Mass

Cornwall [Phys. Rev. D**26**, 1453 (1982)]

Gluons massless in the Lagrangian
Couplings and nonperturbative effects
→ Dynamical mass



$$m^2(q^2) = m_0^2 \left[\frac{\ln\left(\frac{q^2+4m_0^2}{\Lambda^2}\right)}{\ln\left(\frac{4m_0^2}{\Lambda^2}\right)} \right]^{-12/11}$$



Gluon ∼ massive spin 1 → 3 polarization states

Non relativistic models for the low-lying glueballs

$$H_{gg} = 2m + \frac{\boldsymbol{p}^2}{m} + V(r)$$

Cornwall's model for two-gluon glueballs

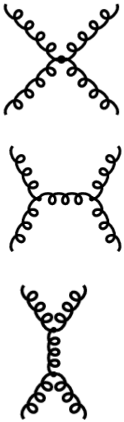
Cornwall and Soni [Phys. Lett. B **120**, 431 (1983)]

$$H^0 = \frac{p^2}{m} + 2m(1 - e^{-r/r_0}).$$

Degeneracy between states with different S
$\rightarrow$ OGE with corrections of $\mathcal{O}(1/m^2)$

$$V_{\text{oge}} = \lambda\Bigg[\left(\frac{1}{4} + \frac{1}{3}\boldsymbol{S}^2\right) U(r) - \frac{\pi}{m^2}\delta(\boldsymbol{r}) \left(\frac{5}{2}\boldsymbol{S}^2 - 4\right)$$
$$\frac{3}{2m^2}\frac{U'(r)}{r}\boldsymbol{L}\cdot\boldsymbol{S} - \frac{1}{6m^2}\left(\frac{U'(r)}{r} - U''(r)\right) T\Bigg]$$

with $\lambda = -3\alpha_S$ and $U(r) = \exp(-mr)/r$.



Every gluons on the same footing
Massive exchanged gluons

Cornwall and Soni [Phys. Lett. B **120**, 431 (1983)]

$C = +$ and $P = (-1)^L$

$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ with $S = 0, 1, 2.$

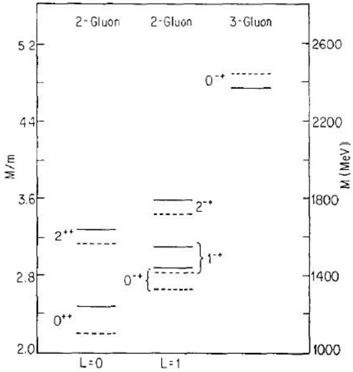
Spin-1 gluons $\rightarrow SU(2)$

$$\boldsymbol{S} : \quad \mathbf{1} \otimes \mathbf{1} = \mathbf{0}_S \oplus \mathbf{1}_A \oplus \mathbf{2}_S$$

$\rightarrow$Vector states 1^{-+}
Lowest three-gluon glueball 0^{-+}
Hierarchy with OGE:

$$0^{++} \quad 0^{-+} \quad 2^{++} \quad 1^{-+} \quad 2^{-+}$$

No lattice results at that time

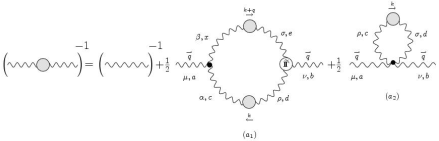


Other models for 2-gluon glueballs using (massive) spin-1 gluons:
Simonov (1990), Kaidalov and Simonov (2000)
Brau and Semay (2004),
Abreu and Bicudo (2007),...

Dynamical mass but WI satisfied

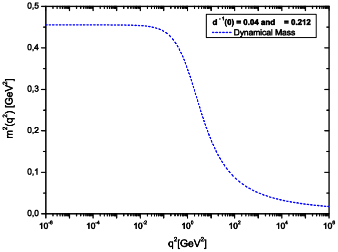
$$k^{\mu}\Pi^{ab}_{\mu}(k) = 0$$

Gauge invariance → transverse gluons
with no longitudinal component



Gluon massless in the UV region

Other possibility:
Gluon with only 2 polarization states
even thought dynamical mass



$\boldsymbol{J} \neq \boldsymbol{L} + \boldsymbol{S}$ only $\boldsymbol{J}$
Same hamiltonian but
other construction for the basis

BARNES [Z. Phys. C **10**, 275 (1981)]

$$H = \frac{p^2}{m} + \sigma r - 3\frac{\alpha_S}{r}$$

Transverse gluons with 2 helicities $\{-1, +1\} \to \boldsymbol{J} \neq \boldsymbol{L} + \boldsymbol{S}$
Other formalism by Jacob and Wick (1959)

General geometrical construction (with $\Lambda = \lambda_1 - \lambda_2$)

$$|J, M; \lambda_1, \lambda_2\rangle = \mathcal{N} \int dU \mathcal{D}^{J*}_{M,\Lambda}(\phi, \theta, -\phi) R(\phi, \theta, -\phi) \left|\Psi_{\lambda_1,\lambda_2}\right\rangle$$

State with total J in term of usual (L, S) states:

$$|J, M; \lambda_1, \lambda_2\rangle = \sum_{L,S} \left[\frac{2L+1}{2J+1}\right]^{1/2} \langle L0S\Lambda|\, J\Lambda\,\rangle \langle s_1\lambda_1 s_2 - \lambda_2|\, S\Lambda\,\rangle \left|{}^{2S+1}L_J\right\rangle$$

Not eigenstates of the parity and of the permutation operator.

$$\begin{aligned} \mathtt{P}\,|J, M; \lambda_1, \lambda_2\rangle &= \eta_1\eta_2(-1)^J\,|J, M; -\lambda_1, -\lambda_2\rangle\,, \\ P_{12}\,|J, M; \lambda_1, \lambda_2\rangle &= (-1)^{J-2s_i}\,|J, M; \lambda_2, \lambda_1\rangle\,, \end{aligned}$$

Construction of symmetric states (bosons) and with a good parity → selection rules.

$$
\begin{aligned}
\left|S_+;(2k)^+\right\rangle &\Rightarrow 0^{++},2^{++},4^{++},\dots\\
\left|S_-;(2k)^-\right\rangle &\Rightarrow 0^{-+},2^{-+},4^{-+},\dots\\
\left|D_+;(2k+2)^+\right\rangle &\Rightarrow 2^{++},4^{++},\dots\\
\left|D_-;(2k+3)^+\right\rangle &\Rightarrow 3^{++},5^{++},\dots
\end{aligned}
$$

Hierarchy [Barnes, 1981]:

$$0^{\pm+} \quad 2^{\pm+} \quad 3^{++} \quad 4^{\pm+} \dots$$

States expressed in term of the usual basis (useful to compute matrix elements!)

$$
\begin{aligned}
\left|S_+;0^+\right\rangle &= \sqrt{\frac{2}{3}}\left|{}^1S_0\right\rangle+\sqrt{\frac{1}{3}}\left|{}^5D_0\right\rangle,\\
\left|S_-;0^-\right\rangle &= \left|{}^3P_0\right\rangle,\\
\left|D_+;2^+\right\rangle &= \sqrt{\frac{2}{5}}\left|{}^5S_2\right\rangle+\sqrt{\frac{4}{7}}\left|{}^5D_2\right\rangle+\sqrt{\frac{1}{7}}\left|{}^5G_2\right\rangle,\\
\left|D_-;3^+\right\rangle &= \sqrt{\frac{5}{7}}\left|{}^5D_3\right\rangle+\sqrt{\frac{2}{7}}\left|{}^5G_3\right\rangle.
\end{aligned}
$$

Degeneracies survive at the OGE in hyperfine splittings

Szczepaniak, Swanson, Ji, and Cotanch [Phys. Rev. Lett.**76**, 2011 (1996)]

Transverse gluons with 2 helicities $\{-1,+1\}$
Gluon mass given by gap equation

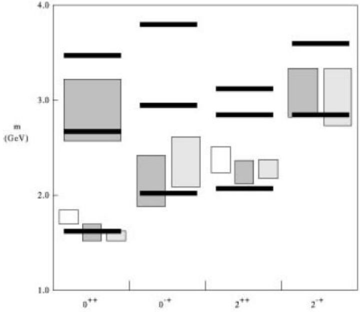
$$\omega^2(k) = k^2 + m^2 e^{-k/\kappa}$$

Relativistic Hamiltonian with

$$V(r) = \frac{9\sigma}{4} r (1 - e^{-\Lambda r}) - \frac{3\alpha_s}{r}$$

No vector state (Yang's theorem)
Agreement with (preliminary) lattice results
without OGE

Small gap, 250 MeV, between 0^{++} and 0^{-+}



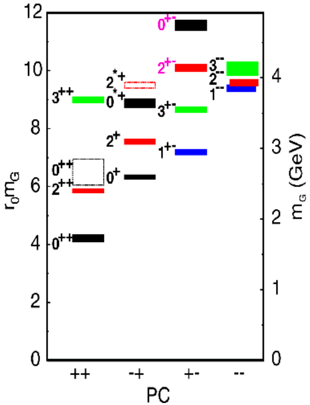
Lattice QCD

Morningstar and Peardon [Phys. Rev. D**60**, 034509 (1999)]

Glueball spectrum (pure gluonic operators) on a lattice
Identification of 15 glueballs below 4 GeV

$$M(0^{++}) = 1.730 \pm 0.130 \text{ GeV}$$
$$M(0^{-+}) = 2.590 \pm 0.170 \text{ GeV}$$
$$M(2^{++}) = 2.400 \pm 0.145 \text{ GeV}$$

Quenched approximation (gluodynamics)
→ mixing with quarks is neglected



Lattice studies with $n_f = 2$ exist. The lightest scalar would be sensitive to the inclusion of sea quarks but no definitive conclusion.

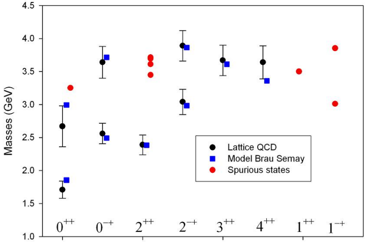
J^{PC}	(L,S)		
0^{++}	(0,0)	(2,2)	$(0,0)^*$
0^{-+}	(1,1)	$(1,1)^*$	
2^{++}	(0,2)	(2,0)	(2,2)
2^{-+}	(1,1)	$(1,1)^*$	
3^{++}	(2,2)		
4^{++}	(2,2)		
1^{++}	(2,2)		
1^{-+}	(1,1)	$(1,1)^*$	

Brau and Semay [Phys. Rev. D**70**, 014017 (2004)]

Massive gluons $\to \boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$
Spin-spin and spin-orbit interactions to split degeneracies
Hamiltonian with OGE

$$H = 2\sqrt{\boldsymbol{p}^2} + \frac{9}{4}\sigma r - \frac{3\alpha_s}{r} + V_{\mathrm{OGE}}$$

Good agreement with lattice QCD
Gluons = spin 1 $\to$ spurious $J = 1$ states and indetermination of states
For instance, 0^{++} can be $(L,S) = (0,0)$ or $(2,2)$



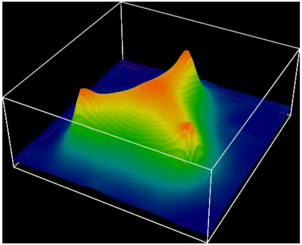
Three-gluon glueballs $C = -$

V.M., Semay, and Silvestre-Brac [Phys. Rev. D**77**, 094009 (2008)]

Extension of the model to three-gluon systems

$$H = \sum_{i=1}^{3} \sqrt{\boldsymbol{p}_i^2} + \frac{9}{4}\sigma \sum_{i=1}^{3} |\boldsymbol{r}_i - \boldsymbol{R}_{\text{cm}}| + V_{\text{OGE}}$$

Same parameters $(\sigma, \alpha_S, \gamma)$ as for two-gluon glueballs



$$V_{\text{OGE}} = -\frac{3}{2}\alpha_S \sum_{i<j=1}^{3} \left[\left(\frac{1}{4} + \frac{1}{3}\boldsymbol{S}_{ij}^2 \right) U(r_{ij}) - \frac{\pi}{\mu^2}\delta(\boldsymbol{r}_{ij}) \left(\beta + \frac{5}{6}\boldsymbol{S}_{ij}^2 \right) \right]$$

$$- \frac{9\,\alpha_S}{4\mu^2} \sum_{i<j=1}^{3} \boldsymbol{L}_{ij} \cdot \boldsymbol{S}_{ij} \frac{1}{r_{ij}} \frac{d}{dr_{ij}} U(r_{ij}) \quad \text{with } U(r) = \frac{e^{-\mu r}}{r}$$

Eigenvalues found thanks to a Gaussian basis

Three-gluon glueballs $C = -$

V.M., Semay, and Silvestre-Brac [Phys. Rev. D**77**, 094009 (2008)]

$$d_{abc} A^a_\mu A^b_\nu A^c_\rho \qquad [(\mathbf{88})_{\mathbf{8}_s}\mathbf{8}]^{\mathbf{1}} \qquad C = -$$

$$f_{abc} A^a_\mu A^b_\nu A^c_\rho \qquad [(\mathbf{88})_{\mathbf{8}_a}\mathbf{8}]^{\mathbf{1}} \qquad C = +$$

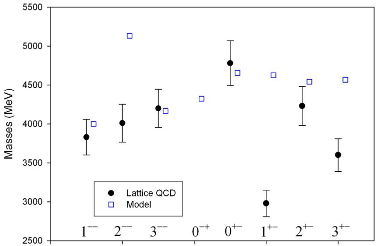
Gluons with spin $\rightarrow \boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$
Good results for 1^{--} and 3^{--} but higher $2^{--} \leftarrow$ symmetry

Disagreement with lattice QCD for $PC = +-$
Impossible to explain the splitting ~ 2 GeV between 1^{+-} and 0^{+-}

$0^{+-}, 1^{+-}, 2^{+-}, 3^{+-}$ are $L = 1$ and degenerate in a model with spin

S	$S_{\rm int}$	Symmetry	J^{PC}
0	0	1 A	0^{-+}
1	0, 1, 2	1 S, 2 MS	1^{--}
2	1, 2	2 MS	2^{--}
3	2	1 S	3^{--}

J^{PC}	(L,S)	J^{PC}	(L,S)
1^{--}	(0,1)	0^{+-}	(1,1)
2^{--}	(0,2)	1^{+-}	(1,1)
3^{--}	(0,3)	2^{+-}	(1,1)
0^{-+}	(1,1)	3^{+-}	(1,2)



Solution: Implementation of the helicity formalism for three transverse gluons.

Three-gluon glueballs $C = -$

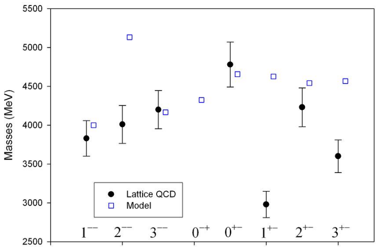
S	S_{int}	Symmetry	J^{PC}
0	0	1 A	0^{-+}
1	0, 1, 2	1 S, 2 MS	1^{--}
2	1, 2	2 MS	2^{--}
3	2	1 S	3^{--}

Gluons with spin → $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$
Good results for 1^{--} and 3^{--} but higher 2^{--} ← symmetry

Disagreement with lattice QCD for $PC = +-$

$0^{+-}, 1^{+-}, 2^{+-}, 3^{+-}$ are $L = 1$ and degenerate in a model with spin

J^{PC}	(L, S)	J^{PC}	(L, S)
1^{--}	(0,1)	0^{+-}	(1,1)
2^{--}	(0,2)	1^{+-}	(1,1)
3^{--}	(0,3)	2^{+-}	(1,1)
0^{-+}	(1,1)	3^{+-}	(1,2)



Other models for 3-gluon glueballs with massive gluons by:
Hou and Soni (1984),
Kaidalov and Simonov (2000 and 2006),
Llanes-Estrada, Bicudo and Cotanch (2005),...
But not the same conclusion !!!

Two Transverse Gluons

V.M., Buisseret and Semay [Phys. Rev. D **77**, 114022 (2008)]
Gluons with 2 polarization states → Jacob and Wick formalism

$$
\begin{aligned}
|S_+;0^+\rangle &= \sqrt{\frac{2}{3}}\,|{}^1S_0\rangle + \sqrt{\frac{1}{3}}\,|{}^5D_0\rangle \\
|S_-;0^-\rangle &= |{}^3P_0\rangle \\
|D_+;2^+\rangle &= \sqrt{\frac{2}{5}}\,|{}^5S_2\rangle + \sqrt{\frac{4}{7}}\,|{}^5D_2\rangle + \sqrt{\frac{1}{7}}\,|{}^5G_2\rangle
\end{aligned}
$$

Application with a simple Cornell potential

$$H^0 = 2\sqrt{\boldsymbol{p}^2} + \frac{9}{4}\sigma r - 3\frac{\alpha_s}{r}.$$

Parameters:

$$\sigma = 0.185 \text{ GeV}^2 \qquad\qquad \alpha_s = 0.45$$

Same matrix elements ($\boldsymbol{L}^2, \boldsymbol{S}^2, \boldsymbol{L}\cdot\boldsymbol{S}$) for 0^{++} and 0^{-+} !
Addition of an instanton induced interaction to split the degeneracy between 0^{++} and 0^{-+}

$$\Delta H_I = -P\,\mathcal{I}\,\delta_{J,0} \qquad \text{with } \mathcal{I} = 450 \text{ MeV}.$$

Instanton attractive in the scalar channel and repulsive in the pseudoscalar and equal in magnitude [Forkel, 2005]

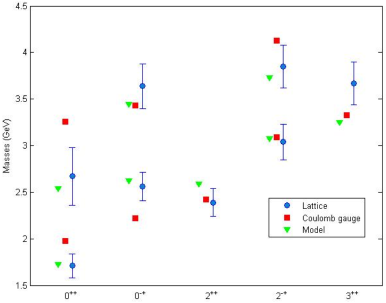
V.M., Buisseret and Semay [Phys. Rev. D **77**, 114022 (2008)]
Szczepaniak and Swanson [Phys. Lett. B **577**, 61 (2003)]

Application with a simple Cornell potential

$$H^0 = 2\sqrt{\boldsymbol{p}^2} + \frac{9}{4}\sigma r - 3\frac{\alpha_s}{r}$$

Parameters:

$\sigma = 0.185\ \text{GeV}^2 \quad \alpha_s = 0.45 \quad \mathcal{I} = 450\ \text{MeV}$



Very good agreement without spin-dependent potential
Instanton interactions required
Extension for three-body systems ?

Boulanger, Buisseret, V.M. and Semay [Eur. Phys. J. A **38**, 317 (2008)]
$\mathcal{SU}(2) \times \mathcal{SU}(3)$ decomposition for two gluons → to the lowest J:

$$F^a_{\mu\nu} \otimes F^b_{\alpha\beta} \quad \rightarrow \quad \delta^{ab}\left(F^c_{\alpha\mu}F^c_{\mu\beta} - \frac{1}{4}g_{\alpha\beta}F^c_{\mu\nu}F^c_{\mu\nu} \oplus F^a_{\mu\nu}F^a_{\mu\nu}\right) \oplus \epsilon^{ab}(\cdots)$$

$$\square^{\,a} \otimes \square^{\,b} \quad = \quad \square\square^{\,(ab)} \oplus \bullet^{(ab)} \oplus \square^{\,[ab]}$$

Lowest J allowed for 3 gluons with helicity:

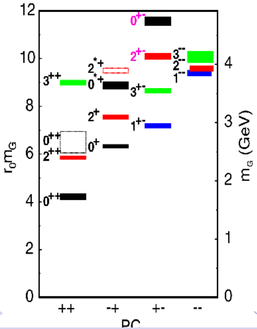
$$\square^{\,a} \otimes \square^{\,b} \otimes \square^{\,c} = \square\square\square^{\,(abc)} \oplus \square^{\,(abc)} \oplus \cdots \oplus \bullet^{[abc]}$$

Low-lying states are $J = 1$ and $J = 3$ with symmetric color function and $J = 0$ with an antisymmetric color function

Low-lying states: the $1^{\pm -}$, $3^{\pm -}$ and the $0^{\pm +}$

$J = 0^{P-}$ are not allowed for three-gluon glueballs
→ 0^{+-} ≡ four transverse gluons
Models with transverse gluons agrees with Lattice

What about experiments ?



Future Developments

Group theory:

$$\square^{\,a} \otimes \square^{\,b} = \square\square^{\,(ab)} \oplus \bullet^{(ab)} \oplus \square^{\,[ab]}$$

$$\square^{\,a} \otimes \square^{\,b} \otimes \square^{\,c} = \square\square\square^{\,(abc)} \oplus \square^{\,(abc)} \oplus \cdots \oplus \bullet^{[abc]}$$

Jacob and Wick formalism for 3-gluon glueballs ?

$$|J, M; \lambda_1, \lambda_2\rangle = \mathcal{N} \int dU \mathcal{D}^{J*}_{M,\Lambda}(\phi, \theta, -\phi) R(\phi, \theta, -\phi) \, |\Psi_{\lambda_1,\lambda_2}\rangle$$

$$|J, M; \lambda_1, \lambda_2\rangle = \sum_{L,S} \left[\frac{2L+1}{2J+1}\right]^{1/2} \langle L0S\Lambda |\, J\Lambda\,\rangle \, \langle s_1\lambda_1 s_2 - \lambda_2 |\, S\Lambda\,\rangle \, \left|^{2S+1}L_J\right\rangle$$

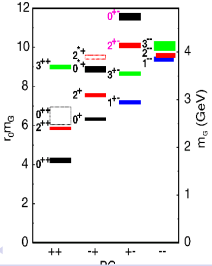
Existing developments :

Giebink (1985),

Stadler, Gross and Frank (1997),

Klink (1998),...

But still no application to glueballs for now



LEVAI AND HEINZ [Phys. Rev. C **57**, 1879 (1998)]

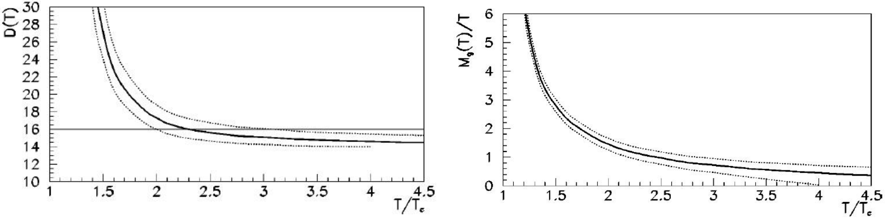
Equations of state for Gluon Plasma

Number of effective gluonic degrees of freedom $D(T)$

Fitted thanks to lattice data for $P_g(T)$ and $\epsilon_g(T)$

$$D(T) = 14.5 \pm 1$$

Close to 8 colors $\times$ 2 helicities



Support the transverse gluon picture

Spin-1 gluons Models for $C = +$

Agreement with lattice but with OGE and unwanted vector states

Spin-1 gluons Models for $C = -$

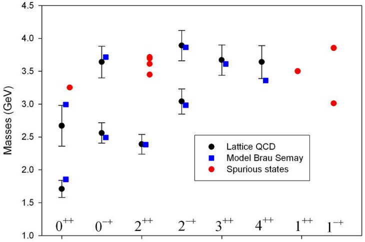
cannot reproduce the lattice data

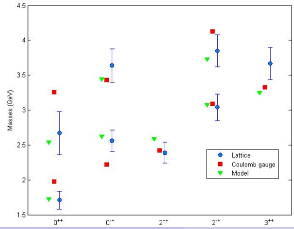
Helicity-1 gluons Models for $C = +$

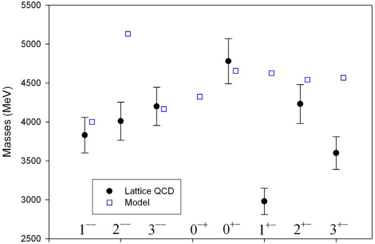
Agreement with lattice without OGE and but instanton interaction

helicity formalism for three transverse gluons would solve the hierarchy problem ?

Pure gauge but what about experiments



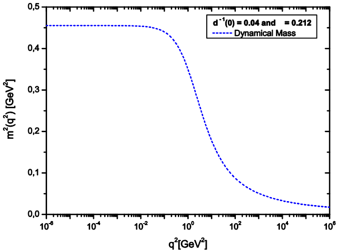




How many degrees of freedom has the gluon ?

Helicity-1 particle $\{+1, -1\}$?

Spin-1 particle $\{+1, 0, -1\}$?



A smooth function from 2 (UV) to 3 (IR) ?

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