Large Representation Polyakov Loop in Hot Yang-Mills Theory

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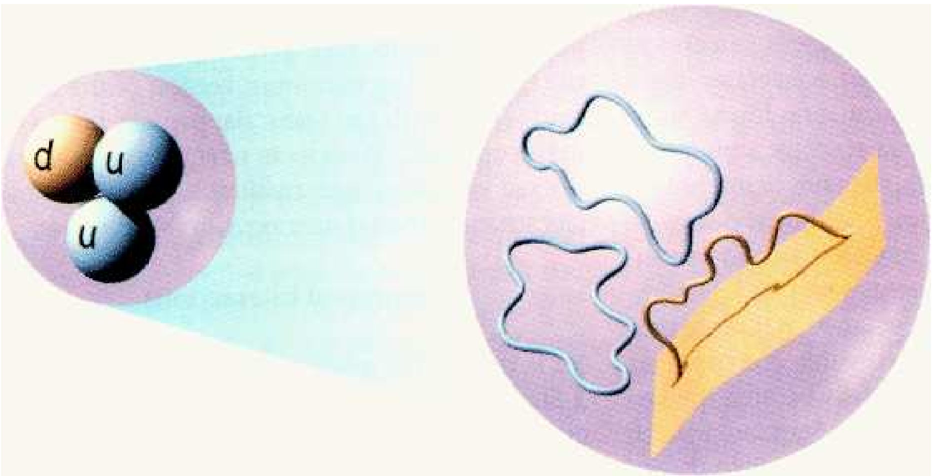

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Polyakov Loop

$$\left\langle \text{Tr } \mathcal{P} e^{i \int_0^\beta d\tau A_0(\tau,\vec{x})} \right\rangle$$

Free energy of classical quark

$$e^{-\beta F_Q} = \frac{\int [dA(x)...] e^{-\int F_{\mu\nu}^2(x)+...} \text{ Tr } \mathcal{P} e^{i \int_0^\beta d\tau A_0(\tau,\vec{x})}}{\int [dA(x)...] e^{-\int F_{\mu\nu}^2(x)+...}}$$

Center symmetry: (in gauge theory with adjoint rep. matter)

$$g(\tau + \beta, \vec{x}) = c \cdot g(\tau, \vec{x})$$

$$\text{Tr } \mathcal{P} e^{i \int_0^\beta d\tau A_0(\tau,\vec{x})} \rightarrow c \cdot \text{ Tr } \mathcal{P} e^{i \int_0^\beta d\tau A_0(\tau,\vec{x})}$$

Confinement-Deconfinement

F_Q finite only if center symmetry is spontaneously broken.

Deconfinement phase transition = breaking of center symmetry

Finite temperature Yang-Mills theory on $S^3 \times S^1$

- Cutoff infrared using a spatial 3-sphere
- R=radius of S^3, T=temperature, Λ_{YM} mass scale RT
- Weak coupling when $R\Lambda_{YM} << 1$, study behavior as a function of RT
- $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, has conformal symmetry - tuneable dimensionless parameters RT and $\lambda = g_{YM}^2 N$.
- AdS/CFT duality – "planar" large N 'tHooft limit ($N \to \infty$ $\lambda = g_{YM}^2 N$ fixed) dual to tree level string theory on $AdS_5 \times S^5$ with radii $R = \sqrt{\lambda^{\frac{1}{2}} \alpha'}$, N units of RR-flux.
- Confinement/deconfinement phase transition of **planar** $\mathcal{N} = 4$ SYM on S^3 is dual to Hawking-Page transition in IIB supergravity. Deconfined phase has AdS black hole.

Weak coupling $\mathcal{N}=4$ SYM on $S^3\times S^1$, all fields gapped.
Integrate out to find an effective action $S_{\rm eff}[U,TR,\lambda,1/N]$.

$$\frac{\int [dA(x)...]e^{-\int F^2_{\mu\nu}(x)+\dots}\mathrm{Tr}\ \mathcal{P}e^{i\int_0^\beta d\tau A_0(\tau,\vec{x})}}{\int [dA(x)...]e^{-\int F^2_{\mu\nu}(x)+\dots}}=\frac{\int [dU]e^{-S_{\rm eff}[U]}\ \mathrm{Tr}\ U}{\int [dU]e^{-S_{\rm eff}[U]}}$$

Perturbative computation of effective action in $\mathcal{N}=4$ SYM:

B.Sundborg, hep-th/9908001, O.Aharony et.al. hep-th/0310285

$$S_{\rm eff}[U]=-\sum_{n=1}^{\infty}\left[z_B(x^n)+(-1)^{n+1}z_F(x^n)\right]\frac{|\mathrm{Tr}U^n|^2}{n}+\dots$$

$$x=e^{-\frac{\beta}{R}}\ ,\ z_B(x)=\frac{6x+12x^2-x^3}{(1-x)^3}\ ,\ z_F(x)=\frac{16x^{\frac{3}{2}}}{(1-x)^3}$$

Gauge invariance: $S_{\rm eff}[U]=S_{\rm eff}[WUW^\dagger]$
Center symmetry: $S_{\rm eff}[U]=S_{\rm eff}[cU]\ \ ,\ \ S_{\rm eff}\sim N^2$
Deconfining phase transition where $T_CR\simeq 0.38$.

Large N limit

The unitary matrix model is an eigenvalue model:

$$\frac{\int[dU]e^{-S_{\rm eff}[U]}\ {\rm Tr}\ U}{\int[dU]e^{-S_{\rm eff}[U]}} = \frac{\int[d\phi_a]\prod_{a<b}|e^{i\phi_a}-e^{i\phi_b}|^2 e^{-S_{\rm eff}[e^{i\phi_a}]}\ \sum_a\ e^{i\phi_a}}{\int[d\phi_a]\prod_{a<b}|e^{i\phi_a}-e^{i\phi_b}|^2 e^{-S_{\rm efff}[e^{i\phi_a}]}}$$

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$$= N \int d\phi \rho(\phi) e^{i\phi}$$

Characterized by Eigenvalue density: $\rho(\phi)$

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$$\rho(\phi) = \frac{\cos\frac{\phi}{2}}{\pi(2-2p)} \sqrt{(2-2p) - \sin^2\frac{\phi}{2}} \quad p = \in [\frac{1}{2}, 1]$$

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Study large representation loops $< \text{Tr}_R U >$ in the deconfined phase.

Giant Wilson loop: $S^3 \to R^3$, then zero temperature

Wilson loop for large representations R in $\mathcal{N}=4$ SYM theory

$$\left\langle \mathrm{Tr}_R \mathcal{P} e^{\oint d\tau \left(iA_\mu(x(\tau))\dot{x}^\mu(\tau) + \phi^1(x(\tau))|\dot{x}(\tau)| \right)} \right\rangle$$

Can be computed exactly in $\mathcal{N}=4$ SYM for C = circle,

$$W[\circ] = \frac{\int dM e^{-\mathrm{Tr} M^2/2g_{YM}^2} \ \mathrm{Tr}_R e^M}{\int dM e^{-\mathrm{Tr} M^2/2g_{YM}^2}} \qquad \left(= \frac{1}{2\sqrt{\lambda}} I_1(\sqrt{\lambda}) \text{ for } R = F \right)$$

J. Erickson, G. S., K. Zarembo, hep-th/0003055

V.Pestun, arXiv:0712.2824[hep-th]

Can extrapolate to strong coupling and compare with conjectured string theory duals.

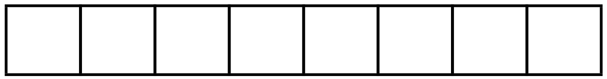
N.Drukker and B.Fiol, hep-th/0501109

S.Yamaguchi, hep-th/0601089,hep-th/0603208

K.Okuyama G.S. hep-th/0604209

J.Gomis and F.Passerini,hep-th/0604007,hep-th/0612022

The Wilson loop in $\mathcal{N} = 4$ SYM theory

- Can be computed and extrapolated to large λ and compared with AdS/CFT prediction for strong coupling.
- For small representation (center charge $<< N$), agrees with dual which is a disc amplitude with boundary of the disc located on a curve $C \in$ boundary of AdS_5.
- For large rep, R, (center charge $k \sim N$), the string theory dual is a D-brane
- For a symmetric rep  with center charge k, it is a D3-brane whose worldvolume is $AdS_2 \times S^2 \subset AdS_5$
- For an antisymmetric rep. it is a D5-brane with worldvolume $(AdS_2 \subset AdS_5) \times (S^4 \subset S^5)$

Is the giant Polyakov loop at finite T

$$< \mathrm{Tr}_{\mathcal{R}}\ \mathcal{P}\ e^{\int_0^\beta d\tau \left(iA_0(\tau,\vec{x})+\phi^1(\tau,\vec{x})\right)} >$$

dual to a D-brane embedded in AdS black hole geometry?

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S.Hartnoll and S.Kumar hep-th/0603190

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G.Grignani, J.Karczmarek and G.S., arXiv:0904.3750 [hep-th]

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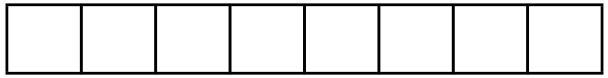
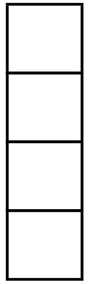
Symmetric representation $\mathcal{S}_k =$ 

Antisymmetric representation $\mathcal{A}_k =$ 

can be obtained from generating functions

$$\mathrm{Tr}_{\mathcal{S}_k}\, U \;=\; \left\langle \sum_{a_1 \leq \ldots \leq a_k} e^{i\phi_{a_1} + \ldots + i\phi_{a_k}} \right\rangle = \int \frac{dt}{2\pi i t^{k+1}} \left\langle \prod_{a=1}^{N} \frac{1}{1 - t e^{i\phi_a}} \right\rangle$$

$$\mathrm{Tr}_{\mathcal{S}_k}\, U \;=\; \int \frac{dt}{2\pi i t^{k+1}} \left\langle e^{-\mathrm{Tr}\ln(1 - tU)} \right\rangle$$

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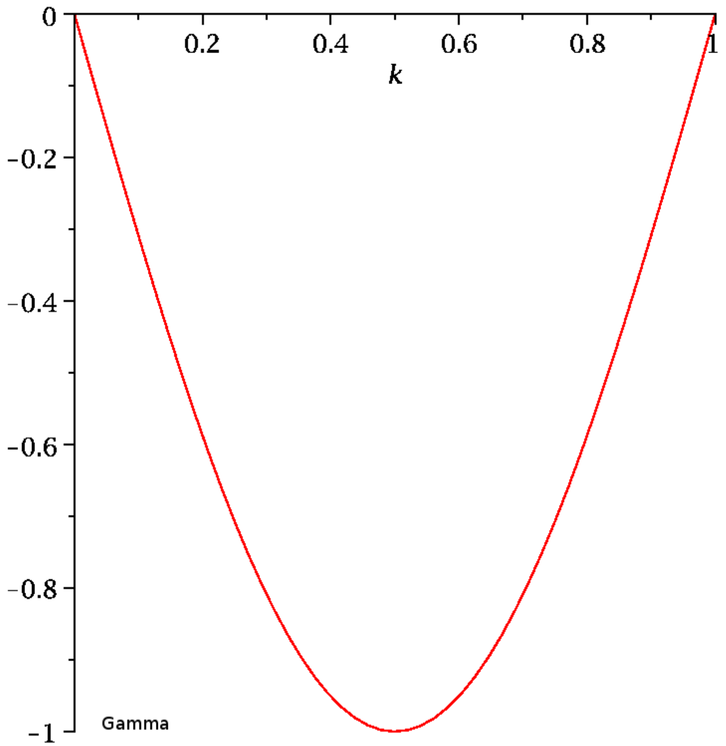
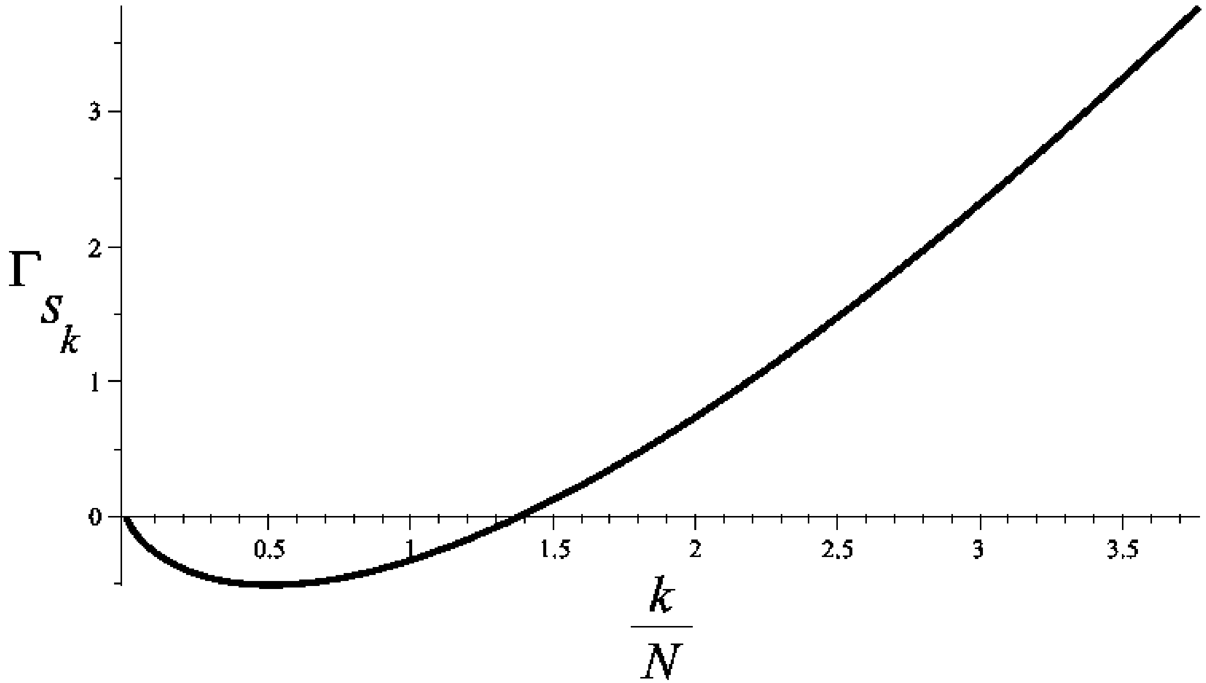
Antisymmetric Representation



$< \text{Tr}_{\mathcal{A}_k} U >= e^{-N\Gamma}$ is always of the form $e^{N\#>0}$ for the entire range of $\frac{k}{N}$

$k \to N - k$ duality

Symmetric Representation



$< \mathrm{Tr}_{\mathcal{S}_k} U >= e^{-N\Gamma}$ vanishes when $\frac{k}{N} > \frac{k}{N}_{\mathrm{crit.}}$

Phase transition – large symmetric representation quarks are confined even in the deconfined phase of SYM.

Revisit the D3-brane

$$ds^2 = R^2 \left[f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 \left(d\chi^2 + \sin^2\chi d\Omega_2^2 \right) + d\Omega_5^2 \right]$$

$f(r) = (r^2 - r_+^2)(r^2 + r_+^2 + 1)/r^2$, $k_B T = 2\pi r_+/(2r_+^2 + 1)$
On brane world volume, $t = t, r = r, \chi = \chi(r), \Omega_2 = \Omega_2$

$$S = \frac{2N}{\pi} \int dtdrr^2 \sin^2\chi \left[\sqrt{1 + r^2 f(r)(\chi')^2 - 4\pi^2 F^2/\lambda} - r^2\chi' \right]$$

Solve for F: $k = \delta S/\delta F$

$$S = \frac{2N}{\pi} \int dtdr \left[\sqrt{(r^2 \sin^2\chi)^2 + \kappa^2} \sqrt{1 + r^2 f(r)(\chi')^2} - r^4 \sin^2\chi\chi' \right]$$

An approximate solution: if $r_+ = 0$ eq.motion is solved by $\chi = -\arcsin\frac{k}{r}$. Only good for $\kappa < r$, suggests $\kappa < r_+$.

Conclusions:

- The question: is there a discernable difference between symmetric and antisymmetric Giant Polyakov loops?
- Large symmetric loops have a maximum size. $e^{k(+\#)} \longrightarrow e^{k(-\#)}$
- AdS/CFT implies that this phase transition goes to small values of $\frac{k}{N}$ at strong coupling.
- Can one see a phase transition in the Polyakov loop as a function of $\frac{k}{N}$ on the lattice?